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SUMMARY KEYWORDS

square root, equal, natural logarithm, root, property, natural log, limit, squared, exponential function, π , nth root, approaches, correspond, work, positive integer, plug, power, function, write, tells

SPEAKERS

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Welcome. In this lecture, I want to go through like just a few more limit properties. So this one has to do with roots. So let's go ahead and get started with this. So the first one is property 10. Okay, and property 10 is that if I take the limit as X approaches A of, and actually here, let me kind of first set up here, we're going to suppose for this property, we want to suppose that, so suppose we have that, so whatever our suppositions for this property, which is that N is a positive integer. So N is a positive integer. And our other kind of supposition is that A is going to be, so and we have that A is going to be greater than zero if N is even. So A is greater than zero, if we know that N is even. Okay? Okay, so that's kind of the set up. And then we have that the limit as X approaches A of the N th root, so we have the N th root here of X , right, of this X , right, what do we get out? So maybe I'll even put the parentheses here to kind of emphasize that, that what we get out is going to be the N th root. So that same N th root of this value A here, so of the value A . And then let's go ahead and do an example. So here's my example. Which is that I'm going to have, so in this example, I'm going to have the limit as X approaches, why not do something like π squared, why not, of X to the $1/4$? This is, I'm just going to do this, this is our entire function. So let's just write that here, this is what we're taking the limit of. And then if we look at that, so what do we get, if we look at that, that this is going to equal the limit as X approaches, so why does this fall under the circumstances, this is equal to this limit. And then another way to write that is that this is actually the fourth root of X . Okay, so this is actually the fourth root of X here. Okay, so just using this property, now, you can kind of see how these are exactly the same and correspond. So I can get here that this is equal to the fourth root, of and then I can put π squared in here.

And then let's just kind of simplify this down a little bit. So this is good practice at this, which is part of why we're, I'm doing this. So we're doing π squared. A fourth root is the same thing as $1/4$. Okay, then what happens if you take a power to a power? You're going to multiply them, so you get 2 times $1/4$, right, which is just π to the $1/2$, or the square root of π . Okay? Great. Now for, so all I did there, right, and there was an extra step in here, right, which is I had to remember that the $1/4$ power is the same thing as a fourth root. And then I could just plug this right in, right for X , its kind of what the property tells me. And then I want to be able to reduce this down. So then I want to put this back to be, because this is a power like that, it's going to help to put that to be a power like that. It's a fourth root, so that's going to be $1/4$. So now I have π , but these are a power to a power, so you multiply them so get π and then 2 times $1/4$. So I get π to the $1/2$, because 2 times $1/4$ is $1/2$, which is the

same thing as the square root of pi. Okay. Here's the next property. This was the first property was up here. Well, that suppose actually goes there. And then we have, oh, by the way to work out here, we have in this correspondence, this is $N = 4$ and $A = \pi^2$. If you're trying to work that out, useful. Okay. Let's do the next one. So we have, this property 11, which is going to be that this is going to work for more general functions. Okay, so we have the limit as X approaches A of the N th root of F of X . And this is going to equal, and so it's going to work kind of, well, it is a generalization of that, but let's just go ahead and, and write it. So this is actually going to equal, what I'm able to do here is I'm able to take the N th root to the outside, which is kind of the same thing as I'm doing there, actually. So I'm going to be able to take this N th root out, so I'm going to take the N th root of the limit.

So let's go ahead and do that of the limit as X approaches A , of the function, right? So I'm kind of able to pull out the root, which is nice. Okay, where here we have, so kind of our setup like that one had a set up up there, is that we have, so this is where, so what do we need to be true? Which is that N is a positive integer. So N is a positive integer. And we're going to have that the other thing that we have going on here is that we need that the limit, this limit as X approaches A of F of X has to be greater than zero, if N is even. Right? And this actually kind of makes sense. Because you can't take, right, you can't take an even root of a negative value, right? You can't take the square root of negative 3, right? Or something like that. That's kind of the situation that this is dealing with. Okay, so this is the other property. This, actually, I'll put the where in it too.

And then we'll go through an example, I'm going to go through an example that's going to kind of emphasize something else that we've been looked at when we looked at limits, I guess I am going to kind of say, this actually goes with this property. So it should be in its box. There, it's kind of like a tagged on box, but I think you understand what I mean. Okay. So to do this example, let's recall how I think about the natural logarithm. So recall how to think about the natural logarithm. Okay, so this is how I think about the natural logarithm, we went over this when we're talking about limits. Okay. So here's kind of the key point, which is that saying that and these colors kind of don't correspond to any other colors, I just want to be able to use them like this. So if I want to say that the natural log of X equals Y , like, what does that correspond to in terms of the exponential function, which is the function that we can understand a lot better? And it tells me that e^Y is going to equal X . Okay, so these two things are like equivalent statements, they correspond to each other. So that tells me then, for example, when I say that, then, when I you know, the natural log of one equals zero this actually corresponds to the statement, which is one that we understand a lot better. So if you're trying to figure this out, we could use the fact that e^0 actually equals one, okay? Because anything to the zero equals one. So you could have used that to kind of figure out what's going on here. And then also, the natural log of e equals one. One could think of is this coming from, because this is an equivalent statement to though where the statement where we have that e^1 equals,

right, anything to the one equals itself. Okay, so these are kind of useful when you're trying to think about natural logarithms to kind of think through what its relationship with the exponential function is going to tell you, because you're often going to need to use that. So here we go. So let's look at an example for this property. So example for this property. Okay, what I want to do for this one is I want to try to figure out what is the limit as X approaches e of the square root of the natural log of X ? It can be tricky, like, I don't, you know, how do you figure that out? Okay. So this is what you can do. Well,

this is equal to, so what does that equal to? This is equal to the limit as, and then we're going to go x approaches E . Well, the square root is actually the same thing as, and this is one of the things I want to emphasize here, saying the square root is like having a 2 there, okay, of the natural log of x .

And then we can bring this right, now we can bring the limit inside by this over here. So this is going to equal, and we do need to check here, 2 is a positive integer. But the natural log, sorry, the 2 is a positive integer, but the natural log at E is going to be positive. So we're good. Okay? So we're going to have the square root on the outside, right, and then we're going to take on the inside the limit as x approaches E of this natural log of x . Okay, well, now we can just plug E into here, like you can look at the graph to see, but this is just a continuous function that's defined there. So you can just plug it in. Okay. So this is going to equal the square root of the natural log of E .

Okay? So what is that? Well, the natural log of E , you can kind of check from up here is 1, right? And if you're trying to figure that out, you can remember how do I get out E , what power? I need to have 1 there, okay? So this is just equal to the square root of 1, what is the square root of 1, that's just 1, and we're done. Okay? So these all kind of used, I mean, these are just two new properties, they basically tell you, you can pull out the root. And I specifically did this with a natural logarithm to help get you more used to the natural logarithm and how it works. And a lot of it is just with the pairing with the exponential function. But we're able to pull out this square. And then another key thing to remember. So what are some other key things I tried to highlight in the examples? One is that if I'm taking the square root, that's actually the same thing as having a 2 there in the root. Okay? The other thing, so I've told you two things I was trying to give you practice with in this example. Another one is going back and forth between the power written as an exponent like that, and then written as a root like this. And the fact that when you take something to a power to a power, you can multiply those powers. Okay, so all those things I was just trying to give you a little bit of extra practice with, because I know that these are things that a lot of times somehow we just kind of miss them. And so the more you see them, the more likely it is that it will stick. Or you can ask yourself, wait, what was this? I missed this why? And you can kind of go back through and know that it's something you should look at it again. But part of that is just giving you more practice so that you know that. Okay so, I hope that made some sense and I will see you in the next lecture.