PfaffModule3L06

Thu, 1/20 3:02PM 17:41

SUMMARY KEYWORDS

limit, approaches, equal, function, rule, multiply, product, positive integer, quotient rule, power, behave, plugging, divide, cubed, combine, quotient, wording, fact, true, suppose

SPEAKERS

Catherine Pfaff

Welcome, in this lecture, we're going to go through some kind of more of these useful limit properties. Limits are going to behave generally, as well as you could possibly want things to behave, there are going to be things that are true. But limits that are not going to be true about derivatives and integrals, and particularly the way they behave with products and quotients. So, you know, enjoy it a while last, but you know, these things are very useful about limits. So let's go ahead and show what they are. And then I'm going to have over on this side, kind of, I'm going to go through all of this with an example. So or, sorry, for the first two, I'm going to use the same example. So let's just kind of set up the example. So we're going to suppose that we have to, we're going to have three things here, we're going to have F of X is going to equal the function X, we're going to have that G of X is going to equal the function E to the X. And finally, we're going to have the A equals zero. Okay, so we're going to suppose these are the first two examples we're going to do. So I'm just going to have these over here, as you can kind of keep them in mind as we go. And looking at these, it's going to be useful to know, so what are the important limits there? Okay, that we know that the limit as, right, the limit as X approaches A of F of X, so the X approaches this A, which is this zero of F of X. So, right, what is that, in this circumstance? Well, I'm taking the limit as the function X approaches zero, and so that's just going to be zero. Okay, so that's one thing that we know, we also know that so let's go ahead with the other ones. So we're going to take the limit as X approaches X. So as X approaches A of the other function, which is G of X, right, which is E to the X. So what happens is that approaches zero? Well, we put zero in, you know, as the exponent on anything, you get out 1, okay? So this is going to equal 1. Okay, so we're going to use those, but let's just go. So our first rule that we want to go through is the product rule. So what does that say? So this is the product rule.

And what does the product rule say? The product rule says that if I take the limit as X approaches A, and this is going to be the function that's a product of, so we have the function, that's a product of F of X and G of X, so we're just multiplying the outputs when I input X and multiply the outputs of F and G. Okay, that's actually going to equal the limit as X approaches A of F of X, we can actually multiply these two limits, which is kind of like the dream come true, that I can just do these limits one at a time and then multiply them, as long as these limits exist. Okay, so the limit as X approaches A, so multiplying these two limits for these two functions should be orange. Okay, so let's show the example over here of this. So this is the example for four with a product rule. So over on the example side, I'm going to get that this is saying that if I take the limit as X approaches 0 of X E to the X.

Right? I'm just plugging in exactly for my F, G and A over there. So this is X E to the X. Well, if I want to compute this, I could just multiply these two limits. So let's kind of see about that. So this is going to be the limit as X approaches zero, just of this function X, right, multiplied by whatever I get there, the limit as X approaches zero of, and then E to the X is my G, so I just write that. Okay, so I could, instead of taking the limit of the product of these two, I can multiply the two limits. Okay? The first one is zero, the second one is 1, I get that from right up there. Okay, so this is equal to zero times 1, which is equal to zero. Okay, which I got from right there. Okay, it didn't need to be true that these two limits existed, that, if they don't exist, it gets more complicated. Okay. So that's how the product world works, you can just actually multiply the limits of the two functions. Okay, quotient rule. So very useful. So here's the quotient rule.

Okay, so we're going to have the limit as X approaches A, and now you can guess this is the quotient rule. So you can guess I'm going to divide the two functions. So I have F of X on top, and then on bottom, I have G of X. And then you're going to be able to do exactly what you want to do again, and again, this is going to work out so nicely, because I could just take right, I can take the limits of the top and bottom separately. Okay, and again, I'm, you know, I'm drawing these on, red, so you can see what we're taking the limit towards doesn't change. And we have F of X on top, and we have G of X on bottom.

Okay, and then let's go ahead and apply this over in this circumstance. So we are in this is the fifth rule. Okay, and so we want to take the limit as X approaches again, I'm just plugging in those things for my A, F and G. So I'm going to take on top, I have X, okay, X divided by E to the X. Right, so this is a new function, but it's a function that's a quotient. And so luckily, I can use the quotient rule. And we'll go ahead and do that. Okay, and so we're going to get for this, we're going to get that this is going to equal, right, and we have the limit as X goes to zero on the top and the bottom. Both of these are the limit as X approaches zero. And then I want to take my top function, which is X, I want to take that limit, and then I want to divide it by, whoops, it's a little bit funny how I wrote that. I forgot my limit sign. So we get the limit as X approaches zero, and we're going to get, I'll make that a little clearer to see, okay, and then we're going to get on the bottom function. So we have E to the X, but we know these two limits. So I can take this limit from here. So the top is this limit, which is zero, the bottom is this limit, which is 1, so I'm going to get zero divided by 1, which is again going to be zero. Not every functions limit is zero, but in all these examples somehow or another this worked out well. Well, I also chose these examples, because I wanted you to get used to these functions. Okay. Now, the next one I'm going to show you is called the power rule. So we're going to be, here's how the power goes. The power rule.

Okay, and how the power rule goes is we're going to take the limit, I'm going to get us a new blue marker, because this one's getting a little difficult to see. So the limit as X approaches A, should be easier to see, I think. Okay, so the limit as X approaches A of, and then we have, I have some kind of function inside of here. And then I'm going to take its power. So let's do this. Right? So if I have a function, right, and it's going, you know, it's a function that's to a power, but this is all inside the limit, I'm taking a limit of this whole thing. Okay, then what am I going to get, I'm actually able to do the power afterwards. So this is going to be the limit as X approaches A of, and then I have, I have my F of X inside of here. And then I can do my power on the outside, so I can take the limit and then do the power. Okay, where this N is a positive integer. Okay, so this N is a positive integer, it has to be a positive integer.

Okay, so let's do an example over here. And I'm going to give you kind of a new suppose. So we're going to suppose that, so now we're supposing for the next example that we're going to do, we're going to suppose that we have, so we have F of X equals X. And we have N equals 3, and we have A equals 2. Okay, so then I'm going to have that this is going to give me that, so that I have, that the limit as X approaches A, right, of F of X is going to equal, so this is the limit of, this is the limit as X approaches 2 of X, which is just going to equal 2. In this circumstance, I'm just able to plug it in. Okay, so then this is going to give me, what like in my example, you know, as an example, the product, the power rule, sorry. This is going to give me the limit as X approaches 2 of, right, so my F of X to the N, so I have my X cubed. So X cubed, and that's a positive integers. So that's an okay example to use. So this is going to equal, well, right now this is like an X cubed is what I'm taking the limit of, but I could just take the limit as X approaches 2 of X and then cube it. So let's go ahead and do that. So the limit as X approaches 2 of X and then cube it. So just using the fact that we've computed that limit there already, we know that we're just getting from here, we're getting 2, right? This 2 goes in here. So we just get 2 cubed, which is going to equal 8.

Okay, so again, I can just pull that power basically to the outside. Okay? So we used in our example, right, so this example we had here, so we used in the example, so we used in this example, the fact, right, we used that, so here's rule eight, which is actually that the limit as X approaches A of X is going to equal A. It's kind of easy to see if you actually look at the graph of the function. Okay? But it's also a rule. So that's another rule that we have here. And the power rule example actually shows how to generalize the power rule, okay? So, this is in, well in the power rule example, in the power rule example over here, we use that fact. And then this example is going to show us how to generalize the power rule. Okay, so, and the power rule example shows how to generalize the power rule to, so here's the rule, which is that we get the limit as well. Yeah, okay as X, well, it's not actually a generalizing it. And the power rule example shows how to generalize. You know how to, this is actually bad wording, so how to combine, I don't like my wording there, so let's just kind of erase that. So, okay, so how to combine, so we're combining really, six and eight. So six, right, so six, and eight. To get, so here's a new rule, which is, you know, you can almost guess what it is at this point, which is going to be the limit as X approaches A of X to the N, right, where this has to be a positive integer.

It has had to be. So, as long as that's a positive integer, that this is actually just going to equal A to the N. So this is going to equal this A that's coming from there to this N power, okay, where you're just plugging A in for X is what you're really doing. Okay? So let's just kind of go through these. So new rules, they basically say, when it comes to limits, you can do everything you want to do. If I'm taking the limit of a product, I get out the product of the limit. If I'm taking a limit of a quotient, I get out the quotient of the limit. If I'm taking the limit of a power, I take the limit, then I take the power. And then the other key thing that kind of comes into play here, when I've just got X, I can put A in for X. And then you can kind of see, you know, combining these two things that like, as we did in here, that actually gives you this rule here, where I can do that with any power of X also. Okay, so I hope that made some sense, and I'll see you in the next lecture.