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limit, summation, function, minus infinity, constant, subtraction, rule, pi, subtracting, equal, properties, outputs, infinity, linear, negative infinity, satisfy, emphasize, setup, circumstance, operation

SPEAKERS

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Welcome. In this lecture, I'm going to talk about how taking limits is something called a linear operation. I know that this is a little bit advanced terminology to say linear operation. But I kind of want to emphasize because you're going to see that there are certain things like limit, summations, derivatives, integrals that all satisfy the same list of properties. And that's really because they are a linear operation. And so knowing when you have this list of properties is useful, and also kind of knowing that you can just look at, you just need to know that it's a linear operator and know it satisfies all these and you don't have to kind of look at them individually. There's actually a reason why a lot of these things all share properties. And it's because they're all related. You'll look at it in the definitions. As you'll look in the definition of a derivative, you'll see limits, and then the definition of an integral, you'll see well, you'll see limits and summations, okay? And so that's going to give you a sense of why these all have the same properties. But here they are. So let's just kind of get going with it. And I will show you in certain examples, I know that sometimes looking at things notationally can be a bit overwhelming. So there we go. So we're going to suppose so here's kind of our set up, so we're going to suppose that we have,

so what do we have here? We're going to have that C is a constant, right? Like a number like π , or negative E , or 5. Okay, so we're in the setup, we had that C is a constant, and then we're going to have that, so that's one thing that is going to be happening. And then we're also going to have that when we take the limit. So if I take the limit as X goes to A , so we're going to have a function F of X so that when I take this limit, and we have the limit as X goes to an A , you know, I'm coloring these, so that you can kind of to emphasize, so these A are the same, these functions are not the same, I just don't have enough colors. But they're both functions. But those A 's do have to be the same. Both exist. So what these limits exist. Okay, so this is kind of our setup. So in this setup, then, so here's kind of the first property I'll denote it, you know, with a Roman numeral one, okay? And this is the summation rule, and what does it say? So this is the summation rule. And what this says is if I take the limit as X goes to A , the sum of these two functions, okay, so of F of X plus G of X , so I get a function where I add together my, my outputs for a given input. Okay, so when I take this, I'm going to, I could actually do we kind of, we like to be able to assume that we would actually get just this sum here.

The limit as X goes to A of F of X , and then we are lucky that this does work out that then I can add this to get this as the limit as X goes to A of G of X . Okay, so that works with summation. So let's kind of, we'll start we'll kind of do these one at a time. And so I'm going to kind of get my example set up and look at that one in there. So here's the example that we're going to be using, how far out can I go and still have you see. Maybe about this far? Okay, so this is the example this kind of kind of carry through all of them. So I'm going to have, so there's kind of four parts to it. So I have F of X is going to equal E to the X . And then I have G of X is going to equal the function, you know, 1 over X . Okay, so those are both functions, I have that A is equal to negative infinity. I need to know what I'm taking the limit as X goes towards, so that's equals negative infinity. And then the last part is that I have a constant C is going to equal π . Okay?

So this is kind of my setup. It happens to be true. I kind of played for a little while with trying to find some other limits and then decided, you know, functions that gave different limits, I decided this is okay. I didn't want to complicated things. So the limit as X goes to A of F of X in this circumstance, so what is that in this particular circumstance? I'm just going to put in the A equals minus infinity and F of X equals E to the X . So this is going to equal the limit as X goes to. Now we have the X is going to minus infinity, I put that in for A , my F of X , I put an E to the X . And what is this? So this is actually going to end up equaling zero. So this is something I've kind of gone through. Okay, now, so and then the other one we know is that if I take, right, we need to work out or other limit before we can even get started. So for this of G of X , this is equal to and then I'm again, just putting things in from up here. So this is going to equal the limit as X goes to minus infinity. And now we're looking at G of X . That's right, there's this 1 over X .

This one also is zero. This is what I mean, I thought about kind of trying to mix it up a little. But this is, this is, it's fine. Okay, so now let's look at the summation rule in this circumstance. Okay, so now we're going to actually, with these functions, and our A and our C , we're actually going to use that for this formula to kind of see how this formula plays out. So I'm just going to plug into that exact formula over there, I'm going to plug in, so I have, instead of a I'm going to have minus infinity, so I'm going to be taking the limit as X approaches, and then I'm going to have minus infinity here. And then I have so my F of X and G of X , I'm going to add those together. So I'm going to get the function. Well, my F of X , right from up here is E to the X . And then I want to add the function G of X , which is 1 over X . So we've got a new function, which is the sum of these two functions. Okay, and this is going to equal the limit. And now I can, right, I can, I can take the limit separately of each of these terms and add them together. So let's go ahead and do that. So I'm going to take the limit as X goes to minus infinity, clean that up a little bit. And then I can just do the first function, which is E to the X . And then I can add to this, the limit as X goes to minus infinity. And we do the second function, which is 1 over X . Right? And we know that both of these limits are zero, right? Because I've got this here, the limit as X goes to minus infinity of E to the X is zero, the limit as X goes to minus infinity of 1 over X is zero. So I know that both of these are zero. So this is zero. This is zero. So all I'm doing is I'm adding together zero plus zero, which is equal to zero.

So that's my answer, I can get it by just adding together those elements. So that's what the summation rule tells me. And now we have another one, which is quite similar, except that it's with subtraction. And actually, it's going to be you could have gotten it from the first one and the third one, but we're going to be happy with, you know, just to be clear that it also holds. Might as well just write it as a rule. So here's the subtraction rule. So what is the subtraction rule, say it looks a lot like

this, except I'm going to be subtracting the functions. So I'm going to take the limit as x goes to minus infinity. And then I've got, now I want to take, right, I want to take the function, which is going to subtract the outputs of F of x and G of x . So F of x minus G of x , if I input x , I get out the, the difference of the outputs. And I can do the same thing is, is up here, except that now this is a minus, right? Because there was a minus in there. This is not a minus infinity here, I got ahead of myself over there, this isn't A , right. Okay, so we've got the limit as x goes to a limit as x goes to A of F of x . And then we have minus the limit as x goes to A of G of x . Okay, so same thing as kind of up there, I'm able to break them apart. I'm able to move this minus sign, okay, do these limits separately. So let's kind of do this in the example here, it's going to look like very similar. All we're doing again, we're going to take our functions F of x and G of x from over here as well as our A .

And so I take the limit, as, and then we have x is going to now it goes to minus infinity. And again, I'm looking at the function that I get, but now it's going to be subtracting the outputs, right, so I subtract the output. So this is a new function, I take, if I put an x , I get out E to the x minus 1 over x . I know these things can be tricky, that's why I kind of emphasize them. Okay, and then we get the limit as x goes to, and we have minus infinity of, and then we can do these again, one at a time. So I can go ahead and do the limit for E to the x . And then I'm going to subtract what happens when I take the limit as x goes to minus infinity. And then I have 1 over x , I don't really need the parentheses there. So I'll just kind of drop them there. Okay, so I'm subtracting out this limit, right? And then this one. So what do I get here? Again, each of these is zero. So what I'm getting out is going to be zero minus zero, which is just zero. So again, I'm just subtracting the limits that I get for each of those functions. Great. So now let's do one more.

So in this one I've got this is the constant multiple rule. Okay? So in this one, I'm finally going to use this constant I have here, which is just kind of a number that staying fixed, right. So I have that the limit as x approaches A , of, and now I have this constant multiplied by the function F of x , that this is going to actually equal, so be able to pull out the constant to the front, kind of like we always wish we can do and sometimes we can and sometimes we can't. For example, you can't if your function is cosine, you can't just pull that out. But when you're taking limits, you can do this and pull it out.

Okay, so let's kind of do this with this example. Now, our constant is π . Okay, so I've got again, I'm just plugging in the values for F of x and A that I'm going to get from F of x and A . So I've got the limit as x approaches minus infinity. And I have π E to the x . So I have the constant in front of the of E to the x . So I get a new function, which is π E to the x , and you've seen kind of functions like that before. And then this is going to equal the limit of what I get when I pull this out, right? So we've got a π , okay, times the limit as x goes to minus infinity of E to the x , this is equal to remember again, that this is zero, right? This is zero here. So this is going to equal, I have that π , and then I need to multiply that by that limit, which is zero. And I get out zero. Okay, I get out zero for the output of all these, but I think that that doesn't take away from understanding what's going on. Okay. So in each of these rules, and sometimes you can kind of pull everything outside of the limit, whether it's addition or subtraction, or multiplication by a constant. And this can be very useful and helpful. And in fact, we'll kind of show some examples of where we use that. So, I hope that made some sense, and I'll see you in the next lecture.