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natural logarithm, graph, negative infinity, natural log, equal, limit, infinitely, exponential function, negative, infinity, understand, values, approaches, positive, output, function, give, logarithmic growth, fact, draw

SPEAKERS

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Welcome. In this lecture I want to go through the limits of the exponential function and the natural logarithm. These are particularly important functions, so it's good to kind of give you a sense of how they work. I know that a lot of students get confused by the natural logarithm. And I'll kind of also go through that, really, the easiest way to kind of think about it, is to kind of use your understanding of the exponential function and its relationship to it. Okay, so to get started, we're going to have, I'm going to draw the graph for you. So we have here, we're going to do the exponential function first. And we're going to have, and you can get Desmos, or whatever you'd like, really to draw this graph for you, if you ever want. There's lots of tools for this even online these days. So you have, this is X and Y. And then I'll just draw this, actually with blue what the actual graph looks like, it kind of goes like this. Okay, and this would be then Y equals E to the X. So this is Y equals, and then we have E to the X, right? Which is also the function we're looking at here. What are a few things to notice here? What happens if I put in zero into this, right? Just to kind of give you, you know, orient your mind as to what's going on. So to think about that, it's kind of like, what is the point here? So what is this point? Well, this point is going to be, so where's this going to come from? So this is going to come from, right, I need zero and then kind of G of zero, so I'm going to get zero and then I'm going to get E to the zero, to the zero, which is going to equal, so this point is going to actually equal zero, and then 1 is my output. Right, so that's that point, there is one point on the graph. Another one to kind of look at is what happens when I put in 1.

If I put in one, so you get a point, right, up here. So what do we know about that point that's there? Well, we know that if I put in, so what does this have to look like? The first coordinate is one, and then the second coordinate is what I get when I put it into the function. So it's going to be E to the 1. And that's just going to equal so that's going to give me the point. That's going to be 1, and then what is E to the 1 that's just E. Okay, so my second coordinate is E. So here are two points that are on this graph to give ourselves a little perspective. The other thing is that if I'm looking at X goes to infinity, I'm looking at right, what happens is my X values go here to the graph. And then if I'm looking at X goes to negative infinity, right, I'm looking at what happens as my X values go in that direction, what's happening to the graph, and you can kind of see it looks like here it's going to be approaching zero, it looks like here it's going to be approaching infinity, it's going infinitely high. Okay, so that's going to give me, so here we go. We have the limit. So the first limit I'm going to look at is the one

that you can really guess from the graph. Which is that, if I wanted to have X go to infinity, then I'm going to have, so what's going to happen with E to the X is that this is also going to infinity. We can tell because the graph kind of goes up infinitely. But another kind of explanation for this one, right, could be the fact that the outputs just get infinitely big. So outputs get infinitely big positive.

As you know we go, or kind of another way, is that, right, the graph goes infinitely high as we go to the right. So the graph goes infinitely high as we kind of move to the right, okay? So that's kind of going on with this one. Why is this true that the outputs are getting infinitely big positive? Or we kind of look at the graph and as I move to the right It goes infinitely far up, right? This is the output getting infinitely big. Okay? Um, so let's do another one. So we have the limit as X goes to negative infinity. Right? And what's happening with this one? So this is sorry, again of E to the X. Okay? And this one is actually going to give us an output of zero. So to kind of understand why, so why does this happen? Why?

Okay, so let's say so, so what is something that we know? We know that E to the minus X equals, right, because we're putting in negative values. So let's let's, we need to kind of understand what happens when we have a minus X here, or something negative in the exponent. This is the same thing as one over E to the X. Okay? So then what happens?So if I'm putting into X, really big negative values, okay, so if I make this orange, if I input for X, really big, right because this is the X negative values, so I made it orange. Negative values, right, then what's happening is that I'm going to get, so as an output, I'm going to get, what happens is output I'm going to get 1 over very, very big, negative, but I know that that converges to zero. Or sorry, I get 1 over very, very big positive, actually. Because this is always positive, right? So I can get something 1 over something very, very big, it doesn't matter whether it's positive or negative, because that always goes to zero. That's kind of what's happening with that one. So now let's look at the natural logarithm. So we'll go over here for that. So here we go. So we have our X and then our Y.

This is our Y. And I'm going to, there's kind of a key fact that's going to happen here before I draw it that I think is important, as I said that it's difficult to understand the natural logarithm function. Or at least, so many students have asked me, I can't understand what's going on with it. Honestly, I don't understand either, I just kind of understand the exponential function, and then I understand this key fact. So here we go, what's a key fact? If I have Y equals the natural log of X, right, this is an if and only if statement. So these correspond that Y equals the natural log of X precisely when I'm going to use that for here to mean precisely when X equals E to the Y. So X is going to equal E to the Y. And we're going to use that a lot when we're looking at the limits. And honestly, if I'm trying to compute any limits dealing with the natural logarithm, I'm absolutely translating them over to what they mean about E to the X. Okay? So I can understand what's going on. So this graph is going to look something like this. Okay, so this is Y equals the natural log of, and then I'm going to input this X.

So that's Y equals the natural log of X. And we have, I guess so this is how we go. So going in this direction is, is X going to infinity? This graph is never actually going to end up going negative Y, precisely using this and the fact that this graph never goes negative. Because when you take an exponent of a positive number, you're never going to get out a negative number. Okay? So that's what's going on there. And what's the, this is the X goes to zero from the positive, right, the right hand limit. So I write on there, okay, so let's look at, like we did over here, so there's going to be kind

of paired points here. So if I have the point 0, 1 over here I have the point 1, 0, right, because they're inverses. So that tells me, well, that tells me where I intersect the axis, right. So this is going to give me the point here. Because I know I have the point 0, 1 over there, now I know I have the point 1, 0 over here, because this is how things correspond. Right? We're just swapping. Okay? The other point is, I have the point 1, E here, so that's going to tell me that I'm going to have some point that's going to be E, 1 here. So at E, and I have the point E, 1, I'm going to be of height one there. So I have E, and then 1. It's useful sometimes so we kind of know that, right? That's at height one there. Okay, so let's go ahead and try to compute some limits. So the first one I want to look at, so let's look at the limit, as, and we have X approaches infinity, the natural log of X. And kind of even looking at the graph, well, you might not, it looks like it's, it does go, grow much slower, but this is actually going to be infinity. It's a little harder to tell than in the other case, because logarithmic growth is really slow exponential growth is really fast. That's something important to know in life.

Okay, um, so why is this true? So why? Okay, well, now we're going to kind of use this correspondence over here. So if we have Y is going to equal the natural log of, I'm supposed to look at the natural log of very, very big, like very, very big positive. So we're going to look at very, very big, positive. Right, because I'm approaching infinity. Well, that's going to correspond to having, this is going to look a little funny, but this is really the way to think of it, is to having very, very big positive is going to equal, and then I have E to the Y. Okay, well how do I get E to the Y to be really, really big? I have to make, I have to make a, sorry, yes, how do I get, because I'm trying to figure out what Y is in this circumstance. So I know that I had to get very, very big, so Y has to get really, really big. That's the only way for that to happen. Okay? So Y must get really, really big. So we know that Y must get really, really big, which is precisely going to tell us the limit, right, i.e., we have that it goes to infinity, okay? Just like we have there. Okay, now let's look at another one. So now we want to look at the limit as now we're going to have X is going to approach, right, we're going to look at this limit here. It's the only other useful one to look at because my, we can only put in positive input values because of this. Okay, um, so this is going to be of the natural log of X is going to equal minus infinity.

Okay, so how do we get this? So why, right? Well, to get E to the Y to be close to zero. So to get, so how do we make E to the Y close to zero? So to get that close to zero, we need that either Y is going to be equal to 1 over something very, very small. It doesn't matter whether it's positive or negative, so I'll just write that, right? So that's how we make this close to zero, is that this ends up equaling something that's 1 over something very, very small. Well, how do we do that? We need that Y is negative and big. Okay? Which is kind of the pairing of what's, what's going on over here, okay? So we have, so we have that Y goes to negative infinity, which is what gives me that. So something actually kind of to notice here, is that these, these are paired, okay, these two are actually paired. That is X approaches negative infinity, this becomes zero is just like, how do we get to negative infinity, right? Or how do we, what happens when we go to zero, we have to go to negative infinity. This is kind of this pairing that we have here that's actually going to tell us these, okay? So these are just kind of a few examples, I really think a lot of the takeaway of these, well part is, you know, remember the explanation so that you can remember the the results. Look at the graphs, look at, you know, this is, if I'm saying X is going to infinity, that's like going infinitely far to the right, if I'm saying that X is going to negative infinity, that's like going infinitely far to the left. And then I'm trying to look at what happens with my output values. So the graph, if it goes infinitely far up, then my limit is going to be positive infinity, if it goes infinitely far down, that's going to be negative infinity. Maybe it approaches zero. We'll go into that actually a little bit more in the future, but I wanted to kind of do these ones specifically, because these are such important function. Also, to kind of show that, you know, how do you play with, you know, the natural log to figure out what's going on? And so the kind

of work, you know, this is how I think about it. I, you know, this isn't anything I read in any book. This is just the way that I think about it. If I was trying to figure out limits of natural logarithms. I'm sure it's in a book, but I am just taking this from my head and how I figured out how to think about it. Okay, so I hope that makes some sense, and I'll see you in the next lecture.