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SPEAKERS

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Welcome. In this lecture, I want to talk about something that students often struggle with I've found, is when you have the, you're taking limits and there's something on the bottom. And so we kind of want to focus in this lecture on how to deal with that. So we're going to go through several different examples. So here's the first one. So this is example one. And in this example, what I want to look at, is I want to take the limit as X goes to infinity, of 1 over X. Okay, and maybe actually, I'll make this part orange. So we have X going to infinity, and then 1 over X, so X is in the bottom there. Okay, and what we need to do, so what does this tell me I need to do? First off, is that we need to check what happens. Okay, so we're going to check what happens, so check what happens as we sub in, what we want to sub in are kind of larger and larger positive X values. So we're going to sub in larger and larger, larger and larger, positive X values.

Okay, so let's kind of look at what one might do with that. Okay, so I'm going to be kind of plugging in bigger and big numbers on the bottom. So what if I'm kind of looking at, you know, if you can start thinking about that's like having 1 over 1, it's a pretty small number. But now we're gonna go 1 over 10., and then we're going to, we're going to make these bigger and bigger, 1 over 100. Right, but as the bottoms get bigger and bigger, what's actually kind of happening, so 1 over 1000, and we can keep going, right, and the picture that you kind of have going on here is that you have, so your two axes, maybe I'll even kind of, so this is my function, 1 over X. So this is kind of, that's the function part there. And we have our axes here, and just as like, just kind of a small, little picture of what's going on here. And you would notice that what's happening is, you're going to get things that are going to drop off very, very, very fast. If I'm, you know, maybe 1 is at 1, right, and then right, as I'm taking 1 over, right, so things are going to kind of this graph is going to kind of drop off like this. Okay, so what's kind of happening, and you can see this part is, if I'm saying X goes to infinity, I'm looking at my X kind of heading off in this direction, and you can actually see what's happening here. Is that, right, so the 1 over X values are getting very small, close to zero, right? So 1 over X values are getting really close to zero.

Right? So, you know, and if we could go infinitely far, then you could say they reach there, which is what we're doing with this limit. Okay? So, and another way to kind of think of this, like, what are these numbers look like? Like, how do you know these numbers are really small, right? You can

imagine that you have a pie, and I'm chopping it into well, one piece, now I'm going to, and then I'm going to chop it into 10 pieces, and I'm going to chop it into 100 pieces, right? And each of those is 100th or a 10th, or whatever. And if I've had to divide that pie into 1000 pieces, the slices are really, really small. So 1/1000 of the pie is really small. Okay? And so what this is going to tell me like looking at this, is that the limit, so the answer that I get is the limit as X goes to infinity of 1 over X is going to equal zero. Okay, because as I put in those bigger and bigger numbers on, into the bottom, then I would actually get things that are smaller and smaller. Okay, so let's look at another example. So this is example two. We'll go through four of them. Okay, so in this one, what I want to look at is I want to look at the limit as, now I have X goes to infinity. And then I have 1 over X again, sorry, this is not infinity anymore. This is negative infinity.

We already did infinity. So now we want to do negative infinity. So this is X goes to negative infinity of, and then we have 1 over X. Right? This is, this the thing. You can think of plugging in negative infinity, that doesn't make sense. So what are we actually doing? So again, now what we're going to do. So what does this tell me I need to do, X is going to negative infinity, is that I need to actually check what happens as I sub in, so check what happens as I sub in, so check what happens as I sub in larger and larger negative X values, okay? So now we're going larger and larger negative X values. Larger and larger negative X values, negative X values. Okay, so let's kind of go ahead and do this. And now, right, so here I was plugging in, so it's like 1, these were positive X values that were getting bigger and bigger and bigger. And now I'm going to do bigger and bigger and bigger negative X values. So like 1 over minus 1, right, which is equal to minus 1. And then we have 1 over minus 10. Right, so this is like negative 1/10. And then I go 1 over minus 100, which is equal to negative 100. Right, you can keep on going like you want. So 1 over negative 1000. I can choose other numbers, I'm just kind of choosing clean numbers. Again, we're slicing the pie smaller and smaller, like into more and more pieces, but with a negative there. And then our picture here, so we have, right, maybe I'll even label this axis. And then we get, so then let's do this axis here. So now we're looking at on the other side, right. Now when we're looking at kind of the graph, and these values on here, I need to look at what happens as I go out in this direction. Oops, let's see, we're putting in negative values, we're getting out something negative here. So this actually needs to be underneath. This, this, I kind of drew this, forgetting that this was going to come out negative here, but that's okay.

Okay, so, right, so what's gonna happen is I go to negative infinity, right? So you can now picture I'm taking my X values, and they're going towards negative infinity, right? And again, I'm approaching zero. Okay, so the 1 over X values as I go towards negative infinity over here, the 1 over X values are getting really close to zero. So 1 over X values get really close to zero. These values get really close to zero. Okay? So, and then that's going to give me that the limit as X approaches negative infinity of 1 over X is again zero. Even though the values that were close by were negative, it doesn't matter because they're approaching zero, whether they're approaching it from a positive direction or a negative direction, they're both approaching zero. Okay, so now let's look at example three. Maybe I'll start that up here. Example three, give myself a little bit more room. And here we've got the limit as X approaches the limit. And now we've got X approaching zero from the positive side. And we've talked a little bit about left and right hand limit, so hopefully that will help you here. Okay, oh, this is this the function, right that we're graphing and so I'm okay, um, so now I have the limit here. And I've got 1 over X, so 1 divided by X. So it's the same function, I'll just box it each time so you can see that this is the same function we're talking about. Okay, but now what do we do so in this circumstance, what am I going to do?

So what I want to do is I check what happens as I sub in smaller and smaller positive X value. So there they were larger and larger. Now they're smaller and smaller, right? Because there's this picture, we should always keep this in mind, if I'm approaching, right, so what's going on is if I'm approaching the, when I have zero plus there, it means oops, I keep on swapping plus and minus today, it seems to be a problem for me. Um, but we're gonna approach from the other side, we approach from this side. Okay, so we have positive values that are very close to this. Okay, so we're going to check what happens as we sub in. So what do we do here we check what happens. What happens as we sub in, as we sub in, so we want to get smaller and smaller, because we want to approach zero. So smaller and smaller, and then we're coming from that side, so they should be positive. So smaller, and smaller positive X values. Okay? So they should be positive, they should be getting smaller and smaller, because we're approaching zero like that. So we can kind of go ahead and do this. So we have 1 over, so like 1/10. So what are some values that approach zero? This might be kind of tricky to think about. But I'm actually going to kind of reuse the fact that I know that these kind of approach zero, we've already talked about that. So I'm going to go 1 over 1/10, right, which is going to equal 10. And then 1 over 1/100. Right? If something's on the bottom, we flip it, so we get 100.

And we've already know that these are getting smaller, smaller, remember, on the bottom, because we've cut, it's like cutting it into more and more pieces, right? This gives me 1000, and so on. So you can kind of see what I'm, actually happening is I'm getting bigger and bigger positive as my output. I'm inputting smaller and smaller positive as something that goes on the bottom, as my input, and I'm getting as my output something that's larger and larger, positive. Okay? So there's kind of a principle here. And I've heard people describe it like this before, and I kind of like this. So I'm going to use this for you, give this to you also. Okay, which is, what happens if I get very, very small positive on the bottom, like I have here, that's actually going to approach positive infinity. Okay, so let's kind of, I'm just going to kind of write out what I mean by this is that I'm getting, I haven't been using red, really, let's use that a little bit more. So very, very small.

Okay, so so if I have very, very small positive that approaches positive infinity. So the picture that I'm going to have here, I cheated because I sometimes drop pens, so I brought an extra set here so I don't have to keep on reaching down for them. Okay. And so we have here we have, but for some reason or another, the orange ones when they're brand new, they're they're not very easy to see. You can see it's kind of light, I'm not exactly sure why that is. Okay, so there's my axis there. This is my X axis. And then I've got my Y axis here. And my function, right, so I've got 1 over X. And we've already kind of seen, so over here, this is what my 1 over X looks like. So as I'm approaching here, you can actually see this happening. So this is what when I mean when I have X, right goes to the right hand limit there of zero, then you can actually see what's going on here is that the 1 over X gets really you know, gets infinitely big. So 1 over X gets infinitely big. Like our output here is getting infinitely big as our X is closing in on zero from the right hand side. Okay? This is a little bit small. Okay, so we can see here that actually, what's kind of happening here, right is 1 over X, right, is going towards infinity. Sometimes you write positive infinity, sometimes you just write infinity, we mean infinity when we do that, okay? So what we get then, in this circumstance, is that the limit as X approaches zero from the right hand side of 1 over X is going to equal infinity. Or you can put a plus in front, just to extra emphasize that if you want, but we know that without a plus too it is plus positive infinity, okay? And then the last example I want to go through, so it's example four. Okay, so here, what I want to do is I want to approach, I want to take the left hand limit. So I have X is going to, X is going to zero. And we're doing the left hand limit here. Okay, and this is going to equal, so we're going to get that this is the limit as X approaches from this side of 1 over X, 1 over X.

And I'm really unhappy with this orange, so I'm picking up the other. Okay, so 1 over X. I don't know why that one, these are all brand new, but for some reason that one, just this it just came out of the machine just not as strong. Okay, so when we're taking this limit here, so what do we do now? So this tells me that I need to, right, well first off, always good to draw the picture. So we're approaching zero. And we use red this time to say that we're, right, this is the zero minus side, right? Like over here, right? If I was doing it from the right, that would be the zero plus side, but I'm doing it from the left. So that's the zero minus side. Okay? Meaning I'm looking at negative values, right? So this is going to tell me that I need to check what happens as I sub in smaller and smaller negative X values. Okay? So I want to check what happens. So check what happens as I sub in, so as I sub in, so again, it's still smaller and smaller. But now it's negative, so smaller and smaller negative X values, so negative X values. Okay, so let's go ahead and do that. So it's the same kind of deal. I'm just putting minuses there. So I have 1 over, so I have 1 over, so we have a minus here, 1/10. Okay, and then, so that's going to equal minus 10. And then I have to go 1 over, and then I have 1 divided by minus 100. So I get out minus 100.

And you can kind of figure out the pattern here pretty quickly. So I'm going to go 1 over a minus, I guess that minus more typically would go right there. Okay, so minus 1 over 1000, which is going to equal minus 1000. Okay, and then what, so what do we end up getting? So there's kind of an analogue to this here. So what I have going on is I have one over, so now it's going to be very, very small, but negative. So it's, we can write it like that, again, just kind of emphasize this is very, very small, negative. And when we have 1 over very, very small negative, then we end up getting that this goes to minus infinity. Right, because we have this picture, and I didn't draw the picture there. But you can actually see this picture here. So maybe we'll share this picture. These are right next to each other anyway, that what I'm doing here is I'm approaching, right? So if I approach this instead, so at first I was going as X goes to minus infinity, but now I'm going as X goes to zero plus, not going to show the picture I feel like it's way too crowded. So let's not do that. Let's just draw it new over here. So we have like this. And then we have like this, and this is my X axis. You can see that, and now we're, our X is approaching zero from the left hand side. So we put the zero minus there, and our 1 over X values are going to get, they're going to get infinitely big negative, right? So 1 over X gets infinitely big, but negative.

Okay, so putting all that together, I'm going to get that the limit as X approaches zero from the left hand side, okay, of 1 over X is going to equal minus infinity. Okay, let's box this, and then we can kind of recap everything we did, because a lot went on here. Okay, so we're looking at when we have something in the denominator, it doesn't need to be X. But with X, it's easiest to see kind of the principles going on. Okay? And the principles going on are, well, we know if we put bigger and bigger things in the bottom, which is the same thing as X going to infinity, if we put bigger and bigger things in the bottom, then just like slicing up a pie into more and more pieces, the pieces get smaller and smaller. So what we're getting out is approaching zero, and so our limit is zero. If we get bigger and bigger things in the bottom, but they're negative, it actually doesn't impact it, because we're just getting smaller and smaller negative things, but they're still approaching zero. Okay, so that's what's going on. In those two circumstances, you're gonna see those circumstances showing up with other functions, which is why I kind of wanted to go through that. So what happens? Okay, so now I'm approaching zero from the right hand side. So that's where we have this kind of plus sign there. And in that circumstance, we're putting in smaller and smaller positive X values. Well, what happens when you put in smaller and smaller values on the bottom, it's kind of like having 1 over something on the

bottom, you know, and so you flip it over and it gets really big, right? You flip over small, you get big, you flip over big, you get small, you can kind of think of it a little bit like that. And so that what's happening is on the bottom, you have very, very small positive, and this kind of limit goes to plus infinity. Okay, and you can kind of see that in the picture. If we add a negative to that instead, right, so as we approach from the left hand side, here, we're looking at negative values, but they're negative, very, very small. And so when you flip them, you get negative really, really big, which is going to lead you to minus infinity. And we're kind of going to abbreviate that situation. Sometimes it's like VVSN, or very, very small negative on the bottom. But when you have 1 over very, very small negative, that is actually going to approach minus infinity. The last thing I kind of want to emphasize is that while we do put a minus in front of minus infinity, sometimes we put a plus in front of the plus infinity and sometimes we leave it out. Okay, so I hope that made some sense, and I'll see you in the next lecture.