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SPEAKERS

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Welcome. In this lecture, we're going to talk about left limits versus right limits, which is really about, you know, you're taking the limit as X approaches A . And the question is, are you approaching A from the left side, or are you putting A from the right side? And you will ultimately need that you get the same thing, but these are on their own important kind of understanding the left hand limit and the right hand limit. So we're going to kind of go through that in this lecture. So sometimes the limit doesn't exist. So sometimes a limit doesn't exist. Right, this is does not exist, does not, right, that's where that comes from. So it does not exist, right? Because it depends on the direction of approach. Right? Because it depends on the direction of approach. So it depends on the direction of approach, right? And specifically, we're talking about, is this from the left side or the right side. So we'll get into this, okay. So suppose, so here's kind of our setup. So suppose that we have that we have this function, so we have G is going to be a function of R . Okay, and it's gonna equal, so this is going to, this is a piecewise-defined function. So it's going to equal R if we have the R happens to be less than or equal to 2, okay. So if R is less than or equal to 2. And then it's going to be minus R , so for the same value R , so it's going to be like minus R , if we're in a circumstance where R is greater than 2. So if R is greater than 2.

Okay, so let's kind of look at the picture of what's going on here. Okay, so for the first one, we're looking at the circumstance, you know, we have our R right, and we're looking at, so we have our, you know, we're looking at a circumstance where R is less than or equal to 2 right, so this could be, this would be 2. And then we've got, we're looking at values that are over here. Okay, um, but when we're looking at the limit, we're kind of looking at, and we would write 2 minus here, okay? We'll kind of we'll get into that. And then the picture for this one is going to be, so we have R here, okay, and now we've got open because this is greater than, you know, this is, there's no equals here. And again, at 2, and this is going to turn out to be right, we're gonna be approaching from this side, this is going to turn out to be, this is the 2 plus way to approach that. But we'll we'll get into that a little bit, okay? Um, so what happens at 2? So, let's look at the graph of this function.

So this is my output, this is my Y , and my input is R , this is my input R . And we're going to focus in on what's happening at 2, specifically, so let's just kind of graph this. So okay, so we've got at 2 we're looking at the values, right, on this side. So, right, we get, it looks like R , right, so it's just going to

look like a slope of 1 here. And I know the arrow points that way, but this is just a slope of 1. So it really does just kind of look like R. And then we're going to have a slope of minus 1, because we put a minus in front of it here. So we're going to be, kind of start at minus 2, and then, right, so and then this is going to go down with this slope, right minus R. So this, this whole graph that I drew here is $Y = G \text{ of } R$, right? Okay, so let's look at what's happening here. Um, so what happens as I approach from the left hand side like this? Well, if I approach from the left hand side, I'm looking at this piece, which is the same thing as here, and I actually get, so approaching from this side, okay, I'm actually going to get this value of 2, right, because that's what I approach as these input values approach 2, that my output is going to approach this value, which is actually 2. Okay?

So what I get here is that my limit, my limit as R approaches, and I'm going to approach 2, but I'm going to approach 2 from the left side. So I denote that with a little minus there of G of R, right? When I'm in that circumstance, I'm using this function here, I'm on this side, okay? I'm looking at values that are less, and this is going to equal 2. Okay, so that's the left limit, comes from values that are less than that. And so we have to use the part of the function that gives us the values that are left. And then for the right hand side, we're going to get, so now we're approaching from the right hand side. So this is like the 2 plus side for my inputs. And then what happens with the input as I approach that? Well I get this here, which is what happens if you put 2 into this one, you get out minus 2, so this is minus 2 here. So this here is going to be minus 2, which is going to equal, so this is going to equal right, this is the limit as I, as this R, right, would approach 2 from the plus, right, from the right hand side, so G of R.

Okay, so to kind of, you know, summarize what's going on here, kind of, so I drew these kinds of pictures of it. So as R, right, if I'm coming from this side, what am I doing? Well, right, as I have in this picture over here, I've got right, I've got that I'm approaching 2 from the left side, that's what that's going to indicate to us. Okay. So I'm going to be approaching 2 from that side. So this is a, this is what I mean when I've got this going on. Okay, um, but in more generally, what it kind of says is that I'm looking at values for G of R. So I look at values for G of R when I have that, okay, so if I'm taking this limit, with the 2 with the little minus sign there means I'm coming from the left, so I'm looking for values of G of R, where I have that R is really close to 2, so R is really close to 2, but less than 2, so I'm on this side, so really close but less okay? But less than 2.

Here I kind of explain all this to you in multiple ways, so you have all these pictures and how they fit together, okay. And now let's look at the other one. So R approaching this side like this, okay? So this is like having, so I have, right, here's my R, what I'm doing is I'm looking at, you know, I'm looking at this this 2 here, but I'm looking at, right, I'm coming from this side, this is what I mean when I write that my R is approaching like this, I'm looking at the, when I output G as I approach from this side, so I'm, I'm bigger, I'm greater than 2, but I want to get really close to 2 and then look at my outputs. Kind of like I did over here, I get really close to 2, but greater than 2 and I look at my outputs and they're down close to minus 2, okay? So I look at values. So I look at values for G of R when I have that, when I'm looking for the outputs when R is really close to two, but now greater than 2, okay? So when my R, okay, so when R is really close to 2. So when R is really close to 2, but instead of being less than like I had there, it's actually going to be greater than. Okay?

Right, but greater than 2, okay? Okay, so where does all of this come into play? Okay, then we have,

so we have these kinds of pictures. So we kind of understand what the left hand and right hand limits are. And then there's something key going on here, I'm going to make myself a little bit of room by scooting this over a little bit, kind of need that, put this R , put a little box here to talk about something in here. So what I want to talk about in here, which is that having that the limit as X approaches A of, and now I'm going back to more standard notation, but it's good to be able to translate back and forth in your mind. So having that this equals L has the same meaning, as we have that, if I take the limit as X approaches A from the left hand side of F of X , I'm having troubles with dropping markers, that this is going to equal L , and, right, so I would have two things, is I need to have both this being true. So I get the same thing, I get this out when I come from the left hand side, and when I come on from the right hand side. So there's kind of a lot going on here, as in it has to, both of these limits, they have to exist, and they have to equal this L . So they have to equal the same thing, which is this L . Okay. And so I get both from the left hand side and the right hand side, I have to get L . Okay, so this is what it would mean for that limit to equal L , okay. So the takeaway from that for us, right, if this is what it means for that limit to equal L , is that the left hand and the right hand limits both equal L . In our circumstance, our left hand and right hand limits are different, right? These are not equal, okay? So 2 and -2 are not equal. Okay, so what this is all giving me is that, so, using this, that we have right here, we know that the limit as R , right, the limit as R approaches 2 , it does not exist of this function G of R , it does not exist.

Right? Because I would get different things, right, I would get 2 if I approach from the left, and I would get -2 if I approach from the right, okay, and that's not a lot, you have to get the same thing. Okay. So what's going on with left hand right hand limits? We do have to have that those are the same. And so we have to keep in mind what those actually mean. Okay, what does this mean? It means I'm approaching, my inputs are approaching from the left, right, and I'm trying to get close to here, but less than here, okay. And I'm looking at my outputs when my inputs are less than 2 , but close to 2 . Here, I'm looking when my inputs are close to 2 but greater than 2 , and then I'm looking at my output, and that's going to give me the right hand limit, okay? And so if you're actually looking at a particular function, what does that kind of mean for us? If I'm looking from the left hand limit, I need to look at what part of the function is going to tell me, you know, is going to work for values that are less than 2 , right? I need the values to be less than 2 . So this part doesn't work. And in the picture, you can kind of see that, if you're coming from this direction, it's this part of the graph, okay? And then for the right hand one, you're looking at values that are greater than 2 . So that's where this part kicks in. Okay, so I hope that made some sense, and I'll see you in the next lecture.