

Global, instantaneous, centimetre, satellite-based positioning with Precise Point Positioning

by

Nacer Naciri

Dissertation Submitted in Partial Fulfillment of the
Requirements for the Degree of
Doctor of Philosophy

in the
Department of Earth and Space Science
Faculty of Graduate Studies

© Nacer Naciri 2023
YORK UNIVERSITY
April 2023

Copyright in this work is held by the author. Please ensure that any reproduction
or re-use is done in accordance with the relevant national copyright legislation.

Abstract

Real-Time Kinematic (RTK) has been the reference technique when it comes to precise (centimetre-level) positioning for Global Navigation Satellite Systems (GNSSs). The fact that RTK performance depends heavily on the distance between the receiver and the base station has led to the technique being deployed only regionally, in areas where GNSS reference station infrastructure is present. Precise Point Positioning (PPP) is a global solution by design, though requiring tens of minutes to reach RTK levels of performance.

In this research, attempts are made to improve PPP performance to reach RTK-type performance. Given that PPP is a measurement-dependent technique, this research starts with making use of signals from all four major GNSS constellations: GPS, GLONASS, Galileo, and BeiDou-2/3. These constellations are coupled with the use of up to four frequencies, as opposed to only two frequencies as is typical for PPP. Carrier-phase ambiguities are resolved to their integer values on all processed signals as well. A model is derived from first principles to process all these measurements and fix their ambiguities.

Results show that incorporating four frequencies has great benefits in improving user positions and increasing the likelihood of correct fixing of ambiguities due to the correlation that exists between ambiguities from the same satellite. Results demonstrate that PPP can reach regional RTK levels of performance with only using global corrections, as instantaneous convergence to 2.5 cm error is achieved consistently, making of PPP a truly global precise positioning technique.

With these results in mind, corrections from Galileo's High Accuracy Service (HAS) are analyzed, where, for the first time on a global scale, corrections are being transmitted by a GNSS constellation. Thanks to early access to these signals, an in-depth analysis of the corrections and user performance is carried out. Good performance is found to be achieved with these limited test signals, showing that PPP can become the de facto global precise positioning technique.

In cases where PPP needs to be augmented, a proof-of-concept is proposed where a global PPP solution is augmented with ionospheric corrections generated from an NRTK network. The generated corrections are found to improve PPP solution by reducing convergence time.

Acknowledgements

I would like to express my gratitude to my supervisor, Professor Sunil Bisnath. When I decided to move to a continent that I never thought I would ever visit, my hope was to get more out of it than a degree; and Professor Bisnath made sure that I did. Through his mentorship, guidance, and beneficence, not only did I discover the research and GNSS worlds, he also thought me soft and life skills. For all those reasons (and more), I will always be grateful. I am now closer than ever to being able to independently make decisions.

I would also like to thank Professors Jianguo Wang and Spiros Pagiatakis for being in my supervisory committee and for all the guidance and advice throughout my Ph.D. journey. Thank you to Professor Regina Lee for stepping in at the last minute to allow for a timely defense, and to the examining committee in general.

None of the work in this dissertation would have been possible without contributions and support from members of the GNSS Laboratory, who have gone from being strangers, to colleagues, to friends. My sincere thanks to Sudha, John, Garrett, Ding, Jiahuan, and others. The group lunches, meetings, discussions at the lab, and group activities will be missed.

I am grateful to my family for giving me the education to be where I am now, and for their constant support throughout this degree. An ocean separates us, but they are always present with me. I am also grateful to the Niri family for being a family away from home, especially Fatima Zahra for being the best.

I will remember my time at the GNSS Laboratory as a time of growth, and there is no quote to describe it better than: "Si on ne raisonne pas on a l'éternité devant nous" by Roger Chomeaux.

Table of Contents

Abstract	ii
Acknowledgements	iii
Table of Contents	iv
List of Tables	viii
List of Figures	x
List of Acronyms	xx
1 Introduction	1
1.1 The evolution of GNSS-based precise positioning	1
1.2 Problem statement	2
1.3 Dissertation statement	4
1.4 Research contributions and significance	4
1.5 Dissertation outline	7
2 GNSS precise positioning techniques	9
2.1 Relative positioning: differential GNSS (DGNSS), Real-Time Kinematic (RTK), and network RTK (NRTK)	9
2.1.1 Differential GNSS	10
2.1.2 Real-Time Kinematic and network RTK	11
2.2 Precise Point Positioning (PPP)	13
2.2.1 Observation equations	15
2.2.2 Common combinations	15
2.2.3 Error models	17
2.2.4 Ambiguity resolution	20
2.3 Recent advances in GNSS and their impact on PPP	23
2.3.1 Effects of ambiguity resolution	23
2.3.2 Effects of modern constellations	27

2.3.3	Modernized and new GNSS signals and frequencies	34
2.4	Summary	42
3	Multi-constellation, multi-frequency PPP ambiguity resolution	44
3.1	Introduction to the decoupled clock model	45
3.2	Original decoupled clock model	46
3.3	Uncombined, dual-frequency, decoupled clock model	47
3.3.1	Reference satellite	48
3.3.2	Non-reference satellites	49
3.3.3	Complete dual-frequency model	50
3.4	Uncombined, triple-frequency, decoupled clock model	50
3.4.1	Third frequency pseudorange	51
3.4.2	Third frequency carrier-phase	51
3.4.2.1	Reference satellite	51
3.4.2.2	Non-reference satellites	52
3.4.3	Complete triple-frequency model	52
3.5	Reference satellite selection and multi-GNSS considerations	53
3.6	Model parameters and position performance analysis	56
3.6.1	Galileo-only results	57
3.6.1.1	Dual-frequency AR results	58
3.6.1.2	Triple-frequency AR results	62
3.6.1.3	Kinematic processing	71
3.6.2	Multi-GNSS results	74
3.6.2.1	Performance comparison with dual-, triple-, and mixed dual- /triple-frequency processing	76
3.6.2.2	Mixed dual-/triple-frequency performance analysis	80
3.6.3	Multi-GNSS, triple-frequency, kinematic automotive results	85
3.7	Summary	89
4	RTK-quality positioning with global PPP corrections	91
4.1	Background and motivation	92
4.2	Expansion of the decoupled clock model to four frequencies	93
4.2.1	Fourth frequency pseudorange	94
4.2.2	Fourth frequency carrier-phase	95
4.2.2.1	Reference satellite	95
4.2.2.2	Non-reference satellites	96
4.2.3	Summary of the complete four-frequency model	96
4.3	Redundancy analysis	98

4.3.1	Redundancy of the decoupled clock model	99
4.3.2	Redundancy of the classic PPP model	100
4.3.3	Analysis of PPP redundancy	101
4.4	Processing and analysis strategies	105
4.5	Position performance analysis	108
4.5.1	Average solution performance	108
4.5.2	Benchmarking against RTK-quality	113
4.5.3	Analysis and improvement of single-epoch results	120
4.5.3.1	One-, two-, and three-epoch PPP	120
4.5.3.2	Augmentation with Global Ionospheric Map (GIM)	123
4.5.3.3	Discrepancy between stations	125
4.6	Summary	128
5	Self-serve RTK: mixing global and regional products	131
5.1	Introduction and motivation	132
5.2	NRTK corrections for PPP processing	134
5.2.1	Analysis of NRTK corrections	135
5.2.2	Effect of corrections on user performance	138
5.2.3	PPP positioning with NRTK corrections	141
5.3	Combination of global and regional corrections	143
5.3.1	Method description	145
5.3.2	Application to a sample dataset	147
5.3.3	Adaptation for real-time processing	152
5.4	Summary	158
6	RTK from space: Galileo HAS	161
6.1	Galileo HAS description	163
6.2	Galileo HAS Signal-in-Space	164
6.3	Assessment of HAS corrections	166
6.3.1	Availability analysis	166
6.3.2	Satellite orbit and clock corrections	168
6.3.3	Satellite code bias corrections	173
6.4	PPP performance with Galileo HAS corrections	179
6.4.1	Processing strategies and data	179
6.4.2	Positioning results	180
6.4.3	Effect of simulated correction latency on HAS position results	183
6.5	Summary	184
7	Conclusions and recommendations for future research	187

7.1	Conclusions	187
7.1.1	Four-constellation, four-frequency PPP-AR (Chapters 3 and 4) . . .	188
7.1.2	Epoch-by-epoch PPP and RTK-like quality (Chapter 4)	189
7.1.3	NRTK corrections for PPP (Chapter 5)	190
7.1.4	Galileo HAS and precise SPP (Chapter 6)	191
7.2	Recommendations	192
7.2.1	Five-frequency PPP-AR	192
7.2.2	Partial ambiguity resolution (AR)	193
7.2.3	Ambiguity validation	193
7.2.4	Dissection and improvement of epoch-by-epoch results	193
7.2.5	Analysis of full HAS service	194
7.2.6	Maturity of global and regional mixing proof-of-concept	194
7.2.7	DCM for atmospheric modelling	194
7.2.8	Application to low-cost hardware	195
	References	196

List of Tables

Table 2.1	Convergence time, horizontal and vertical root mean square error (rms) statistics corresponding to the results in Figure 2.15.	40
Table 3.1	States and number of states to be estimated for each constellation and frequency. A cross in the state dependency column (One per ...) means that a state term is either receiver-, constellation-, or satellite-dependent. A cross in the measurement type column specifies whether pseudorange or carrier-phase measurements are used to estimate the state. A cross in the frequency band column means that measurements on the selected frequency are used to estimate the state term.	55
Table 3.2	Processing strategies for the estimated parameters.	57
Table 3.3	Horizontal convergence time and rms statistics for the results in Figure 3.4.	60
Table 3.4	Vertical convergence time and rms statistics for the results in Figure 3.5.	62
Table 3.5	Horizontal and vertical convergence time and rms statistics for results in Figure 3.15.	71
Table 4.1	Number of satellites with one, two, three, and four frequencies for each constellation (QZSS Cabinet Office, 2022).	102
Table 4.2	Processing strategy for the estimated parameters.	106
Table 5.1	rms statistics comparison using both approaches described in sections 5.3.2 and 5.3.3 for the different ranges and extracted biases of interest. These statistics relate to the dataset that was used in the current section for station JARG on day 273 of year 2021.	156
Table 6.1	Range and specific indicators of the HAS satellite orbit, clock, and code bias corrections. Validity and update interval values are specific to test campaigns. The indicators are specific to the NovAtel decoder format, and binary values for High Accuracy Service (HAS) can be found in European Union (2022).	165

Table 6.2	rms of the radial, along-track, cross-track, 3D orbit, and clock errors of HAS relative to Center for Orbit Determination in Europe (CODE) final products on day 247 of year 2022. The rms are computed taking into account all satellites from each constellation. Values are in cm.	171
Table 6.3	PPP processing strategy for the HAS processing.	182
Table 7.1	Summary of the main performance demonstrated in the dissertation, and magnitudes of the achieved convergence times, as well as means of achieving the performance.	189

List of Figures

Figure 2.1	Single Point Positioning (SPP) and Differential GNSS (DGNSS) for station GOPE in the Czech Republic on Day Of Year (DOY) 245, 2022. The DGNSS solution uses a base station with a baseline of 33 km.	11
Figure 2.2	Position performance of station VILL in Spain, processed with Network Real-Time Kinematic (NRTK) corrections generated from the network in 2.2a on Day Of Year (DOY) 81, 2022. Station VILL is the red marker on the map.	14
Figure 2.3	Diagram of the PPP-AR estimation and carrier-phase fixing process (Teunissen et al., 1995). The mathematical notation in the diagram follows the same notation as the reference.	24
Figure 2.4	Locations of the global set of stations	26
Figure 2.5	Average horizontal position error at the 100 th percentile with and without Ambiguity Resolution (AR) based on processing over 2000 datasets on two frequencies for GPS, GLONASS, Galileo, and BeiDou-2.	26
Figure 2.6	Dependence of the 100 th percentile average solutions on the mean number of satellites and mean Position Dilution Of Precision (PDOP). Black lines and symbols refer to float solutions, while red lines and symbols refer to fixed solutions.	28
Figure 2.7	Distribution of the PDOP and number of satellites for three different constellation combinations.	29
Figure 2.8	100 th percentile horizontal (a) and vertical (b) average position solutions. Italicized constellations with dashed lines are float Precise Point Positioning (PPP) (without AR), while bold underlined constellations with solid lines are fixed PPP (with AR).	30
Figure 2.9	100 th percentile convergence time and rms statistics corresponding to the results shown in Figure 2.8.	30

Figure 2.10	Distribution of stations used in the user processing of ultra-rapid products in red. Black dots represent stations for which some of the results are shown in Section 2.3.3.	31
Figure 2.11	Average static convergence time (a), average epoch-by-epoch number of satellites (b), average static and kinematic horizontal rms (c), and static and kinematic vertical rms (d) for different constellation combinations at the 100 th , 95 th and 67 th percentiles.	33
Figure 2.12	Difference between the estimated BeiDou-2 and BeiDou-3 receiver clocks for station TLSE in Toulouse, France on DOY 183, 2020 between UTC hours 8 and 15.	37
Figure 2.13	BeiDou-2 pseudorange PR (a) and carrier-phase CP (b) residuals and BeiDou-3 pseudorange PR (c) and carrier-phase CP (d) residuals for station TLSE on DOY 183, 2020 with separate receiver clocks for each generation.	38
Figure 2.14	BeiDou-2 pseudorange PR (a) and carrier-phase CP (b) residuals and BeiDou-3 pseudorange PR (c) and carrier-phase CP (d) residuals for station TLSE on DOY 183, 2020 with a common receiver clock for both generations.	38
Figure 2.15	Average horizontal (a) and vertical (b) errors for station CRO1 on DOY 186, 2020 between UTC hours 6 and 12. GEC constellations are processed in kinematic mode on L1,E1,B1/L5,E5a,B2a frequencies, with and without the application of GPS L5 corrections.	39
Figure 2.16	Difference in the estimated GPS-related states when using the GPS L1/L2 and L1/L5 clocks for station BRST in Brest, France on DOY 187 (a, c, d, e, f), and applied L5 corrections (b).	41
Figure 3.1	Reference satellite choice strategy when processing a combination of dual- and triple-frequency satellites.	55
Figure 3.2	World map of the stations used in the processing.	59
Figure 3.3	(a) Receiver code clock $\overline{dt_r}$, receiver phase clock $\overline{\delta t_r}$ and (b) receiver L2 phase bias δt_{12} for station YELL on DOY 344, 2019.	60
Figure 3.4	Static dual-frequency average horizontal error based on all datasets at the (a) 100 th , (b) 95 th , (c) 67 th and (d) 50 th percentiles. The horizontal dashed lines represent the convergence threshold.	61
Figure 3.5	Static dual-frequency average vertical error based on all datasets at the (a) 100 th , (b) 95 th , (c) 67 th and (d) 50 th percentiles. The horizontal dashed lines represent the convergence threshold.	61

Figure 3.6	Static dual-frequency average horizontal convergence times and rms for each station - 56 datasets per station.	63
Figure 3.7	(a) Receiver L3 phase bias δt_{13} and (b) Inter-Frequency Bias $IFB_{r,3}$ for station YELL on DOY 344, 2019.	64
Figure 3.8	Kinematic (a) dual-frequency and (b) triple-frequency horizontal error and fixed ambiguities for station YELL on DOY 344, 2019 between 21:00 and 23:00 UTC. The ambiguities jumping from zero to non-zero values mean that a satellite starts being processed. Ambiguities jumping from non-zero values to zero mean that a satellite stops being processed.	65
Figure 3.9	Static dual- and triple-frequency average horizontal error based on all datasets at the (a) 100 th , (b) 95 th , (c) 67 th and (d) 50 th percentiles. Horizontal dashed lines represent the convergence threshold.	68
Figure 3.10	Static dual- and triple-frequency average vertical error based on all datasets at the (a) 100 th , (b) 95 th , (c) 67 th and (d) 50 th percentiles. Horizontal dashed lines represent the convergence threshold.	68
Figure 3.11	Histograms comparing the Galileo horizontal convergence times of the static dual-frequency fixed solutions with the triple-frequency fixed solutions.	69
Figure 3.12	Distribution of (a) the number of Galileo satellites and (b) PDOP based on all epochs from all datasets.	69
Figure 3.13	(a) Average horizontal convergence times and (b) rms for both dual-frequency and triple-frequency solutions at different percentiles in static mode.	70
Figure 3.14	(a) Average vertical convergence times and (b) rms for both dual-frequency and triple-frequency solutions at different percentiles in static mode.	70
Figure 3.15	Static (a) horizontal and (b) vertical dual- and triple-frequency float and fixed solutions for datasets with an average of over six satellites during the first hour. Horizontal dashed lines represent the convergence threshold.	72
Figure 3.16	Kinematic dual-frequency and triple-frequency horizontal vertical error based on all datasets at the (a) 100 th , (b) 95 th , (c) 67 th and (d) 50 th percentiles. Horizontal dashed lines represent the convergence threshold.	73

Figure 3.17	Kinematic dual-frequency and triple-frequency average vertical error based on all datasets at the (a) 100 th , (b) 95 th , (c) 67 th and (d) 50 th percentiles. Horizontal dashed lines represent the convergence threshold.	73
Figure 3.18	(a) Average horizontal convergence times, (b) horizontal and (c) vertical rms for both dual-frequency and triple-frequency solutions at different percentiles in kinematic mode.	75
Figure 3.19	IGS stations used in processing.	75
Figure 3.20	Time series comparison of average 100 th , 95 th , and 67 th percentile horizontal and vertical errors for dual-frequency, triple-frequency, and mixed dual-/triple-frequency combinations both with and without ambiguity resolution. Horizontal dashed lines represent convergence threshold.	77
Figure 3.21	Horizontal convergence time for dual-frequency, triple-frequency, and mixed dual-/triple-frequency results with and without AR at 100 th , 95 th , and 67 th percentiles. Convergence time statistics correspond to results in Figure 3.20.	78
Figure 3.22	Histogram of number of satellites being processed in dual-frequency, triple-frequency, and mixed dual-/triple-frequency processing. The graph is based on all epochs from all datasets. “dual” (green) and “dual/triple” (yellow) histograms are overlapping in the dark yellow histogram.	79
Figure 3.23	100 th , 95 th , and 67 th percentile horizontal and vertical convergence times and rms for solution combining dual- and triple-frequency satellites, with and without AR.	81
Figure 3.24	Float, dual-frequency fixed, and mixed dual-/triple-frequency fixed histogram of horizontal convergence time of all processed datasets.	83
Figure 3.25	(a) Dual-frequency, triple-frequency, and mixed dual-/triple-frequency 3D errors for station GODS, on DOY 72, 2021 between UTC hours 5 and 6, and (b) number of satellites with two and three frequencies at station GODS on DOY 72, 2021.	84
Figure 3.26	Ground tracks of the datasets used in kinematic processing	86
Figure 3.27	Float and fixed dual-frequency and dual-/triple-frequency horizontal (a) and vertical (b) errors, number of satellites per frequency (c), and ambiguity success rate (d) for the first kinematic dataset collected near York University on DOY 337, 2020.	87

Figure 3.28	Float and fixed dual-frequency and dual-/triple-frequency horizontal (a) and vertical (b) errors for the open-sky kinematic dataset collected near York University on DOY 151, 2021. The car is static during the first 7 minutes, and kinematic for the rest of the dataset.	87
Figure 4.1	Daily average number of visible GPS, GLONASS, Galileo and BeiDou-2/3 satellites on day 305 of year 2021.	102
Figure 4.2	Effect of the number of satellites, constellations, and frequencies on the redundancy / degrees of freedom of the PPP model. (b) is a more realistic version of (a), as it takes into account the limited number of satellites observed with a limited number of constellations. . . .	103
Figure 4.3	Map of the stations used in the processing.	106
Figure 4.4	Kp and f10.7 indices for the period of processing, on November 2021.	109
Figure 4.5	Time series of the average horizontal and vertical errors at the 100 th , 95 th , and 67 th percentiles for the dual-frequency, dual- and triple-frequency and all-frequency fixed, and dual-frequency float solutions. The figure focuses on the first 20 minutes of processing and on the decimetre-level errors. The Xth percentile is computed by taking, for each epoch, the average of the X% datasets with the lowest error, with $X \in \{100, 95, 67\}$	110
Figure 4.6	Average 100 th , 95 th , and 67 th percentile (a) convergence times to 10 cm horizontal error, (b) horizontal rms, and (c) vertical rms for the dual-, dual- and triple-, and all-frequency solutions, with and without AR. These statistics correspond to the time series in Figure 4.5. . .	111
Figure 4.7	95 th percentile average convergence times to 10 cm per station. Each bar is computed by 1) computing the convergence times of all datasets, 2) only keeping the 95% smallest convergence times, and 3) computing the mean of these smallest convergence times.	114
Figure 4.8	67 th percentile average convergence times to 2.5 cm per station. Each bar is computed by 1) computing the convergence times of all datasets, 2) only keeping the 67% shortest convergence times, and 3) computing the mean of these shortest convergence times. . .	115
Figure 4.9	(a) 100 th and (b) 67 th percentile distributions of the epoch-by-epoch horizontal errors using all datasets. Each bin corresponds to 5 cm. The i th percentile distribution, with $i \in \{100, 67\}$, means that the i % epochs with the lowest horizontal errors are plotted.	116

Figure 4.10	Percentage of epochs with horizontal errors below 2.5 cm per station in the epoch-by-epoch solutions at the (a) 100 th and (b) 67 th percentiles. Each bar represents the percentage of a station's epochs below 2.5 cm horizontal error over one week. At the 67 th percentile, each bar is based on the 67% epochs with the lowest horizontal errors. The horizontal dashed lines represent the average across all stations.	119
Figure 4.11	Time series of the multi-constellation all-frequency ambiguity-fixed epoch-by-epoch solutions of one well-performing station (ARTH00ATA), one average-performing station (GOPE00CZE), and one bad-performing station (BOGT00COL) on day 305 of year 2021 between 6 AM and 9 AM UTC. The table shows the percentage of epochs (based on all epochs over three hours) which are below 10 cm and 2.5 centimetre errors.	120
Figure 4.12	Percentage of epochs with horizontal errors below 2.5 cm per station at the 67 th percentile for three solutions: processing each epoch, every two epochs, and every three epochs independently. All solutions use all frequencies on all constellations. Each bar represents the percentage of a station's epochs below 2.5 cm horizontal error over one week. At the 67 th percentile, each bar is based on the 67% epochs with the lowest horizontal errors. The horizontal dashed lines represent the average across all stations.	122
Figure 4.13	67 th percentile distributions of the one-, two-, and three-epoch solutions' horizontal errors using all datasets. Each bin corresponds to 5 cm. The figure is based on the 67% epochs with the lowest horizontal errors.	122
Figure 4.14	Percentage of epochs with a horizontal position error below 2.5 cm error at the 67 th percentile with and without the use of Global Ionospheric Map (GIM).	124
Figure 4.15	Correlation figure between the percentage of epochs below 2.5 cm errors at the 67 th percentile in an epoch-by-epoch solution and the average number of satellites (top), and of PDOP (bottom). Each dot is generated based on all data for that station.	126
Figure 4.16	Bar graphs of the 67 th percentile performance of the epoch-by-epoch solutions. The percentage values are the same as in Figure 4.10b. Figure 4.16a is color-coded based on the receiver type, while Figure 4.16b is color-coded based on the receiver antenna type.	127

Figure 4.17	Location of the stations, color-coded based on the percentages of epochs below 2.5 cm horizontal error for the 67 th percentile epoch-by-epoch multi-Global Navigation Satellite System (GNSS) multi-frequency solution, as shown in Figure 4.10b.	129
Figure 5.1	Global and regional satellite radial, along-track, cross-track, clock, and L1/E1 and L2/E5b code bias corrections for GPS and Galileo satellites on day 230 of year 2021. Each color corresponds to a different satellite.	137
Figure 5.2	Correlation figures between regional corrections and the GPS and Galileo satellites' elevation angles on day 230 of year 2021. Each color corresponds to a different satellite. The mean values for each satellite are removed to highlight trends.	138
Figure 5.3	Effect of satellite code and phase biases on float PPP solutions using both global and regional products on station VILL in Spain on day 81 of year 2022. The solutions are generated based on dual-frequency observations for GPS and Galileo.	139
Figure 5.4	Effect of regional atmospheric corrections on float and fixed PPP solutions for station VILL in Spain on day 81 of year 2022. The solutions are generated based on dual-frequency observations for GPS and Galileo.	141
Figure 5.5	Average position results for PPP processing with NRTK corrections generated from a network in France. <i>in-loop</i> results are based on processing the same stations that were used to generate the corrections (triangles in Figure 5.5a), while <i>out-of-loop</i> results are based on processing stations within the NRTK network's boundaries, but not used in the correction generation (red circles in Figure 5.5a).	142
Figure 5.6	Global ionosphere (top left), regional ionosphere (top right), their differences (bottom left), and difference between global and regional troposphere (bottom right) for station JARG in France on day 273 of year 2021. Each colour represents a different satellite, with satellites from Galileo and GPS being shown.	145
Figure 5.7	Diagram of the strategy used to generate precise atmospheric corrections that are compatible with global products from NRTK corrections.	148
Figure 5.8	Error in the extracted ionospheric delays (top), and difference between estimated global ranges and corrected regional ranges (bottom) for station JARG on day 273 of year 2021.	148

Figure 5.9	Between-satellite, between-source double-differences of the full Observation State Representation (OSR) ranges for station JARG on day 273 of year 2021. Constant satellite-dependent biases are removed from Figure 5.9a in Figure 5.9b.	150
Figure 5.10	Between-satellite differences of the error in the extracted ionospheric delays without (top) and with (bottom) the removal of the between satellite-differences. The latter were estimated as shown in Figure 5.9.151	
Figure 5.11	Ambiguity-fixed position performance for station JARG on day 273 of year 2021 using regional corrections, global corrections, and global corrections together with the extracted regional ionospheric corrections. All solutions use dual-frequency Galileo and GPS data. . . .	153
Figure 5.12	Swimlane diagram of the strategy to mix regional and global State Space Representation (SSR) products in real-time. Green boxes and arrows refer to SSR corrections, light blue boxes and arrows refer to estimates, and red boxes refer to OSR ranges computed with the SSR corrections/estimates.	154
Figure 5.13	Comparison of the estimated biases with the average and real-time approaches for station JARG on day 273 of year 2021.	156
Figure 5.14	Between-satellite, between-source double-differences of the full OSR ranges for station JARG on day 273 of year 2021 after the removal of the biases with the real-time approach (5.14a), and between-satellite difference of the error in the extracted ionosphere with and without biases removal with the real-time approach (5.14b).	157
Figure 5.15	Ambiguity-fixed position performance for the global solution without atmospheric augmentation, and with two global solutions that use ionospheric corrections derived using the average and real-time approaches for station ISLA in France on day 272 of year 2021. . .	159
Figure 6.1	Availability of HAS orbit and clock corrections on day 247 of year 2022 for Galileo and GPS satellites.	167
Figure 6.2	Percentage of time with available HAS orbit and clock corrections per Galileo (top) and GPS (bottom) satellite between days 242 and 248 of year 2022.	169
Figure 6.3	Along-track (top left), cross-track (top right), radial (center left), and 3D (center right) orbital errors and clock (bottom) errors of Galileo and GPS HAS corrections relative to final CODE orbits and clocks on day 247 of year 2022.	172

Figure 6.4	Orbital Signal-In-Space Ranging Error (SISRE) (top) and total SISRE (bottom) of Galileo and GPS HAS corrections relative to final CODE orbits and clocks on day 247 of year 2022. The horizontal dark red lines represent the Galileo rms, while the black lines represent the GPS rms. Satellites G19 and E12 are excluded from the rms computation.	174
Figure 6.5	Daily average orbital SISRE (orange) and total SISRE (blue) per Galileo and GPS satellite of HAS corrections relative to final CODE orbits and clocks on day 247 of year 2022.	175
Figure 6.6	Daily average difference between HAS code biases and final Differential Code Bias (DCB) products from Deutsches zentrum für Luft- und Raumfahrt (DLR). The graphs are for the period between days 242 and 248 of year 2022 for Galileo and GPS satellites/signals. Each colour represents a different satellite. The x-axis is in the format “day/month”. On September 4th, satellite E25 goes beyond the y-axis and reaches values of -1.3 ns, -1.4 ns, and -1.4 ns for C1C-C5Q, C1C-C7Q, and C1C-C6C, respectively.	177
Figure 6.7	Mean difference and standard deviation per Galileo and GPS satellite of HAS code biases relative to final DCB products from DLR. The statistics are based on one week of data, between days 242 and 248 of year 2022.	178
Figure 6.8	Map of the stations used in the PPP processing and HAS analysis.	179
Figure 6.9	Average PPP horizontal (top row) and vertical (bottom row) performance at the 95 th (left column) and 67 th (right column) percentiles. The solutions include Galileo-only, GPS-only, and Galileo+GPS HAS results in both static and kinematic modes. Horizontal dashed lines represent HAS service specification.	181
Figure 6.10	Convergence time statistics for Galileo-only, GPS-only and Galileo+GPS HAS solutions in both static and kinematic modes. The statistics correspond to the results in Figure 6.9. Horizontal convergence corresponds to convergence below 20 cm horizontal error, and vertical convergence corresponds to convergence below 40 cm vertical error.	183
Figure 6.11	rms statistics for Galileo-only, GPS-only and Galileo+GPS HAS solutions in both static and kinematic modes. The rms is computed over the whole three hours of processing. Green and orange bars represent kinematic and static mode results, respectively. The 95 th percentile vertical rms are 30.4 cm and 52.9 cm in static and kinematic modes, respectively.	184

Figure 6.12	Effect of simulated Galileo HAS correction latency on PPP performance at the 95 th percentile based on processing all datasets described in the previous section. The data are processed in kinematic mode using Galileo and GPS satellite orbits, clocks and code biases. The horizontal lines represent the service specification.	185
-------------	---	-----

List of Acronyms

ANTEX	ANTenna EXchange
APC	Antenna Phase Centre
AR	Ambiguity Resolution
ARP	Antenna Reference Point
CAS	Chinese Academy of Sciences
CDMA	Code Division Multiple Access
CLAS	Centimetre-Level Augmentation Service
CNES	Centre National d'Etudes Spatiales
CODE	Center for Orbit Determination in Europe
DCB	Differential Code Bias
DCM	Decoupled Clock Model
DGNSS	Differential GNSS
DGPS	Differential GPS
DLR	Deutsches zentrum für Luft- und Raumfahrt
DOP	Dilution Of Precision
DOY	Day Of Year
ECEF	Earth-Centred Earth-Fixed
EDCM	Extended Decoupled Clock Model
EUSPA	EU Agency for the Space Programme
EWL	Extra-widelane
FCB	Fractional Cycle Biases
FDMA	Frequency Division Multiple Access
GEO	Geostationary Earth Orbit
GFZ	GeoForschungsZentrum
GISCAD-OV	Galileo Improved Services for Cadastral Augmentation Development On-field Validation
GIM	Global Ionospheric Map
GNSS	Global Navigation Satellite System
GPS	Global Positioning System

HAS	High Accuracy Service
ICD	Interface Control Document
IERS	International Earth Rotation and Reference Systems Service
IFB	Inter-Frequency pseudorange Bias
IGS	International GNSS Service
IGSO	Inclined GeoSynchronous Orbit
INS	Inertial Navigation System
IRC	Integer Recovery Clock
LAMBDA	Least-squares AMBiguity Decorrelation Adjustment
LEO	Low Earth Orbit
MLAMBDA	Modified Least-squares AMBiguity Decorrelation Adjustment
MEO	Medium Earth Orbit
MW	Melbourne-Wübbena
NL	Narrowlane
NRTK	Network Real-Time Kinematic
OSB	Observable-specific Signal Biases
OSR	Observation State Representation
PCO	Phase Centre Offset
PCV	Phase Centre Variation
PDOP	Position Dilution Of Precision
PPP	Precise Point Positioning
QZSS	Quasi-Zenith Satellite System
RINEX	Receiver INdependent EXchange
RTK	Real-Time Kinematic
rms	root mean square error
SA	Selective Availability
SiS	Signal-in-Space
SISRE	Signal-In-Space Ranging Error
SPP	Single Point Positioning
SSR	State Space Representation
TGD	Timing Group Delays
TTF	Time To First Fix
UTC	Universal Time Coordinated
WL	Widelane

Chapter 1

Introduction

Satellite positioning has become a critical part of many industries since the conception of the Global Positioning System (GPS) in the late 20th century. Applications from Global Navigation Satellite Systems (GNSSs) vary from timing, to navigation, cadastral surveying, remote sensing, etc. Despite this large range of applications, estimating a receiver's position - the original purpose for all GNSSs - is still the main use of the many signals being constantly broadcast by these mostly Medium Earth Orbit (MEO) satellites. As such, GNSS-based positioning is the theme of this dissertation, with a focus on precise positioning, where centimetre-level user positioning is sought.

1.1 The evolution of GNSS-based precise positioning

When it comes to outdoor positioning, GNSSs have been the go-to technology thanks to the well-known GPS and, more recently, other constellations in the form of the Russian GLONASS, European Galileo, and Chinese BeiDou. Although GPS was initially designed to allow many metre positioning - tens of metres while Selective Availability (SA) was enabled, and a few metres after it was disabled in 2000 (GPS, 2021) - its potential to provide precise decimetre- or centimetre-level positions was quickly unveiled through measurement differencing. The first iteration came in the form of Differential GPS (DGPS), which relies on the presence of a base station with known coordinates near the rover and achieves decimetre-level positioning using pseudorange measurements only. Incorporating carrier-phase measurements into DGPS leads to Real-Time Kinematic (RTK), which became capable of centimetre-level positioning thanks to the possibility of resolving carrier-phase ambiguities to integers (Van Diggelen, 1997; Zhodzishsky et al., 1999; Gao et al., 1997; Bisnath, 2020b). However, the performance of RTK relies on the distance to the base station, as atmospheric corrections start degrading performance the further the rover is from the base. In an attempt to resolve this issue, NRTK came to life, where a network of base stations is used to interpolate and send corrections to the rover, which is present within the

boundaries of the network (Jensen and Cannon, 2000; Townsend et al., 1999; Geisler, 2006). Given the number of corrections to be sent to the rover, a transition from an Observation State Representation (OSR) to a State Space Representation (SSR) was necessary to limit the bandwidth required to transmit all of the corrections (Wübbena and Willgalis, 2001; Wübbena et al., 2005). Indeed, instead of transmitting range corrections for each signal from each base station to the rover, NRTK estimates different error components as states using the network of reference stations and transmits the states to the rover. The error components include satellite orbits, clocks, biases, and ionospheric and tropospheric delays.

However, both RTK and NRTK assume the presence of nearby receivers that can be used as calibration stations. Unfortunately, such networks are not widely available as they are concentrated only in major cities/metropolises, or densely populated areas in rich countries. In order to provide precise positioning for users outside of NRTK networks, PPP was invented (Zumberge et al., 1997; Muellerschoen et al., 2001; Kouba and Héroux, 2001). The principle for PPP is similar to NRTK, in the sense that SSR satellite products are computed from a network of stations and broadcast to the user. The main differences in terms of performance between PPP and NRTK lie in the presence/absence of precise atmospheric corrections due to the size of the networks, as PPP with regional atmospheric corrections has been shown to provide NRTK-like level of performance (Wübbena et al., 2005; Banville et al., 2014; Teunissen et al., 2010). Indeed, in order for PPP to provide global coverage, it uses global reference stations spread around the world and does not focus on specific areas. Having a worldwide network of stations allows for satellite products to be estimated for all GNSS satellites, regardless of their location, allowing users anywhere to access the corrections for the satellites visible to them. In addition, the global reference station network is sparse (as little as tens of stations (Zumberge et al., 1997)) compared to NRTK, allowing for better decoupling and decorrelation between the various GNSS error components. However, a consequence of such sparse networks is that the atmospheric (troposphere and ionosphere) delays cannot be estimated precisely. Therefore, the atmospheric delays at the user side in PPP have to be estimated without a priori information, significantly increasing the number of states to be estimated. These factors lead to PPP being heavily depending on the quality and number of measurements to be processed. Indeed, the high number of states, together with the absence of precise atmospheric corrections, leads to the original GPS PPP requiring minutes to tens of minutes to achieve centimetre-level positioning (Bisnath and Gao, 2009b).

1.2 Problem statement

PPP has historically been known for its long convergence times. Since the technique’s conception in a GPS-only context, long convergence times were observed and were attributed

to the convergence of the state estimation filter, as it initially relies on pseudorange measurements and the improved estimation of ambiguities over time leads to convergence of the user positions (Kouba and Héroux, 2001; Héroux et al., 2004). Over the years, much of the research has been focused on reducing PPP’s convergence time from tens of minutes to a few minutes or seconds through various means. For instance, in an attempt to reduce the PPP convergence time and achieve NRTK levels of performance, i.e., centimetre-level positioning near-instantaneously, research has been done to augment the global corrections with regional atmospheric corrections by generating precise atmospheric information from a dense network and having the rover be present within the network (Zhang et al., 2011; Psychas and Verhagen, 2020; Nadarajah et al., 2018). However, while such an approach allows for RTK-like PPP performance, it still relies on the use of local reference stations, therefore not solving the issue of having a global precise positioning technique with instantaneous centimetre-level positioning. Other research has been done to improve PPP through the use of Ambiguity Resolution (AR), the use of multiple frequencies, as well as the use of multiple constellations. Indeed, ambiguity resolution has proven to be crucial in reducing PPP convergence time and having the technique reach a few centimetres within a few minutes (Collins et al., 2008; Naciri and Bisnath, 2020, 2021a; Laurichesse et al., 2009; Ge et al., 2008; Bertiger et al., 2010; Teunissen et al., 2010; Geng et al., 2012; Mervart et al., 2008). However, AR alone is proving to not be enough to reach RTK levels of performance, as it still requires a few minutes to reach RTK’s level of accuracy. Recent advances in GNSS constellations have opened the door for improvements in the performance of a technique as measurement-dependent as is PPP. Indeed, recent years have seen the emergence and completion of additional GNSS constellations to complement GPS, with all of these constellations broadcasting signals on as many as four different frequency bands (Naciri et al., 2021a).

So far, in the literature, there has been limited research that has been successful in allowing PPP to achieve RTK levels of performance, i.e., near-instantaneous, centimetre-level positioning, without the use of any local augmentation. Achieving such performance without reliance on local infrastructure would mean that PPP would become a truly global precise positioning technique, as it would be able to achieve RTK-type performance anywhere in the world. Additionally, with the advent of services such as Galileo’s High Accuracy Service (HAS), where corrections are directly transmitted by the GNSS constellation on a global scale, PPP could be similar to Single Point Positioning (SPP) in principle and similar to RTK in performance: the user would only use information from the GNSS signals to estimate their position, with that position being precise to the centimetre-level near-instantaneously (Bisnath, 2020a). It is worth noting that the definition of (*near-*) *instantaneous* cannot be universal and that different applications and users would have different definitions. In this dissertation, *instantaneous* refers to convergence within a few seconds up

to one minute, while *near-instantaneous* refers to convergence within a couple of minutes at most.

1.3 Dissertation statement

The initial convergence time being one of the main hurdles stopping PPP from becoming the overall reference precise positioning technique, and from having truly global precise positioning, it can still be reduced mainly thanks to recent updates to the GNSS constellations and in the different processing techniques. Subsequently, ***the aim of this work is to achieve global centimetre-level positioning with PPP within a few seconds to minutes***. Such performance would be achieved using PPP processing based only on corrections that are available worldwide, without a dependence on user location or local infrastructure. It is believed that being able to do so would open new opportunities for PPP, not only in replacing RTK in certain applications, but also in opening up to new applications. Based on the conclusions from this research, an analysis of what the future of precise positioning may look like is performed.

1.4 Research contributions and significance

While the goal from this dissertation is to reach RTK levels of performance using only global corrections and without local augmentation, the methodology involves achieving the best possible PPP solution using as many measurements as possible and available, and comparing the performance to RTK thresholds and requirements. Additionally, it is not only necessary to use as many constellations and frequencies as possible, but it is also important to fix ambiguities on these additional signals, as fixing ambiguities correctly typically shortens the PPP convergence time. Parallel work is performed as well, with an in-depth first look at the Galileo HAS corrections, where PPP corrections are broadcast by the Galileo constellation - a first of its kind technology on a global scale.

The following objectives and research questions are defined and answered in this dissertation:

1. From first principles, derive a four-constellation, four-frequency, uncombined Decoupled Clock Model capable of processing measurements from satellites regardless of the number of frequencies that they broadcast on, and capable of fixing ambiguities on all processed frequencies.
2. Assess possibility of reaching convergence to centimetre-level positioning within a few seconds, as well as possibility of centimetre-level epoch-by-epoch positioning, both using only global PPP products.

3. Analyze corrections and user performance using global PPP products broadcast within Galileo’s E6-B signal.
4. Study possibility of using NRTK corrections for PPP processing, and feasibility of mixing NRTK corrections with global PPP corrections, as NRTK corrections do not retain their physical meaning and cannot be mixed easily with global PPP products.

To put this research to context, comparison to published research in the literature is presented. The focus of this research is in obtaining centimetre-level solutions instantaneously without the use of any local augmentations. Such research has been attempted in the literature with limited results. For instance, Laurichesse and Banville (2018) showed that instantaneous centimetre-level positioning can be achieved using only global products with dual-frequency GPS and four-frequency Galileo measurements. However, such performance was demonstrated on a limited sample size using data from only one receiver over one hour. In this research, the study is broadened to more signals and constellations and is demonstrated on a global set of data. Additionally, other research has been published that focused on instantaneous PPP convergence, but the research was focused on achieving decimetre-level solutions, rather than centimetre-level ones (Geng et al., 2019; Li et al., 2020b), the latter being much harder to achieve. On the other hand, the research that managed to achieve instantaneous centimetre-level positioning did so by augmenting the PPP solutions with precise regional atmospheric corrections (Gao et al., 2021). Conceptually, using these regional corrections defeats the purpose of having PPP perform globally regardless of the user’s location, and is an approach that is avoided in the research presented in this dissertation. Finally, Hexagon, a commercial company, announced a new service in November 2020 by which global instantaneous precise positioning can be achieved (Hexagon, 2021). The service as announced is able to achieve centimetre-level positioning in approximately one minute at the 95th percentile, with some locations requiring up to 3 minutes. Details on how such performance are not disclosed, though mention of the multiple GNSS frequencies being the catalyst was made.

When it comes to epoch-by-epoch PPP, limited research has been published on the topic due to the typically long PPP convergence times. For instance, Geng and Guo (2020); Geng et al. (2019) analyzed the feasibility of single-epoch PPP, but with a target of decimetre positioning, and with only the fixing of extra-widelane and widelane ambiguities, which are easier to fix than narrowlane ambiguities. Gao et al. (2021) analyzed the possibility of epoch-by-epoch PPP by fixing all ambiguities, but relied on regional atmospheric modelling, making the proposed solutions dependant on the user’s location.

Finally, when it comes to analyzing Galileo HAS corrections, the analysis is novel given that early access to the corrections and signals has been provided through the Galileo Improved Services for Cadastral Augmentation Development On-field Validation (GISCAD-OV)

project (GEOWEB S.p.A., 2022). The analysis presented in this research presents an in-depth analysis to corrections' availability and performance. In the literature, such detailed analysis has not been published yet. For instance, Hauschild et al. (2022) analyzed the corrections' performance with a focus on orbit determination for satellite sentinel-6A. Fernandez-Hernandez et al. (2022) analyzed earlier versions of the signals, but did not go into details of availability and performance. The position tests were performed on a limited number of datasets as well. Finally, when it comes to combining global PPP products with regional NRTK products, no research was found on the topic in the literature. In the literature, most instances of augmenting PPP solutions with precise atmospheric corrections estimate the atmospheric corrections with a regional network after fixing the satellite-dependent state terms to globally-estimated values (Gao et al., 2021; Psychas and Verhagen, 2020; Nadarajah et al., 2018). This approach leads to different corrections compared to estimating all corrections (including satellite-dependent ones) with the same regional network.

A large part of the research in this dissertation has been published in the literature through journal and conference proceedings publications. The first-authored publications are listed hereafter:

- Naciri, N., & Bisnath, S. (2023). RTK-Quality Positioning With Global Precise Point Positioning Corrections. *NAVIGATION: Journal of the Institute of Navigation*, 70(3).
- Naciri, N., Yi, D., Bisnath, S., de Blas, J., & Capua, R. (2023). Assessment of Galileo High Accuracy Service (HAS) test signals and preliminary positioning performance. *GPS Solutions* 27, 73.
- Naciri, N. (2022, September). RTK-quality Positioning with Global PPP Corrections. In *Proceedings of the 35th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2022)* (pp. 2546-2562).
- Naciri, N., Yi, D., Bisnath, S., de Blas, F. J., & Capua, R. (2022, September). Validation of a European High Accuracy GNSS Service for Cadastral Surveying Applications. In *Proceedings of the 35th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2022)* (pp. 381-396).
- Naciri, N., & Bisnath, S. (2021). Approaching Global Instantaneous Precise Positioning with the Dual-and Triple-Frequency Multi-GNSS Decoupled Clock Model. *Remote Sensing*, 13(18), 3768.
- Naciri, N., Vana, S., Seepersad, G., & Bisnath, S. (2021, September). Rapid Position Initialization for Automated Automotive Applications. In *Proceedings of the 34th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2021)* (pp. 2718-2732).

- Naciri, N., Hauschild, A., & Bisnath, S. (2021). Exploring signals on L5/E5a/B2a for dual-frequency GNSS precise point positioning. *Sensors*, 21(6), 2046.
- Naciri, N., & Bisnath, S. (2021). An uncombined triple-frequency user implementation of the decoupled clock model for PPP-AR. *Journal of Geodesy*, 95(5), 1-17.
- Naciri, N., & Bisnath, S. (2020, September). Multi-GNSS Ambiguity Resolution as a Substitute to Obstructed Satellites in Precise Point Positioning Processing. In *Proceedings of the 33rd International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2020)* (pp. 2960-2971).

1.5 Dissertation outline

Chapter 2 starts with a description of the various positioning techniques, starting with SPP, through DGPS, RTK, and NRTK, and ending with PPP. Special attention is given to PPP, being at the core of this dissertation, with a description of the observation equations and models that the technique relies on, as well as a literature review of PPP-AR. The chapter continues with an analysis of the effect of the recent GNSS advancements on PPP performance, with an analysis of the effect of the new constellations and frequencies on PPP user performance, as well as the effect of PPP-AR on user performance in the dual-frequency case.

Chapter 3 focuses on the mathematical derivation of a three frequency, four-constellation Decoupled Clock Model (DCM), which is a PPP-AR model that has been derived from first principles. Special considerations related to, e.g., reference satellite selection and different biases' treatment are provided, before presenting the results. The results section ranges from analysis of model-specific parameters, to analysis of user performance from a large sample of global datasets processed in static mode with the Galileo constellation as a test-bed. The results are expanded by using all four major constellations, and with processing being performed in kinematic mode, without assuming the receiver to be static. Finally, results from kinematic datasets that have been collected in a real-life environment with the receiver and antenna being driven in a suburban environment are presented.

Chapter 4 constitutes the focal point of the dissertation, as it focuses on attempting to reach RTK-like performance and on single-epoch PPP processing. The chapter starts with an expansion of the DCM to four frequencies to include the fourth Galileo frequency that was not used in Chapter 3. Following the expansion, a redundancy analysis is performed, where the effect of the additional frequencies and satellites on the degrees of freedom of the system is analyzed, and where comparison of the derived model's redundancy to the classic float PPP model is performed. Finally, results are presented, where typical PPP-type analysis is performed, followed by a more strict comparison against RTK-like performance

thresholds. The results section ends with an analysis of the performance of the epoch-by-epoch solution on a large set of global data, with the analysis of the performance per station, and with an analysis of potential ways to improve the proposed solutions.

Chapters 5 and 6 are two parts of the same chapter, as each explores a different way in which PPP will most probably be used in the near future. Chapter 5 presents the first part, where the possibility of augmenting global PPP solutions with NRTK corrections from a regional network is assessed. To do so, first, the feasibility of using NRTK corrections in PPP processing is analyzed, with a comparison of the corrections between PPP and NRTK being performed, followed by an assessment of user performance using NRTK corrections. Next, a proof-of-concept is proposed, where receiver-satellite ranges are computed based on PPP and NRTK corrections in order to extract biases, which are caused by the difference in the networks used to generate NRTK and PPP corrections. The estimated biases are used to extract ionospheric corrections, which are compatible with the global PPP solution, and the method is demonstrated on a limited dataset. Chapter 6 focuses on another potential way of PPP usage in the future in the form of Galileo HAS. The chapter starts with a description of the service and of the corrections being broadcast, followed by an analysis of the availability of the corrections. The corrections are analyzed next by comparing them to well-established PPP products, before using the HAS corrections to generate PPP solutions to assess the level of performance that is achievable with the service. The chapter ends with an analysis of the effect of correction latency on HAS user performance.

Chapter 7 summarizes the research that was presented in the earlier chapters, and provides recommendations for future work that can be performed in the near future based on the presented work and on potential applications of PPP.

Chapter 2

GNSS precise positioning techniques

When it comes to satellite navigation, GPS was initially designed for military use before finding its way into civilian use by the 1980s (Office of Science and Technology Policy, 2000). The initial goal of the system was to provide positions accurate to the tens of metres level, as well as timing information. Such performance was to be achieved with what is known as Single Point Positioning (SPP), in which any receiver tracking at least four GPS satellites can estimate its position using only GPS signals (Enge, 1994; Gov, 2015). Indeed, the signals transmit broadcast ephemeris which can be used to recover the satellites' position, clock and bias errors. Using this information, together with (pseudo)range measurements from the user receiver, the user can apply trilateration - intersection of ranges, to estimate their position to a few metres of accuracy. The system had more potential though as it could be used to estimate user position even more precisely, which has led to the emergence of different techniques that managed to leverage the GNSS signals to provide centimetre-level solutions. The following chapter describes the precise positioning techniques that use GNSS signals, with a focus on Precise Point Positioning (PPP), which constitutes the core of this dissertation.

2.1 Relative positioning: differential GNSS (DGNSS), Real-Time Kinematic (RTK), and network RTK (NRTK)

As a way to improve position accuracy, relative positioning came into existence, where data from a nearby base station is used to augment user measurements and improve position estimates. Relative (or differential) positioning evolved from GPS-only to multi-GNSS, from a pseudorange-based approach to a carrier-phase based approach, and from a single-baseline technique to a wide area augmentation (Bisnath, 2020b). The different relative (differential) techniques are discussed in the current section.

2.1.1 Differential GNSS

One of the main reasons for the development of differential positioning in general, and Differential GPS (DGPS) specifically, was the presence of Selective Availability (SA). Indeed, when the GPS constellation was deployed, SA was turned on in order to limit the accuracy achieved by civilian users by intentionally degrading the quality of the broadcast ephemeris. In order to circumvent this limitation, DGPS was devised to eliminate this intentional degradation by using between-receiver differencing that eliminates most spatially-correlated error sources (Janeau et al., 2001; Kremer et al., 1990). The main error sources that are eliminated are the satellite orbit and clock errors, as well as the ionospheric and tropospheric delays. This elimination of the errors is possible thanks to the presence of a nearby base station with a known location. The station generates corrections for the spatially-correlated states, and transmits them to the user (Parkinson et al., 1996; Gao et al., 1997). The user, present in the surrounding local or regional area, takes the difference between these corrections and their measurements to eliminate spatially-correlated error sources, in addition to estimating non-spatially-correlated errors such as the receiver clock, or the receiver hardware biases. By doing so, the user can achieve accuracies that can range from the few-decimetre-level, to the many metre-level, depending on the distance to the reference station. An example of such DGNSS networks is what is referred to as Wide Area DGNSS (WADGNSS) in the case of, for instance, the American Wide Area Augmentation System (WAAS) (FAA, 2023), or the European Geostationary Navigation Overlay Service (EGNOS) (EUSPA, 2023). In such instances, corrections are generated based on a country/continent-wide network of stations, and are broadcast to users using geostationary satellites.

In case the reference station and the user use multiple constellations rather than only GPS, the technique is referred to as DGNSS. An alternative technique exists as well, where, instead of using one reference station, multiple stations are used to generate corrections over a local, regional or wide area. Thanks to the presence of multiple reference stations covering a large area, it becomes possible to generate models to fit the spatially-correlated errors to. These model parameters can then be transmitted to the user who can correct for such errors, regardless of their location, as long as they are within the network's bounds. Naturally, the quality of the corrections and of the accuracy that can be achieved by the user depend on the quality of the model, as well as the density of the network and proximity to the reference stations in the area surrounding the user.

Traditionally, differential positioning uses pseudorange measurements in both the correction generation and at the user side. Another approach exists that uses carrier-phase measurements to smooth the pseudorange measurements, leading to improvements in the accuracy. Additionally, another alternative exists where the carrier-phase measurements are used as measurements in addition to pseudorange measurements, though this formulation is

very similar to Real-Time Kinematic (RTK) and Network Real-Time Kinematic (NRTK), which are detailed in the next subsection.

To illustrate the performance that DGNSS achieves and compare it to SPP, Figure 2.1 shows the performance for station GOPE in the Czech Republic over one hour. The SPP solution uses observation data from the station with broadcast GPS, GLONASS, Galileo, and BeiDou orbits and clocks. The DGNSS solution uses the user's observations on the same four constellations, as well as corrections from a base station 33 km away. The figure shows the expected performance, as the SPP solution is at the many-metres level due to the use of the broadcast ephemeris without any augmentation, while the DGNSS solution is an order of magnitude more accurate thanks to the use of corrections from a nearby base station. The DGNSS is at the decimetre-level rather than centimetre-level, due to its reliance on pseudorange measurements, which are noisy. Improvements are expected with the inclusion of carrier-phase measurements, which is the principle behind RTK/NRTK described next.

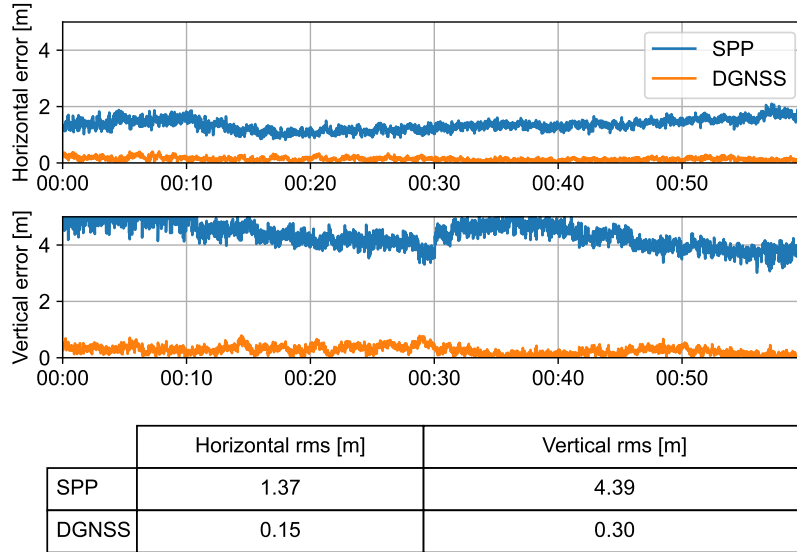


Figure 2.1: SPP and DGNSS for station GOPE in the Czech Republic on DOY 245, 2022. The DGNSS solution uses a base station with a baseline of 33 km.

2.1.2 Real-Time Kinematic and network RTK

The principle behind Real-Time Kinematic (RTK) is similar to the one for differential positioning explained in the previous subsection. In the case of RTK, in addition to pseudorange corrections, the user also receives carrier-phase corrections from the base station. Using these corrections, the user can perform between-satellite and between-receiver differencing to remove most error sources, assuming that both the user and the base station are close enough for the spatially-correlated errors to be similar in both locations, and for the

same satellites to be in view. There are different ways of performing RTK processing: for example, the network can either provide the raw measurements to the user, or it can provide corrections to the true range; or the user can either explicitly form the double-differences, or they can estimate individual errors, while using measurements from both their receiver and the base station's. Nevertheless, the principle is the same: the base station with known location transmits corrections in an Observation State Representation (OSR) to the user, and the user uses these corrections, as well as their measurements to eliminate most error source, assuming proximity to the base station and elimination of the spatially-correlated errors through differencing.

Naturally, the performance of RTK depends on the distance to the base station, as spatial correlation of the corrections / error sources between the user and the base station decreases with baseline (distance between user and base station) length. To reduce the effect of the baseline and increase the coverage to a whole area, similar to wide area DGNS, network RTK was developed, where a network of stations is used instead of a single base station (Townsend et al., 1999; Jensen and Cannon, 2000). By using a network of base stations, having corrections being sent from each base station to each user drastically increases the number of OSR corrections to be sent to the user, increasing the computational power required by the user to process all the corrections. Instead, it is natural to move to a State Space Representation (SSR), where state-space corrections are estimated by the network and provided to the user, requiring a reduced amount of information to be sent to the user. The corrections include satellite orbits, clocks, and hardware biases, as well as interpolated ionospheric and tropospheric corrections for the user's location. The atmospheric information is sent to the user for the approximate position that they provide to the network. The network can either transmit the atmospheric corrections directly to the user's location, or it can model the atmosphere in the network's coverage area, and provide the model's parameters to the user.

With both RTK and NRTK, on short to medium baselines of a few tens of kilometres, the user can reach centimetre-level positioning near-instantaneously thanks to the ability to eliminate most error sources with double-differencing, as well as the estimation of carrier-phase integer ambiguities through Ambiguity Resolution (AR) (Gao et al., 1997; Van Diggelen, 1997). The latter will be discussed in more detail in subsection 2.2.4.

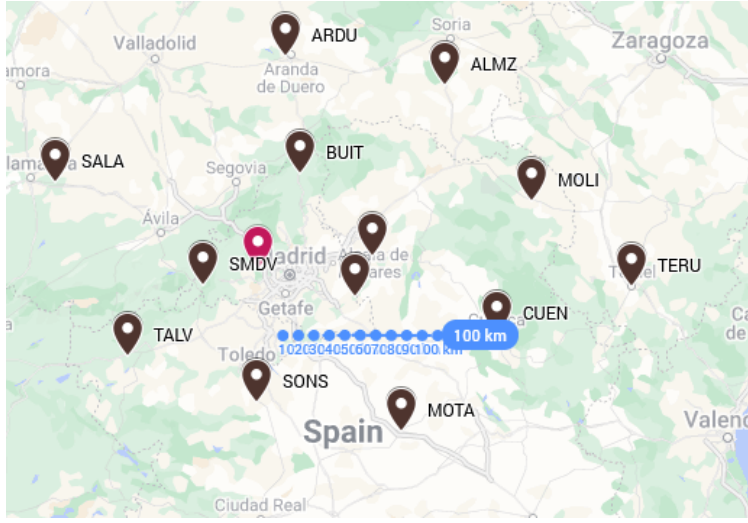
However, as the distance between the user and closeby base station(s) increases, the performance degrades, as the spatial correlation of the atmospheric corrections is lost. As a consequence, in areas without base stations, or where base stations are too far, RTK and NRTK cannot be used and the user has to resort to Single Point Positioning. Precise Point Positioning comes as an alternative that can be used to augment SPP in areas without NRTK coverage.

Figure 2.2 shows a sample performance that is typical in the case of NRTK. In this example, corrections are generated from reference stations located over a region in Spain surrounding Madrid. Data from the stations is processed to generate NRTK corrections, which are broadcast to the user for processing. Using their approximate position, the user receives corrections for their location including satellite orbits, clocks, and code and phase biases, as well as ionospheric and tropospheric corrections. In this example, a station located close to Madrid is chosen as a rover. The station is approximately 36 km away from the closest reference station. Figure 2.2b shows the typical performance that is expected from such processing, and the importance of AR in NRTK processing. The float solution (without AR) is at the decimetre-level and is subject to variations, with the errors decreasing over time. The decrease in error is due to the use of continuous carrier-phase measurements, coupled with sequential filtering. Fixing ambiguities to their integer values removes the convergence trend and leads to centimetre-level solutions from the beginning of the processing. The overall accuracies (rms) are approximately 1 cm in the horizontal component and 2 cm in the vertical component.

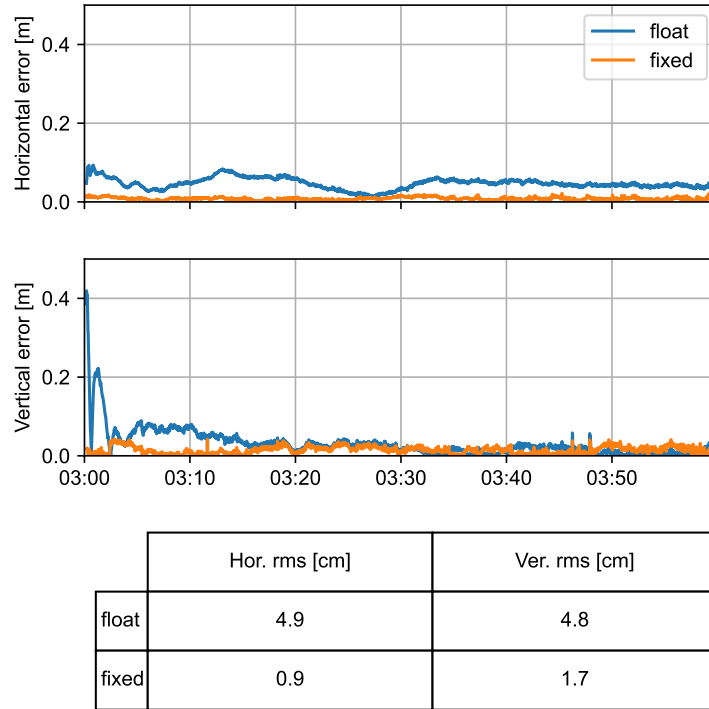
2.2 Precise Point Positioning (PPP)

Precise Point Positioning (PPP) relies on the use of a global network of stations, which is used to estimate precise satellite corrections in a state space representation (Malys and Jensen, 1990; Kouba and Héroux, 2001). As opposed to RTK and NRTK which rely on the use of one or multiple regional base stations with baselines less than 100 kilometres, PPP uses a global network of stations, with distances between stations ranging from kilometres to thousands of kilometres. The fact that the network of stations in PPP covers a large part of the Earth allows for spatial and temporal decorrelation of the satellite corrections, allowing for precise estimation of the satellite orbits and clocks. There are many advantages to the move to PPP, one of which being the reduction in computation burden and bandwidth requirements to transmit corrections to users. Indeed, in an Observation State Representation, the correction provider has to transmit a range correction for each measurement. In a multi-constellation, multi-frequency world, this representation becomes cumbersome as the number of corrections to transmit to the user increases drastically. Instead, a State Space Representation, as used in PPP, becomes a cheaper alternative in terms of bandwidth. Additionally, while RTK and NRTK require bi-directional communication for the user to send their approximate coordinates to the base station / control centre and for the latter to send corrections to the user, PPP only requires one-way communication, from the control centre to the user.

While PPP requires a network of stations to estimate corrections, it can still be considered a point positioning technique, as it only requires the use of one receiver from the user,



(a) Location of the rover station within the network.



(b) Position performance.

Figure 2.2: Position performance of station VILL in Spain, processed with NRTK corrections generated from the network in 2.2a on Day Of Year (DOY) 81, 2022. Station VILL is the red marker on the map.

and a communication link with a corrections provider - Internet, geostationary satellites, GNSS satellites, etc. As opposed to SPP, PPP is able to achieve decimetre- to centimetre-level positioning through four main avenues: 1) the use of precise satellite corrections instead of the broadcast ephemeris; 2) additional error modeling to mitigate various error sources; 3) the use of precise but ambiguous phase measurements; and 4) sequential filtering which allows the use of previous information in each epoch's processing. The combination of the four tools allows for precise positions to be obtained, though requiring minutes to tens of minutes to reach such precise positions, due to the sequential nature of the process and having to wait for various state terms' estimation to be refined - that is, to converge.

2.2.1 Observation equations

Considering receiver r and satellite s , the pseudorange and carrier-phase measurements on frequency f_i , $i \in \{1, 2, \dots, n\}$, are defined as:

$$\begin{cases} P_{r,i}^s &= \rho_r^s + c(dt_r - dt^s) + \gamma_i I_1^s + M_r^s \cdot T_r + (b_{r,i} - b_i^s) + \epsilon_{P_i} \\ \Phi_{r,i}^s &= \rho_r^s + c(dt_r - dt^s) - \gamma_i I_1^s + M_r^s \cdot T_r + \lambda_i (N_i^s + B_{r,i} - B_i^s) + \epsilon_{\Phi_i} \end{cases} \quad (2.1)$$

where $P_{r,i}^s$ and $\Phi_{r,i}^s$ are, respectively, the pseudorange and carrier-phase measurements in metres, ρ_r^s is the geometric range between the receiver and the satellite, c is the speed of light in vacuum, dt_r is the receiver clock offset from GPS time and dt^s is the satellite clock offset from GPS time. $\gamma_i = \frac{f_1^2}{f_i^2}$ is the ratio of frequencies that is applied to the first-frequency ionospheric delay I_1^s to recover the ionospheric delay at frequency i , and M_r^s is the mapping function that is used to map the zenith troposphere wet delay T_r to the satellite-receiver line-of-sight troposphere delay. $\lambda_i = \frac{c}{f_i}$ is the signal's wavelength, and N_i^s is the integer ambiguity on frequency i . $b_{r,i}$ and b_i^s are, respectively, the pseudorange receiver and satellite hardware biases and $B_{r,i}$ and B_i^s are the receiver and satellite phase biases, respectively. ϵ_{P_i} and ϵ_{Φ_i} contain any residual unmodelled errors such as multipath and noise in the pseudorange and carrier-phase measurements, respectively.

All the parameters in Equation (2.1) cannot be simultaneously estimated directly due to a rank deficiency of the system, as not all states are directly observable. Therefore, it is necessary to re-parameterize the equations by incorporating the receiver and satellite biases into other parameters in order to limit the number of estimated parameters.

2.2.2 Common combinations

Historically, PPP has been using the ionosphere-free combinations of measurements in order to eliminate the ionosphere state terms. To form the ionosphere-free combination, measurements on two frequencies are required. Considering frequencies f_1 and f_2 , the ionosphere-free

combinations are defined as:

$$\begin{cases} P_{IF,r}^s &= \frac{f_1^2 P_{r,1}^s - f_2^2 P_{r,2}^s}{f_1^2 - f_2^2} \\ \Phi_{IF,r}^s &= \frac{f_1^2 \Phi_{r,1}^s - f_2^2 \Phi_{r,2}^s}{f_1^2 - f_2^2} \end{cases} \quad (2.2)$$

As can be seen from Equations (2.1) and (2.2), the ionosphere state terms are eliminated from the equations thanks to the combination, as the ionosphere state term is inversely proportional to the square of the frequency. Additionally, in the classic PPP model, not only are the ionosphere-free combinations formed, certain parameters are absorbed by other state terms as well to simplify the estimation model (Kouba and Héroux, 2001). The complete model for a dual-frequency receiver is used as:

$$\begin{cases} P_{IF,r}^s &= \rho_r^s + c(\overline{dt}_r - dt^s) + M_r^s.T_r + \epsilon_{P_i} \\ \Phi_{IF,r}^s &= \rho_r^s + c(\overline{dt}_r - dt^s) + M_r^s.T_r + \lambda_i \overline{N}_i^s + \epsilon_{\Phi_i} \end{cases} \quad (2.3)$$

The state terms marked with an overline are state terms which have absorbed other quantities, and are therefore tainted. For instance, the ambiguities are not integer in Equation (2.3), as they have absorbed non-integer parameters.

Other combinations that are commonly used, especially when it comes to Ambiguity Resolution (AR), are the Widelane (WL), Narrowlane (NL), and Melbourne-Wübbena (MW) combinations. The WL combination is used to form measurements with higher wavelengths and lower noise than the individual measurements, respectively, while the MW combination is generally used to estimate the WL ambiguities directly, as the combination eliminates the geometric range and other parameters. The expression of the various combinations are shown in Equation (2.4).

$$\begin{cases} P_{NL,r}^s &= \frac{f_1 P_{r,1}^s + f_2 P_{r,2}^s}{f_1 + f_2} \\ \Phi_{NL,r}^s &= \frac{f_1 \Phi_{r,1}^s + f_2 \Phi_{r,2}^s}{f_1 + f_2} \\ P_{WL,r}^s &= \frac{f_1 P_{r,1}^s - f_2 P_{r,2}^s}{f_1 - f_2} \\ \Phi_{WL,r}^s &= \frac{f_1 \Phi_{r,1}^s - f_2 \Phi_{r,2}^s}{f_1 - f_2} \\ \Phi_{MW,r}^s &= \Phi_{WL,r}^s - P_{NL,r}^s \end{cases} \quad (2.4)$$

In equation 2.4, $P_{NL,r}^s$ and $\Phi_{NL,r}^s$ are the Narrowlane (NL) pseudorange and carrier-phase combinations, respectively, $P_{WL,r}^s$ and $\Phi_{WL,r}^s$ are the Widelane (WL) pseudorange and carrier-phase combinations, respectively, and $\Phi_{MW,r}^s$ is the Melbourne-Wübbena (MW) combination. Note that, for certain frequency combinations with a large enough wavelength, i.e., GPS L2 and L5, the WL combination is referred to as an Extra-widelane (EWL) combination, as the combined wavelength can reach well above one metre.

2.2.3 Error models

In order to perform PPP processing, several corrections have to be taken into account. Some of the corrections are common to other processing techniques, for example atmospheric delays which have to be considered in SPP, RTK, and NRTK, although they are dealt with differently in each technique. Other corrections, such as satellite antenna corrections (which are described in the following section) tend to not be needed by other techniques, but they are crucial in PPP. Some of the error sources in PPP can be modelled, while others have to be estimated, either directly by the user, or by the network provider before being disseminated to the users. A list of error sources and their mitigation techniques is provided in the remainder of this subsection.

- **Satellite orbit errors** In SPP, or even RTK/NRTK, users can compute the satellite orbital locations using the correction parameters in the broadcast ephemeris. The latter are precise to a few metres and, in high-precision applications (RTK and NRTK), the residual errors are eliminated through between-receiver differencing. However, in PPP where only one receiver is used and between-receiver differencing cannot be performed, broadcast satellite orbits are not precise enough to provide satisfactory PPP solutions. Therefore, precise satellite orbits are required. Such precise orbits are generated by using a global network of stations with known locations. Processing observation data from all the stations simultaneously while holding the satellite orbits as unknowns allows for the precise estimation of the latter. These satellite orbits can either be disseminated in real-time as corrections to the broadcast orbits (Wübbena et al., 2005), or they can be provided to the user in Receiver INdependent EXchange (RINEX) files (International GNSS Service, 2018). Typical accuracy of precise satellite orbits is a few centimetres.
- **Satellite clock errors** Similar to the satellite orbits, the satellite clocks need to be precisely estimated by the network for PPP processing, as residual errors in the broadcast clocks affect PPP performance. In fact, the clock errors have more effect on PPP than orbit errors, as the clock errors directly correlate to the line-of-sight with a 1:1 ratio, while orbit errors are projected onto the line-of-sight, depending on the satellite's orientation. Additionally, precise satellite clocks need to be estimated at a higher frequency than satellite orbits, as the former generally vary quickly over a short period of time. Precise orbit corrections tend to be updated every 5 minutes, while precise clock corrections tend to be updated every 30 seconds, depending on the network provider (Prange et al., 2020; Uhlemann et al., 2015). Similar to the orbits, the precise satellite clocks are estimated from a global network of stations, and disseminated to the user either as corrections to the broadcast clocks or as precise

clock corrections. Satellite clock corrections have a magnitude of several hundreds of metres, and their estimates are typically accurate to the centimetre level.

- **Relativistic effects** Since GNSS satellites are subject to gravitational potentials which are different from the user's, and satellites are moving relative to the user, relativistic effects from Einstein's theory of relativity need to be taken into account (Ashby, 2003). A constant component of the relativistic correction is applied by the GNSS satellites by shifting their transmitted frequencies to make sure that users on Earth receive signals at the desired frequencies without needing to correct for the constant component. A periodical component needs to be computed by the user by either using parameters that are broadcast in the satellite ephemeris, or by using the satellite positions and velocities. Typical magnitudes of the periodical component are a few tens of metres. In addition, the rotation of the Earth as the signal travels between the satellite and the receiver needs to be taken into account. As a consequence, the user's position on Earth at the time of reception of the GNSS signal is different from their position at the time of transmission. This difference is significant enough to lead to tens of metres of error in the user position. This Earth rotation correction is referred to as the Sagnac effect, and can be corrected for as shown in Kaplan and Hegarty (2017).
- **Ionospheric delays** GNSS signals travel through the atmosphere after being broadcast by the satellites in order to reach the user. Given the altitude that the satellites are on, the signals have to traverse the ionosphere, which is located at altitudes between 70 km and 1,000 km above the Earth's surface. This atmospheric layer contains free electrons, which affect the propagation of signals through it, and lead to refraction of the GNSS signals. The amount of particles in the ionosphere, and therefore the effect of the latter on the signal delay, is dependent on the amount of solar activity. Given the latter, it is generally hard to precisely model the ionosphere. While there are models to correct for the effect of the ionosphere (Klobuchar, 1975; European Commission, 2016), they are generally not precise enough for PPP applications. Therefore, the common strategies are either to 1) estimate the ionospheric delays as part of the PPP model (Pengfei et al., 2011; Laurichesse and Privat, 2015), or 2) estimate the ionosphere from a dense network of nearby stations, and provide the estimates to the user as corrections (Banville et al., 2014). Another strategy takes advantage of the fact that the ionosphere is a dispersive medium and that GNSS signals' refraction depends on their frequency. The strategy consists of forming ionosphere-free combinations of GNSS measurements to eliminate the ionospheric delays, as the latter are frequency-dependent and can be eliminated by using signals on different frequencies. The ionosphere-free combinations have been detailed in equation 2.2. Typical mag-

nitudes of the ionospheric delays are a few tens of metres, with an accuracy of the estimated delays by the user at the centimetre level.

- **Tropospheric delays** In addition to the ionosphere, GNSS signals have to travel through the troposphere, which is the non-dispersive, lower part of the atmosphere with an altitude up to approximately 10 km. The troposphere is electrically neutral to radio waves, and it gets denser the closer to the surface of the Earth. Similar to the ionosphere, the troposphere refracts GNSS signals and leads to delays in their propagation between satellites and receivers. In reality, GNSS signals travel through the stratosphere as well; however, the tropospheric and stratospheric delays tend to be lumped together and called tropospheric delay in the GNSS context. The tropospheric delay can be decomposed into two components: a dry component, which makes up about 90% of the total zenith tropospheric delay, and a wet component, which makes up about 10% of the total zenith tropospheric delay (Hopfield, 1969). While the dry component can be modelled accurately using various models (Saastamoinen, 1972; Hopfield, 1969), the wet component is hard to predict, as it depends, among other factors, on the local water vapor. The wet component therefore needs to be estimated in the PPP model. The dry component has a typical magnitude of a few metres, while the wet component has a typical magnitude of a few decimetres. Both components are estimated / modelled to the centimetre level.
- **Satellite and receiver hardware delays** Between its generation in the satellite and its transmission from the satellite antenna, the GNSS signal is subject to various processes that cause delays to its transmission. Given that signals on different frequencies, and on different modulations within the same frequency, follow different paths before their transmission, the hardware delays are different between frequencies and modulations. These satellite hardware delays are usually estimated and provided to the user in the form of, e.g., Timing Group Delays (TGD) corrections, Differential Code Bias (DCB) corrections, or Observable-specific Signal Biases (OSB) corrections. Hardware delays exist at the user level too, though they tend to be ignored, as they get absorbed by other state terms during the estimation, or they can be eliminated through between-satellite single-differencing.
- **Satellite and receiver antenna corrections** When estimated with a global network, satellite orbits tend to refer to the satellite's antenna centre of mass. However, signals are broadcast from the antenna phase centre of the antennas, which means that users have to correct for the lever arm between the satellite's centre of mass and its antenna phase centre. On the other hand, receiver locations usually refer to a point at the antenna referred to as Antenna Reference Point (ARP), therefore requiring to

account for the level arm between the receiver Antenna Phase Centre (APC) and its ARP. The lever arm values for both satellite and receiver ends are made available through files called ANTenna EXchange (ANTEX) (Schmid et al., 2016), which are regularly updated to account for new receivers, as well as changes in the satellites. The magnitude of the antenna corrections can range from a few centimetres to a few metres, though their uncertainty is usually of only a few centimetres.

- **Carrier-phase wind-up** The wind-up is a phenomenon that appears when an antenna is rotating, leading to a variation in the carrier-phase measurements. The phenomenon only affects carrier-phase measurements, and not pseudorange measurements. A phase wind-up correction is needed because of the satellites' motion, as they need to rotate to keep their solar panels facing the Sun, while the antenna keeps pointing towards the Earth. Therefore, the rotation of the satellites leads to a wind-up that needs to be compensated for by the user. The wind-up correction can be computed and applied by the user to their carrier-phase measurements, as shown in Wu et al. (1992). The carrier-phase wind-up has typical magnitudes of a few decimetres.
- **Solid Earth tides** The Earth is subject to gravitational forces from other celestial bodies, mainly the Moon and the Sun. Since the Earth's surface is not rigid, it tends to move depending on the gravitational forces that are applied to it. These movements of the Earth's surface need to be accounted for in high-precision applications, as they vary in time, depending on the gravitational forces. The solid Earth tides can be modelled and corrected for using models described in McCarthy et al. (2004). Its magnitude can reach a few decimetres, though it is modelled to the millimetre level.
- **Ocean loading** Along with the solid Earth tides, another form of tides that affects PPP is the ocean loading. The loading is due to ocean tides, which lead to displacements of the Earth's crust (Agnew, 1997; Francis and Mazzega, 1990; Wunsch and Stammer, 1997). As a consequence of being related to the ocean tides, the ocean loading correction is only impactful near the oceans, though its impact is limited - generally at the few millimetres level and can be ignored for centimetre-level applications.

2.2.4 Ambiguity resolution

PPP has been the subject of much research throughout the past years, which has led to many developments. Though the technique is capable of providing highly accurate positions, it still suffers from relatively long initial convergence times (Kouba and Héroux, 2001; Bisnath and Gao, 2009b). Many efforts have been made to reduce PPP initial convergence time, and Ambiguity Resolution (AR) has proved to be a very promising avenue. The reason for PPP's long convergence times is that the model is based on pseudorange and

carrier-phase measurements. Carrier-phase measurements are known to be ambiguous, as they correspond to a count of the number of cycles in the signal coming from the satellite and received by the user. When the receiver locks onto the satellite signal at the start of signal tracking, the number of cycles is unknown. This unknown number of cycles is referred to as *ambiguity* and needs to be estimated in the user filter. There exists one ambiguity per carrier-phase measurement. In terms of PPP processing, initially, the model mainly relies on the pseudorange measurements to estimate the various states, including the position estimates; therefore, the initial solution is at the many-decimetres level. In subsequent epochs, thanks to sequential filtering and to the use of continuous carrier-phase measurements, the latter overtakes the estimation process and gains more weight in the estimation. As a consequence, as more data are processed over time, the ambiguity estimates' quality improves, as they slowly converge to a steady state.

The idea behind AR in PPP is to make use of this additional information that the ambiguities are integer numbers, and to therefore add this constraint to the processing. Instead of estimating the ambiguities as float numbers and waiting for them to converge over time, one can include the integer constraint in the processing and hasten the convergence. However, to be able to fix ambiguities to their integer values, the classic PPP model has to be modified to remove any biases from the ambiguity estimates. Receiver-related biases are eliminated through between-satellite single-differencing, while satellite-dependent biases are estimated using global networks of stations before being disseminated to the user.

The ability to remove biases from PPP ambiguities and then fix the ambiguities to their integer values has shown to improve both convergence time and accuracy, especially in a multi-GNSS context (Collins et al., 2008; Ge et al., 2008; Laurichesse et al., 2009; Teunissen et al., 2010). Three main methods can be utilized to achieve PPP-AR: the Fractional Cycle Biases (FCB) method (Ge et al., 2008; Geng, 2011), the Decoupled Clock Model (DCM) (Collins, 2008; Collins et al., 2008) and the Integer Recovery Clock (IRC) method (Laurichesse et al., 2009; Laurichesse, 2011; Loyer et al., 2012). Geng et al. (2010); Shi and Gao (2014a); Teunissen and Khodabandeh (2015) have studied the relationship between the models. They have demonstrated equivalence in the ambiguity-fixed user positioning results and that the models are related through transformations. Seepersad and Bisnath (2017); Banville et al. (2020) have studied the interoperability of AR products from different analysis centres and the latter assessed the capacity to combine different products into a combined product.

The PPP-AR model used in this research is the Decoupled Clock Model (DCM). as it does not make assumptions on the behaviour of various biases. The model was first introduced in Collins (2008) and it is based on the assumption that the pseudorange and carrier-phase measurements are not synchronized to the same level of precision. This difference in precision is assumed to be due to the hardware delays that affect differently

measurements on the same frequency. Therefore, rather than estimating the same receiver clock for both sets of pseudorange and carrier-phase measurements, the DCM estimates a separate receiver clock for the pseudorange and carrier-phase measurements. Doing so "decouples" both measurements and allows for integer estimation of the ambiguities. These ambiguities are single-differenced relative to a reference satellite for which the ambiguities are set to arbitrary integer values.

The original DCM was based on an ionosphere-free combination of GPS pseudorange and carrier-phase measurements, as well as the Melbourne-Wübbena (MW) combination, allowing for fixing of Widelane (WL) and Narrowlane (NL) ambiguities. The DCM was later expanded into an extended version - the Extended DCM - for which the Melbourne-Wübbena combination is split into two separate observables: a narrowlane pseudorange and a widelane carrier-phase (Collins et al., 2012). Doing so allows for the estimation of the ionospheric delays, which in turn makes the constraining of said delays possible. This reformulation enables so-called "PPP-RTK", for which regional ionospheric maps can be used to improve PPP performance and approach RTK levels of accuracy and convergence time (Banville et al., 2014; Nadarajah et al., 2018), as well as the use of global ionospheric maps (GIM), which are less accurate (Aggrey and Bisnath, 2019; Banville et al., 2014).

The procedure that is generally followed to fix ambiguities to integer values is shown in Figure 2.3. Using the satellite products and the receiver observation data, the user applies one of the models described above to estimate float ambiguities, which are free of non-integer biases. Once the float ambiguities are estimated along with their covariances, they can be used to fix to integer values. Once the ambiguities are fixed to their integer values, the state terms are updated to reflect the integer ambiguities. This fixing process occurs every epoch. The most widely-used method to fix the ambiguities to integers is Least-squares AMBiguity Decorrelation Adjustment (LAMBDA) (Teunissen, 1993). The idea behind the method is that, taking the float ambiguities and their covariances as input, it performs a transformation of the ambiguities in order to decorrelate them. Indeed, there is a high correlation between the ambiguities, which leads to a large search space to find the set of integer candidates that fits the least squares problem. Therefore, decorrelating the ambiguities allows to reduce the size of the search space, and to identify a set of possible integer candidates through a least-squares adjustment. Once the candidates are identified, a back-substitution is performed to update the remaining state terms, e.g. position states, after the fixing of ambiguities, and a validation procedure is followed to select one set of integer ambiguities. Based on this set of integer ambiguities and their covariances, the rest of the float estimated state terms, including the receiver coordinates, are updated to account for the integer nature of the fixed ambiguities. This procedure is repeated every epoch following the float estimation process. While LAMBDA is the most popular method, other methods exist to fix ambiguities to integers, such as the Best Integer Equivariant (BIE) (Teunissen, 2003).

In this dissertation, the LAMBDA method is selected, as it is widely used in the literature. However, it was found that the validation step is not reliable to identify correct and incorrect sets of ambiguity candidates, as correctly fixed sets would not pass the validation step, while incorrectly fixed sets would pass the validation step. While improvements in the validation step are still needed to improve its reliability it was opted, in this dissertation, to fix all estimated ambiguities (with some exceptions, for example satellites with elevation angles below a 15 degree threshold). In terms of validation, a simple ratio test is used, though with a loose threshold to ensure that the validation step accepts candidates earlier in the processing. This brute-force approach is opted for to assess the impact of the various proposed factors on the reliability of the fixed ambiguities and positions.

2.3 Recent advances in GNSS and their impact on PPP

PPP augmentation has noticeably evolved since its early conception. The technique was initially intended as a precise positioning alternative that would reduce the computational burden in network static relative baseline processing, and would only require a stand-alone receiver (Zumberge et al., 1997; Kouba and Héroux, 2001), as opposed to, e.g., RTK/NRTK, which require the presence of and communication with nearby base station(s) (Takasu and Yasuda, 2010; Odolinski et al., 2015). Because of the latter techniques requiring local infrastructure to provide precise positioning, PPP is becoming the de facto global precise positioning technique, as it only requires globally estimated satellite corrections (Bisnath, 2020a). However, the technique is known to suffer from relatively long convergence times (Bisnath and Gao, 2009b; Nadarajah et al., 2018) due to its large number of states to be estimated, especially the satellite-dependent ionospheric and ambiguity states. Fortunately, thanks to recent advancements in the GNSS constellations, as well as in measurement processing techniques, improvements can be seen in PPP performance. The following subsections highlight some of the advancements and their impact on PPP performance.

2.3.1 Effects of ambiguity resolution

Various research has been performed to reduce the PPP initial convergence time through different means. Ambiguity Resolution (AR) has proven to be promising in reducing the convergence time. AR takes advantage of additional information that is not used in the classic PPP model: the integer nature of the ambiguities. Indeed, in the classic PPP model, ambiguities are affected by various biases, which make them lose their integer nature. In AR, the biases are extracted from the ambiguities, which are fixed to integers after being estimated as float values. When this process is done correctly, it greatly reduces convergence time as it eliminates the need to wait for ambiguities to converge to integers and can directly fix them to the correct integer. Various AR research has been performed and proved its

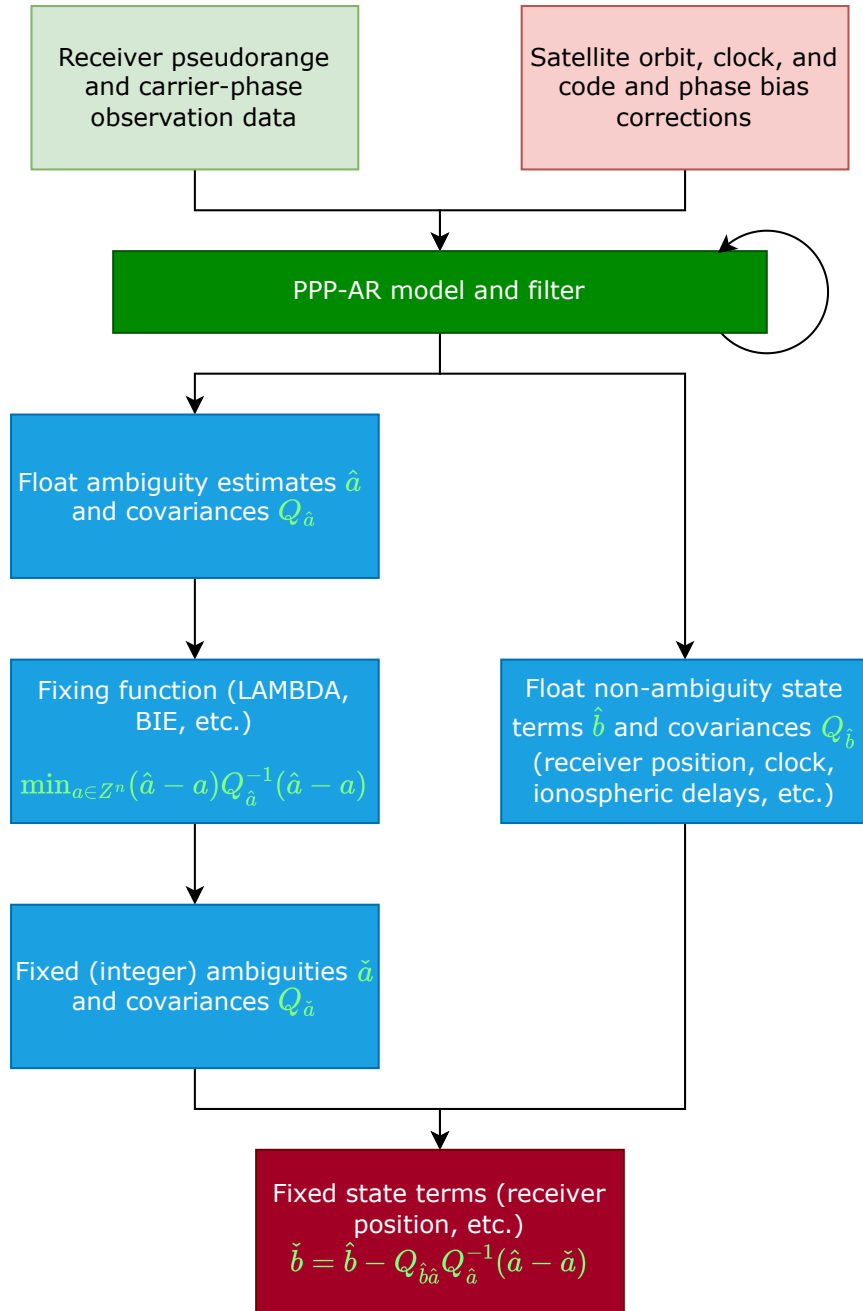


Figure 2.3: Diagram of the PPP-AR estimation and carrier-phase fixing process (Teunissen et al., 1995). The mathematical notation in the diagram follows the same notation as the reference.

benefits. For example, dual-frequency AR has been shown to decrease convergence time by an average of 50%, from 17.5 minutes for the float solution to 9.2 minutes for the fixed one using GPS, Galileo, BeiDou and QZSS (Geng et al., 2020). Liu et al. (2020b) has been able to reduce convergence time from 17.6 minutes to 11.9 minutes with GPS and Galileo AR. Collins et al. (2008) has shown that, within 20 minutes of processing, AR can lead to two-centimetre solutions 70% of the time, as opposed to less than 40% without AR.

In order to illustrate the importance of AR and its impact on PPP solutions, data from a global set of 39 International GNSS Service (IGS) stations have been processed using the York-PPP engine. A map with the locations of the stations is shown in Figure 2.4. The observation data were processed over 7 days between DOY 60 and DOY 66, 2019. The datasets were split into 3-hour sessions, thereby resulting in over 2000 3-hour-long datasets in total. Observations were processed in uncombined mode with ambiguities on both primary frequencies being fixed for GPS (G), Galileo (E) and BeiDou (C). GLONASS (R) was also included in the processing but without ambiguity resolution, as its Frequency Division Multiple Access (FDMA) signal structure makes AR challenging (Banville, 2016). Additional details on the processing can be found in Naciri and Bisnath (2020).

Results from processing the observation data with and without AR are shown in Figure 2.5. The figure shows the average results from processing all datasets, and shows typical performance that is expected and achieved in PPP/PPP-AR. The float (without AR) solution behaves as expected, as it slowly converges to better accuracies due to improved estimation of the ambiguities over time. The solution achieves sub-decimetre positioning within 11.5 minutes with a total rms of 8.4 cm. After convergence, the rms reaches 1.5 cm. Fortunately, the solution can be improved by fixing ambiguities. Indeed, the figure shows that, after fixing ambiguities, convergence is achieved sooner in the processing, as lower errors are observed compared to the float solution. When fixing ambiguities, convergence to the sub-decimetre level is achieved within 5.5 minutes, showing a 52% improvement from the float solution. Overall accuracies are improved as well with AR, as the overall rms is 6.8 cm, and the rms after convergence is 0.8 cm. These results show typical performance when resolving ambiguities: due to the ability to fix ambiguities to integer values sooner, and not having to wait for them to converge, the convergence of the PPP solution is shortened. Not only does the convergence time (defined as time to reach and settle below 10 cm horizontal error) decrease, the position errors decrease as well throughout the processing, as residual non-integer errors are pushed out of the estimated ambiguities when the latter are fixed to integers.

In order to show additional benefits of AR and its impact on PPP convergence time, Figure 2.6 summarizes the dependence of the 100th percentile average solutions on mean PDOP and mean number of satellites through a correlation plot. Each mean value is based on the average results of all 2000 processed datasets. Each mean value is generated by

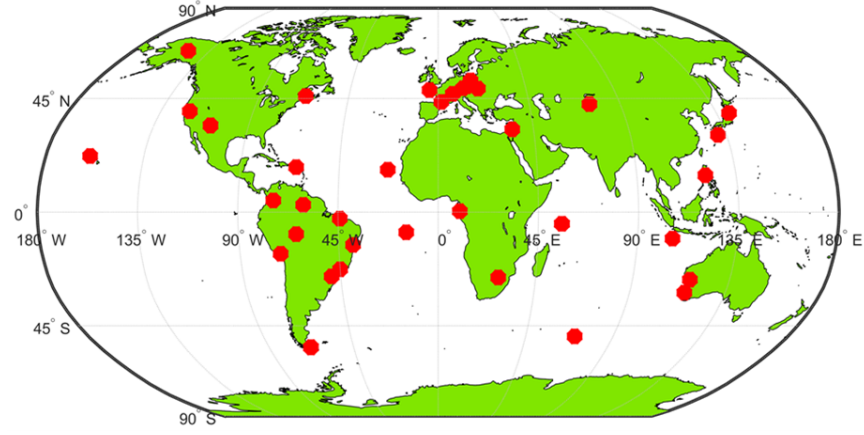


Figure 2.4: Locations of the global set of stations

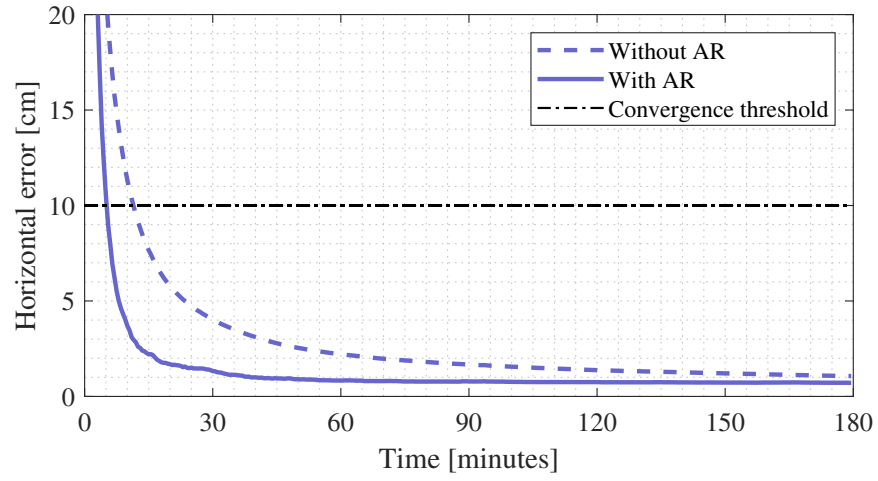


Figure 2.5: Average horizontal position error at the 100th percentile with and without AR based on processing over 2000 datasets on two frequencies for GPS, GLONASS, Galileo, and BeiDou-2.

imposing a mask based on the satellites' elevation angle to reduce the number of visible satellites and worsen the satellite geometry (Naciri and Bisnath, 2020).

As can be seen from Figure 2.6, increasing obstructions with the elevation mask decreases the number of satellites, and consequently increases the PDOP, leading to longer convergence times. Both the float and fixed solutions follow similar trends, and the improvement that the fixing brings to the float solution appears to be relatively constant, regardless of the number of satellites/PDOP. On the other hand, it appears that the number of satellites has less effect on the convergence time as the former increases. Indeed, having a mean number of satellites of 12.5 instead of 8.7 (around 4 additional satellites) leads to a 43% to 46% decrease in the convergence time for both the float and fixed solutions. These additional 4 satellites lead to a decrease of 0.7 in the PDOP, from 1.8 to 1.2. On the other hand, although there are almost 5 additional satellites between the two solutions with a $\text{PDOP} < 1$, their PDOP is still relatively similar, and so is the convergence time. Therefore, extrapolating the plot for a higher number of satellites, we can expect the curve to become more and more horizontal, as adding more satellites does not provide much new information or improvement to the satellite geometry, leading to a stagnation of the PPP performance. This plot is important in highlighting the benefits of PPP-AR, as it shows that processing an ever-increasing number of satellites might not be the answer to improving the PPP performance, but it is rather important to look at other techniques to improve it, such as ambiguity resolution in which additional information is used in the form of the integer nature of the ambiguities. In that case, the graph gets shifted downwards and leads to improved PPP performance.

2.3.2 Effects of modern constellations

GPS has historically been known as the pioneer when it comes to satellite positioning, but recent years have seen the emergence and modernization of other constellations: the Russian GLONASS, European Galileo and Chinese BeiDou. With these constellations being completed or close to completed, it is now possible to take advantage of the increasing number of visible satellites, as well as all the additional frequencies that the new constellations broadcast. Numerous studies have shown the advantages of combining the various GNSS constellations and using all available signals (Jiao et al., 2019; Cai et al., 2015; Laurichesse and Blot, 2016; Geng et al., 2020). The research is conclusive: including more constellations in processing leads to more satellites (and therefore measurements) being processed, leading to a better satellite geometry above the user. This improvement in geometry and increase in number of measurements lead to improvements in PPP performance. This conclusion can be deduced from Figure 2.6, where increasing the number of processed satellites decreases the convergence time.

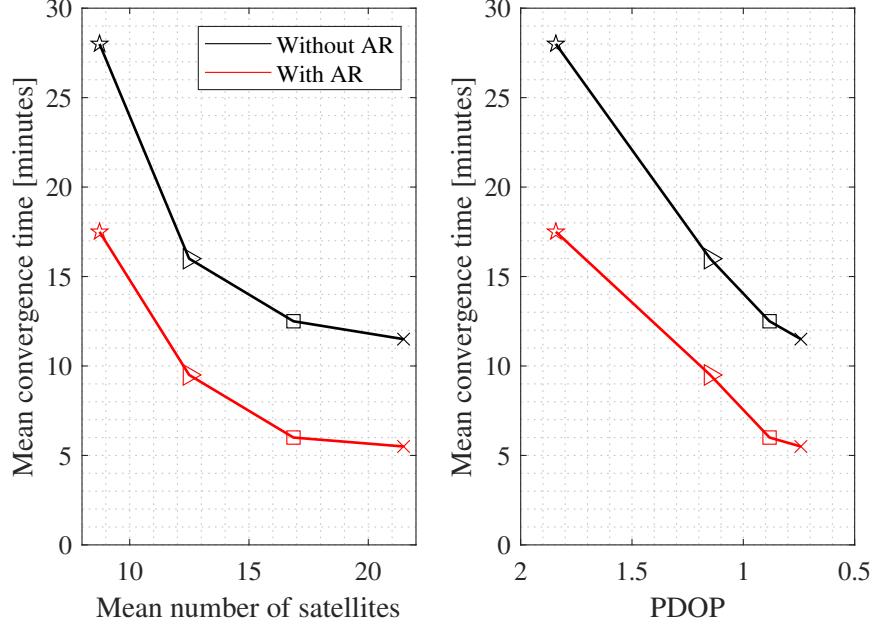


Figure 2.6: Dependence of the 100th percentile average solutions on the mean number of satellites and mean PDOP. Black lines and symbols refer to float solutions, while red lines and symbols refer to fixed solutions.

In order to highlight the effect of additional constellations on the number of visible satellites and satellite geometry, Figure 2.7 is generated for three constellation combinations: *G* uses only GPS, *GR* uses both GPS and GLONASS, and *GREC* uses GPS, GLONASS, Galileo, and BeiDou-2. The figure uses all epochs in the ~ 2000 datasets described in Figure 2.4, amounting to over 700,000 data points for each constellation combination. The bin sizes for PDOP and number of satellites histograms are 0.05 and 1, respectively.

It is clear from Figure 2.7 that, as expected, the number of processed satellites increases with the increase in the number of constellations. The average number of satellites in *G*, *GR* and *GREC* is 9, 14 and 22 satellites, respectively. It is interesting to note that the average PDOP for the *GREC* combination of 0.74 is relatively close to the average PDOP for the *GR* combination of 0.99, as opposed to the big improvement that adding GLONASS brings to the GPS PDOP of 1.97. This relatively small improvement is despite the fact that adding Galileo and BeiDou to the *GR* combination adds 7 satellites on average to the positioning engine. The reason behind this PDOP behaviour is that the satellite geometry from the *GR* combination is already very good, which means that adding more satellites only marginally improves the satellite geometry, leading to the relatively similar PDOP – the law of diminishing returns.

The impact of the additional constellations and PDOP on the PPP solutions can be seen in Figure 2.8. The figure shows the 100th percentile average based on all datasets for the

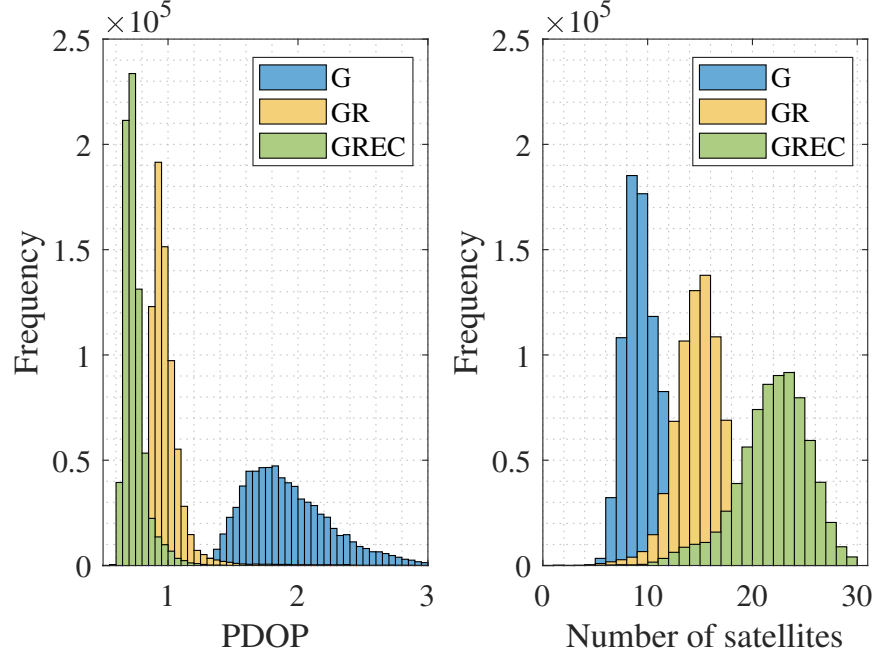


Figure 2.7: Distribution of the PDOP and number of satellites for three different constellation combinations.

different constellation combinations. The dashed graph lines represent the float solutions and the solid ones are their fixed counterparts. Figure 2.9 contains the convergence time and rms for each graph in Figure 2.8. The convergence time is defined as the time it takes for the solution to settle below 10 cm horizontal error or the time at which the horizontal error graph line appears in the figure. The horizontal and vertical rms are computed between minutes 60 and 180 to assess the accuracies after convergence.

First, focusing on the float solutions in Figures 2.8 and 2.9, one can see the effect of the additional constellations on typical float PPP performance. Figure 2.8 shows a clear correlation, as adding constellations leads to improvements in position solutions with shifts towards lower position errors. These downward shifts lead to improvements in the convergence time and accuracy, as shown in Figure 2.9. Indeed, adding GLONASS to GPS leads to a decrease in convergence time from 17.5 minutes to 14.5 minutes, which is further reduced to 11.5 minutes when adding Galileo and BeiDou-2. Similar levels of improvements are observed as well in accuracy post-convergence. These improvements are attributed to two factors: the improvement of satellite geometry and the addition of measurements thanks to the additional satellites from the additional constellations.

However, one cannot ignore that, even with all four constellations, the float solution still requires 11.5 minutes to converge to the sub-decimetre level. Fortunately, as discussed in the previous section, AR can be used to further improve performance and reduce convergence

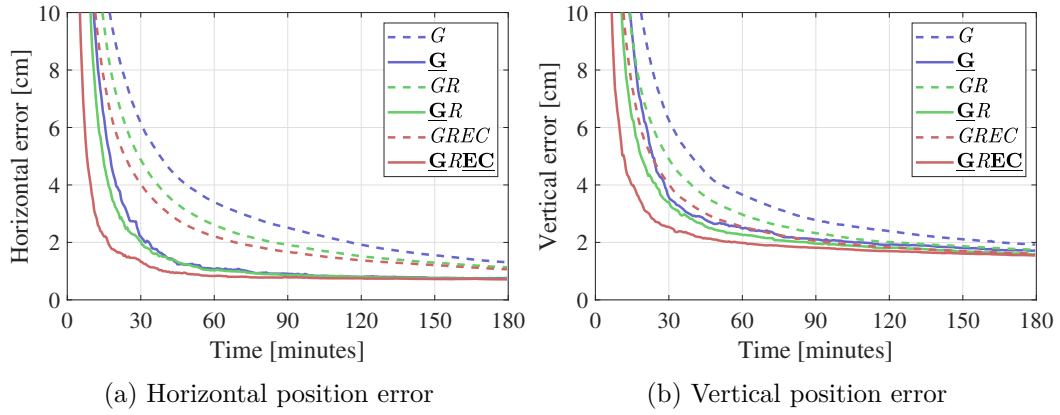


Figure 2.8: 100th percentile horizontal (a) and vertical (b) average position solutions. Ital-icized constellations with dashed lines are float PPP (without AR), while bold underlined constellations with solid lines are fixed PPP (with AR).

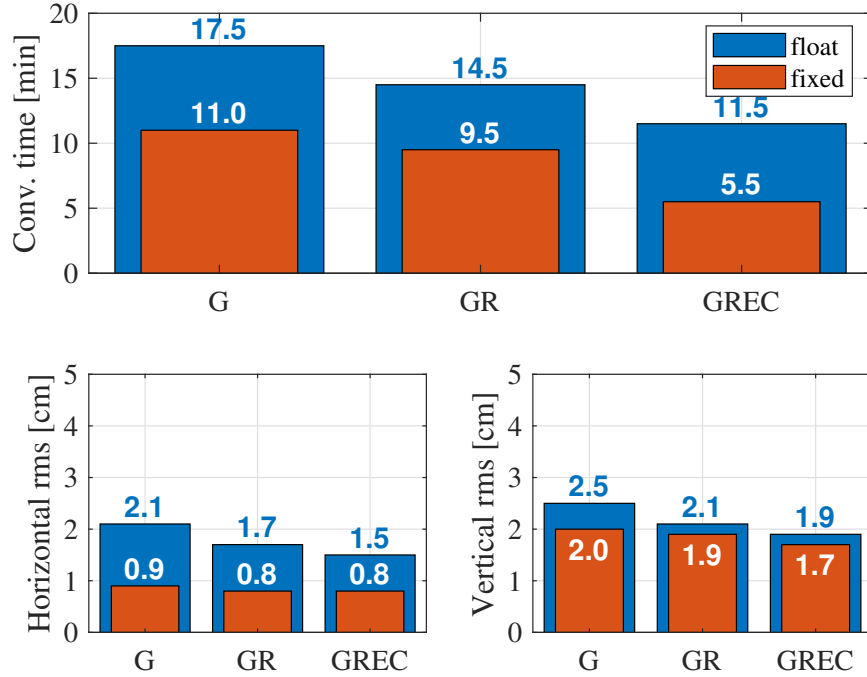


Figure 2.9: 100th percentile convergence time and rms statistics corresponding to the results shown in Figure 2.8.

time. It is interesting that the fixed solutions with fewer constellations can converge faster than float solutions with more constellations. For instance, the fixed G solution converges 3.5 minutes faster than the GR float solution and is able to reach 49% better horizontal accuracies, respectively, even though GLONASS adds an average of 6 additional satellites. The same conclusion holds for the fixed GR solution compared to the float $GREC$ solution. It is worth noting that, in this case, even the GPS-only fixed solution converges half a minute faster than the full float GREC solution, even though the latter has 13 more visible satellites and over half the PDOP of the former. The horizontal accuracies are also smaller in the fixed GPS solution, 0.9 cm, compared to all the float solutions, 2.1 cm, 1.7 cm and 1.5 cm. However, the latter conclusion does not hold all the time, as these results are based on averaged results from multiple datasets. Such observation of fixed solutions with fewer satellites converging faster than float solutions with more satellites can be attributed to the power of AR in accelerating convergence. Indeed, correct fixing of ambiguities - even with a lower number of satellites - is found to shorten convergence time more than the additional satellites with no fixing being performed.

The results shown in Figures 2.8 and 2.9 are generated by using rapid satellite products. These products' delivery is delayed by a few tens of hours, as it requires the time-consuming computation of precise satellite orbits, as well as observation data from a large network of reference stations. These products are therefore not suitable for real-time applications. Instead, ultra-rapid products can be used, which are products that use predicted satellite orbits and which can be available in real-time or with a latency of a few hours (<https://igs.org/products/>). The ultra-rapid products are generally of worse quality (approximately double the error) than the rapid products, but PPP performance is relatively comparable between both sets of products (Banville et al., 2021). The rest of this subsection will focus on performance of different constellation combinations using ultra-rapid products. A detailed description of the ultra-rapid products that were used can be found in Naciri et al. (2021a).

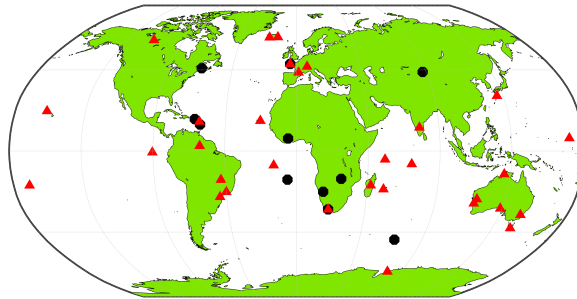


Figure 2.10: Distribution of stations used in the user processing of ultra-rapid products in red. Black dots represent stations for which some of the results are shown in Section 2.3.3.

The performance using GPS, Galileo, BeiDou-2 and BeiDou-3 is analyzed using different constellation combinations, and using ultra-rapid products. Observation data from 33 worldwide IGS stations, shown in Figure 2.10, are processed and analyzed over 7 days on the L1 and L2 frequencies for GPS, E1 and E5a for Galileo, and B1-2/B3 for BeiDou-2 and BeiDou-3. In addition to GLONASS, which was not included in the processing, some constellation combinations are omitted in the analysis. For instance, Galileo-only and BeiDou-only solutions have not been included in the combinations due to the limited number of satellites that can be seen at the chosen ground stations from these constellations. Indeed, there are numerous epochs where less than four satellites from the individual constellation are tracked, rendering PPP unfeasible. Therefore, five constellation combinations are analyzed: GPS-only (*G*), GPS and Galileo (*GE*), GPS and BeiDou-2/3 (*GC*), Galileo and BeiDou-2/3 (*EC*), and GPS, Galileo and BeiDou-2/3 (*GEC*). Fig. 2.11 summarizes the convergence time and rms statistics of the processing results using different constellation combinations at the 100th, 95th, and 67th percentiles. The convergence time is defined as the time it takes for the horizontal error to reach and settle below 10 cm. The rms error is computed from the first hour to the 24th in order to highlight accuracy after convergence.

When it comes to the constellation combinations containing GPS, these results indicate that the GPS-only solution has higher rms and convergence time compared to *GE*, *GC* and *GEC*. This behaviour is expected, as shown previously, as adding constellations improves the geometry and increases the number of measurements, which is helpful in a technique that is as measurement-dependent as PPP. On the other hand, *GE* results have lower horizontal and vertical rms and convergence time compared to *GC*. The most probable reason behind this difference is that BeiDou has a lower number of visible satellites globally compared to Galileo. Fig. 2.11(b) shows the epoch-by-epoch average of the number of processed satellites from all three constellations, based on all processed datasets. Although this number of satellites time series does not represent the number of satellites at a specific location or on a specific day, it is helpful to show that, on average, only 3 BeiDou satellites are processed compared to averages of 7 and 9 for Galileo and GPS, respectively. Note that BeiDou Geostationary Earth Orbit (GEO) satellites have not been included in the processing, and that some stations have a low number of visible BeiDou satellites. This low number of BeiDou satellites would explain the *GC* performance compared to *GE*, as well as the fact that *GEC* has similar performance compared to *GE*, as only limited information is added by including BeiDou as well.

When only processing Galileo and BeiDou (*EC*), it appears that both horizontal and vertical components at the 100th percentile are noisy and have several jumps, leading to relatively high rms and a convergence time which reaches 146.5 minutes. This behaviour is due to some outlier datasets having a very low number of visible satellites because of, e.g., the location of the station, time of day, low elevation angle, etc. Such datasets have high

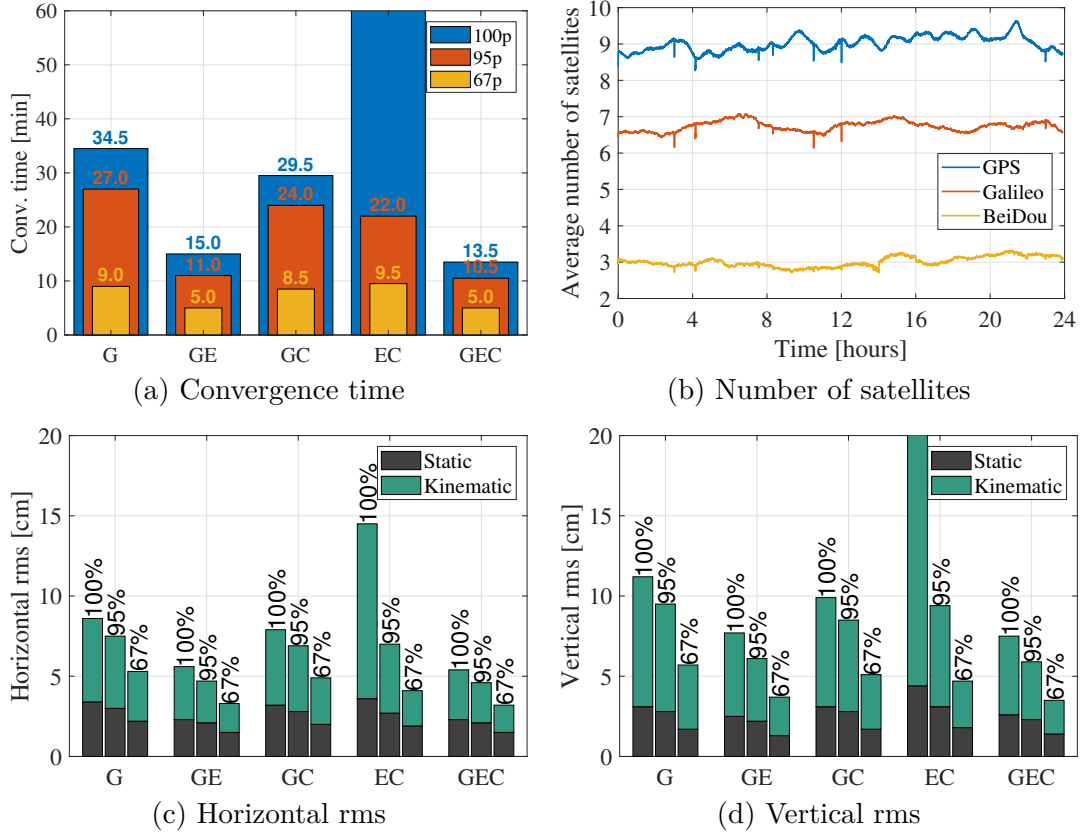


Figure 2.11: Average static convergence time (a), average epoch-by-epoch number of satellites (b), average static and kinematic horizontal rms (c), and static and kinematic vertical rms (d) for different constellation combinations at the 100th, 95th and 67th percentiles.

errors, which adversely affect the average solutions. These outliers are eliminated in the 95th and 67th percentiles, where the *EC* combination is comparable to the other combinations.

The datasets have also been processed in kinematic mode with position state process noise typical of a car (120 km/h), in order to assess the performance when the position is not averaged out throughout the processing. Doing so leads to more variations in the position estimates, and therefore slightly higher rms. The rms from the kinematic processing are summarized in Fig. 2.11, along with the static rms. It is clear that kinematic processing leads to slightly higher rms in both horizontal and vertical components. When it comes to comparing the different constellation combinations, the same conclusions hold compared to the static case. One interesting observation is that, in static mode, the horizontal and vertical rms have very similar values. In some instances, the vertical rms is smaller than the horizontal one, which goes against the fact that the vertical component cannot be estimated as well as the horizontal one due to the satellite geometry, as all satellites are above the receivers. Such behaviour could be due to the averaging power of static processing. However, in kinematic processing, the vertical components are a few centimetres higher than the horizontal ones, which is more realistic and expected. The results in subsection show the importance of multi-GNSS processing in improving PPP performance.

2.3.3 Modernized and new GNSS signals and frequencies

With the rapid evolution of GNSS constellations, new frequencies and signals are being broadcast, which have a positive impact on PPP performance (Bisnath and Gao, 2009a; Bisnath, 2020a). GPS originally broadcasted signals on two frequencies L1 and L2 at 1575.42 MHz and 1227.60 MHz, respectively. However, recent years saw the addition of a third GPS frequency on L5 at 1176.45 MHz, with higher power and aimed towards safety-of-life applications. 18 satellites from the newer generations GPS-IIF and GPS-III have been launched since 2010, all of which broadcast on all three frequencies (NAVCEN, 2023). Galileo, on the other hand, has been designed to broadcast on four frequencies (E1, E6, E5a, E5b) with the option to track E5a+E5b together as AltBOC (Alternative Binary Offset Carrier) (Zaminpardaz and Teunissen, 2017), a modulation combining E5a and E5b with reduced measurement noise. The Galileo E1 and E5a frequencies coincide with the GPS L1 and L5 frequencies, respectively, while the E5b, E5, and E6 signals are at 1207.140 MHz, 1191.795 MHz, and 1278.75 MHz, respectively. With launches that began in 2011, Galileo is almost complete with the exception of satellites E14 and E18, which are set as unhealthy because of their eccentric orbits. Including these two satellites in the processing leads to a total number of usable satellites of 24 (GSC, 2020). BeiDou has also been designed to broadcast on multiple frequencies with satellites in Medium Earth Orbit (MEO), Geostationary Earth Orbit (GEO) and Inclined GeoSynchronous Orbit (IGSO). The BeiDou-2 generation

is designed to broadcast signals on three frequencies B1-2, B2b and B3 at 1561.098 MHz, 1207.140 MHz and 1268.52 MHz, respectively. The newer BeiDou-3 generation has been designed to broadcast on five frequencies in total (B1, B2a, B2b, B2 and B3), with B1 and B2a coinciding with the GPS L1 and L5 frequencies, respectively, B2b and B3 coinciding with BeiDou-2, and B2 being at 1191.795 MHz. The BeiDou-3 generation has been recently completed, with the last satellites being launched in June 2020 (Elizabeth, 2020). As of March 2021, the constellation has 44 operational satellites (CNSO, 2020). GLONASS also broadcasts on multiple frequencies, though the signal structure is different from other constellations: the constellation uses a Frequency Division Multiple Access (FDMA) technique, leading to each satellite broadcasting on a separate frequency. The constellation is being modernized, with new satellites being launched that broadcast on three Code Division Multiple Access (CDMA) frequencies: the same technique used by other GNSS constellations. All of these new GNSS signals and constellations were shown to have direct positive impacts on PPP performance (Naciri et al., 2021a; Cai et al., 2015; Li et al., 2020b; Geng and Guo, 2020; Nadarajah et al., 2018).

Given all the frequencies being broadcast by an ever-growing number of satellites, it is necessary to move away from a combined view of dual-frequency measurement processing into an uncombined method to simplify models, limit the number of possible combinations, and make processing scalable. Combined here refers to forming a linear combination of measurements, while uncombined refers to processing the raw measurements without combining the measurements. Additionally, part of the products required for PPP processing are satellite (code and phase) bias corrections. These corrections have been historically provided in a DCB format, where the difference in code biases between two observables is provided (Montenbruck et al., 2014a; Wang et al., 2016). Given the existence of multiple code modulations on each GNSS frequency (International GNSS Service, 2018), not only do inter-frequency DCBs have to be provided, intra-frequency corrections have to be as well, as, despite tracking the same frequencies, receivers can track different modulations. This large number of observables (signals and modulations), coupled with the fact that the DCBs are differential values between two observables, leads to an enormous number of DCBs to be broadcast to the user. As a consequence, DCB providers tend to only provide a limited and select number of combinations to the users, which cover the most common observables (Montenbruck et al., 2014a; Wang et al., 2016). Not only does this situation limit the observables that users can use to be compatible with the DCB products, it also complicates the application of the biases at the user side. Fortunately, and to solve this complexity due to the large number of available signals and modulations, an Observable-specific Signal Biases (OSB) format has been proposed (Schaer, 2018) and started being adopted by various providers (Villiger et al., 2019; Deng et al., 2021; Su et al., 2022; Geng et al., 2022). The advantage of using OSB products is that they directly provide the biases on each observable type, eliminating

the need for the user to know the model that was used in the estimation of the biases on the network side, nor do they need to know the types of the reference observables. This situation is a major advantage in moving towards interchangeability of the products, and simplifying the application of satellite biases in the current multi-constellation, multi-frequency context. For instance, Banville et al. (2020) opted for the OSB format to assess the interoperability of AR products from different analysis centres. With OSB corrections, both code and phase biases are applied at the observable level, similar to the satellite orbits and clocks, instead of applying the phase biases at the ambiguity level, and without having to form linear combination of biases, as is the case with DCBs. This approach is believed to be more compatible and in-line with the PPP philosophy, as all the SSR corrections are signal-dependent and applied directly to the measurements.

With all these new satellites and signals come new challenges. For instance, Wanninger and Beer (2015) demonstrated that BeiDou-2 suffers from the existence of satellite-induced, elevation-dependent group delay variations that affect the pseudorange measurements. They have provided a satellite-, elevation- and frequency-dependent look-up table to correct for these pseudorange variations. Not accounting for these group delay variations leads to degradations in PPP performance. Additionally, BeiDou-2 and BeiDou-3 suffer from the existence of clock and time delay biases between both generations. Jiao et al. (2020) have shown the existence of these biases between BeiDou-2 and BeiDou-3, and they have noticed that the biases are receiver-dependent, as some receivers suffer from significant biases, while others do not. Improvements have been noticed when estimating the BeiDou-2 and BeiDou-3 Inter-Signal Biases (ISB) as random constants. Zhang et al. (2020) have shown that not taking these ISBs into account affects the Signal-In-Space Ranging Error (SISRE) of the satellite orbits, while Li et al. (2019) have shown their effect on both receiver and satellite DCB estimation. To remedy to the issue, one can either directly apply these values as corrections, or process both BeiDou generations as two different constellations by estimating a different receiver clock for each. Applying the corrections at the network side leads to a decrease of BeiDou-3 SISRE. Fig. 2.12 shows an example of the difference between the BeiDou-2 and BeiDou-3 receiver clocks when estimated separately for station TLSE in Toulouse, France. The difference between both clocks contains the biases between the two BeiDou generations, which can reach relatively high values of 20 ns or 6 m. If unaccounted for, these TGD biases go into the residuals, as they cannot be fully absorbed by the other parameters of the PPP model.

In order to illustrate the effect of inter-constellation biases, Figures 2.13 and 2.14 show the residuals from processing GPS, Galileo, and BeiDou-2/3 observations from station TLSE on DOY 183, 2020. Figure 2.13 estimates separate clocks for BeiDou-2 and BeiDou-3, while Figure 2.14 estimates a common receiver clock for all BeiDou satellites. It is clear that using a common clock for both BeiDou generations leads to biases in the residuals: the BeiDou-2

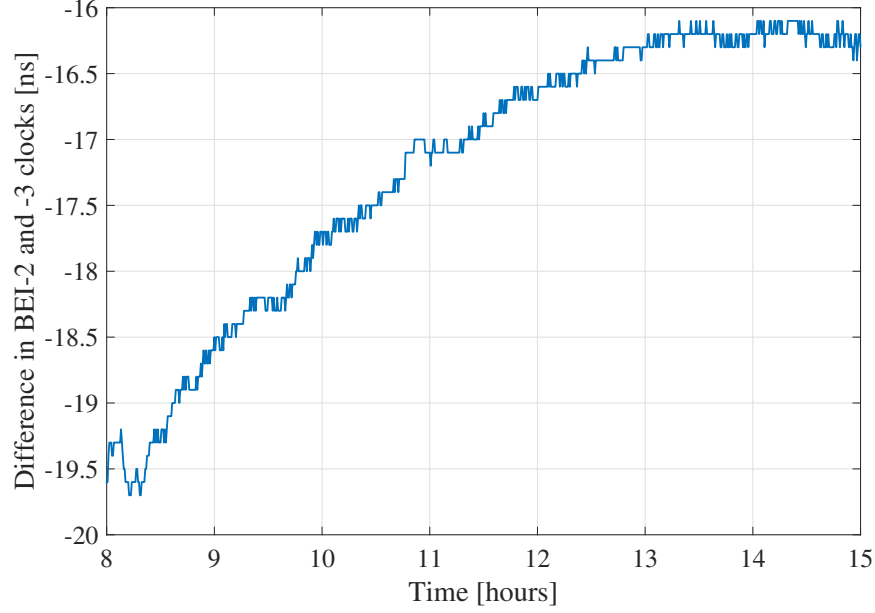


Figure 2.12: Difference between the estimated BeiDou-2 and BeiDou-3 receiver clocks for station TLSE in Toulouse, France on DOY 183, 2020 between UTC hours 8 and 15.

pseudorange residuals are not centered around zero but rather around 4/5 m. On the other hand, the BeiDou-3 pseudorange residuals are centered around zero until hour 9 where the residuals start shifting and end up being centered around 2 m after hour 15. Therefore, it is necessary to account for these TGD biases as they affect the PPP model since they do not only appear in the residuals, but they can also affect the estimation of the PPP parameters.

Another consideration with the modern frequencies and constellations relates to the GPS L5 signal. Indeed, GPS Block-IIIF L5 signals suffer from clock variations as first noticed by Montenbruck et al. (2012). These variations are caused by a thermal effect depending on the orbital plane orientation with relation to the Sun. Different strategies are adopted to eliminate these biases: Tegedor and Øvstedal (2014) proposed the estimation of separate L5 clocks along with the typical L1/L2 clocks in an offline setting after the first three GPS Block-IIIF satellites were launched. Guo and Geng (2018) used a similar approach with a focus on triple-frequency processing; Montenbruck et al. (2012) proposed a harmonic approximation of the variations; Li et al. (2013) and Li et al. (2016) estimated the biases based on satellite- and epoch-differenced measurements, where the focus was triple-frequency processing. Liu et al. (2020b) showed that the variations can be absorbed and disseminated to the user through the satellite L5 phase bias corrections. In the results for the remainder of this chapter, the first strategy is adopted, where separate L5 clocks are estimated for GPS IIF satellites, along with the L1/L2 clocks. In the following, the performance of GPS, Galileo and BeiDou-3 on the same two frequencies is assessed, which is only possible thanks

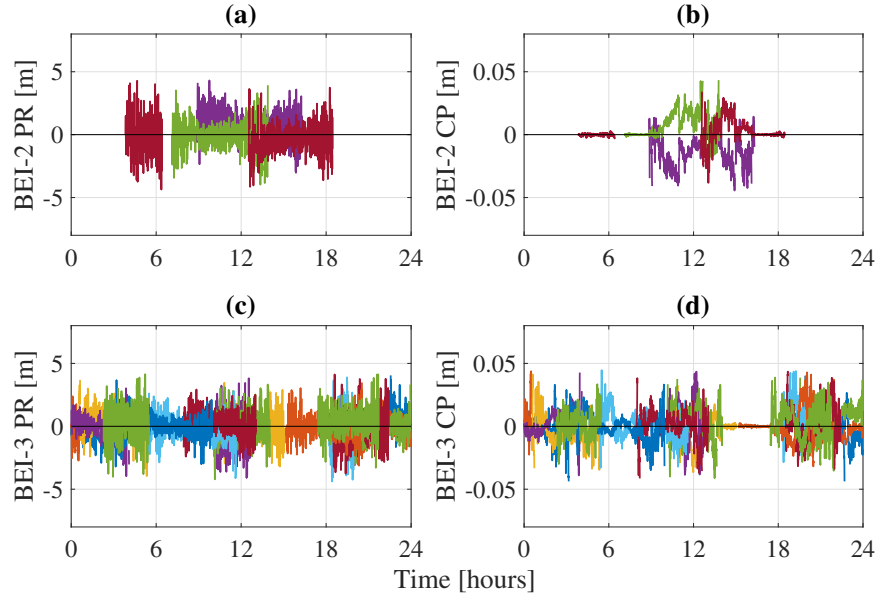


Figure 2.13: BeiDou-2 pseudorange PR (a) and carrier-phase CP (b) residuals and BeiDou-3 pseudorange PR (c) and carrier-phase CP (d) residuals for station TLSE on DOY 183, 2020 with separate receiver clocks for each generation.

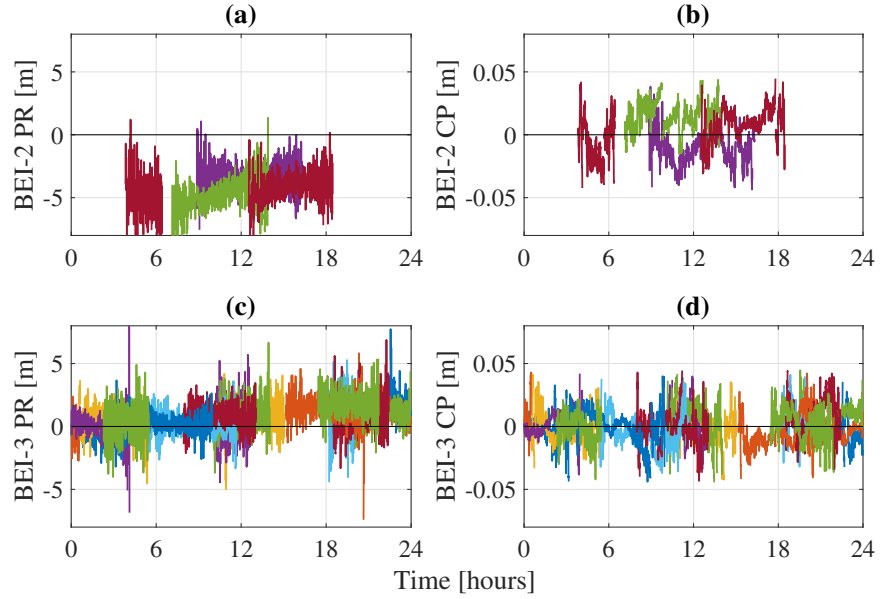


Figure 2.14: BeiDou-2 pseudorange PR (a) and carrier-phase CP (b) residuals and BeiDou-3 pseudorange PR (c) and carrier-phase CP (d) residuals for station TLSE on DOY 183, 2020 with a common receiver clock for both generations.

to the modernization of GNSS constellations. The two frequencies are L1, E1, and B1 at 1575.42 MHz and L5, E5a, and B2a at 1176.45 MHz. In order to highlight the effect of GPS L5 variations on the PPP results, data from global stations were processed by ensuring that they specifically log measurements from the desired frequencies for all three constellations, and splitting the observation data into 6 hour-long datasets. The 6 hour-long sessions are further filtered and only those with more than four visible GPS IIF satellites are selected and used in the processing. This condition is set in order to have enough GPS satellites to influence the solutions and, more specifically, to assess the effect of the GPS L5 bias on the estimation of the parameters. However, the minimum number of GPS satellites condition does not mean that all four GPS satellites can be used in the processing as some would have low elevation angles, while others could be rejected by quality control measures.

First, the effect of the known GPS L5 biases on the positioning solution is assessed on a sample dataset. To do so, GPS measurements on the L1/L5 frequencies are processed using two sets of satellite clocks: L1/L2 clocks on which L5 corrections are not applied, and L1/L5 clocks, which are the same L1/L2 clocks to which the L5 corrections are applied. Due to the number of GPS satellites occasionally dropping below four satellites and in order to stabilize the positioning solution, Galileo and BeiDou-3 have been processed as well. Figure 2.15 and Table 2.1 show the effect of accounting or not for the GPS L5 biases on station CRO1 in the Virgin Islands, U.S.A, which was processed with GPS, Galileo and BeiDou-3 on DOY 186, 2020 between 6h and 12h Universal Time Coordinated (UTC) in kinematic mode. Galileo and BeiDou-3 are processed on the same frequencies and with the same setting in both cases.

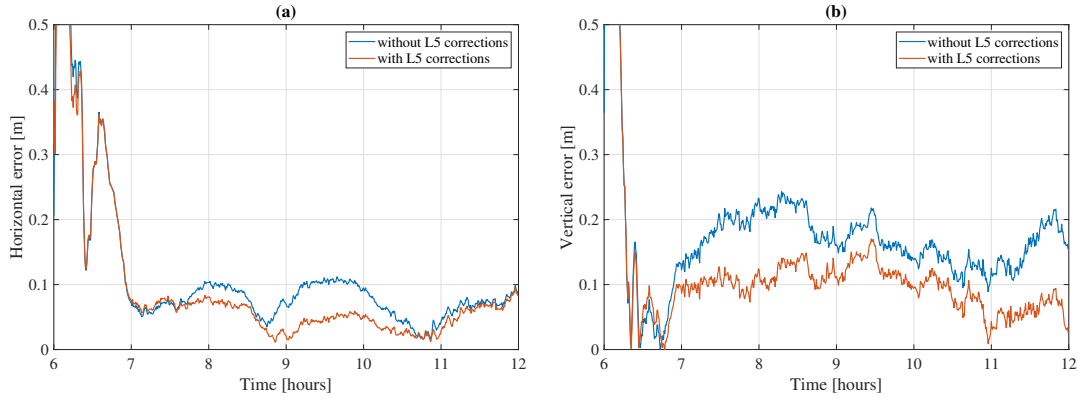


Figure 2.15: Average horizontal (a) and vertical (b) errors for station CRO1 on DOY 186, 2020 between UTC hours 6 and 12. GEC constellations are processed in kinematic mode on L1,E1,B1/L5,E5a,B2a frequencies, with and without the application of GPS L5 corrections.

Looking at the station CRO1 results, it is clear that accounting for the GPS L5 biases and correcting for them improves the positioning solution. Both horizontal and vertical

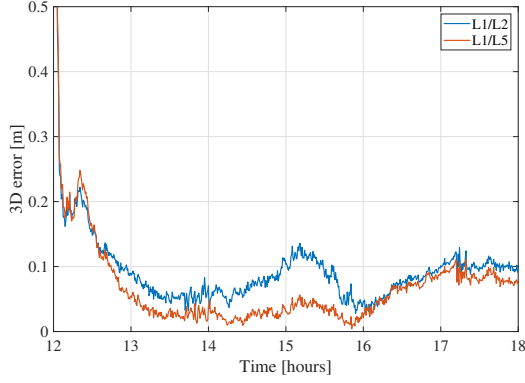
Table 2.1: Convergence time, horizontal and vertical rms statistics corresponding to the results in Figure 2.15.

	Conv. time [mins]	Hor. rms [cm]	Vert. rms [cm]
L5 corr. not applied	235.5	18.9	17.3
L5 corr. applied	56.5	11.6	10.2

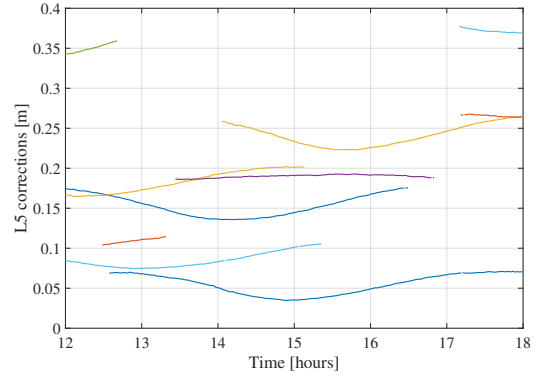
components are improved as demonstrated in the time series as well as the rms. Indeed, when correcting the L5 biases, the horizontal rms decreases from 18.9 cm to 11.6 cm, while the vertical rms decreases from 17.3 cm to 10.2 cm. The convergence time, defined as the time it takes for the horizontal error to settle below 10 cm, is much longer (235.5 minutes) when using L1/L2 clocks as opposed to 56.5 minutes when using L1/L5 clocks. This big convergence time is due to the horizontal solution going slightly above the 10 cm threshold between hours 9 and 10.

Figure 2.16a gives another example of the effect of L5 biases for station BRST in Brest, France on DOY 187 between UTC hours 12 and 18. The same observations hold as for Figure 2.15, as 15% and 20% decreases in the horizontal and vertical rms, respectively, are noticed when applying L5 corrections on *GEC* processing. Therefore, it is clear that the L5 biases have an effect on the position estimates. In order to better understand the effect of L5 biases on the PPP estimation as a whole, the BRST dataset is further analyzed. Comparing the *EC* results with and without applying the L5 corrections, the residuals are not affected as they are the same for both solutions, with only millimetre-level differences being seen. Therefore, when not accounting for L5 biases, they do not get absorbed by the residuals. Figure 2.16 shows the 3D error for station BRST with and without accounting for the L5 biases, the L5 corrections, as well as the difference in all the GPS-related state terms between both solutions. The assumption in these plots is that the only difference between the two solutions is whether the L5 corrections are applied or not. Thus, taking the difference in the estimated states would show the effect of the L5 biases on them.

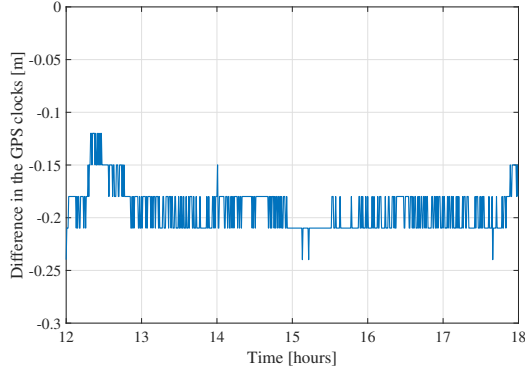
For instance, the Galileo- and BeiDou-related states are very similar, to a few millimeters. However, this similarity is not the case for the GPS-related states as Figure 2.16 shows. First, the position estimates (Figure 2.16a) are affected, as applying the L5 corrections (Figure 2.16b) reduces the 3D rms from 8.1 cm to 5.4 cm. The GPS receiver clock (Figure 2.16c) is affected as well, as the difference in the clock between both solutions is 0.2 m on average. This difference is not the case for the wet zenith tropospheric delay (Figure 2.16d), for which there is less than 2 cm difference between both solutions. The tropospheric estimate is common to all constellations and is estimated as a random walk process. Therefore, it is more robust to biases than, e.g., the GPS clock or the GPS ambiguities. On the other hand, the slant ionospheric delays (Figure 2.16e) and the ambiguities (Figure 2.16f) are clearly affected, as decimetre-level satellite-dependent differences are noticeable. It is interesting to



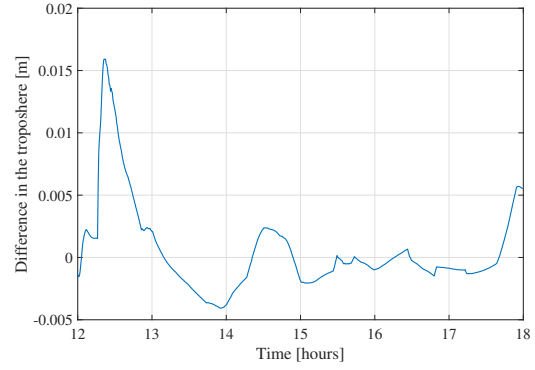
(a) 3D position error.



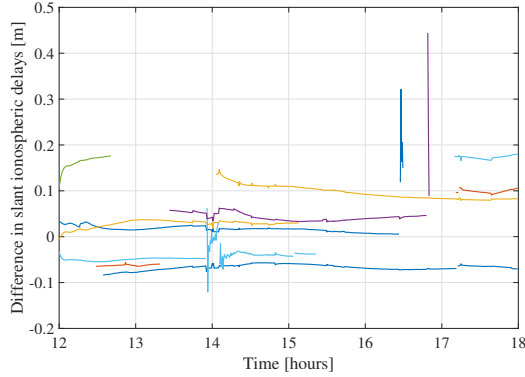
(b) L5 corrections.



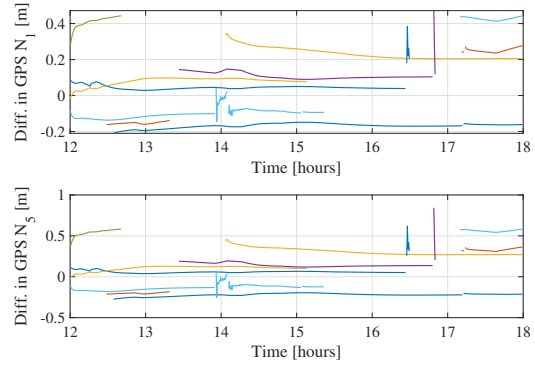
(c) Diff. in the receiver GPS clocks.



(d) Diff. in estimated troposphere.



(e) Diff. in the GPS slant ionospheric delays.



(f) Diff. in GPS L1 and L5 ambiguities.

Figure 2.16: Difference in the estimated GPS-related states when using the GPS L1/L2 and L1/L5 clocks for station BRST in Brest, France on DOY 187 (a, c, d, e, f), and applied L5 corrections (b).

note that they follow the same trends as the L5 corrections, shown in Figure 2.16*b*. Each colour in the correction, ionosphere and ambiguity sub-figures represents a satellite, with each satellite's colour being the same in all sub-figures. These results show that GPS L5 corrections have to be taken into account, as their effect not only degrades the position estimates when ignored, it also affects the estimation of other estimated state terms.

2.4 Summary

Since the deployment of the first GPS satellites, the field of satellite navigation in general, and satellite-based positioning specifically, has seen a great deal of evolution. While the satellite systems were designed to be used in SPP mode by using the GPS signals together with broadcast ephemeris data, the user processing technique evolved with the advent of differencing. Indeed, between-receiver differencing was used from early on to eliminate intentionally-introduced errors in the broadcast satellite clock corrections, known as Selective Availability (SA). While SA was disabled in May 2000, improving users' accuracy from several tens of metres to a few metres, differential techniques remained in use due to their capacity to eliminate spatially-correlated errors such as the signal delays caused by the propagation through the atmosphere.

While different techniques exist within differential positioning, RTK and NRTK remain the most widely used techniques due to their capacity to reach centimetre-level accuracies quickly. In the case of RTK, one reference station with known coordinates is necessary, together with a communication link between the rover and the base; while, for NRTK, a network of nearby reference stations with known coordinates is necessary instead of a single station. In the case of both techniques, the user's performance depends on the distance to the closest reference station, as well as the quality of the interpolation used to generate corrections at the user's location in the case of NRTK. Thus, in areas where the closest base station is tens of kilometres away, the performance of both RTK and NRTK gets degraded significantly.

Fortunately for high precision applications in areas without (N)RTK coverage, PPP came into existence as a counter to relative double-differencing. While NRTK uses a regional network of reference stations to estimate corrections for the user in either SSR or OSR, PPP is designed to use a global network of stations to estimate SSR corrections to be transmitted to the user without the user needing to provide their location. The advantage of this approach is that it can be used anywhere and only requires a stream of SSR corrections, which can be transmitted over various mediums, i.e., Internet, radio link, satellite link, etc. The drawback is that the precise atmospheric information that RTK and NRTK provide in short and medium baselines is unavailable in PPP due to the sparsity of the network. Therefore, PPP has to rely on the quality and quantity of GNSS measurements to estimate

precise positions, which are known to require tens of minutes to reach centimetre-level accuracy.

Over the past decades, there have been many advancements both in terms of processing techniques, as well as in the GNSS constellations. One of the most promising avenues to reduce PPP convergence time has been Ambiguity Resolution (AR), where, instead of waiting for carrier-phase ambiguities to converge, the user can make use of the knowledge that they are integer values to shorten their convergence and constrain them to integers. By doing so, the convergence period of the position estimates can be shortened. As was shown in Section 2.3.1, fixing ambiguities is shown to decrease convergence time by approximately 50% on a basic PPP solution. While PPP is a measurement-dependent technique, it was shown that ever-increasing the number of satellites - and consequently number of measurements - yields lower returns in terms of reduction of the convergence time due to the law of diminishing returns. As a consequence, AR was shown to be a promising avenue in further reducing convergence time. In addition to AR, recent advancements that affect PPP include modernization of GNSS constellations, whether through new modern constellations, such as Galileo and BeiDou-3, or through new and improved signals being broadcast on different frequencies, such as GPS L5 and Galileo E6. This chapter has demonstrated that the additional constellations lead to more satellites being visible to users worldwide, improving their PPP performance. It additionally showed that new GNSS signals can be used for PPP processing, although some of them need to be carefully handled, such as L5 signals on GPS Block-IIIF satellites that suffer from clock variations due to a thermal effect, or the existence of biases between BeiDou-2 and BeiDou-3, leading to both constellations having to be processed as different constellations. Fortunately, carefully handling these various signals and constellations allows for proper PPP processing. As a consequence, these additional signals and constellations open the door for multi-constellation, multi-frequency processing, which can be used, together with AR to further improve PPP performance.

Chapter 3

Multi-constellation, multi-frequency PPP ambiguity resolution

This chapter focuses on the Decoupled Clock Model, and extends the original combined DCM to an uncombined, triple-frequency variant at the user side. The original combined DCM is first introduced. Then, an uncombined dual-frequency variant of the DCM is derived from first principles and the dual-frequency uncombined model is then extended to a triple-frequency uncombined model. Special considerations have to be taken into account when processing multiple constellations (for instance, not all satellites broadcast on three frequencies), therefore details on such considerations are presented before introducing results from the processing. The derivation of a triple-frequency multi-GNSS uncombined DCM is novel, as the DCM has traditionally been a GPS-only dual-frequency combined model.

Performance of the model is shown through dual- and triple-frequency processing of multiple worldwide datasets using Galileo as the test constellation. The behaviour of some of the model's parameters are also shown for specific datasets as an example. The analysis is followed by multi-GNSS processing results, where the proposed algorithm is tested on a network of IGS stations for which hourly datasets were processed over one week. A comparison of performance between a solution with only dual-frequency satellites, only triple-frequency satellites, and a mix of dual- and triple-frequency satellites is introduced first. The comparison is followed by an analysis of the mixed dual-/triple-frequency results through a convergence time and accuracy analysis, as well as a typical performance of a sample dataset. Finally, the algorithm is tested on kinematic automotive data that show the possibility of instantaneous centimetre-level positioning with a moving receiver. The work in this chapter has been published in Naciri and Bisnath (2021a,b).

3.1 Introduction to the decoupled clock model

This research focuses on the DCM, which was originally introduced by Collins (2008). The main reasons behind choosing the DCM is that it does not make a priori assumptions about receiver biases, which are estimated as clock-like terms instead of being eliminated; as well as its ability to free the ambiguities from biases and to directly estimate them as integers. For instance, Abdelazeem et al. (2016); Zhang et al. (2017, 2019a) have studied the temporal variability of receiver biases and have found that they can vary by several nanoseconds over a few hours. They have found that the variations are due, in part, to the temperature variations of the receiver, its quality and its location. Wang et al. (2020) have analyzed the effect of long-term and short-term variations of receiver biases on the stability of the ambiguities and on positioning performance by extracting the receiver biases from the ambiguities and estimating them. They conclude that doing so leads to more stable ambiguities and therefore faster and more reliable PPP-AR.

It is therefore important to not make assumptions about the receiver biases, which is one of the main principles of the DCM. The model is based on the fact that the pseudorange and carrier-phase measurements are not synchronized to the same level of precision, which means that different clocks need to be used for the pseudorange and carrier-phase measurements. The model is based on the GPS ionosphere-free combination, and therefore to perform AR, it uses the MW combination. Forming the MW combination allows for the estimation of the WL and NL ambiguities and, consequently, their fixing.

The DCM removes the receiver biases from the ambiguities through implicit differencing by the selection of a reference satellite. Implicit differencing is a consequence of the decoupling of the receiver clocks, and the datum in the phase measurements moves from the receiver clock to the reference satellite's ambiguities. As a consequence, all estimated ambiguities are relative to the reference satellite's, and the receiver biases are estimated instead of being eliminated as is the case with explicit differencing. The advantage of the DCM approach is that it does not assume any particular behaviour of the biases, which are estimated as white noise.

The DCM was further developed into the Extended DCM by Collins et al. (2012), where the narrowlane pseudorange and widelane carrier-phase measurements were used as two separate measurements instead of forming a single MW measurement. Doing so allows for estimation of the ionospheric delays, even in combined mode, which offers the possibility of ionospheric constraining.

Shi and Gao (2010) analyzed the integer nature of the ambiguities and the characteristics of the code and phase clocks using the combined DCM. The authors concluded that the DCM is able to recover the integer nature of the ambiguities, and they demonstrated that the receiver phase clock is improved compared to the receiver code clock. They also found

that receiver biases can be time-varying and they therefore need to be estimated as white noise.

PPP is typically used by forming ionosphere-free combinations of the raw measurements in order to eliminate the ionosphere from the estimation (Collins et al., 2010; Katsigianni et al., 2019). However, with multiple frequencies and multiple constellations contemporarily being used, the number of possible combinations increases drastically. Therefore, uncombined processing is becoming much more convenient and practical than forming combinations. Zhang et al. (2011) proposed an uncombined processing concept for PPP-RTK, which was also later used in Zhang et al. (2018). Geng and Bock (2013) analyzed the triple-frequency performance on simulated triple-frequency GPS measurements. Since then, GPS, Galileo and BeiDou have started broadcasting three frequencies or more. Xiao et al. (2019) and Geng et al. (2020), among others, proposed multi-frequency AR models, which are based on processing of the raw uncombined measurements on their individual frequencies. For clarity, in this dissertation, *uncombined* refers to not using linear combinations of observables, but rather directly using the raw measurements on each frequency, while *combined* processing refers to forming linear combinations of observables.

3.2 Original decoupled clock model

The original DCM is based on ionosphere-free pseudorange and carrier-phase measurements and the MW widelane observable (Collins, 2008). The MW combination is formed of a narrowlane pseudorange observable and a widelane carrier-phase observable.

In the DCM, the receiver clocks are decoupled between the pseudorange and carrier-phase measurements. This separation is due to the fact that the pseudorange observables are not synchronized with the carrier-phase observables at the level of precision of the latter, as described by Collins et al. (2012). The equations, from Collins (2008), are summarized as:

$$\begin{cases} P_{r,3}^s &= \rho_r^s + c(dt_r - dt^s) + M_r^s.T_r + \epsilon_{P3} \\ \Phi_{r,3}^s &= \rho_r^s + c(\delta t_r - \delta t^s) + M_r^s.T_r - \lambda_3(17N_1^s + 60N_4^s) + \epsilon_{\Phi3} \\ MW_{r,4}^s &= b_{r,MW} - b_{MW}^s - \lambda_4 N_4^s + \epsilon_{A4} \end{cases} \quad (3.1)$$

where $P_{r,3}^s$ is the ionosphere-free pseudorange, $\Phi_{r,3}^s$ is the ionosphere-free carrier-phase and $MW_{r,4}^s$ is the Melbourne-Wübbena widelane combination. These combinations are introduced in Equation (2.4). The receiver and satellite clocks are decoupled into pseudorange clocks: dt_r and dt^s , respectively, and carrier-phase clocks: δt_r and δt^s , respectively. The narrowlane N_1^s and widelane $N_4^s = N_1^s - N_2^s$ ambiguities are lumped into the ionosphere-free ambiguity with a wavelength λ_3 . $\lambda_4 = \frac{c}{f_1 - f_2}$ is the widelane wavelength, with f_1 and f_2

being the first and second frequencies, respectively. In this model, the receiver and satellite biases are moved from the pseudorange and carrier-phase observables to the MW observable. The biases are defined as:

$$\begin{cases} b_{r,MW} &= \frac{c}{f_1-f_2}B_{r,1} - \frac{c}{f_1-f_2}B_{r,2} - \frac{f_1}{f_1+f_2}b_{r,1} - \frac{f_2}{f_1+f_2}b_{r,2} \\ b_{MW}^s &= \frac{c}{f_1-f_2}B_1^s - \frac{c}{f_1-f_2}B_2^s + \frac{f_1}{f_1+f_2}b_1^s + \frac{f_2}{f_1+f_2}b_2^s \end{cases} \quad (3.2)$$

The model in Equation (3.1) was later expanded to a variant referred to an Extended Decoupled Clock Model (EDCM) by Collins et al. (2012). In this case, the MW observable in Equation (3.1) is split into its narrowlane pseudorange and widelane carrier-phase components, and each form a separate observable. This process allows for the estimation and constraining of the ionospheric delays. Therefore, the EDCM relies on the use of four observables: the ionosphere-free code and phase measurements, the narrowlane code measurements and the widelane phase measurements. This model is designed in the dual-frequency case.

3.3 Uncombined, dual-frequency, decoupled clock model

Seepersad (2018) derived the uncombined dual-frequency DCM through matrix transformations between combined and uncombined observables. In this section, the uncombined dual-frequency DCM is derived from Equation (2.1). The derivation is made from first principles to obtain the components of each parameter. First, the clocks are decoupled by estimating a separate parameter for the receiver pseudorange and carrier-phase clocks. For simplicity, in the following derivation, the satellite clocks will be omitted from the equations, as they are corrected by using external products; as well as the tropospheric delay, which is common to all observables. Equation 2.1 can be simplified to:

$$\begin{cases} P_{r,i}^s &= \rho_r^s + cdt_r + \gamma_i I_1^s + (b_{r,i} - b_i^s) + \epsilon_{P_i} \\ \Phi_{r,i}^s &= \rho_r^s + c\delta t_r - \gamma_i I_1^s + \lambda_i(N_i^s + B_{r,i} - B_i^s) + \epsilon_{\Phi_i} \end{cases} \quad (3.3)$$

where dt_r is the receiver pseudorange clock and δt_r is the receiver carrier-phase clock. The first step is to focus on the pseudoranges on both frequencies and try to assimilate the receiver pseudorange biases $b_{r,i}$ into the receiver pseudorange clock $\bar{dt}_r$ and ionospheric delays $\bar{I}_1^s$ so that the system is as follows:

$$\begin{aligned} P_{r,i}^s &= \rho_r^s + cdt_r + \gamma_i I_1^s + (b_{r,i} - b_i^s) + \epsilon_{P_i} \\ &= \rho_r^s + c\bar{dt}_r + \gamma_i \bar{I}_1^s - b_i^s + \epsilon_{P_i} \end{aligned} \quad (3.4)$$

In order to calculate $\overline{cdt_r}$ and $\overline{I_1^s}$, Equation (3.4) is written on both frequencies as a system. The terms that are similar on both sides of the equalities are simplified leading to Equation (3.5).

$$\begin{bmatrix} 1 & \gamma_1 \\ 1 & \gamma_2 \end{bmatrix} \begin{bmatrix} \overline{cdt_r} \\ \overline{I_1^s} \end{bmatrix} = \begin{bmatrix} 1 & \gamma_1 & 1 & 0 \\ 1 & \gamma_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} cdt_r \\ I_1^s \\ b_{r,1} \\ b_{r,2} \end{bmatrix} \quad (3.5)$$

Solving the system in Equation (3.5) for the unknowns $\overline{cdt_r}$ and $\overline{I_1^s}$ leads to the following expression:

$$\begin{cases} \overline{cdt_r} &= cdt_r + \frac{f_1^2}{f_1^2 - f_2^2} b_{r,1} - \frac{f_2^2}{f_1^2 - f_2^2} b_{r,2} \\ \overline{I_1^s} &= I_1^s - \frac{f_2^2}{f_1^2 - f_2^2} (b_{r,1} - b_{r,2}) \end{cases} \quad (3.6)$$

By substituting $\overline{cdt_r}$ and $\overline{I_1^s}$ from Equation (3.6) into Equation (3.3), the code and phase measurements can then be written as:

$$\begin{cases} P_{r,1}^s &= \rho_r^s + \overline{cdt_r} + \gamma_1 \overline{I_1^s} - b_1^s + \epsilon_{P_1} \\ P_{r,2}^s &= \rho_r^s + \overline{cdt_r} + \gamma_2 \overline{I_1^s} - b_2^s + \epsilon_{P_2} \\ \Phi_{r,1}^s &= \rho_r^s + c\delta t_r - \gamma_1 \overline{I_1^s} + \lambda_1 (N_1^s + B_{r,1} - B_1^s) - \frac{f_2^2}{f_1^2 - f_2^2} (b_{r,1} - b_{r,2}) + \epsilon_{\Phi_1} \\ \Phi_{r,2}^s &= \rho_r^s + c\delta t_r - \gamma_2 \overline{I_1^s} + \lambda_2 (N_2^s + B_{r,2} - B_2^s) - \frac{f_1^2}{f_1^2 - f_2^2} (b_{r,1} - b_{r,2}) + \epsilon_{\Phi_2} \end{cases} \quad (3.7)$$

One of the consequences of the decoupling of receiver clocks is that the carrier-phase measurements lose their datum. Indeed, in PPP, the datum in the carrier-phase measurements is usually set by pseudorange measurements through the receiver clock. However, since, in the DCM, the receiver clocks are decoupled, the datum in the carrier-phase measurements is lost and needs to be recovered. In order to perform this recovery, one satellite is selected as a reference satellite. Therefore, proceeding further and being able to eliminate the receiver biases from the carrier-phase measurements requires a closer look at the reference satellite's measurements.

3.3.1 Reference satellite

In order to eliminate the receiver biases from the phase equations, one satellite is chosen as reference and used for between-satellite single-differencing. In the decoupled clock model, the reference satellite's ambiguities are fixed to arbitrary integer values. These ambiguities are not estimated and they set the datum for the carrier-phase measurements. Indeed, typically

in PPP, the carrier-phase measurements' datum is set by the receiver clock, but because the receiver clocks are decoupled in the DCM, the phase measurements lose the receiver clock datum and therefore require a new datum: the reference satellite's ambiguities. This situation means that the receiver phase clocks are not absolute but rather relative to the ambiguity datum.

Refer to the reference satellite as *ref* and set $N_1^{ref} = N_{01}^{ref} + \delta N_1^{ref}$ and $N_2^{ref} = N_{02}^{ref} + \delta N_2^{ref}$, where N_{0i}^{ref} are the arbitrarily set integer ambiguities on frequency i and δN_i^{ref} are the differences between the true integer ambiguities and the arbitrarily set integer ones on frequency i . Re-parameterizing the receiver phase clock so that it absorbs any receiver biases in the first-frequency carrier-phase observable leads to:

$$c\overline{\delta t}_r = c\delta t_r + \lambda_1(\delta N_1^{ref} + B_{r,1}) - \frac{f_2^2}{f_1^2 - f_2^2}(b_{r,1} - b_{r,2}) \quad (3.8)$$

The phase measurements for the reference satellite can then be expressed as:

$$\begin{cases} \Phi_{r,1}^{ref} &= \rho_r^s + c\overline{\delta t}_r - \gamma_1 \overline{I}_1^{ref} + \lambda_1(N_{01}^{ref} - B_1^{ref}) + \epsilon_{\Phi_1} \\ \Phi_{r,2}^{ref} &= \rho_r^s + c\overline{\delta t}_r - \gamma_2 \overline{I}_1^{ref} + \lambda_2(N_{02}^{ref} - B_2^{ref}) + \delta t_{12} + \epsilon_{\Phi_2} \end{cases} \quad (3.9)$$

where $\delta t_{12} = \lambda_2(\delta N_2^{ref} + B_{r,2}) - \lambda_1(\delta N_1^{ref} + B_{r,1}) - (b_{r,1} - b_{r,2})$ will be referred to as the receiver widelane bias to be consistent with the combined model. This term is receiver dependent, as it is only made of receiver biases and the differences between the arbitrarily set integer ambiguities and the unknown integer ones. These differences are constant unless a cycle-slip occurs or a change in the reference satellite happens.

3.3.2 Non-reference satellites

Based on the definitions of $\overline{I}_1$ and $\overline{\delta t}_r$ in Equations (3.5) and (3.6), the first frequency's phase measurement for a given satellite s in Equation (3.7) can be rewritten as:

$$\begin{aligned} \Phi_{r,1}^s &= \rho_r^s + c\overline{\delta t}_r - \gamma_1 \overline{I}_1^s + \lambda_1(N_1^s + B_{r,1} - B_1^s) \\ &\quad - \frac{f_2^2}{f_1^2 - f_2^2}(b_{r,1} - b_{r,2}) - \lambda_1(\delta N_1^{ref} + B_{r,1}) + \frac{f_2^2}{f_1^2 - f_2^2}(b_{r,1} - b_{r,2}) \\ &= \rho_r^s + c\overline{\delta t}_r - \gamma_1 \overline{I}_1^s + \lambda_1(\overline{N}_1^s - B_1^s) \end{aligned} \quad (3.10)$$

where $\overline{N}_1^s = N_1^s - \delta N_1^{ref}$. The second frequency's phase measurement for satellite s can be rewritten from Equation (3.7) as:

$$\begin{aligned} \Phi_{r,2}^s &= \rho_r^s + c\overline{\delta t}_r - \gamma_2 \overline{I}_1^s + \lambda_2(N_2^s + B_{r,2} - B_2^s) \\ &\quad - \frac{f_1^2}{f_1^2 - f_2^2}(b_{r,1} - b_{r,2}) - \lambda_1(\delta N_1^{ref} + B_{r,1}) + \frac{f_2^2}{f_1^2 - f_2^2}(b_{r,1} - b_{r,2}) \\ &= \rho_r^s + c\overline{\delta t}_r - \gamma_2 \overline{I}_1^s + \lambda_2(\overline{N}_2^s - B_2^s) + \delta t_{12} \end{aligned} \quad (3.11)$$

where $\bar{N}_2^s = N_2^s - \delta N_2^{ref}$. Therefore, the carrier-phase measurements on both frequencies for the reference satellite and non-reference satellites have very similar expressions. Note how important it is to set the reference satellite's ambiguities to integer values, as these arbitrary values (N_{01}^{ref} and N_{02}^{ref}) set the datum for the phase measurements and they get lumped into the ambiguities for all the satellites as δN_1^{ref} and δN_2^{ref} , which are integers. Having arbitrary float values would mean non-integer δN_1^{ref} and δN_2^{ref} , and therefore non-integer $\bar{N}_1^s$ and $\bar{N}_2^s$.

3.3.3 Complete dual-frequency model

Following the phase measurement derivations in Sections 3.3.1 and 3.3.2 and Equation (3.7), the complete uncombined dual-frequency model, including the satellite clocks and tropospheric wet delays, can be summarized in Equation (3.12).

$$\begin{cases} P_1^s &= \rho_r^s + c(\bar{dt}_r - dt^s) + \gamma_1 \bar{I}_1^s + M_r^s \cdot T_r - b_1^s + \epsilon_{P_1} \\ P_2^s &= \rho_r^s + c(\bar{dt}_r - dt^s) + \gamma_2 \bar{I}_1^s + M_r^s \cdot T_r - b_2^s + \epsilon_{P_2} \\ \Phi_1^s &= \rho_r^s + c(\bar{\delta t}_r - dt^s) - \gamma_1 \bar{I}_1^s + M_r^s \cdot T_r + \lambda_1(\bar{N}_1^s - B_1^s) + \epsilon_{\Phi_1} \\ \Phi_2^s &= \rho_r^s + c(\bar{\delta t}_r - dt^s) - \gamma_2 \bar{I}_1^s + M_r^s \cdot T_r + \lambda_2(\bar{N}_2^s - B_2^s) + \delta t_{12} + \epsilon_{\Phi_2} \end{cases} \quad (3.12)$$

with:

$$\begin{cases} \bar{cdt}_r &= cdt_r + \frac{f_1^2}{f_1^2 - f_2^2} b_{r,1} - \frac{f_2^2}{f_1^2 - f_2^2} b_{r,2} \\ \bar{I}_1^s &= I_1^s - \frac{f_2^2}{f_1^2 - f_2^2} (b_{r,1} - b_{r,2}) \\ \delta t_{12} &= \lambda_2(\delta N_2^{ref} + B_{r,2}) - \lambda_1(\delta N_1^{ref} + B_{r,1}) - (b_{r,1} - b_{r,2}) \\ \bar{c\delta t}_r &= c\delta t_r + \lambda_1(\delta N_1^{ref} + B_{r,1}) - \frac{f_2^2}{f_1^2 - f_2^2} (b_{r,1} - b_{r,2}) \\ \bar{N}_1^s &= N_1^s - \delta N_1^{ref} \\ \bar{N}_2^s &= N_2^s - \delta N_2^{ref} \end{cases} \quad (3.13)$$

3.4 Uncombined, triple-frequency, decoupled clock model

The triple-frequency model can be extended from the dual-frequency one. Such extension has not been done in the literature. The third frequency, f_3 , comes with an additional pseudorange and carrier-phase measurement, which have to be arranged from Equation (2.1) to an equation similar to (3.12). Ignoring the satellite clocks and tropospheric zenith wet delay, the third frequency's measurements' general expressions are:

$$\begin{cases} P_{r,3}^s &= \rho_r^s + cdt_r + \gamma_3 I_1^s + (b_{r,3} - b_3^s) + \epsilon_{P_3} \\ \Phi_{r,3}^s &= \rho_r^s + cdt_r - \gamma_3 I_1^s + \lambda_3(N_3^s + B_{r,3} - B_3^s) + \epsilon_{\Phi_3} \end{cases} \quad (3.14)$$

3.4.1 Third frequency pseudorange

Replacing the receiver clock and the first frequency's ionospheric delay's expressions in the third-frequency pseudorange observable by those in Equation (3.6) leads to:

$$\begin{aligned}
P_{r,3}^s &= \rho_r^s + cdt_r + \gamma_3 I_1^s + (b_{r,3} - b_3^s) + \epsilon_{P_3} \\
&= \rho_r^s + \overline{cdt}_r + \gamma_3 \overline{I}_1^s + (b_{r,3} - b_3^s) \\
&\quad - \frac{f_1^2}{f_1^2 - f_2^2} b_{r,1} + \frac{f_2^2}{f_1^2 - f_2^2} b_{r,2} + \frac{f_3^2}{f_1^2 - f_2^2} (b_{r,1} - b_{r,2}) + \epsilon_{P_3} \\
&= \rho_r^s + \overline{cdt}_r + \gamma_3 \overline{I}_1^s - b_3^s + IFB_{r,3} + \epsilon_{P_3}
\end{aligned} \tag{3.15}$$

$IFB_{r,3} = \frac{f_3^2 - f_1^2}{f_1^2 - f_2^2} b_{r,1} - \frac{f_3^2 - f_2^2}{f_1^2 - f_2^2} b_{r,2} + b_{r,3}$ is the Inter-Frequency pseudorange Bias (IFB) which arises from the fact that $\overline{dt}_r$ and $\overline{I}_1^s$ cannot absorb all the pseudorange receiver biases on the third frequency. This term only contains receiver biases and it is therefore not satellite dependent. This term varies slowly throughout the day and can be estimated as a constant (Xiang et al., 2020). However, because the DCM does not make any assumptions regarding the behaviour of biases, $IFB_{r,3}$ is estimated as white noise, similarly to the receiver clocks.

3.4.2 Third frequency carrier-phase

Replacing the receiver phase clock and first frequency's ionospheric delay's expressions by those in Equation (3.6) leads to:

$$\begin{aligned}
\Phi_{r,3}^s &= \rho_r^s + c\delta t_r - \gamma_3 I_1^s + \lambda_3 (N_3^s + B_{r,3} - B_3^s) + \epsilon_{\Phi_3} \\
&= \rho_r^s + \overline{cdt}_r - \gamma_3 \overline{I}_1^s + \lambda_3 (N_3^s + B_{r,3} - B_3^s) + \frac{f_2^2}{f_1^2 - f_2^2} (b_{r,1} - b_{r,2}) \\
&\quad - \lambda_1 (\delta N_1^{ref} + B_{r,1}) - \frac{f_3^2}{f_1^2 - f_2^2} (b_{r,1} - b_{r,2}) + \epsilon_{\Phi_3}
\end{aligned} \tag{3.16}$$

Similar to the dual-frequency derivation, the reference satellite's phase measurements are looked into first.

3.4.2.1 Reference satellite

Set $N_3^{ref} = N_{03}^{ref} + \delta N_3^{ref}$ where N_{03}^{ref} is the arbitrarily set integer ambiguity at frequency f_3 and δN_3^{ref} is the difference between the true integer ambiguity and the arbitrarily set integer one. The reference satellite's third frequency phase measurement can be written as:

$$\begin{aligned}
\Phi_{r,3}^{ref} &= \rho_r^{ref} + \overline{cdt}_r - \gamma_3 \overline{I}_1^{ref} + \lambda_3 (N_3^{ref} + B_{r,3} - B_3^{ref}) \\
&\quad + \frac{f_2^2 - f_3^2}{f_1^2 - f_2^2} (b_{r,1} - b_{r,2}) - \lambda_1 (\delta N_1^{ref} + B_{r,1}) + \epsilon_{\Phi_3} \\
&= \rho_r^{ref} + \overline{cdt}_r - \gamma_3 \overline{I}_1^{ref} + \lambda_3 (N_{03}^{ref} - B_3^{ref}) + \delta t_{13} + \epsilon_{\Phi_3}
\end{aligned} \tag{3.17}$$

where $\delta t_{13} = \frac{f_2^2 - f_3^2}{f_1^2 - f_2^2}(b_{r,1} - b_{r,2}) - \lambda_1(\delta N_1^{ref} + B_{r,1}) + \lambda_3(\delta N_3^{ref} + B_{r,3})$ is equivalent to δt_{12} in the second frequency's phase measurements, and will be referred to hereafter as the receiver extra-widelane bias.

3.4.2.2 Non-reference satellites

For any given satellite s , the third frequency carrier-phase can be written as:

$$\begin{aligned}
\Phi_{r,3}^s &= \rho_r^s + c\bar{\delta t}_r - \gamma_3 \bar{I}_1^s + \lambda_3(N_3^s + B_{r,3} - B_3^s) \\
&\quad + \frac{f_2^2 - f_3^2}{f_1^2 - f_2^2}(b_{r,1} - b_{r,2}) - \lambda_1(\delta N_1^s + B_{r,1}) + \epsilon_{\Phi_3} \\
&= \rho_r^s + c\bar{\delta t}_r - \gamma_3 \bar{I}_1^s + \lambda_3(N_3^s + B_{r,3} - B_3^s) \\
&\quad + \delta t_{13} - \lambda_3(\delta N_3^{ref} + B_{r,3}) + \epsilon_{\Phi_3} \\
&= \rho_r^s + c\bar{\delta t}_r - \gamma_3 \bar{I}_1^s + \lambda_3(\bar{N}_3^s - B_3^s) + \delta t_{13} + \epsilon_{\Phi_3}
\end{aligned} \tag{3.18}$$

where $\bar{N}_3^s = N_3^s - \delta N_3^{ref}$ is the implicitly single-differenced third frequency integer ambiguity. Therefore, we are able to recover the same equation for all satellites thanks to δt_{13} absorbing the receiver biases and reference satellite ambiguities.

3.4.3 Complete triple-frequency model

The complete uncombined triple-frequency model can be summarized as follows:

$$\left\{ \begin{array}{llll}
P_1^s &= \rho_r^s + c(\bar{dt}_r - dt^s) + \gamma_1 \bar{I}_1 + M_r^s.T_r & -b_1^s & +\epsilon_{P_1} \\
P_2^s &= \rho_r^s + c(\bar{dt}_r - dt^s) + \gamma_2 \bar{I}_1 + M_r^s.T_r & -b_2^s & +\epsilon_{P_2} \\
P_3^s &= \rho_r^s + c(\bar{dt}_r - dt^s) + \gamma_3 \bar{I}_1 + M_r^s.T_r & -b_3^s & +IFB_r +\epsilon_{P_3} \\
\Phi_1^s &= \rho_r^s + c(\bar{\delta t}_r - dt^s) - \gamma_1 \bar{I}_1 + M_r^s.T_r & +\lambda_1(\bar{N}_1^s - B_1^s) & +\epsilon_{\Phi_1} \\
\Phi_2^s &= \rho_r^s + c(\bar{\delta t}_r - dt^s) - \gamma_2 \bar{I}_1 + M_r^s.T_r & +\lambda_2(\bar{N}_2^s - B_2^s) & +\delta t_{12} +\epsilon_{\Phi_2} \\
\Phi_3^s &= \rho_r^s + c(\bar{\delta t}_r - dt^s) - \gamma_3 \bar{I}_1 + M_r^s.T_r & +\lambda_3(\bar{N}_3^s - B_3^s) & +\delta t_{13} +\epsilon_{\Phi_3}
\end{array} \right. \tag{3.19}$$

with:

$$\begin{cases}
\overline{cdt_r} &= cdt_r + \frac{f_1^2}{f_1^2 - f_2^2} b_{r,1} - \frac{f_2^2}{f_1^2 - f_2^2} b_{r,2} \\
\overline{c\delta t_r} &= c\delta t_r + \lambda_1(\delta N_1^{ref} + B_{r,1}) - \frac{f_2^2}{f_1^2 - f_2^2} (b_{r,1} - b_{r,2}) \\
\overline{I_1} &= I_1 - \frac{f_2^2}{f_1^2 - f_2^2} (b_{r,1} - b_{r,2}) \\
\delta t_{12} &= \lambda_2(\delta N_2^{ref} + B_{r,2}) - \lambda_1(\delta N_1^{ref} + B_{r,1}) - (b_{r,1} - b_{r,2}) \\
IFB_r &= \frac{f_3^2 - f_1^2}{f_1^2 - f_2^2} b_{r,1} - \frac{f_3^2 - f_2^2}{f_1^2 - f_2^2} b_{r,2} + b_{r,3} \\
\delta t_{13} &= \frac{f_2^2 - f_3^2}{f_1^2 - f_2^2} (b_{r,1} - b_{r,2}) - \lambda_1(\delta N_1^{ref} + B_{r,1}) + \lambda_3(\delta N_3^{ref} + B_{r,3}) \\
\overline{N_1^s} &= N_1^s - \delta N_1^{ref} \\
\overline{N_2^s} &= N_2^s - \delta N_2^{ref} \\
\overline{N_3^s} &= N_3^s - \delta N_3^{ref}
\end{cases} \quad (3.20)$$

Compared to the dual-frequency model, the triple-frequency model has an additional receiver-dependent term to absorb the pseudorange receiver biases, as well as a carrier-phase-related receiver bias, which absorbs the phase receiver biases. All three estimated ambiguities are free of biases and should, after convergence, reach integer values. Furthermore, the above triple-frequency model can easily be expanded into a four- or five-frequency model. The additional frequencies' measurements can be separated for the reference satellite and for non-reference satellites as was done for the expansion of the dual-frequency model with the third frequency.

3.5 Reference satellite selection and multi-GNSS considerations

As mentioned in the previous section, one satellite has to be chosen as reference in order to set the datum in the carrier-phase measurements which have lost their datum when decoupling the receiver clocks. Doing so results in additional parameters needing to be estimated in the form of δt_{12} and δt_{13} . When dealing with multiple constellations, the PPP user has to be mindful of inter-system biases. Indeed, all receiver clocks and biases are constellation-dependent due to constellations being processed by different processing units in the receivers, as well as due to the presence of inter-system time biases (El-Mowafy et al., 2016; Håkansson et al., 2017). It is therefore typical to estimate separate clock/bias terms for each constellation, or to choose one constellation as a reference for which the clock/bias is estimated, and then estimate the difference between other constellation clocks/biases and the reference clock/bias (Zhou et al., 2019; Cai et al., 2015). In this research, the first strategy is adopted: one set of receiver clocks and biases is estimated for each constellation, and all those state terms are treated as clock-like terms by being estimated as white noise processes. The complete set of states to be estimated, as well as the number of states to

be estimated depending on the number of constellations and frequencies to be processed is summarized in Table 3.1.

Given the dependency of the receiver clock and bias states on the constellation, and knowing that the reference satellite’s ambiguities are directly correlated with the receiver clocks and biases (as was shown in Sections 3.3 and 3.4), it is necessary to select one reference satellite for each constellation. Fortunately, having as many reference satellites as there are constellations being processed does not negatively impact the degrees of freedom of the system. Consider one constellation with N frequencies being processed. Compared to the classic float PPP model, the DCM contains the additional receiver phase clock (one state term, common to all frequencies), and the receiver phase bias terms δt_{12} and δt_{13} . In dual-frequency mode, the receiver phase clock $\bar{\delta t}_r$ and L2 phase bias δt_{12} need to be estimated, which are compensated for by the fact that the two ambiguities for the reference satellite are not estimated and they are fixed to arbitrary integers. In triple-frequency mode, in addition to the receiver phase clock $\bar{\delta t}_r$ and the receiver L2 phase bias δt_{12} , the receiver L3 phase bias δt_{13} is estimated as well; all three are compensated for by the reference satellite’s three ambiguities not needing to be estimated. These observations apply to the multi-GNSS context, as one reference satellite is selected per constellation, and a set of receiver clocks and biases has to be estimated for each constellation as well.

The fact that the datum of the carrier-phase measurements is set by the reference satellite’s ambiguities means that the reference satellite choice must depend on the number of frequencies available for the constellation in question; e.g., for the GPS constellation, if only dual-frequency satellites are visible to the receiver, then the reference satellite can be dual-frequency. However, as soon as a triple-frequency satellite is included in the processing, the reference satellite must be switched to one of the triple-frequency satellites in order to set the datum on the third frequency. The reference satellite must therefore contain as many frequencies as the maximum number of frequencies in its constellation. A summary of the strategy of reference satellite choice is provided in Figure 3.1. The summary only focuses on the impact of the number of frequencies on the reference satellite choice. Other parameters need to be taken into account when making the choice, such as: choosing a satellite that has been processed for a long enough period of time that its ambiguities are correctly estimated, and a satellite which has the highest elevation angle and signal-to-noise ratio among the available satellites, etc. Note that another approach can be used when setting the datum for the phase measurements, as the datum on each frequency does not have to be set by the same satellite. As a consequence, instead of requiring the reference satellite to have as many frequencies as are processed for its constellation, the datum on the first two frequencies can be set by a dual-frequency satellite, while the datum on the third frequency can be set by one of the triple-frequency satellites’ third frequency phase measurement.

Table 3.1: States and number of states to be estimated for each constellation and frequency. A cross in the state dependency column (One per ...) means that a state term is either receiver-, constellation-, or satellite-dependent. A cross in the measurement type column specifies whether pseudorange or carrier-phase measurements are used to estimate the state. A cross in the frequency band column means that measurements on the selected frequency are used to estimate the state term.

	One Per ...			Measur. Type		Freq. Band		
	Rec.	Const.	Sat.	Code	Phase	1	2	3
Receiver coordinates	X			X	X	X	X	X
Wet tropospheric delay	X			X	X	X	X	X
Receiver pseudorange clock		X		X		X	X	X
Receiver carrier-phase clock		X			X	X	X	X
Receiver L2 phase bias		X			X		X	
Receiver L3 phase bias		X			X			X
Receiver IFB		X		X				X
Slant ionospheric delay			X	X	X	X	X	X
L1 ambiguity			X		X	X		
L2 ambiguity			X		X		X	
L3 ambiguity			X		X			X

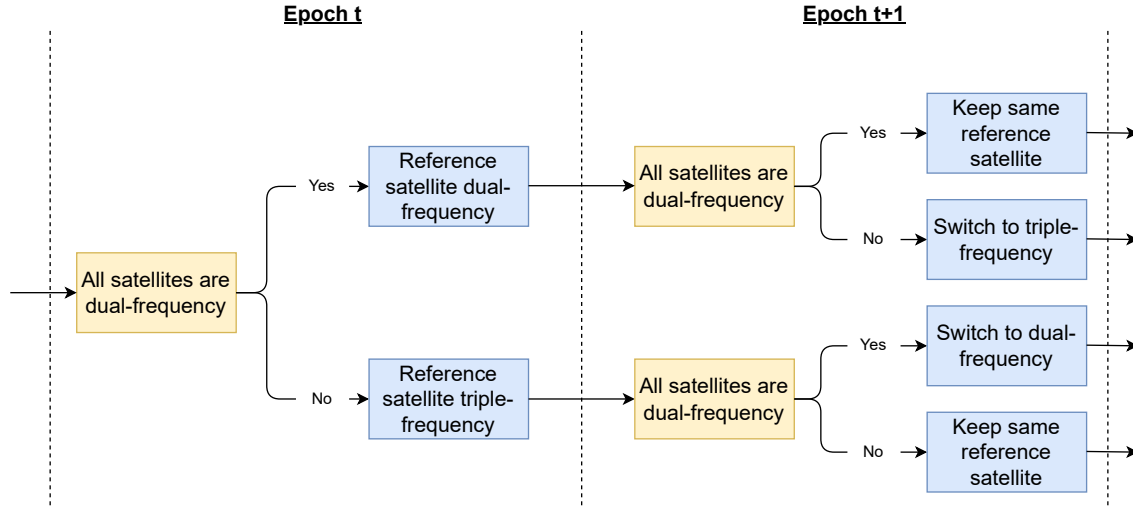


Figure 3.1: Reference satellite choice strategy when processing a combination of dual- and triple-frequency satellites.

3.6 Model parameters and position performance analysis

The algorithms described in Section 3.3 and 3.4 are used to process a set of datasets from the IGS, as well as kinematic automotive data that were collected around York University’s Keele campus. A description of the datasets is provided in each subsection. This section is divided into three subsections: first, results using only observations from the Galileo constellation are presented, in order to highlight the impact of the third frequency on the processing, as well as to highlight DCM-specific parameters. Second, multi-GNSS results are shown, followed third with results from a kinematic automotive test. For all three subsections, the processing strategies described in Table 3.2 are used for the parameters’ estimation.

As shown in Sections 3.3 and 3.4, δt_{12} , δt_{13} and $IFB_{r,3}$ consist of receiver biases. These biases are constant within a few nanoseconds over the course of a day, though their behaviour depends on the temperature, location, and quality of the receiver, as they can contain relatively high variations (Abdelazeem et al., 2016; Zhang et al., 2017, 2019a). To be cautious, and in order to avoid such over-constraining, the DCM does not make assumptions on the behaviour of biases and treats them as clock-like terms, leading to all receiver-related states being treated as clock-like terms and estimated as white noise processes. Because the three quantities are common to all satellites, they can be estimated successfully.

The satellite Phase Centre Offset (PCO) and Phase Centre Variation (PCV) errors are corrected for using values from the ANTEX file (Schmid et al., 2016), provided by the IGS. Phase wind-up, Earth rotation, solid Earth tides, and relativistic effects are corrected for following the International Earth Rotation and Reference Systems Service (IERS) conventions (Kouba and Mireault, 1998). Satellite code and phase bias corrections are used from different sources for each subsection. However, all the satellite biases corrections that were used are in the OSB format. Details on the OSB format and its advantages are provided in subsection 2.3.2.

The LAMBDA method introduced by Teunissen (1995) is a widely used method for ambiguity fixing in GNSS. However, it is also known to be one of the slow methods (Xu et al., 2012). In this work, a modified version of LAMBDA is used: Modified Least-squares AMBiguity Decorrelation Adjustment (MLAMBDA), introduced by Chang et al. (2005), is a faster and more computationally efficient method. For each dataset, the fixing of ambiguities is attempted from the first epoch of processing, with the fixing only being attempted on satellites with elevation angles above 15° . The reason for eliminating satellites below that threshold is that such satellites would be expected to have more multipath, ionospheric refraction and measurement noise, which would affect the estimation of those satellites’ ambiguities. In order to fix the ambiguities through mLAMBDA, the uncombined float ambiguities are first estimated based on the uncombined DCM models in Equations (3.12)

Table 3.2: Processing strategies for the estimated parameters.

Parameter	Strategy
Receiver coordinates	Static mode: estimated as constants Kinematic mode: estimated with process noise equivalent to 100 km/h
Reference coordinates	IGS SINEX positions
Receiver clocks (code and phase)	Estimated as white noise processes
Receiver L2 and L3 phase biases	Estimated as white noise processes
Receiver IFB	Estimated as white noise process
Tropospheric delay	Dry: GMF model and mapping function (Kouba, 2009). Wet: estimated as a random walk process with process noise of $0.05 \text{ mm}/\sqrt{h}$
Ionospheric delays	Estimated as white noise processes
Ambiguities	Estimated as constants on each continuous arc
Weighting strategy	Elevation dependent weighting: $\sigma = \frac{\sigma_{90}}{0.15+0.85\sin\lambda}$ with σ_{90} equal to 0.3 m and 0.003 m for the pseudorange and carrier-phase measurements, respectively, and λ being the elevation angle

and (3.19). These uncombined ambiguities are already single-differenced and free of satellite and receiver biases. They are fed directly into the MLAMBDA function along with their associated float covariances. The MLAMBDA function returns a set of fixed estimates and covariances, which are used to update the state terms. The validation of the ambiguities is done through a combination of a standard ratio test, the residuals, and the reference coordinates. Note that it is possible to form the EWL, WL and NL combinations for the ambiguities and their covariances prior to calling MLAMBDA; however, the results are found to be similar to those without forming the combinations.

3.6.1 Galileo-only results

In order to validate the derived triple-frequency DCM model and assess its performance, datasets from 25 worldwide IGS stations have been used and their data processed over 7 days between DOY 342 and DOY 348, 2019. The stations are shown in Figure 3.2. The 24 hour datasets were split into 3 hour sessions in order to increase the number of datasets. The total number of processed datasets was 1400, with a sampling interval of 30 seconds. The purpose of the processing is to validate the proposed model and assess its various parameters. Therefore, only processing one constellation in the current subsection is opted for in order to lower the complexity of the section by not including inter-constellation

biases. Multi-GNSS processing is the topic of the next sub-section and chapters. Since all Galileo satellites transmit on three frequencies, and because the receivers typically track all three frequencies simultaneously, only Galileo satellites were processed in order to keep the same number of satellites between the dual-frequency and triple-frequency modes. The three Galileo frequencies used in the processing are: E1, E5b and E5a. In addition to the processing strategies shown in Table 3.2, an elevation angle cut-off of 10° is used to eliminate satellites at low elevation angles, which are more prone to multipath and other undesired effects. Note that the 15° cut-off mentioned at the start of the section refers to the minimum elevation angle of satellites for which the fixing is attempted, while the 10° cut-off refers to the minimum angle of satellites processed. Satellite orbits and clocks are corrected for using rapid products from GeoForschungsZentrum (GFZ) (Deng et al., 2017). Satellite code and phase biases, in OSB format, are corrected for using the Centre National d’Etudes Spatiales (CNES) products (Laurichesse and Blot, 2016), which are meant to be used together with the GFZ rapid orbit and clock corrections.

Results in the current subsection are split into three parts: static dual-frequency results and analysis are showed first. They are followed by a static triple-frequency analysis and a comparison of the dual-frequency and triple-frequency results. Finally, a comparison between static and kinematic processing is performed. All the results are generated in simulated real-time, where only forward filtering is performed.

3.6.1.1 Dual-frequency AR results

Figure 3.3 illustrates an example of the variations in the parameters that are specific to the DCM for station YELL in northwestern Canada on DOY 344, 2019 over the course of the day. Both the receiver pseudorange and carrier-phase clocks have a similar slow drift. This similarity is due to the fact that they both contain the actual receiver clock, but they are each affected by different biases. The difference between the two receiver clocks consists of receiver code and phase biases, as well as δN_1^{ref} , which explains the vertical shift between the two graphs. The pseudorange clock is more noisy, as this term is estimated based only on the pseudorange measurements, which are significantly more noisy (~ 100 times) than their carrier-phase counterparts. To quantify the level of noise in the receiver clocks, a linear polynomial has been fit to them and the rms between the estimated clocks and the fitted polynomials was calculated. The pseudorange clock has an rms of 19 dm, compared to 13 dm for the phase clock, proving that pseudorange receiver clocks are indeed more noisy. The receiver biases are also lumped into the receiver L2 phase bias δt_{12} , which is shown in Figure 3.3b. δt_{12} is relatively stable over the course of the day with variations up to 2 nanoseconds. However, it is preferable not to make assumptions on its stability during the estimation, as Abdelazeem et al. (2016); Zhang et al. (2017, 2019a) have shown relatively

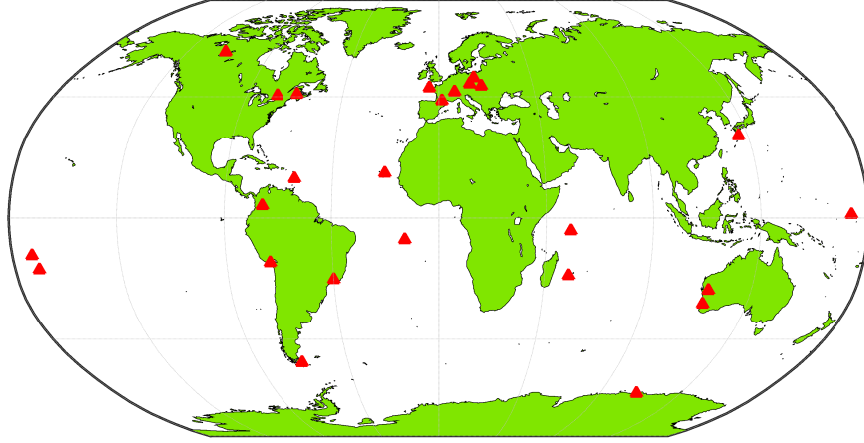


Figure 3.2: World map of the stations used in the processing.

large variations in the receiver biases depending on the receiver's temperature, location and quality. Estimating those receiver biases as clock-like terms, as was done here, ensures that the ambiguities are stable, integers, and free of residual biases; especially when working with lower-quality/cost receivers. The IRC does not assume the behaviour of the receiver biases either, unlike the FCB model (Geng et al. (2010)). It is worth noting that both the receiver carrier-phase clock and the receiver L2 phase bias contain the reference satellite's integer ambiguity. Therefore, these quantities depend on the choice of the reference satellite: a change in the reference satellite results in an integer vertical shift of the receiver carrier-phase clock and receiver L2 phase bias.

Figures 3.4 and 3.5 show the epoch-by-epoch average horizontal and vertical position solutions based on all 1400 datasets, respectively. The plots show the (a) 100th, (b) 95th, (c) 67th and (d) 50th percentiles and highlight the typical performance of the dual-frequency float and fixed solutions. The convergence here and throughout the rest of the chapter is defined as the time it takes for the error to reach and settle below 10 cm. The same definition is valid both for the horizontal and vertical errors. The rms have been computed between minutes 60 and 180 in order to assess the final level of accuracy that the solutions reach. Tables 3.3 and 3.4 provide summary statistics for Figures 3.4 and 3.5, respectively. Under these definitions, the 100th percentile dual-frequency float solution converges within 18.5 minutes for the horizontal component (and 23.5 minutes for the vertical), while the fixed one converges within 14.5 minutes horizontally (and 21.5 minutes vertically). The float solution is able to reach a horizontal rms of 2.7 cm (and vertical of 2.9 cm), while the fixed one has a value of 1.1 cm for the horizontal (and 2.5 cm for the vertical). Therefore, the

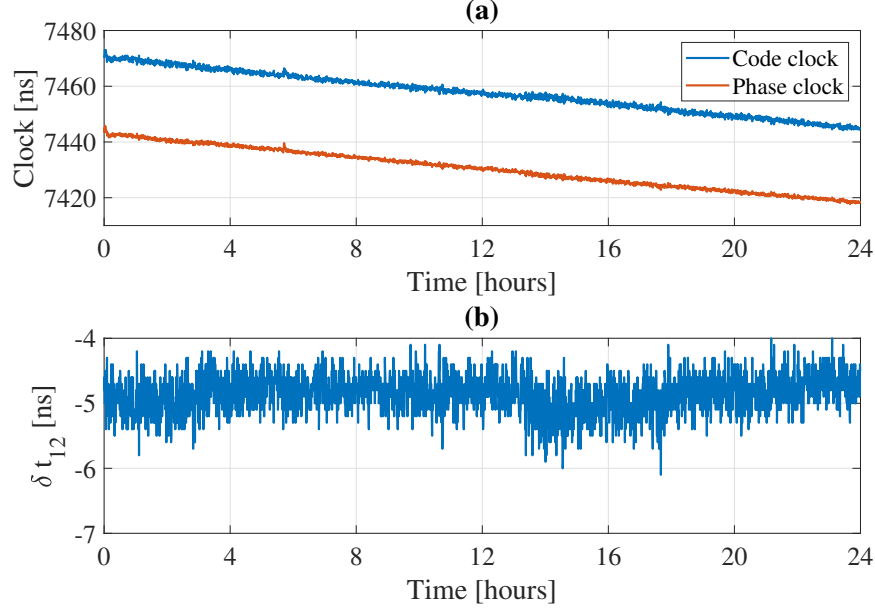


Figure 3.3: (a) Receiver code clock $\overline{dt_r}$, receiver phase clock $\overline{\delta t_r}$ and (b) receiver L2 phase bias δt_{12} for station YELL on DOY 344, 2019.

benefit of fixing the ambiguities on Galileo is more noticeable for the horizontal component than it is for the vertical one. The 100th percentile horizontal convergence time and rms see 22% and 59% decreases, respectively, while the vertical convergence time and rms see decreases of 9% and 14%, respectively. Shi and Gao (2014b) had also noticed the limited benefit of AR on the vertical component, and they tried to remedy it by constraining the troposphere, as the latter limits the estimation of the vertical component. In this work, lower percentiles have the solution converging faster and with higher accuracies, with the convergence times of both float and fixed solutions becoming closer the lower the percentile. This behaviour is due to the solutions already converging fast enough (5.5 minutes for the horizontal 50th percentile solution), which means that AR's improvement is limited in this case (5 minutes for the horizontal 50th percentile solution).

Figure 3.6 is included to highlight the 100th percentile performance of the solutions on each station individually, as opposed to the average from all datasets. Each station's result

Table 3.3: Horizontal convergence time and rms statistics for the results in Figure 3.4.

	Convergence time [min]				rms[cm]			
	Percentile				Percentile			
	100 th	95 th	67 th	50 th	100 th	95 th	67 th	50 th
Float	18.5	15.5	8.0	5.5	2.7	2.3	1.4	1.1
Fixed	14.5	11.5	7.0	5.0	1.1	0.8	0.5	0.4

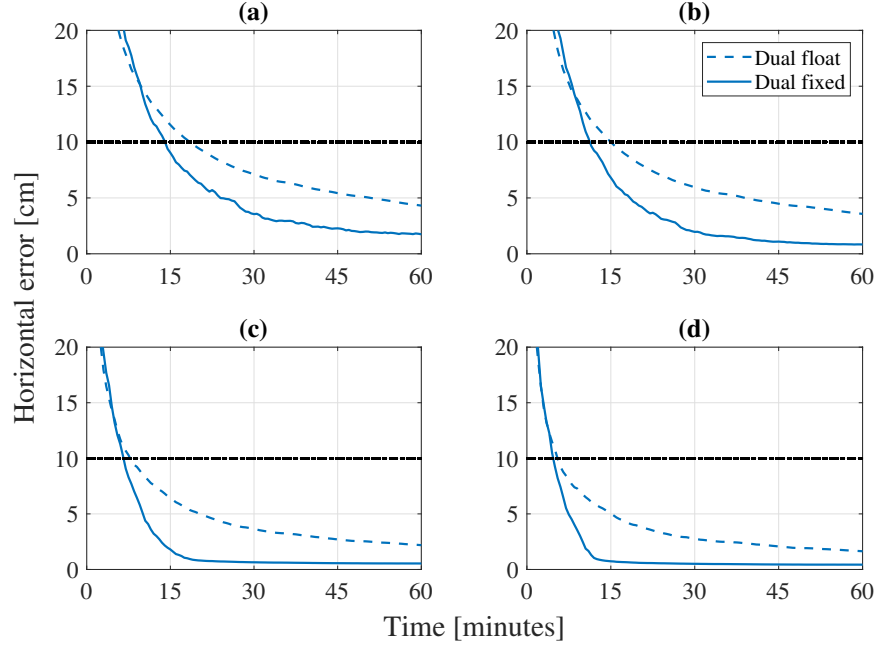


Figure 3.4: Static dual-frequency average horizontal error based on all datasets at the (a) 100th, (b) 95th, (c) 67th and (d) 50th percentiles. The horizontal dashed lines represent the convergence threshold.

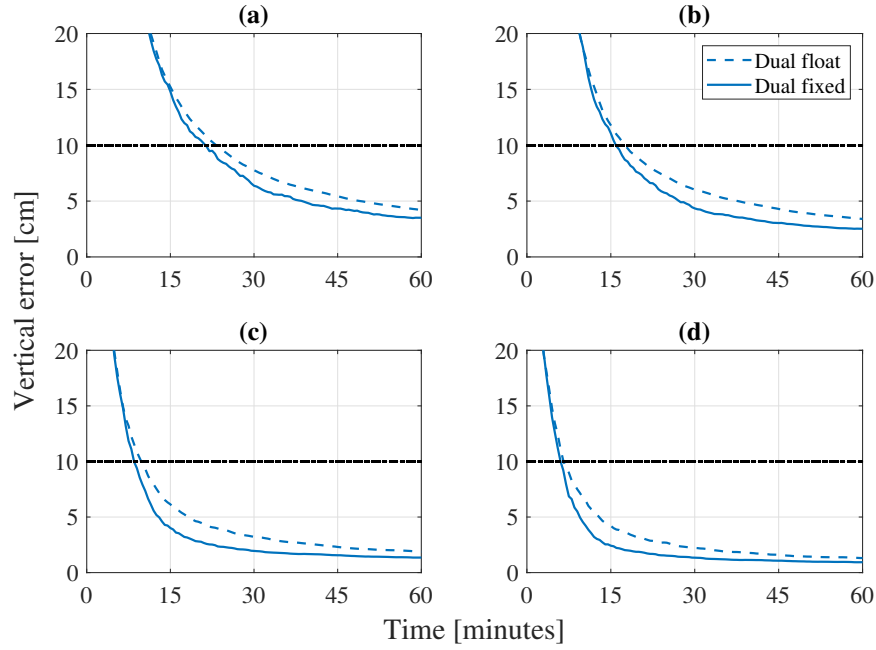


Figure 3.5: Static dual-frequency average vertical error based on all datasets at the (a) 100th, (b) 95th, (c) 67th and (d) 50th percentiles. The horizontal dashed lines represent the convergence threshold.

Table 3.4: Vertical convergence time and rms statistics for the results in Figure 3.5.

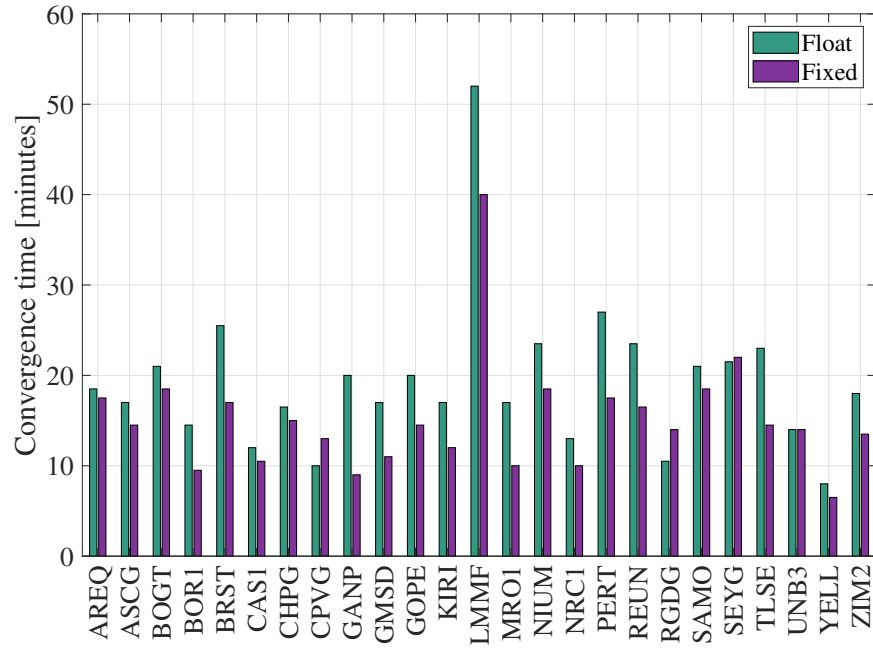
	Convergence time[min]				rms [cm]			
	Percentile				Percentile			
	100 th	95 th	67 th	50 th	100 th	95 th	67 th	50 th
Float	23.5	18.0	10.0	7.0	2.9	2.4	1.3	0.9
Fixed	21.5	16.0	8.5	6.5	2.5	2.0	1.1	0.7

contains the average from 56 datasets: eight three-hour datasets per day, for seven days. The benefit of AR varies depending on the stations, as some see a big improvement, e.g., GANP, while others have relatively the same convergence time, e.g., AREQ. On the other hand, AR significantly improves the accuracy for all stations, as the horizontal rms see an average decrease of one centimetre for all stations.

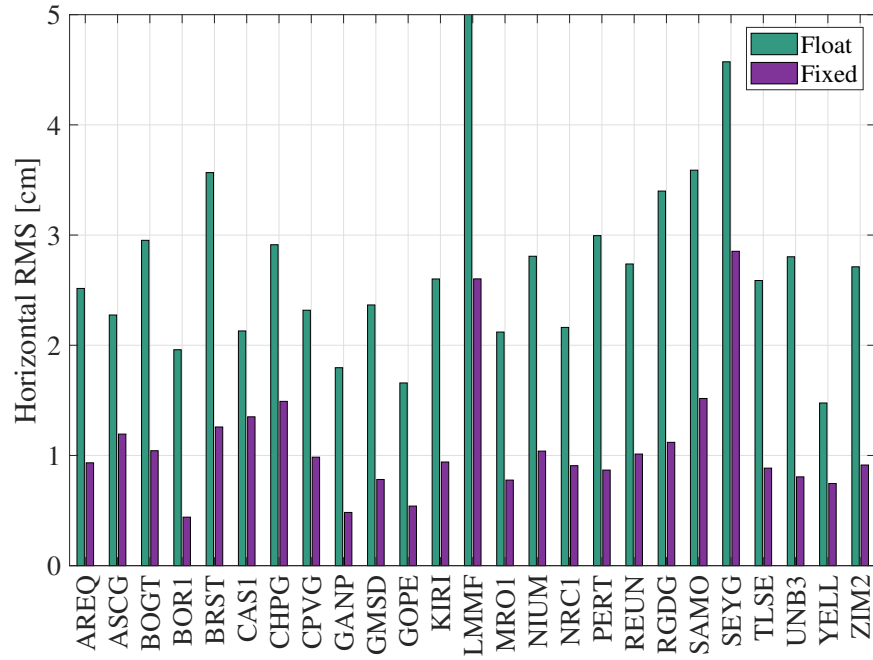
3.6.1.2 Triple-frequency AR results

Figure 3.7 illustrates the trends of both the receiver L3 phase bias and IFB for the same dataset shown in Figure 3.3. Similarly to the receiver L2 phase bias in Figure 3.3, the receiver L3 phase bias depends on the receiver biases, as well as the reference satellite’s ambiguities. It is relatively stable but it is best to not make assumptions regarding its stability, and rather estimate it as white noise. Similarly to the receiver L2 phase bias, δt_{13} depends on the reference satellite’s ambiguities and it therefore depends on the choice of reference satellite: a different reference satellite would lead to the same plot shifted vertically by an integer value. On the other hand, the IFB only depends on the receiver biases and not on the reference satellite’s ambiguities. Consequently, it does not depend on the choice of reference satellite. Although the IFB was estimated as a white noise process as well, it is relatively stable. This result was expected as δt_{12} , δt_{13} and $IFB_{r,3}$ are all made of receiver biases (and additional integer values), which means that they should all follow similar trends, assuming that the receiver biases have similar time-dependence regardless of frequency.

Figure 3.8 highlights the performance of the float and fixed solutions, as well as the fixed ambiguities for both dual- and triple-frequency solutions for station YELL on DOY 344, 2019 for two hours in kinematic mode. As stated earlier, the uncombined ambiguities on each frequency are fixed by the MLAMBDA function. In order to show that, even in this case, the combination properties are still valid after the fixing, the ambiguities have been combined into NL, WL and EWL after the fixing. Looking at the dual-frequency results, it is clear that the WL ambiguities in red are fixed correctly from the first epoch thanks to their large wavelength, and that the initial variations in the NL ambiguities (which have a much smaller wavelength) are causing the variations in the fixed horizontal solution. Once the NL ambiguities start being fixed correctly, the fixed solution converges and remains at the same level of accuracy, while the float one still has a longer convergence period.



(a) Average convergence times per station.



(b) Average rms per station.

Figure 3.6: Static dual-frequency average horizontal convergence times and rms for each station - 56 datasets per station.

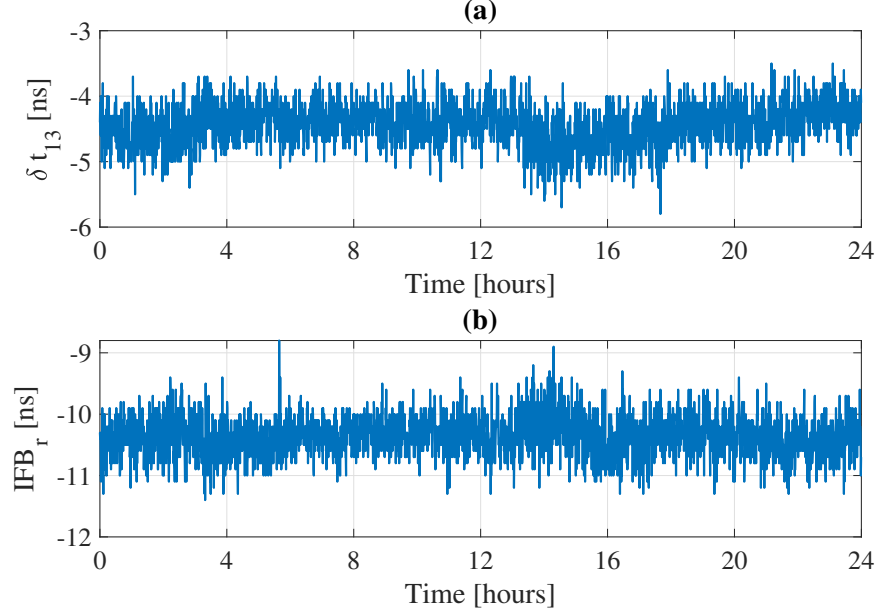
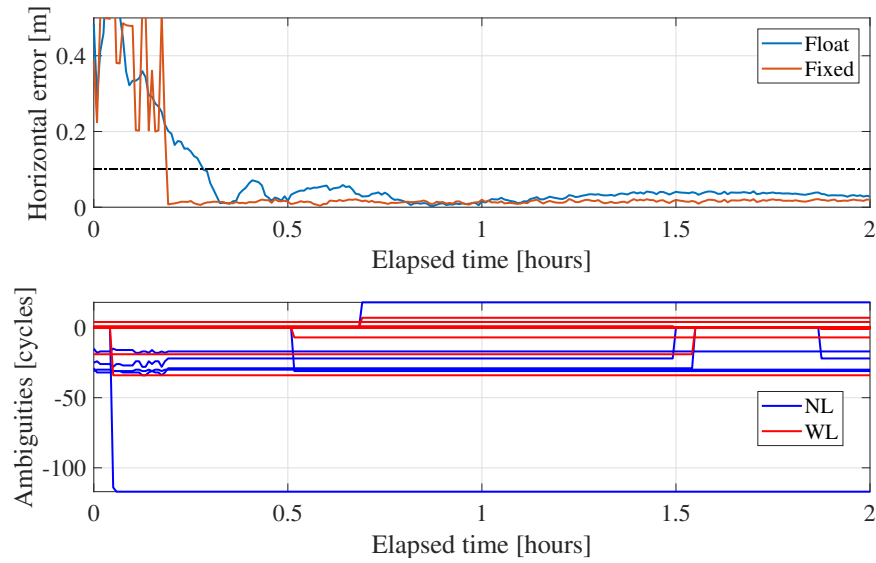
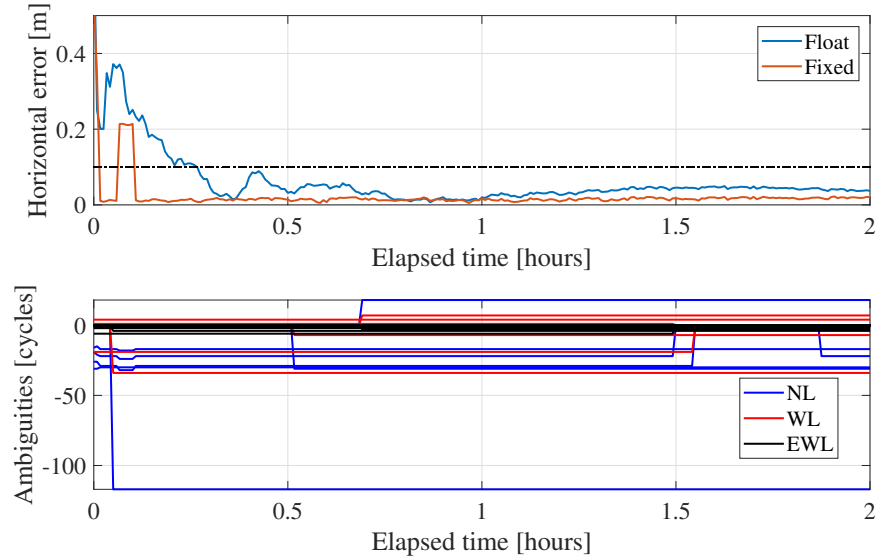


Figure 3.7: (a) Receiver L3 phase bias δt_{13} and (b) Inter-Frequency Bias $IFB_{r,3}$ for station YELL on DOY 344, 2019.

These conclusions hold for the triple-frequency solution as well, although it is interesting to notice that the NL ambiguities are fixed correctly faster than in the dual-frequency case thanks to the EWL ambiguities (instantly fixed) helping in the fixing process. Given the correlation between ambiguities from the same satellite, being able to instantaneously fix the EWL ambiguity leads to faster fixing of the WL ambiguity, which in turn leads to faster fixing of the NL ambiguity. A study on the correctness of NL fixing for dual- and triple-frequency data by Geng and Bock (2013) has shown that more NL ambiguities can be fixed in the first tens of seconds of processing with three frequencies than with two frequencies. When it comes to the float solutions, two observations can be made: on one hand, there is a noticeable difference between the float dual- and triple-frequency solutions during the first few epochs. An explanation is that, if the Dilution Of Precision (DOP) is relatively high during the first few epochs (an average of 2 for this dataset), the third frequency will help improve the positioning solution thanks to its additional measurements. In the case of multi-GNSS processing where the DOP is low, it is expected that both dual- and triple-frequency float solutions will have minimal differences. On the other hand, it can be seen that the dual-frequency solution at 0.4 hours is slightly better than the triple-frequency one. This can be explained by the fact that, unlike the dual-frequency case where residual errors can be absorbed by other states, having three frequencies means that the model is not as lenient towards such residual errors, which can have more of an impact on state estimation. Degradations of the position estimates when using three frequencies as opposed



(a) Dual-frequency horizontal error and fixed ambiguities.



(b) Triple-frequency horizontal error and fixed ambiguities.

Figure 3.8: Kinematic (a) dual-frequency and (b) triple-frequency horizontal error and fixed ambiguities for station YELL on DOY 344, 2019 between 21:00 and 23:00 UTC. The ambiguities jumping from zero to non-zero values mean that a satellite starts being processed. Ambiguities jumping from non-zero values to zero mean that a satellite stops being processed.

to two frequencies have been noticed by Geng et al. (2020) and they were attributed to residual errors.

Figures 3.9 and 3.10 expand on Figures 3.4 and 3.5’s respective results by adding the triple-frequency results as well. Comparing all percentiles dual- and triple-frequency float solutions shows that processing an additional frequency provides a negligible improvement as the 100th percentile triple-frequency float horizontal solution converges in 16.5 minutes (compared to 18.5 for dual-frequency) and reaches a horizontal rms of 2.6 cm (compared to 2.7 cm for dual-frequency). Guo et al. (2016) have noticed similar behaviour with BeiDou triple-frequency processing, where they mentioned that the third frequency only helps in certain scenarios, including low signal strength for one of the signals, or undetected cycle-slips with dual-frequency data. Therefore, this minimal improvement may be due to the fact that the additional frequency does not provide any significant additional independent information compared to, e.g., having an additional satellite, which would improve the satellite geometry. For instance, moving from single-frequency processing to dual-frequency processing improves the PPP performance as it helps estimate the ionosphere more accurately (Lou et al., 2016), but moving to triple-frequency processing does not improve the solution as significantly. An example of the improvement for the float solutions was discussed in Figure 3.8, in which most of the improvement to the float solution came in the initialization errors, but the convergence time and final accuracies were not affected.

On the other hand, there is a larger improvement when it comes to the fixed solution, as the triple-frequency fixed solution converges in 9.5 minutes and reaches a horizontal rms of 1.0 cm. Although the accuracy is similar to the dual-frequency fixed solution, there is a noticeable improvement in the convergence time, which sees a 34% decrease compared to the dual-frequency fixed solution. These results are relatively better than those presented by Xiao et al. (2019), where the 3D float triple-frequency Galileo solution converged in 64.5 minutes (compared to 35 minutes for the 3D convergence in this study) and the fixed solution converged in 56 minutes (compared to 21.5 minutes for the 3D convergence here). The authors in Xiao et al. (2019) down-weighted the observations by a factor of 4, which could explain the differences with these results. Li et al. (2020b) had average horizontal dual- and triple-frequency AR convergence times of 22.3 and 17.4 minutes, respectively; although their convergence time definition was different: time taken for the horizontal accuracy to be better than 5 cm and kept within 5 cm during ten consecutive epochs. Though it is important to keep in mind that this work and the cited research use different datasets, which could explain the differences in results. These results mean that the most benefit from triple-frequency processing comes from fixing the ambiguities on three frequencies instead of two. The possible reason behind this result could be that fixing three frequencies adds robustness to the fixing thanks to the EWL ambiguity, which can be fixed instantaneously

- meaning that the ambiguities are fixed correctly earlier and more often, leading to the aforementioned improvement.

In order to show the improvement of triple-frequency AR over dual-frequency AR at the dataset level, the histograms for the convergence times of all datasets are plotted in Figure 3.11. The bin size is 30 seconds. The figure shows that triple-frequency AR leads to the histogram shifting towards the left compared to dual-frequency AR, as more datasets converge sooner. It also shows that more datasets converge within the first few minutes: 23% of the datasets converge within the first 5 minutes in the triple-frequency case, compared to 12% in the dual-frequency case, while 50% of the solutions converge within the first 10 minutes in triple-frequency processing, compared to 36% in the dual-frequency counterpart.

Since the number of visible Galileo satellites is currently relatively limited compared to GPS, the performance of both the float and fixed solutions varies. Some datasets which are subject to a low number of satellites or low DOP result in poor float and, consequently, poor fixed solutions. Figure 3.12 paints a picture of the number of Galileo satellites that were processed for all datasets. Sub-figure (a) represents the distribution of the number of satellites based on all epochs from all datasets with a bin size of 1. The histogram shows an average of seven satellites with 15% of epochs having five satellites or less and 7% of epochs having nine satellites or more. It is expected that epochs with low number of satellites will lead to worse float and fixed solutions due to poor satellite geometry. This expectation is confirmed by the Position Dilution Of Precision (PDOP) distribution shown in sub-figure (b) with a bin size of 0.25. The sub-figure shows an average PDOP of 2.5 with 18% of epochs having a PDOP higher than 3. A PDOP of 1 being indicative of a good satellite distribution, the PDOP histogram helps make sense of the improvements that are seen when looking at lower percentiles.

As a summary, Figures 3.13 and 3.14 show the levels of performance of both the dual- and triple-frequency solutions (float and fixed) at different levels of confidence. These statistics correspond to the Figures 3.9 and 3.10 results. Looking at different percentiles helps assess the performance of the solutions without the outliers that are due to the low number of satellites: the lower the percentile, the less outliers are present.

It is interesting to see how much AR improves the horizontal accuracy at all percentiles for both the dual- and triple-frequency cases as AR leads to over 60% improvements to the horizontal rms. On the other hand, improvements to vertical accuracy are not as obvious. At all percentiles, dual-frequency AR decreases horizontal convergence time by 20-30% compared to the dual-frequency float solutions, and triple-frequency AR decreases the horizontal convergence time by 40-50% compared to the triple-frequency float solutions. The vertical components see improvements, but on a smaller scale. The performance of the float dual- and triple-frequency solutions is very similar at all three levels of confidence, which confirms

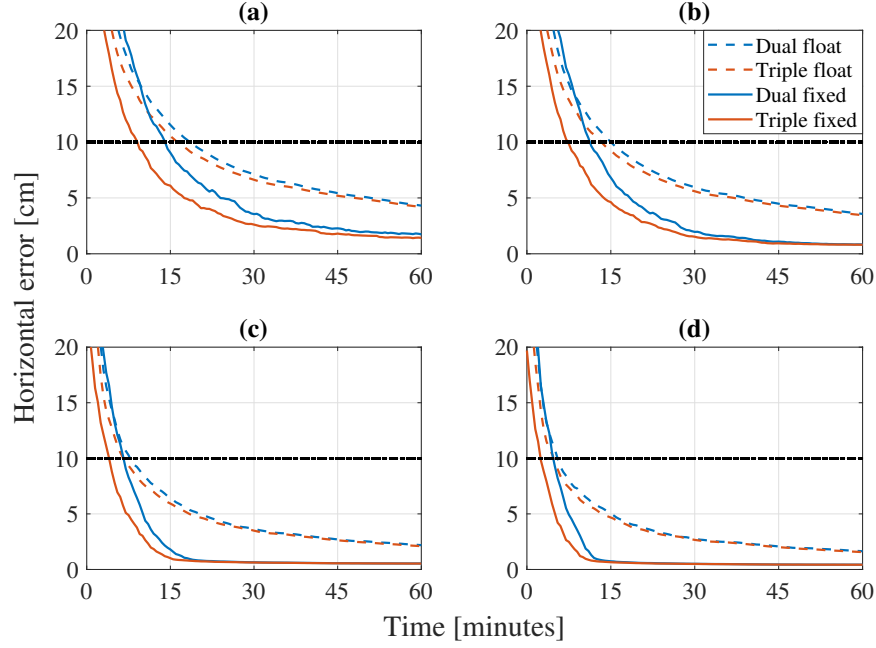


Figure 3.9: Static dual- and triple-frequency average horizontal error based on all datasets at the (a) 100th, (b) 95th, (c) 67th and (d) 50th percentiles. Horizontal dashed lines represent the convergence threshold.

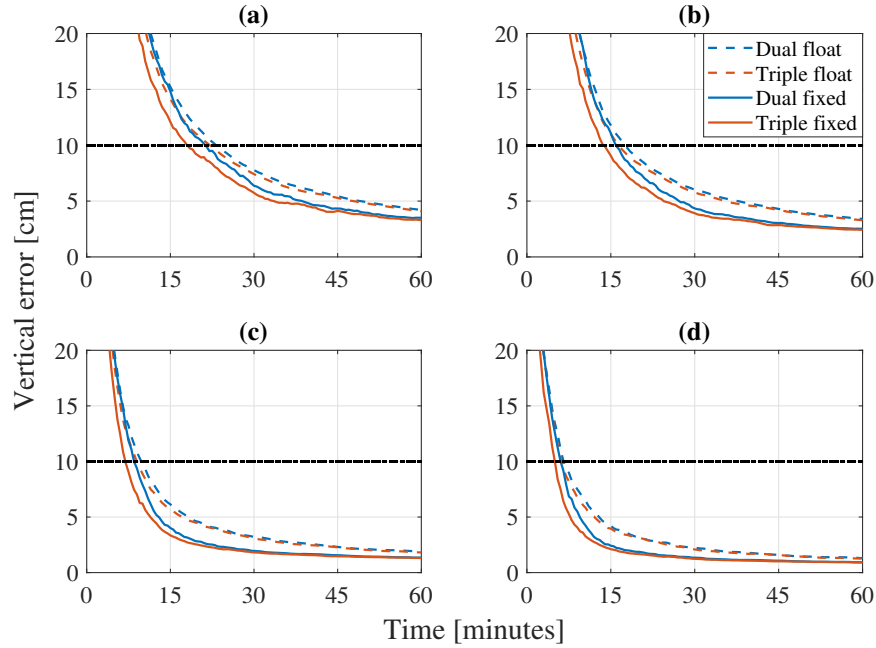


Figure 3.10: Static dual- and triple-frequency average vertical error based on all datasets at the (a) 100th, (b) 95th, (c) 67th and (d) 50th percentiles. Horizontal dashed lines represent the convergence threshold.

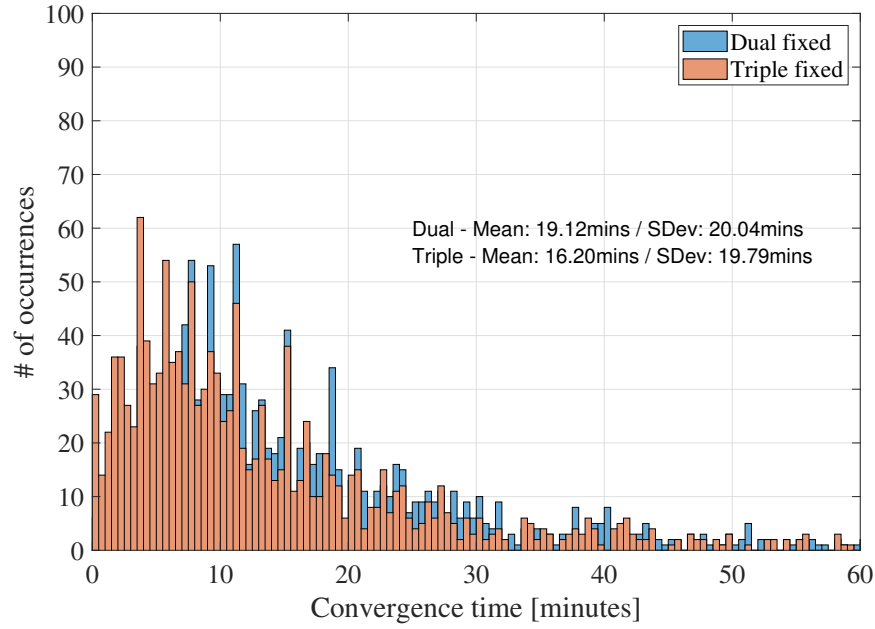


Figure 3.11: Histograms comparing the Galileo horizontal convergence times of the static dual-frequency fixed solutions with the triple-frequency fixed solutions.

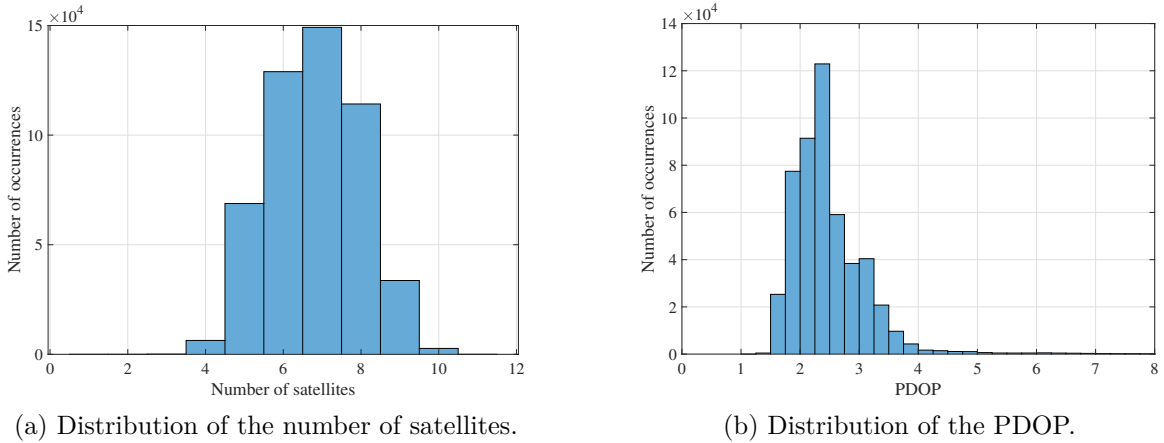
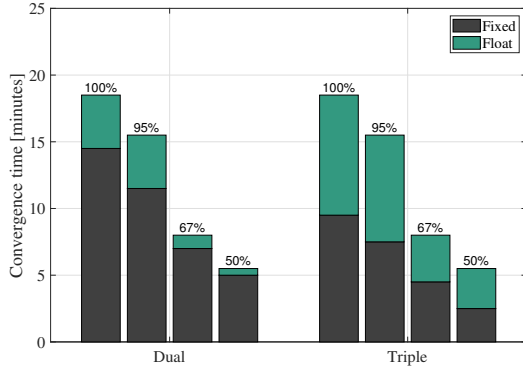
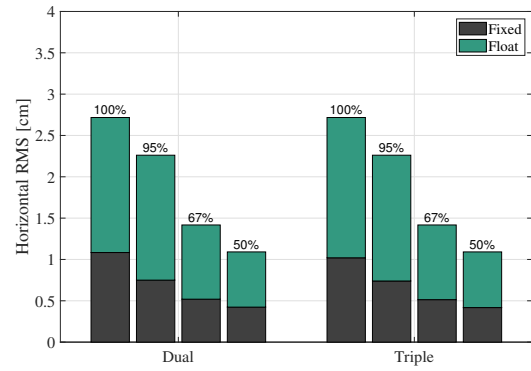


Figure 3.12: Distribution of (a) the number of Galileo satellites and (b) PDOP based on all epochs from all datasets.

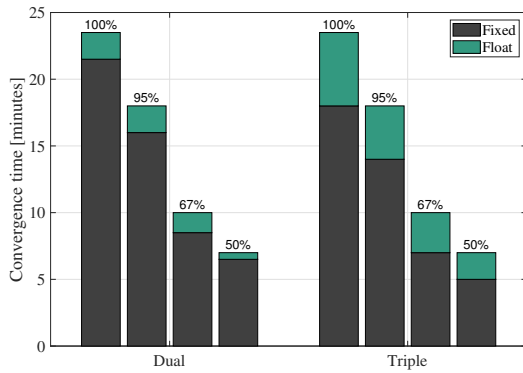


(a) Average horizontal convergence time.

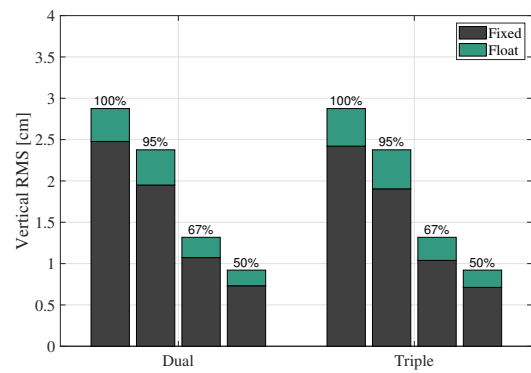


(b) Average horizontal rms.

Figure 3.13: (a) Average horizontal convergence times and (b) rms for both dual-frequency and triple-frequency solutions at different percentiles in static mode.



(a) Average vertical convergence time.



(b) Average vertical rms.

Figure 3.14: (a) Average vertical convergence times and (b) rms for both dual-frequency and triple-frequency solutions at different percentiles in static mode.

the earlier assumption that adding a third frequency does not add much new information to the already used two frequencies.

As mentioned above, compared to GPS, Galileo currently suffers from a relatively low number of satellites, which would impact the solutions. In order to have an idea on the Galileo performance with a more complete constellation, only the datasets with more than an average of six visible satellites during the first hour of processing were used for the generation of Figure 3.15 and Table 3.5. 73% of the datasets fulfill this condition. Both the figure and table show the performances of the float and fixed, dual-frequency and triple-frequency, horizontal and vertical solutions at the 100th percentile. When it comes to the horizontal convergence time, the performance of this solution corresponds roughly to the 95th percentile performance of Figure 3.13, while the vertical accuracy is between the 100th and 95th percentiles.

3.6.1.3 Kinematic processing

The results shown in subsection 3.6.1.1 are generated by considering that the receiver is static and by giving a process noise, in the sequential least-squares filter, for receiver position states of zero. That zero process noise means that the position coordinates are getting averaged across all epochs. In order to have a better idea on the performance of the proposed DCM model for Galileo satellites without such constraints, the same datasets are processed in kinematic mode and presented in the current subsection. In this case, position coordinate states are given process noise that is equivalent to typical speeds of a car (100 km/h). Since no assumption is made on the dynamics of the receiver, the performance of the kinematic processing is expected to be worse than the static one.

Figures 3.16 and 3.17 highlight the performance of the horizontal and vertical errors, respectively, for the kinematic processing. By visually comparing these two figures with Figures 3.9 and 3.10, it is clear that, as expected, the performance is worse in kinematic processing as solutions take longer to converge and have larger errors. Figure 3.18 shows the statistics corresponding to the kinematic processing. At all percentiles, the improvements from AR are more significant than in static processing, due to the fact that the solutions

Table 3.5: Horizontal and vertical convergence time and rms statistics for results in Figure 3.15.

		Conv. [min]		rms [cm]	
		Hor.	Vert.	Hor.	Vert.
Dual	Float	15.5	22.5	2.6	3.0
	Fixed	13.0	21.5	1.1	2.6
Triple	Float	14.0	21.5	2.5	2.9
	Fixed	9.0	18.0	1.1	2.5

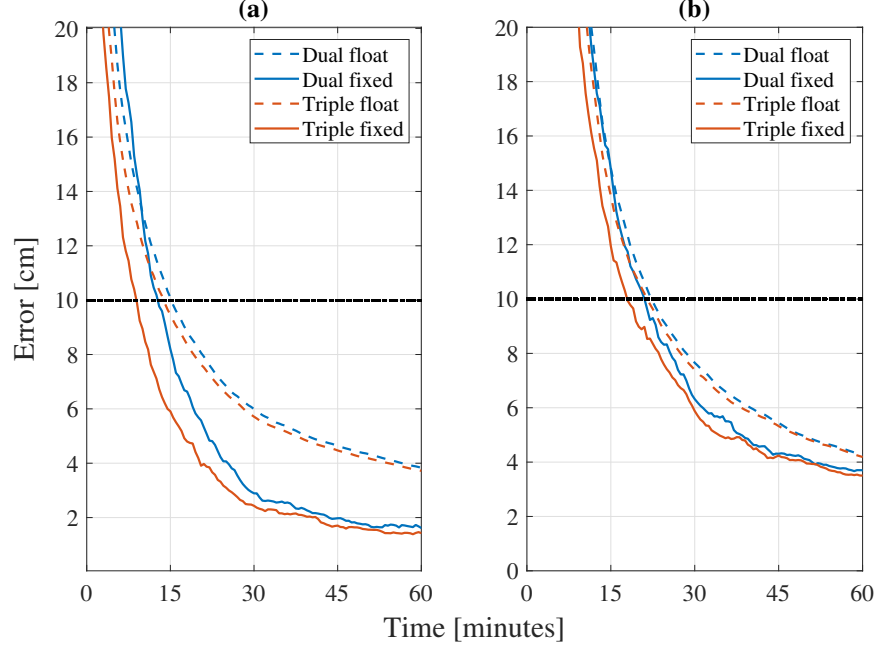


Figure 3.15: Static (a) horizontal and (b) vertical dual- and triple-frequency float and fixed solutions for datasets with an average of over six satellites during the first hour. Horizontal dashed lines represent the convergence threshold.

are significantly worse in kinematic mode, therefore leaving more room for improvement. At the 100th percentile in kinematic mode, the dual-frequency float solution converges in 61 minutes compared to 41.5 minutes for the fixed solution, leading to a 32% improvement. These values are larger than the static case where the float and fixed solutions converged in 18.5 minutes and 14.5 minutes, respectively, with an improvement of 22%. In terms of accuracy, the float and fixed horizontal accuracies in kinematic mode are 7.4 cm and 4.4 cm, respectively, compared to 2.7 cm and 1.1 cm, respectively, for the static processing. With three frequencies, the kinematic float solution converges in 50.5 minutes, while the fixed one converges in 20 minutes, as opposed to 16.5 minutes and 9.5 minutes in static mode. The reduction of the convergence time to 20 minutes is a significant improvement, as it is over half the convergence time of the dual-frequency fixed solution, showing the importance of fixing ambiguities on multiple frequencies.

Looking at Figure 3.18, there are significant improvements between the different percentiles, especially when comparing the 95th percentile and the 67th percentile. The reason for this improvement is that the kinematic processing is more sensitive to the PDOP and the number of satellites than the static processing, especially after convergence. Indeed, in static processing, if the number of satellites drops to a low number leading to a high PDOP, the position estimates would not be significantly affected if such scenario happens after convergence, as the position of the receiver would be averaged based on previous epochs. On

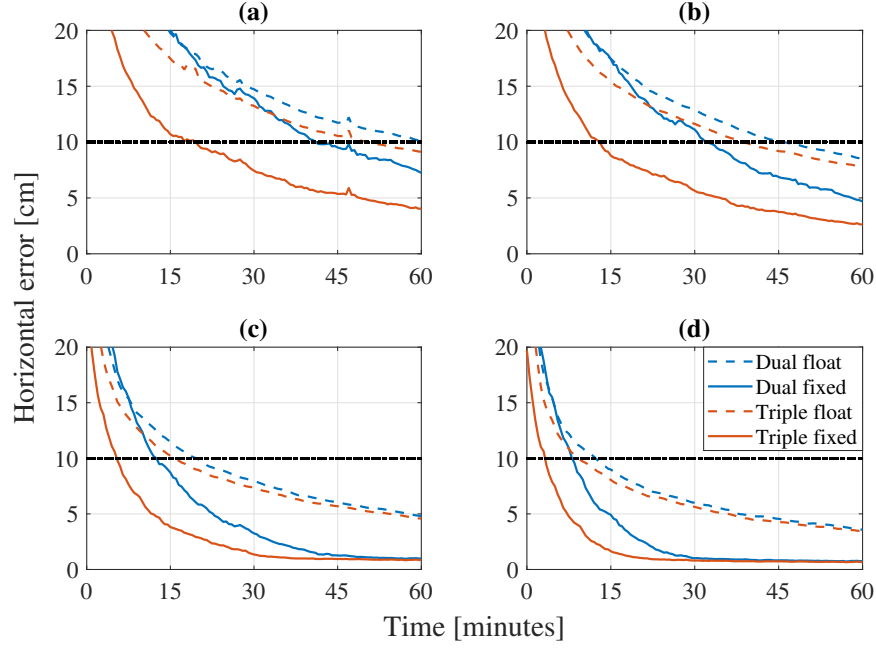


Figure 3.16: Kinematic dual-frequency and triple-frequency horizontal vertical error based on all datasets at the (a) 100th, (b) 95th, (c) 67th and (d) 50th percentiles. Horizontal dashed lines represent the convergence threshold.

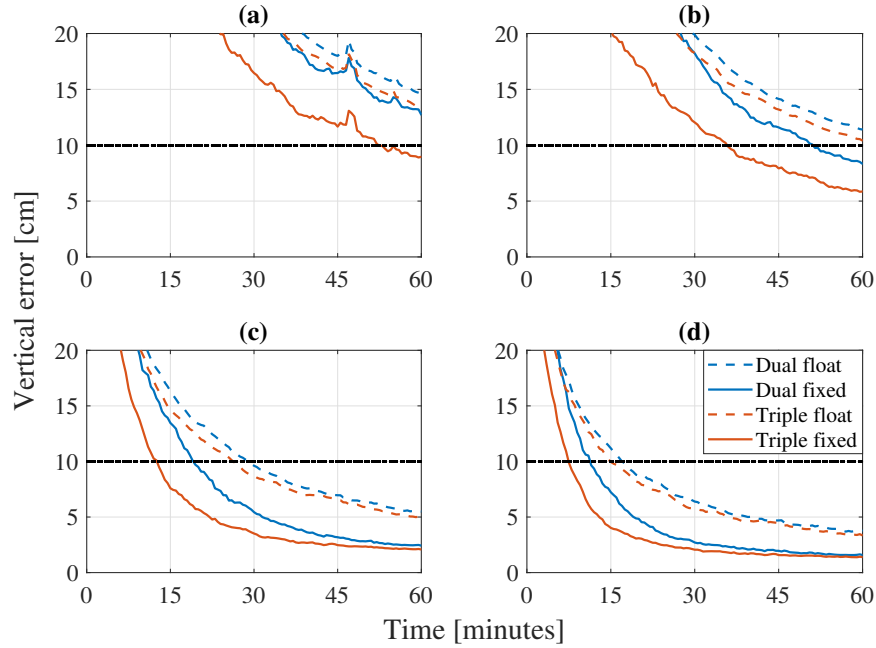


Figure 3.17: Kinematic dual-frequency and triple-frequency average vertical error based on all datasets at the (a) 100th, (b) 95th, (c) 67th and (d) 50th percentiles. Horizontal dashed lines represent the convergence threshold.

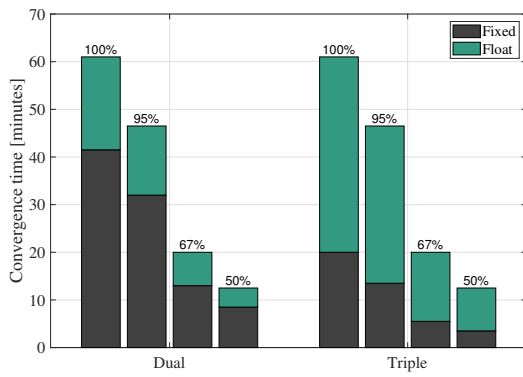
the other hand, in kinematic processing, a drop in the number of satellites would directly lead to a worse position estimate as the position states are estimated every epoch based on the received measurements. Figure 3.12 shows that several epochs have high PDOP, which affect the performance of the kinematic processing. Therefore, when looking at lower percentiles, those epochs when the PDOP is high get eliminated, leading to significant performance improvements. This enhancement is even more significant for the 67th percentile, as an additional 28% of (worse-performing) data are removed compared to the 95th percentile. Given that this issue is caused by the bad geometry on some epochs, it is expected that processing multiple constellations instead of only Galileo would lead to more consistent performance across datasets, as the epochs would have more satellites to process, and therefore better geometry.

3.6.2 Multi-GNSS results

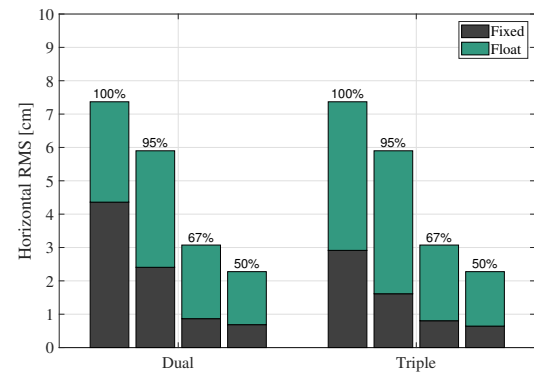
In this subsection, results from multi-GNSS processing of the models described in Sections 3.3 and 3.4 are presented, and are based on work published in Naciri and Bisnath (2021b). The results are generated by processing a worldwide set of stations from the IGS is chosen to assess the global performance of the algorithms, as well as to draw conclusions on areas of improvement. Figure 3.19 shows the network of stations used in the user processing. 32 IGS stations were randomly chosen, with the only constraints being that they should track all four GNSS constellations on up to three frequencies. The stations were processed in one-hour-long independent datasets from DOY 70 to DOY 77, 2021, with a sample interval of 30 seconds, leading to a total of 6,137 individual datasets after filtering out some datasets with missing epochs of observations. This sample size is considered to be significant enough to allow for assessment of the performance of the proposed algorithms and to analyze the impact of the fixing of the third frequency’s ambiguities.

The constellations processed are GPS, GLONASS, Galileo, and BeiDou-2, with ambiguities being resolved for GPS, Galileo, and BeiDou-2. BeiDou-3 was not included in the processing due to satellite bias corrections not being widely available at the time of the processing, making ambiguity resolution impossible. In terms of frequencies, L1, L2, and L5 were processed for GPS, G1, and G2 for GLONASS, E1, E5b, and E5a for Galileo, and B1-2, B2b, and B3 for BeiDou-2. For simplicity, all constellations are given the same weight on all frequencies. Further tuning could be done by adapting the weighting to each constellation and frequency. An elevation angle cut-off of 7° is used to eliminate satellites at low elevations.

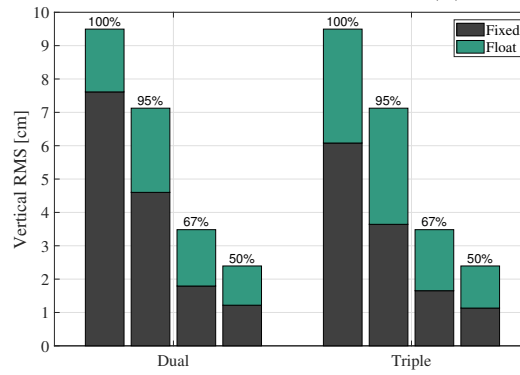
Although the IGS stations are static, they are processed as if they were kinematic without imposing constraints on the receiver’s dynamics. The stations’ position coordinates are estimated using process noise of 100 km/h. Receiver and satellite antenna corrections



(a) Average horizontal convergence time.

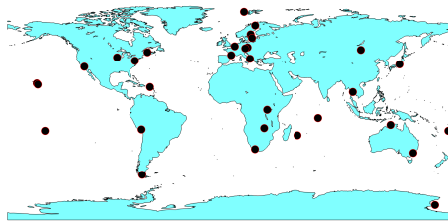


(b) Average horizontal rms.



(c) Average vertical rms.

Figure 3.18: (a) Average horizontal convergence times, (b) horizontal and (c) vertical rms for both dual-frequency and triple-frequency solutions at different percentiles in kinematic mode.



(a) Locations of the stations.

AREG	GOP6	MBAR	TID1
ARHT	GOPE	MKEA	TLSG
BRUX	HAL1	NLIB	ULAB
CRO1	JPLM	NYA2	UNB3
CUSV	KIRU	PARC	USUD
DARW	LAUT	REUN	VIS0
DGAR	MAR6	SUTM	WTZZ
GODS	MATE	THTG	ZAMB

(b) Names of the stations.

Figure 3.19: IGS stations used in processing.

are corrected for using the IGS14 ANTEX file (Schmid et al., 2016). Because the GPS L5 satellite antenna corrections are not available in the ANTEX file, the corrections on L2 are applied on L5 measurements. The same strategy regarding GPS L5 antenna corrections is usually adopted in the literature (Fan et al., 2019; Gong et al., 2020; Xiao et al., 2019; Geng et al., 2022). The rest of the processing strategies adopted for the various estimated parameters is provided in Table 3.2.

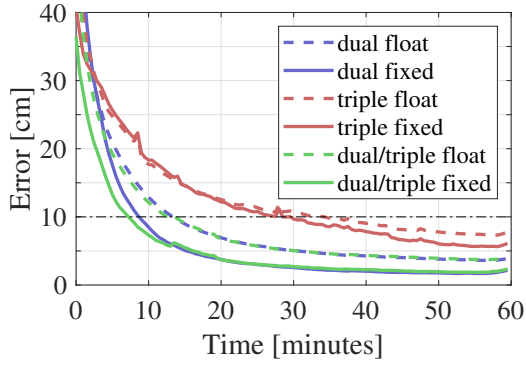
To allow for PPP processing in general, and ambiguity resolution specifically, satellite products are required. The products used in this work are provided by CNES, in RINEX format. The products are ultra-rapid products, meaning that the products would be the same as those used by a real-time engine. However, instead of doing the processing in real-time, it is done in post-processing with real-time products. Therefore, the performance in this subsection is expected to be similar to the performance a real-time user would get. This fact is promising and increases the importance of the results in this subsection, as it allows the assessment of the attainable performance for real-time applications, for example, autonomous vehicles, or any application interested in real-time use of PPP.

The results in this subsection consist of an analysis from IGS stations processing through a comparison of dual-frequency, triple-frequency, and mixed dual-/triple-frequency results. The latter results (simultaneous processing of dual-frequency and triple-frequency satellites) are analyzed in more detail in terms of convergence time and accuracy, with sample results from a single dataset being provided as well.

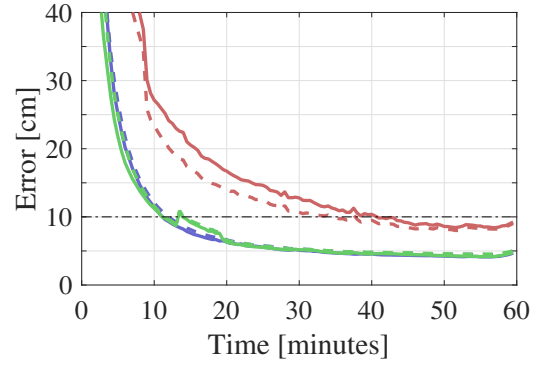
3.6.2.1 Performance comparison with dual-, triple-, and mixed dual-/triple-frequency processing

To assess the impact of the frequency combinations on the PPP and PPP-AR results, the 100th, 95th, and 67th percentiles are computed for both float and fixed horizontal and vertical errors. The results are presented in Figure 3.20 in the form of time series. The horizontal dashed lines refer to the convergence threshold defined as the time it takes to reach and settle below 10 cm horizontal error for the rest of the dataset. The horizontal convergence times corresponding to Figure 3.20 are summarized in Figure 3.21.

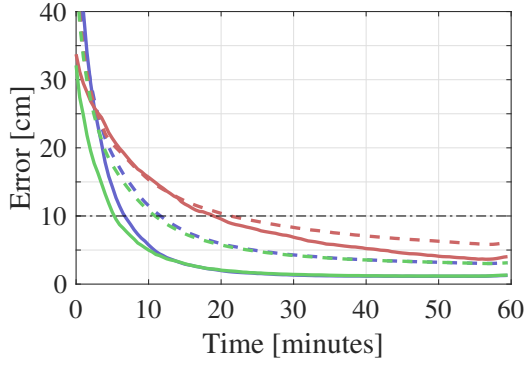
Multiple observations can be made from Figures 3.20 and 3.21. The triple-frequency results appear to perform worse than dual-frequency and dual-/triple-frequency results, for both float and fixed solutions. The reason for this behaviour is that the triple-frequency result excludes any satellite that does not have measurements on all three frequencies simultaneously. This exclusion means that only GPS-IIF and GPS-III satellites are processed for example, and GLONASS is not included in the processing as all of its satellites are dual-frequency. The histogram of the number of satellites for all three frequency combinations is shown in Figure 3.22. The graph is based on all epochs from all datasets, and indicates the distribution of the number of satellites overall. Given that the dual-/triple-frequency



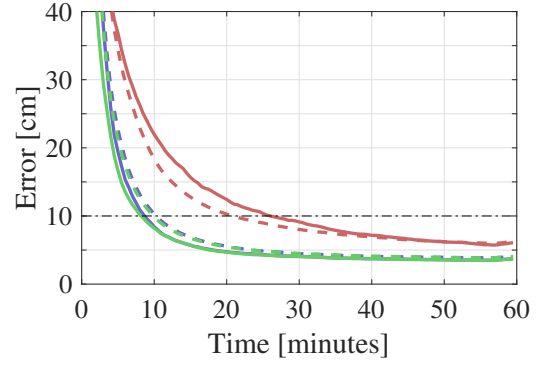
(a) 100th percentile horizontal errors.



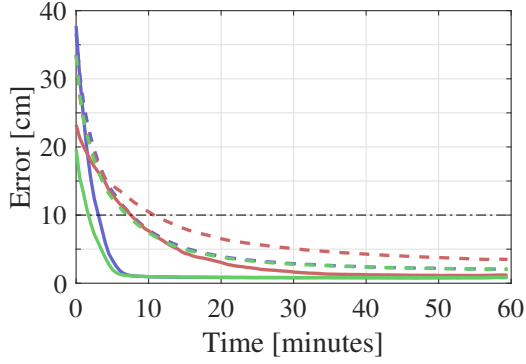
(b) 100th percentile vertical errors.



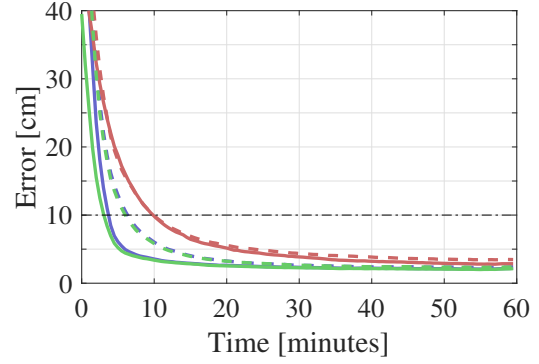
(c) 95th percentile horizontal errors.



(d) 95th percentile vertical errors.



(e) 67th percentile horizontal errors.



(f) 67th percentile vertical errors.

Figure 3.20: Time series comparison of average 100th, 95th, and 67th percentile horizontal and vertical errors for dual-frequency, triple-frequency, and mixed dual-/triple-frequency combinations both with and without ambiguity resolution. Horizontal dashed lines represent convergence threshold.

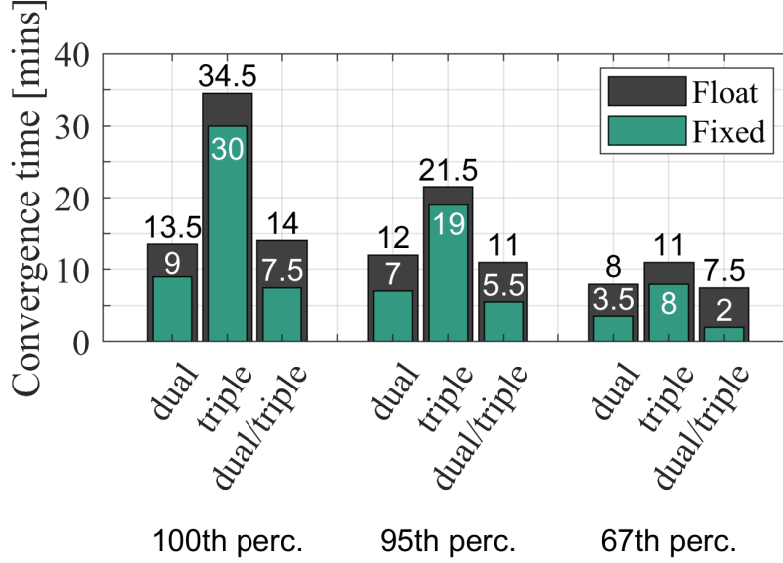


Figure 3.21: Horizontal convergence time for dual-frequency, triple-frequency, and mixed dual-/triple-frequency results with and without AR at 100th, 95th, and 67th percentiles. Convergence time statistics correspond to results in Figure 3.20.

processing mode includes both dual- and triple-frequency satellites, the number of satellites distributions between the dual-frequency mode and dual-/triple-frequency mode are completely overlapping. On average, there are 13 satellites being processed in the triple-frequency mode, as opposed to 23 satellites in dual-(/triple-)frequency mode. This discrepancy in the number of satellites explains the worse performance in the triple-frequency-only results.

The triple-frequency results show worse performance from the AR solution at the 100th and 95th percentiles. The reason is that there are certain datasets for which a very low number of satellites is being tracked on all three frequencies simultaneously, leading to poor satellite geometry as well. Some of the problematic datasets are removed at the 95th percentile results, while most are removed when looking at the 67th percentile. The latter shows a decrease in the horizontal convergence time from 11 minutes for the float solution to 8 minutes for the fixed solution. The convergence time of the vertical solution was not impacted by AR, owing to the limited impact of PPP-AR on the vertical component.

When comparing the dual-frequency and mixed dual-/triple-frequency results, the addition of the third frequency does not seem to have any significant impact on the float solutions. Indeed, when comparing the dual-frequency and dual-/triple-frequency results, both float solutions have centimetre or even millimetre-level differences at all percentiles for both the horizontal and vertical components. This observation hints that the additional pseudorange and carrier-phase measurements do not provide significantly new information

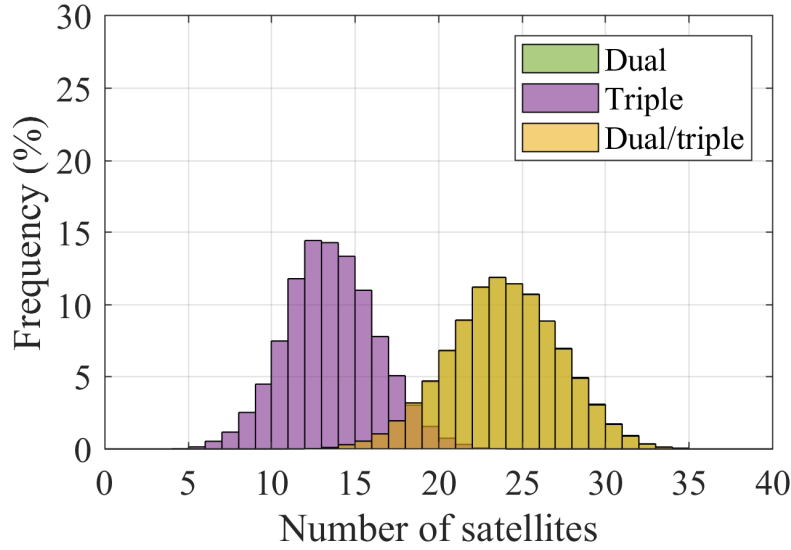


Figure 3.22: Histogram of number of satellites being processed in dual-frequency, triple-frequency, and mixed dual-/triple-frequency processing. The graph is based on all epochs from all datasets. “dual” (green) and “dual/triple” (yellow) histograms are overlapping in the dark yellow histogram.

that would help the estimation of the various PPP states, as opposed to, e.g., adding a satellite to the processing, which would improve the satellite geometry and provide a new set of independent measurements. Similar observations on the limited effect of additional frequencies on the float solution were noticed by Guo et al. (2016) using the BeiDou constellation and the effect of the third frequency was expected to be more significant in the case of poor measurement quality. The limited effect of the third frequency was also mentioned by Naciri and Bisnath (2021a), presented in the previous section, where the Galileo constellation was used, and by Geng et al. (2020) where multiple constellations were processed.

However, despite the third frequency having a limited impact on the float solutions, its impact on the fixed solutions is more noticeable. Adding a third frequency to the dual-frequency fixed solutions improves performance, especially when it comes to the convergence of the solution. At the 100th percentile for example, not only does the horizontal convergence time decrease from 9 minutes to 7.5 minutes, but the initial errors decrease as well from 60 cm to 36 cm on the first epoch. The same levels of improvement are noticed at lower percentiles for the horizontal error, with the convergence time decreasing from 7 minutes to 5.5 minutes at the 95th percentile, and from 3.5 minutes to 2 minutes at the 67th percentile. The initial horizontal error decreases from 55 cm to 32 cm at the 95th percentile, and from 38 cm to 20 cm at the 67th percentile. The improvements in the fixed solution with the addition of the third frequency are attributed to the fixing becoming more reliable.

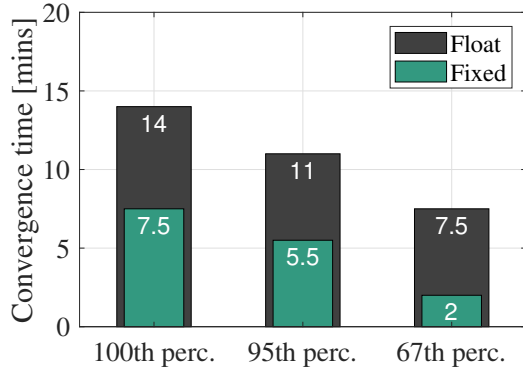
Indeed, the ambiguities from the same satellite are correlated, which means that adding a third ambiguity for a satellite would lead to more reliable and accurate fixing of the ambiguities, as was similarly discussed in Xiao et al. (2019). For instance, the addition of the third frequency allows for the formation of the EWL ambiguity, which has metre-level wavelength and can be fixed instantaneously. Being able to fix the EWL helps in the fixing of the WL ambiguity, which in turn helps the fixing of the narrowlane. Geng and Bock (2013) demonstrated the improvements that the third frequency ambiguities bring to the correct fixing of the narrowlane ambiguities based on simulated GPS data.

In terms of the vertical component, the addition of the third frequency appears to have a limited effect on both the float and fixed solutions. For float solutions, millimetre-level differences are seen between dual-frequency and mixed dual-/triple-frequency solutions. As described above for the horizontal component, this behaviour is attributed to the limited information provided by the third frequency to the state estimation. The improvements from the third frequency come from the addition of another ambiguity to fix, which is correlated to the first and second frequency ambiguities. However, its effect is very limited for the vertical component. The vertical convergence time at the 100th percentile decreases from 11.5 minutes to 11 minutes, while decreasing from 9 minutes to 8.5 minutes at the 95th percentile, and from 4 minutes to 3.5 minutes at the 67th percentile. The limited impact of AR on the vertical component's convergence was also noticed in Naciri and Bisnath (2021a); Shi and Gao (2014b). However, there are still improvements that can be observed in the initial epochs, as the third frequency helps reduce the vertical errors before convergence. For instance, at the 100th percentile, the dual-frequency solution's 124 cm initial error decreases to 90 cm with the dual-/triple-frequency solution. At the 95th percentile, the initial error decreases from 109 cm to 78 cm, while it decreases at the 67th percentile from 62 cm to 40 cm.

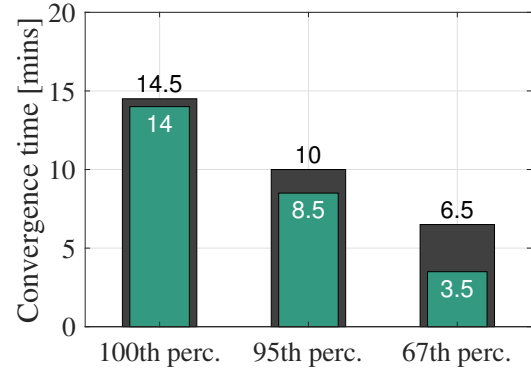
3.6.2.2 Mixed dual-/triple-frequency performance analysis

The previous subsection showed the benefits of simultaneously processing and fixing dual- and triple-frequency satellites. The current subsection focuses on the dual- and triple-frequency solution and analyzes its convergence time and accuracy performance in more detail. The horizontal and vertical convergence time and rms for the dual-/triple-frequency solution are summarized in Figure 3.23.

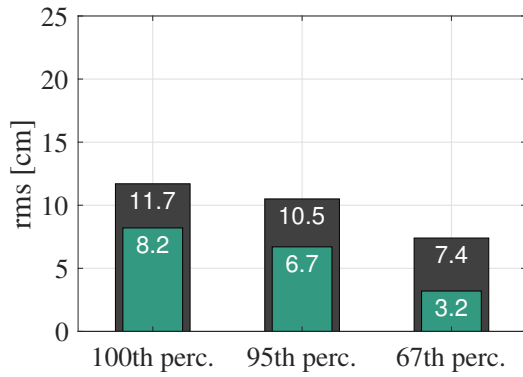
As discussed in the previous section, looking at lower percentiles eliminates outlier solutions and gives a better idea on the distribution of the performance. For instance, the benefits of ambiguity resolution on up to three frequencies are clear on the horizontal convergence time. When fixing ambiguities, at the 1-sigma (67th percentile), the solutions converge to and stay below one decimetre within 2 minutes, with a 73% improvement compared to that



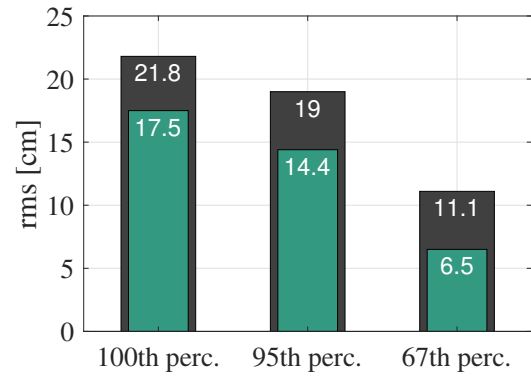
(a) Horizontal convergence time.



(b) Vertical convergence time.



(c) Horizontal rms.



(d) Vertical rms.

Figure 3.23: 100th, 95th, and 67th percentile horizontal and vertical convergence times and rms for solution combining dual- and triple-frequency satellites, with and without AR.

of the float solution. This result shows that it is possible to approach RTK levels of performance in terms of convergence time using only global corrections. Horizontal accuracy is also significantly improved as, at the 1-sigma (67th percentile), the rms reaches 3.2 cm. Note that this rms value is computed over the whole hour of processing and that it includes the initial convergence period. The accuracies after convergence are below 1 cm. These results could be improved with the inclusion of BeiDou-3, as well as the fourth Galileo frequency. However, the effect of AR is limited when it comes to the vertical component when looking at the 100th and 95th percentiles, as the improvements to the convergence time and rms are not as significant as for the horizontal component. Though at the 67th percentile, the vertical convergence time has a 46% improvement to reach 3.5 minutes. Similar conclusions are valid for the vertical rms, as the vertical positions are not as accurate as the horizontal ones. The better performance of the horizontal metrics is attributed to the fact that it is harder to estimate the vertical component due to the visible satellites all being distributed above the antenna (and the surface of the Earth).

To understand the effect of AR and of the third frequency at the dataset level, Figure 3.24 shows the distribution of the convergence time based on all the datasets. Only one float histogram is shown, as the performance of both dual-frequency and mixed dual-/triple-frequency float solutions is very similar.

Figure 3.24 shows clear improvements when fixing ambiguities compared to the float solution: the histograms get shifted to the left of the x-axis, with datasets converging sooner and with the mean of the histogram becoming smaller. A large portion of the fixed results converge within the first few minutes, as opposed to the float results, for which the majority converge within 10 minutes. More specifically, only 0.1% of float solutions converge instantaneously, compared to 7% for the fixed dual-frequency solutions, and 14% for the dual-/triple-frequency solutions. Similarly, only 0.2% of float solutions converge within a minute, as opposed to 12% for the fixed dual-frequency solutions, and 21% for the dual-/triple-frequency solutions. These results show that fixing ambiguities on as many frequencies as possible brings PPP closer to global instantaneous convergence, as fixing more frequencies leads to faster convergence at the dataset level. To further highlight the impact of the third frequency at the dataset level, one sample result is shown in Figure 3.25 for station GODS in Greenbelt, USA.

First, Figure 3.25a shows the similar performance between both dual-frequency and dual-/triple-frequency float solutions. Both solutions are overlapping with minimal differences, as was noticed in the average results using all datasets. The triple-frequency solution appears to perform worse than its dual(/triple)-frequency counterpart. As was explained for Figure 3.22, the reason for this difference is the fewer number of satellites broadcasting on three frequencies. Figure 3.25b shows the number of satellites broadcasting on two and three frequencies and the evolution of the numbers over one day. In the case of triple-frequency

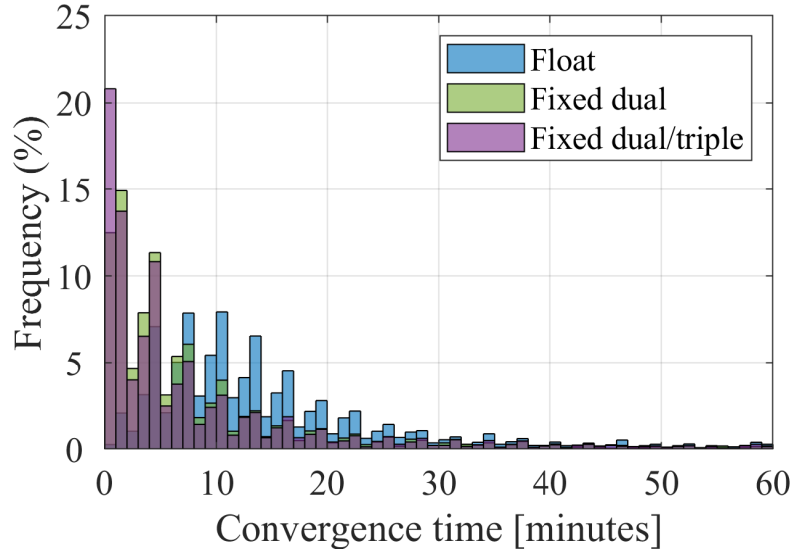
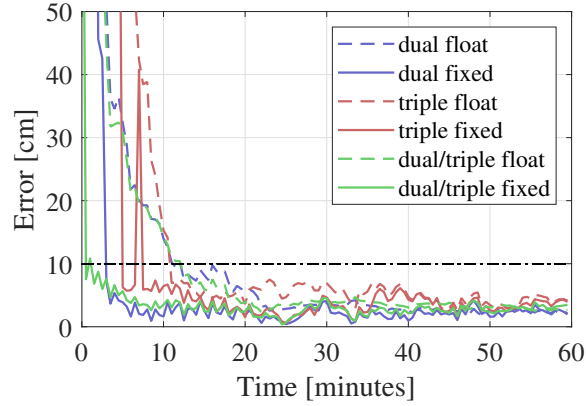


Figure 3.24: Float, dual-frequency fixed, and mixed dual-/triple-frequency fixed histogram of horizontal convergence time of all processed datasets.

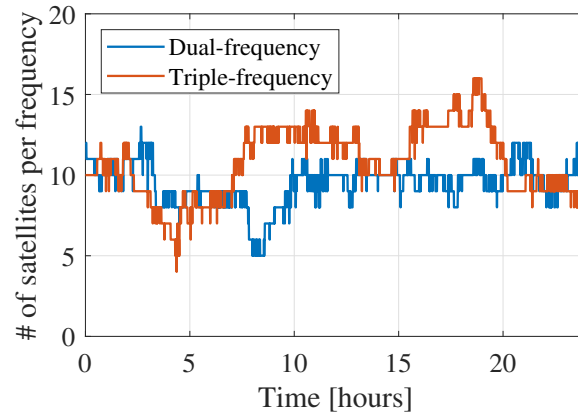
processing, all dual-frequency-only satellites are rejected, leading to the number of satellites corresponding to the orange line and dropping as low as four satellites at around 4 AM. Including both dual- and triple-frequency satellites leads to the number of satellites being the sum of both blue and orange graphs.

When looking at the fixed solutions, the triple-frequency solution performs the worst out of the three, as it uses a lower number of satellites. However, when comparing dual-frequency and dual-/triple-frequency results, the benefits of the third frequency become obvious. The 3D convergence time of the dual-/triple-frequency solution reaches 1.5 minutes, compared to 3 minutes for that of the dual-frequency solution. This reduction in the convergence time is a direct consequence of the third frequency, since it helps quicken the correct fixing of the ambiguities due to the correlation between ambiguities from the same satellite. After convergence, all accuracies are relatively similar, showing that most of the benefits from the additional frequency are in the form of faster convergence and smaller position errors before convergence. The fluctuations in the solutions after convergence are due to the receiver coordinates being estimated in kinematic mode, leading to noise affecting the position estimates.

The analysis of the impact of the third frequency on PPP and PPP-AR with a flexible model was not shown in the literature. Analysis involving multiple frequencies tends to be focused on comparing only satellites with the same number of frequencies (for example Galileo, or GPS-IIF) (Geng et al., 2020; Guo et al., 2016; Geng and Guo, 2020; Li et al., 2020b). In this subsection, a comprehensive performance analysis was done by not rejecting



(a) 3D error in centimetres.



(b) Number of satellites per frequency over 24 hours.

Figure 3.25: (a) Dual-frequency, triple-frequency, and mixed dual-/triple-frequency 3D errors for station GODS, on DOY 72, 2021 between UTC hours 5 and 6, and (b) number of satellites with two and three frequencies at station GODS on DOY 72, 2021.

satellites that only broadcast on two frequencies, allowing for the processing of many healthy satellites that would be otherwise rejected. This flexible processing allows for a realistic assessment of PPP performance in the current multi-GNSS, multi-frequency context, and it proves the benefits of including as many satellites as available, as PPP performance is further improved compared to a solution where only dual-frequency or only triple-frequency satellites are processed.

3.6.3 Multi-GNSS, triple-frequency, kinematic automotive results

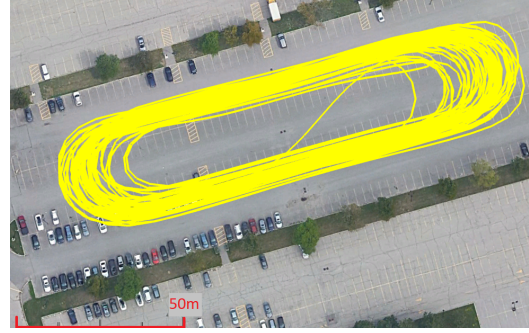
While it is important to assess the performance of the proposed algorithm with static IGS data, it is equally important to do the assessment with various test scenarios. In this case, the focus is on kinematic data using geodetic hardware. A set of kinematic data were collected by placing a geodetic receiver on a car and driving in a suburban environment, where the purpose is to highlight the benefits from the additional frequencies to achieve instantaneous precise positioning in the kinematic case. This kinematic testing is performed with a geodetic NovAtel PwrPak7 SPAN receiver, which was placed, along with its antenna, on a rack on top of the roof of a car. The car was driven to collect two separate datasets, with a sampling rate of 1Hz: one dataset was collected by driving close to the York University campus in Toronto, Canada on DOY 337, 2020, and a second parking lot dataset where the car was driven in an open-sky parking lot on DOY 151, 2021. The ground tracks of the two datasets are shown in Figure 3.26. A second NovAtel PwrPak7 receiver was used as a base station at a surveyed location and an RTK solution was generated as a reference solution using the commercial RTK software NovAtel Inertial Explorer. The baseline for both datasets was kept under 5 kilometres. The test setup was built and the data were collected as a common effort with colleague Sudha Vana, whose help is greatly appreciated.

Similarly to Section 3.6.2, observation data from GPS, GLONASS, Galileo, and BeiDou-2 are processed, with ambiguities being resolved for all except GLONASS. The signals being processed are GPS L1, L2 and L5, GLONASS G1 and G2, Galileo E1, E5b, and E5a, and BeiDou-2 B1-2, B2b, and B3. Since GPS L5 antenna corrections are not available, the corrections on the L2 signals are applied to the L5 signals. Ultra-rapid orbit, clock, and code and code and phase bias products from CNES are used for the processing. The two datasets are processed in dual-frequency and dual-/triple-frequency modes. The triple-frequency processing is omitted as its limitations were established in the previous sections. Figure 3.27 shows the results for the first suburban dataset.

Figures 3.27a and 3.27b show the improvements from both ambiguity resolution and the third frequency. The horizontal float solution (being the same in both dual-frequency and dual-/triple-frequency) converges below a decimetre after 24.5 minutes and has an overall rms of 12.5 cm. These statistics are greatly improved when fixing ambiguities in dual-



(a) Ground track of the first kinematic dataset.



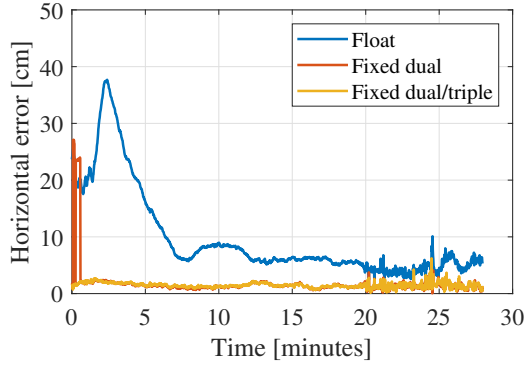
(b) Ground track of the second kinematic dataset.

Figure 3.26: Ground tracks of the datasets used in kinematic processing

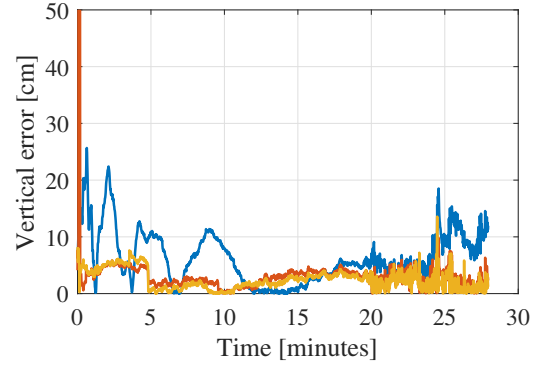
frequency mode, as the convergence time decreases to 0.5 minutes - near-instantaneously for data with a 30 second sample interval, with an overall rms of 3.2 cm. The results improve further when fixing the third frequency, as instantaneous convergence is achieved with an overall rms of 1.5 cm. This level of performance is comparable to RTK and is impressive given that only ultra-rapid global orbits, clocks and biases are used in the processing. The improvements from the third frequency in this case appear in the form of correct fixing in the initial epochs when the ambiguities tend to not be estimated well enough to be fixed correctly. This impact on the correct fixing of ambiguities is shown in Figure 3.27d which shows the ambiguity success rate (Teunissen, 2007) for both frequency combinations: adding a third frequency leads to an increase of the success rate of the ambiguity fixing, especially during the initial epochs of processing, allowing for instantaneous convergence.

As was the case with the IGS data, the vertical component is not estimated as well as the horizontal component. The variations in the float solution, which has an rms of 7.8 cm, are reduced when fixing the ambiguities, as the rms of the dual-frequency fixed solution is reduced to 4.7 cm, to be further reduced to 3 cm for the dual-/triple-frequency fixed solution. The higher variations that are seen in both horizontal and vertical errors after 20 minutes are due to the car being driven in a high-multipath area, with buildings on both sides of the road. The effect of the signal blockages on the number of satellites can be seen in Figure 3.27c.

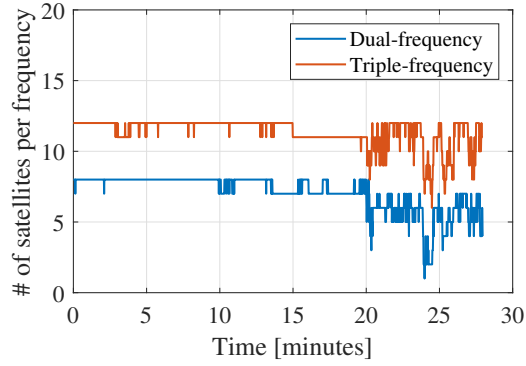
While the effect of the third frequency is relatively limited for this first dataset, it is more noticeable when looking at the second dataset, which was collected in an open-sky parking lot. The results from the processing are summarized in Figure 3.28.



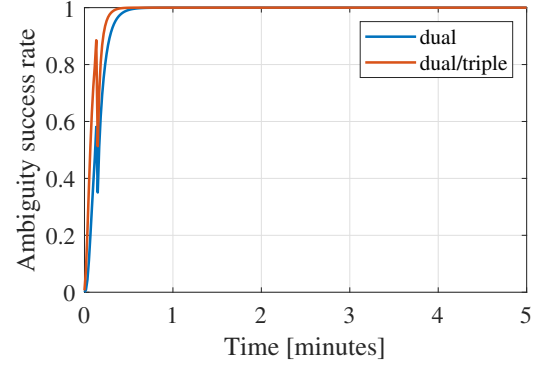
(a) Horizontal error.



(b) Vertical error.

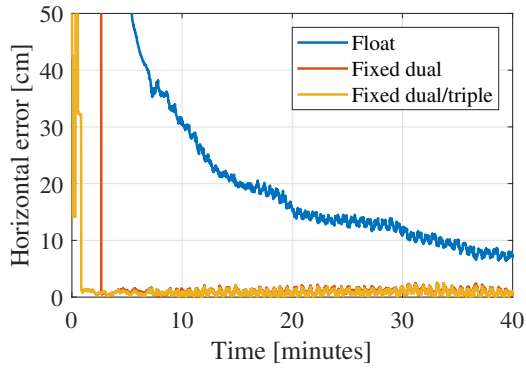


(c) Number of satellites per frequency.

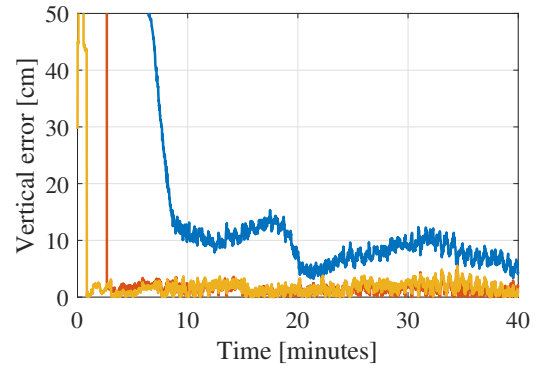


(d) Ambiguity success rate.

Figure 3.27: Float and fixed dual-frequency and dual-/triple-frequency horizontal (a) and vertical (b) errors, number of satellites per frequency (c), and ambiguity success rate (d) for the first kinematic dataset collected near York University on DOY 337, 2020.



(a) Horizontal error.



(b) Vertical error.

Figure 3.28: Float and fixed dual-frequency and dual-/triple-frequency horizontal (a) and vertical (b) errors for the open-sky kinematic dataset collected near York University on DOY 151, 2021. The car is static during the first 7 minutes, and kinematic for the rest of the dataset.

In the same fashion as for the first kinematic dataset, only one float solution is shown due to both float solutions being very similar in performance. The horizontal float solution takes a relatively long time of 35 minutes to converge to the decimetre-level in this case with an overall rms of 36.4 cm. Similarly to the first dataset, fixing the dual-frequency ambiguities allows for better accuracies to be reached sooner, as the solution converges within 2.7 minutes, with an accuracy of 32.4 cm. The relatively high rms value is due to the errors being relatively high before convergence, as the rms after convergence is 1 cm. The convergence time is further decreased to 51 seconds when fixing the third frequency as well, with a total rms of 5 cm, and an rms after convergence of 1 cm. A similar performance is noticed with the vertical component, as the rms after 5 minutes reaches 17 cm, 1.7 cm, and 1.9 cm for the float, fixed dual-frequency, and fixed dual-/triple-frequency solutions, respectively, with convergence times of 34 minutes, 2.7 minutes, and 51 seconds, respectively. These results highlight, not only the importance of the additional frequencies in improving the performance of the PPP-AR, but also the potential of PPP in becoming a standalone precise positioning technique capable of quickly providing centimetre-level position solutions relatively quickly using only corrections available globally.

These vehicle results demonstrate the benefits of processing a mix of dual- and triple-frequency measurements from all four major GNSS constellations. Doing so opens the door to instantaneous precise positioning that only requires globally estimated satellite products. The analysis of the impact of PPP-AR in general, and multi-frequency PPP-AR specifically, on vehicle datasets is limited in the literature. For instance, Geng et al. (2019) analyzed the performance of a triple-frequency, four-constellation solution where only triple-frequency satellites were processed and where the average number of satellites was 12. The solution achieved rms of 0.29 m, 0.35 m, and 0.77 m in the east, north, and up components, respectively, although only widelane and extra-widelane ambiguities were fixed. Zhang et al. (2019c) analyzed the impact of an Inertial Navigation System (INS) on PPP-AR re-convergence using GPS and GLONASS on two frequencies, and was able to achieve convergence (Time To First Fix (TTFF)) in approximately 30 minutes. Li et al. (2020a) achieved 0.23 m, 0.27 m, and 0.46 m rms in the east, north, and up components, though only the BeiDou constellation was used, and only widelane and extra-widelane ambiguities were fixed. This brief literature review highlights the importance of the kinematic results in this chapter, as narrowlane ambiguities are fixed in addition to widelane and extra-widelane, in a model that is able to process and fix dual- and triple-frequency measurements from all major GNSS constellations. This flexibility and capability allow for instantaneous centimetre-level positioning within seconds to tens of seconds.

3.7 Summary

The benefit of ambiguity resolution to PPP has been the subject to recent research and it has proved to improve the float PPP solutions in most cases. In this chapter, an uncombined multi-frequency AR model is proposed that is based on the combined decoupled clock model. The proposed model has been derived step-by-step for both the dual-frequency and triple-frequency cases starting from the general PPP equations. It is derived in uncombined mode, which means that it can be easily scaled to four or five frequencies, as Galileo broadcasts on five frequencies in total. This model and its analysis are novel and have been published in Naciri and Bisnath (2021a,b). In this model, the code and phase clocks are decoupled similarly to the original DCM. The datum in the phase measurements is then set to the reference satellite’s ambiguities instead of the receiver clock. This datum change is due to the receiver biases being eliminated through implicit differencing, as opposed to explicit differencing: no assumptions were made regarding the behaviour of receiver biases that get lumped into an additional phase term receiver L2 phase bias δt_{12} and receiver L3 phase bias δt_{13} . Therefore, variations in the receiver biases are estimated in the additional phase terms. The receiver biases are shown to be relatively stable over the course of a day for a sample dataset, with variations of up to 2 ns. The additional receiver L2 and L3 phase biases were shown to be dependent on the choice of reference satellite, unlike the IFB, which is absolute and only depends on the receiver code biases.

The advantage of AR is shown for both the dual- and triple-frequency cases using the Galileo constellation as a test bed, since all its satellites broadcast on at least three frequencies. Dual-frequency AR improved the horizontal convergence time of the 100th percentile average result from 18.5 minutes to 14.5 minutes for the large global IGS dataset that was assessed, and the three-hour horizontal rms from 2.7 cm to 1.1 cm. These improvements are significant and are also valid for individual stations. These results are improved even further by moving to the triple-frequency mode. Although no significant benefit comes from the addition of the float third frequency, performing triple-frequency AR leads to an average solution that converges within 9 minutes with a horizontal rms of 1.1 cm. Fixing the additional frequency does not provide major improvements to the final accuracy, but does help immensely in decreasing the convergence time which is one of the biggest hurdles preventing the wider use of PPP.

Additionally, given that not all satellites broadcast on three frequencies, as some only support two frequencies, the DCM was altered to be able to process and fix both dual-frequency and triple-frequency satellites simultaneously. This simultaneous processing requires careful handling of the PPP states and of the reference satellite, as the latter has to contain as many frequencies as the maximum number of frequencies being available in all satellites.

The multi-GNSS multi-frequency algorithm was first tested on static IGS data processed in kinematic mode, which showed that convergence below a decimetre for the horizontal component is possible within two minutes of processing at the 1-sigma (67th percentile). After convergence, the horizontal rms reaches 1 cm, while the vertical rms reaches 2 cm. The third frequency is found to have a limited impact on the float solution, as most of its benefits are in the form of more successful ambiguity fixing during convergence, allowing for centimetre accuracy to be achieved faster; though the effect of both ambiguity resolution and the third frequency is relatively limited when it comes to the vertical component. The algorithm was also tested on kinematic data collected by driving a car in two different scenarios. The results show that quasi-instantaneous centimetre positioning is possible even with a moving receiver, as one dataset reached and stayed at 1 cm horizontal error for the whole duration of the dataset, while the other dataset managed to do so after 51 seconds; though it was noticed that the environment can still have an impact on the level of noise in the position estimates, in high-multipath areas, for example.

The work in this chapter is novel in the fact that, rather than discarding satellites broadcasting only on two frequencies as is typically done, those satellites are included in the processing, alongside triple-frequency satellites. Doing so allows for improvements in the geometry, but also more reliable fixing of ambiguities. Having this flexible processing is shown to provide significant improvements compared to a dual-frequency-only or a triple-frequency-only solution. The importance of this work is confirmed by the results, as they show that global instantaneous centimetre-level positioning is possible, even with vehicle data. Indeed, these results are based on ultra-rapid products generated from a global network, without the use of any local information. These results open the door for PPP to be used in real-time applications that require precise positioning, such as autonomous driving.

The results are expected to further improve by adding additional frequencies, such as Galileo's E6 signal or with the addition of BeiDou-3, which are all included in the next chapter. By doing so, it is expected that the results would improve even further, with more datasets being able to converge instantaneously. When it comes to kinematic datasets, the results can be further improved through careful handling of the multipath errors in areas where it would affect the positioning accuracy.

Chapter 4

RTK-quality positioning with global PPP corrections

Global Navigation Satellite System (GNSS) Precise Point Positioning (PPP) processing has been subject to much research in recent years, and the technique has shown potential as an alternative or replacement for Real-Time Kinematic (RTK) processing in certain situations. Indeed, RTK (and network RTK) is known for being able to reach centimetre-level positions instantaneously, or within a few seconds, primarily depending on the length of the baseline. However, local reference stations are required for measurement differencing to eliminate error sources, which is a limitation in areas where such infrastructure is not available or costly. PPP on the other hand does not need such local calibration stations, and only requires global satellite corrections to achieve high precision positions. This chapter aims to produce RTK levels of performance without the need for local information or infrastructure via PPP. Conventional PPP is a measurement-dependent technique requiring tens of minutes to reach centimetre-level positions. Luckily, given recent advances in GNSS with many constellations broadcasting on two, three, or four frequencies, this chapter makes use of the signals currently available by processing signals from GPS, Galileo, BeiDou-2/3, and GLONASS and fixing ambiguities for the first three constellations, on as many frequencies as available. This chapter builds on the triple-frequency Decoupled Clock Model (DCM) derivation from the previous chapter by expanding it to a four-constellation, four-frequency model that is capable of flexible multi-GNSS, multi-frequency PPP-AR processing. A redundancy analysis is performed as well, where equivalence between the DCM and the classic PPP model is established, and where the impact of new signals on the redundancy of the PPP model is studied. The results are tested on a large dataset from stations spread around the world, and typical PPP analysis, as well as novel RTK-quality analysis where an analysis of the possibility of robust centimetre-level positioning in an epoch-by-epoch PPP solution were performed. The chapter assesses whether consistent RTK-like quality can be achieved with PPP processing without the need for local infrastructure or corrections. This

assessment, along with the comprehensive use of all constellations and frequencies, and the redundancy analysis are novel and such analysis and strict comparison again RTK-like performance has not been published in the literature. This work has been published in Naciri (2022) for which it has won a best student paper award, as well as in the NAVIGATION journal (Naciri and Bisnath, 2023).

4.1 Background and motivation

In the current multi-constellation, multi-frequency GNSS context, users have access to many more satellites and frequencies than a few decades or even years ago. This plethora of constellations and signals means that users have access to many more measurements to be used for precise positioning, allowing for improvements in PPP performance (Geng et al., 2020; Naciri and Bisnath, 2021b; Katsigianni et al., 2019; Xiao et al., 2019; Psychas et al., 2021; Xin et al., 2020). For instance, Geng et al. (2020) showed that 6.1 minutes were required to achieve PPP-AR when using triple-frequency data from GPS, Galileo, BeiDou and QZSS, as opposed to 9.2 minutes for dual-frequency data. Naciri and Bisnath (2021b), presented in the previous chapter, showed that 2 minutes were required on average at 1σ to reach and settle below 10 cm horizontal error when fixing ambiguities on GPS, Galileo and BeiDou on up to three frequencies. Hexagon, a commercial company, has released its latest correction service that allows centimetre-level positioning within less than two minutes at 2σ without regional corrections, although details on how such performance is achieved are not provided (Hexagon, 2021). Psychas et al. (2021) showed the importance of increasing frequencies in accelerating convergence, as well as the importance of the separation between frequencies in increasing the fixing success rate. The study was based on Galileo E1/E5a/E5b/E5/E6 signals, and GPS L1/L2 signals. Xin et al. (2020) also showed the importance of the third frequency choice, as they demonstrated that the E1/E5a/E6 combination of Galileo frequencies leads to the best PPP-AR performance in a three-frequency solution. These publications demonstrate the benefits of leveraging additional frequencies and constellations to improve PPP performance.

In this chapter, the goal is to bridge the gap between PPP and NRTK in terms of performance by improving PPP performance in a way that will lead to very short convergence times. This goal is aimed to be achieved without the use of any local augmentation in order to have PPP reach RTK-like performance globally. To achieve this goal, knowing that PPP is a measurement-dependent technique, as many signals as possible from as many constellations and on as many frequencies as available are made use of in order to improve performance. Coupling this approach with Ambiguity Resolution (AR) could help bring PPP to RTK levels of performance. Given the varying number of frequencies being broadcast by each constellation and within each constellation, it is important to have a

flexible PPP-AR model that is capable of adjusting to the different signals available and fix ambiguities to their correct integer values. The novelty in the research presented in this chapter lies in:

- The expansion of the decoupled clock model into an uncombined, four-frequency model,
- The flexible processing of multiple numbers of frequencies, depending on availability, even within the same constellation,
- The processing of all four GNSS (GPS, GLONASS, Galileo, and BeiDou-2/3), and ambiguity resolution on GPS, Galileo, and BeiDou-2/3,
- The use of stricter PPP performance requirements than typically done in the literature, for a better comparison to RTK quality of performance. This analysis is done through: stricter convergence and root mean square error (rms) error thresholds (e.g., convergence to 2.5 cm rather than 10 cm), and epoch-by-epoch processing,
- The analysis of the equivalence in redundancy / degrees of freedom between the DCM and classic PPP, as well as an analysis of the redundancy in the multi-constellation, multi-frequency context.

4.2 Expansion of the decoupled clock model to four frequencies

In order to process measurements on four frequencies, and fix ambiguities on all four frequencies, the PPP-AR model has to be expanded to allow for the additional measurements. As part of the work in Chapter 3, the classic dual-frequency combined Decoupled Clock Model (DCM) was re-formulated into a triple-frequency uncombined model, with the derivation being done from first principles. In this section, the same model will be expanded into a quadruple-frequency model.

As explained in earlier parts of this dissertation, the DCM is based on the assumption that carrier-phase and pseudorange measurements are not synchronized to the same level of precision. In order to solve this difference in levels of precision, two sets of clocks are estimated: pseudorange clocks, and carrier-phase clocks - hence, the clocks are decoupled. Decoupling the clocks leads to the carrier-phase measurements losing their datum, which was originally set by the pseudorange clocks. The phase datum is recovered by selecting one satellite as a reference satellite and setting its ambiguities to arbitrary values, to which other satellites' ambiguities will be implicitly differenced. A version of the DCM where dual-frequency uncombined measurements were used instead of combined measurements has been shown by Seepersad (2018); Naciri and Bisnath (2021a). As part of this

research, the DCM has been expanded to uncombined triple-frequency processing, with the mathematical derivation being detailed in chapter 3 and in Naciri and Bisnath (2021a) for the Galileo constellation. Furthermore, the flexible multi-constellation, dual- and triple-frequency model was introduced in chapter 3 and in Naciri and Bisnath (2021b), where dual-frequency and triple-frequency measurements are processed and fixed simultaneously for GPS, Galileo and BeiDou-2. In the current chapter, simultaneous processing of dual-, triple-, and quadruple-frequency measurements is performed through an upgrade of the model introduced in chapter 3.

The complete triple-frequency model, which is the basis of the derivation in the current chapter, was presented in Equation (3.19). The following derivation will follow the same steps as the triple-frequency derivation, by first focusing on the fourth frequency's pseudorange measurements, before re-arranging the fourth frequency's carrier-phase measurements. If the tropospheric delays and satellite clock errors are omitted from the PPP equations, the fourth frequency's pseudorange and carrier-phase measurements at receiver r from satellite s can be written as:

$$\begin{cases} P_{r,4}^s &= \rho_r^s + cdt_r + \gamma_4 I_1^s + (b_{r,4} - b_4^s) + \epsilon_{P_4} \\ \Phi_{r,4}^s &= \rho_r^s + c\delta t_r - \gamma_4 I_1^s + \lambda_4(N_4^s + B_{r,4} - B_4^s) + \epsilon_{\Phi_4} \end{cases} \quad (4.1)$$

In Equation (4.1), $P_{r,4}^s$ and $\Phi_{r,4}^s$ are the fourth frequency pseudorange and carrier-phase measurements, respectively. ρ_r^s is the geometric range between the receiver and the satellite, γ_4 is the ratio of the square of frequencies between f_1 and f_4 that maps the first frequency ionospheric delay I_1^s to the fourth frequency's. dt_r and δt_r are the receiver code and phase clock errors, respectively, and N_4^s is the fourth frequency carrier-phase ambiguity. $b_{r,4}$ and b_4^s are the receiver and satellite code biases, respectively, and $B_{r,4}$ and B_4^s are the receiver and satellite phase biases, respectively. In order to proceed further, the receiver code bias must be removed from the pseudorange equation, as it leads to rank deficiency of the system.

4.2.1 Fourth frequency pseudorange

In order to eliminate the pseudorange code bias from the fourth frequency pseudorange, the receiver code bias and ionospheric delay expressions in Equation (3.6) are used. Omitting the residual error term ϵ_{P_4} , and replacing the expressions for $\overline{dt_r}$ and $\overline{I_1^s}$ into the fourth frequency pseudorange leads to:

$$\begin{aligned}
P_{r,4}^s &= \rho_r^s + cdt_r + \gamma_4 I_1^s + (b_{r,4} - b_4^s) \\
&= \rho_r^s + c\overline{dt}_r + \gamma_4 \overline{I}_1^s + (b_{r,4} - b_4^s) - \frac{f_1^2}{f_1^2 - f_2^2} b_{r,1} + \frac{f_2^2}{f_1^2 - f_2^2} b_{r,2} + \gamma_4 \frac{f_2^2}{f_1^2 - f_2^2} (b_{r,1} - b_{r,2}) \\
&= \rho_r^s + c\overline{dt}_r + \gamma_4 \overline{I}_1^s - b_4^s + \frac{f_4^2 - f_1^2}{f_1^2 - f_2^2} b_{r,1} + \frac{f_2^2 - f_4^2}{f_1^2 - f_2^2} b_{r,2} + b_{r,4}
\end{aligned} \tag{4.2}$$

In Equation (4.2), the pseudorange measurement has been re-arranged to contain similar receiver code clock and ionospheric state terms as in the other frequencies' pseudoranges in Equation (3.19). However, it can be noticed that, in Equation (4.2), receiver-related state terms are remaining, which have not been absorbed by the other state terms, unlike in the dual-frequency case where these quantities are absorbed by $\overline{dt}_r$ and $\overline{I}_1^s$. In this case, similar to the third frequency case, a new receiver-dependent state term needs to be estimated, referred to as an Inter-Frequency pseudorange Bias (IFB). Setting $IFB_{r,4} = \frac{f_4^2 - f_1^2}{f_1^2 - f_2^2} b_{r,1} + \frac{f_2^2 - f_4^2}{f_1^2 - f_2^2} b_{r,2} + b_{r,4}$, the fourth frequency pseudorange measurement becomes:

$$P_{r,4}^s = \rho_r^s + c\overline{dt}_r + \gamma_4 \overline{I}_1^s - b_4^s + IFB_{r,4} \tag{4.3}$$

4.2.2 Fourth frequency carrier-phase

After having re-arranged the pseudorange equation, we can focus on the carrier-phase measurement's expression. As was done for the pseudorange, the fourth frequency carrier-phase measurement can be re-arranged to include $\overline{\delta t}_r$ and $\overline{I}_1^s$ rather than δt_r and I_1^s . Including the two state terms in the equation leads to:

$$\begin{aligned}
\Phi_{r,4}^s &= \rho_r^s + c\delta t_r - \gamma_4 I_1^s + (N_4^s + B_{r,4} - B_4^s) \\
&= \rho_r^s + c\overline{\delta t}_r - \gamma_4 \overline{I}_1^s + \lambda_4 (N_4^s + B_{r,4} - B_4^s) - \lambda_1 (\delta N_1^{ref} + B_{r,1}) \\
&\quad + \frac{f_2^2}{f_1^2 - f_2^2} (b_{r,1} - b_{r,2}) - \gamma_4 \frac{f_2^2}{f_1^2 - f_2^2} (b_{r,1} - b_{r,2})
\end{aligned} \tag{4.4}$$

In order to proceed further, distinction between the reference satellite's equation and the non-reference satellites' has to be made.

4.2.2.1 Reference satellite

In the DCM, when decoupling the receiver clocks, the datum in the phase measurements is lost, as it is usually set by the pseudorange measurements through the receiver clock. In order to recover the datum in the phase measurements, one satellite is selected as a reference satellite, and its ambiguities are fixed to arbitrary integer values without being estimated. By referring to the reference satellite as *ref* and setting $N_4^{ref} = N_{04}^{ref} + \delta N_4^{ref}$ where N_{04}^{ref} is the arbitrarily set integer ambiguity on the fourth frequency and δN_4^{ref} is the difference

between the true integer ambiguity and the arbitrarily set integer one. Replacing N_4^{ref} with its expression leads to the following expression for the reference satellite's fourth frequency phase measurement:

$$\begin{aligned}
\Phi_{r,4}^{ref} &= \rho_r^{ref} + c\overline{\delta t}_r - \gamma_4 \overline{I}_1^{ref} + \lambda_4(N_{04}^{ref} + \delta N_4^{ref} + B_{r,4} - B_4^{ref}) \\
&\quad - \lambda_1(\delta N_1^{ref} + B_{r,1}) + \frac{f_2^2}{f_1^2 - f_2^2}(b_{r,1} - b_{r,2}) - \gamma_4 \frac{f_2^2}{f_1^2 - f_2^2}(b_{r,1} - b_{r,2}) \\
&= \rho_r^{ref} + c\overline{\delta t}_r - \gamma_4 \overline{I}_1^{ref} + \lambda_4(N_{04}^{ref} - B_4^{ref}) + \lambda_4(\delta N_4^{ref} + B_{r,4}) \\
&\quad - \lambda_1(\delta N_1^{ref} + B_{r,1}) + \frac{f_2^2 - f_4^2}{f_1^2 - f_2^2}(b_{r,1} - b_{r,2})
\end{aligned} \tag{4.5}$$

By setting $\delta t_{14} = \lambda_4(\delta N_4^{ref} + B_{r,4}) - \lambda_1(\delta N_1^{ref} + B_{r,1}) + \frac{f_2^2 - f_4^2}{f_1^2 - f_2^2}(b_{r,1} - b_{r,2})$, the equation can be re-written as shown in Equation (4.6). δt_{14} is referred to as the fourth frequency receiver phase bias, and is a state term that only appears on the fourth frequency's phase measurements.

$$\Phi_{r,4}^{ref} = \rho_r^{ref} + c\overline{\delta t}_r - \gamma_4 \overline{I}_1^{ref} + \lambda_4(N_{04}^{ref} - B_4^{ref}) + \delta t_{14} \tag{4.6}$$

4.2.2.2 Non-reference satellites

In the general case where a satellite is not a reference satellite, Equation (4.4) can be re-arranged to make use of δt_{14} which was derived in the reference satellite case. By ingesting δt_{14} into Equation (4.4), the equation becomes:

$$\begin{aligned}
\Phi_{r,4}^s &= \rho_r^s + c\overline{\delta t}_r - \gamma_4 \overline{I}_1^s + \lambda_4(N_4^s + B_{r,4} - B_4^s) - \lambda_1(\delta N_1^{ref} + B_{r,1}) + \frac{f_2^2 - f_4^2}{f_1^2 - f_2^2}(b_{r,1} - b_{r,2}) \\
&= \rho_r^s + c\overline{\delta t}_r - \gamma_4 \overline{I}_1^s + \lambda_4(N_4^s + B_{r,4} - B_4^s) + \delta t_{14} - \lambda_4(\delta N_4^{ref} + B_{r,4}) \\
&= \rho_r^s + c\overline{\delta t}_r - \gamma_4 \overline{I}_1^s + \lambda_4(\overline{N}_4^s - B_4^s) + \delta t_{14}
\end{aligned} \tag{4.7}$$

In Equation (4.7), $\overline{N}_4^s = N_4^s - \delta N_4^{ref}$ is the difference between the satellite's ambiguity and the integer error in the reference satellite's ambiguity. Therefore, this ambiguity term is an integer and is single-differenced as it contains both the satellite's ambiguity and the reference satellite's. Similarly to the other frequencies' phase equations, this single-difference was done implicitly, without needing to explicitly take the difference in measurements or ambiguities between a given satellite and the reference satellite.

4.2.3 Summary of the complete four-frequency model

Following the derivation, for satellite s and receiver r , the complete uncombined, four-frequency DCM can be summarized as:

$$\begin{cases}
P_1^s = \rho_r^s + c(\overline{dt}_r - dt^s) + \gamma_1 \bar{I}_1 + M_r^s T_r & -b_1^s & +\epsilon_{P_1} \\
P_2^s = \rho_r^s + c(\overline{dt}_r - dt^s) + \gamma_2 \bar{I}_1 + M_r^s T_r & -b_2^s & +\epsilon_{P_2} \\
P_3^s = \rho_r^s + c(\overline{dt}_r - dt^s) + \gamma_3 \bar{I}_1 + M_r^s T_r & -b_3^s & +IFB_{r,3} +\epsilon_{P_3} \\
P_4^s = \rho_r^s + c(\overline{dt}_r - dt^s) + \gamma_4 \bar{I}_1 + M_r^s T_r & -b_4^s & +IFB_{r,4} +\epsilon_{P_4} \\
\Phi_1^s = \rho_r^s + c(\overline{\delta t}_r - dt^s) - \gamma_1 \bar{I}_1 + M_r^s T_r + \lambda_1(\bar{N}_1^s - B_1^s) & & +\epsilon_{\Phi_1} \\
\Phi_2^s = \rho_r^s + c(\overline{\delta t}_r - dt^s) - \gamma_2 \bar{I}_1 + M_r^s T_r + \lambda_2(\bar{N}_2^s - B_2^s) & +\delta t_{12} & +\epsilon_{\Phi_2} \\
\Phi_3^s = \rho_r^s + c(\overline{\delta t}_r - dt^s) - \gamma_3 \bar{I}_1 + M_r^s T_r + \lambda_3(\bar{N}_3^s - B_3^s) & +\delta t_{13} & +\epsilon_{\Phi_3} \\
\Phi_4^s = \rho_r^s + c(\overline{\delta t}_r - dt^s) - \gamma_4 \bar{I}_1 + M_r^s T_r + \lambda_4(\bar{N}_4^s - B_4^s) & +\delta t_{14} & +\epsilon_{\Phi_4}
\end{cases} \quad (4.8)$$

In Equation (4.8), for frequency $i \in \{1, 2, 3, 4\}$, P_i^s and Φ_i^s are the pseudorange and carrier-phase measurements, ρ_r^s is the geometric range between the receiver and the satellite, c is the speed of light, γ_i is the ratio of frequencies between the first frequency and frequency i , and λ_i is the wavelength for frequency i . The satellite corrections from the network side are the satellite orbits in ρ_r^s , the satellite clocks dt^s , the satellite code biases b_i^s , and the satellite phase biases B_i^s . The estimated atmospheric delays are the zenith wet tropospheric delay T_r to which a mapping function M_r^s is applied to map it to the slant direction, as well as the ionospheric delay on the first frequency $\bar{I}_1^s$, which is mapped to the delay on frequency i through the factor γ_i . The receiver biases and clocks are at the core of the DCM. For these state terms:

- $\overline{dt}_r$ and $\overline{\delta t}_r$ are the decoupled pseudorange and carrier-phase receiver clocks, respectively. The estimation of two receiver clocks is due to the decoupling of clocks and, as a consequence, the carrier-phase measurements lose their datum. The datum in those measurements is recovered through selecting a reference satellite, not estimating its ambiguities, and setting them to arbitrary integer values. As a consequence, the datum on each frequency's carrier-phase measurements is set by the reference satellite's ambiguity on that frequency. Therefore, $\bar{N}_i^s$ is the integer single-differenced ambiguity for satellite s relative to the reference satellite. The differencing is done implicitly when choosing one satellite as reference, as there is no need to manually difference measurements or ambiguities,
- δt_{12} , δt_{13} , and δt_{14} are referred to as the second, third and fourth frequency's receiver phase biases. The three state terms are all receiver-related and they contain combinations of receiver biases and of the reference satellite's ambiguities. These three state terms are direct consequences of the DCM and of the implicit differencing, as they show up when arranging the reference satellite's measurement equations. Each of the three state terms is specific to the carrier-phase measurements on a specific frequency,

- $IFB_{r,3}$ and $IFB_{r,4}$ are inter-frequency pseudorange biases. These biases are common to any multi-frequency model, and they arise from the fact that other state terms cannot absorb all of the code biases on all frequencies. Indeed, the first and second frequencies' receiver code biases are absorbed by the pseudorange receiver clock and the ionospheric delays, but the third and fourth frequencies' receiver code biases have to be estimated as additional state terms, as they would otherwise show up in the post-fit residuals.

Given that the DCM does not make assumptions on the nature of biases and their behaviour, all of the receiver bias state terms are estimated as white noise processes, the same way that clock terms are estimated. In addition to the pseudorange $\overline{dt}_r$ and carrier-phase $\overline{\delta t}_r$ clocks, that includes the inter-frequency code biases $IFB_{r,3}$ and $IFB_{r,4}$, and the receiver phase biases δt_{12} , δt_{13} , and δt_{14} .

The model above is described for a quadruple-frequency satellite. In case of a dual-frequency or triple-frequency satellite, the model remains the same, with the exception of the third- and fourth-frequency and fourth frequency measurements being omitted for dual-frequency, and triple-frequency satellites, respectively. As a consequence, some of the state terms do not need to be estimated when processing a lower number of frequencies for a specific satellite. For example, δt_{14} and $IFB_{r,4}$ will not be estimated for a triple-frequency satellite. Another consequence of this flexible processing of multiple frequencies is that careful consideration has to be given to the reference satellite. Indeed, knowing that the datum on each frequency's phase measurements is set by the reference satellite's ambiguity on that frequency, it is important to make sure that the reference satellite has as many frequencies as the maximum number of frequencies being processed. Other strategies can be adopted, such as selecting one reference satellite per frequency, rather than having one reference satellite for all frequencies. A more detailed explanation of the reference satellite choice depending on frequency is provided in Section 3.5. While a summarized version of the four-frequency DCM was published in Naciri (2022), the derivation in this chapter is the first time that the step-by-step derivation of the four-frequency model from the three-frequency model is published.

4.3 Redundancy analysis

It is typical in PPP to estimate constellation-dependent, receiver-related states, meaning that one set of clocks is estimated per constellation. The reason for doing so is because constellations are typically tracked by different processing units in the receivers, and because of the different time scales between constellations and the presence of inter-system time biases (El-Mowafy et al., 2016; Håkansson et al., 2017). The presence of these constellation-related biases means that receiver biases are constellation-dependent as well, in addition to

the receiver clocks. Therefore, in the model described in Equation (4.8), there is one set of receiver clocks and biases per constellation: $(\overline{dt}_r, \overline{\delta t}_r, IFB_{r,3}, IFB_{r,4}, \delta t_{12}, \delta t_{13}, \delta t_{14})$. As described in the previous section, the receiver phase bias state terms are correlated to the reference satellite's integer ambiguities. Therefore, it is also required to have one reference satellite per constellation, as the receiver biases are constellation-dependent. Furthermore, BeiDou-2 and BeiDou-3 suffer from the presence of clock and time delay biases between both BeiDou generations, which means that the biases can either be estimated, or the generations have to be treated as two different constellations (Jiao et al., 2020). The second option is opted for in this work, therefore one reference satellite is chosen for each constellation for which AR is performed: GPS, Galileo, BeiDou-2, and BeiDou-3, with each reference satellite having measurements on as many frequencies as the maximum number of frequencies being tracked by all satellites in its constellation.

Fortunately, having as many reference satellites does not negatively impact the PPP model and its degrees of freedom. Indeed, as explained above, the reference satellite's ambiguities are not estimated and they are set to arbitrary integers. Therefore, the additional receiver-related state terms which are specific to the DCM $(\overline{\delta t}_r, \delta t_{12}, \delta t_{13}, \delta t_{14})$ are compensated for by the reference satellite's ambiguities that do not have to be estimated. In fact, the DCM is equivalent to a classic PPP model in terms of redundancy and degrees of freedom of the two models, as the DCM is a reformulation of the classic PPP model in order to access the integer nature of the ambiguities. This equivalence is demonstrated next. Let N^c be the number of constellations being used, N^s the total number of satellites regardless of constellation, and N^f the number of frequencies being processed. For this demonstration, a subset of satellites broadcasting on the same number of frequencies is considered. In the more realistic scenario where satellites are broadcasting signals on a varying number of frequencies, the demonstration is still valid and the derivation is still the same.

4.3.1 Redundancy of the decoupled clock model

The model that is used in this demonstration is described in Equation (4.8). In the following, the number of states to be estimated is bolded. The number of measurements and of states to be estimated depending on the number of constellations, satellites and frequencies is:

- receiver coordinates X, Y, Z (**3**), and zenith wet tropospheric delay T_r (**1**) are estimated regardless of N^c , N^s , or N^f ,
- receiver code $\overline{dt}_r$ and phase $\overline{\delta t}_r$ clocks are estimated as long as $N^f \geq 2$. One set of states is estimated per constellation, meaning that N^c sets of two clocks are to be estimated,

- third frequency $IFB_{r,3}$ and fourth frequency $IFB_{r,4}$ receiver code biases are only estimated if $N^f \geq 3$ or $N^f = 4$, respectively. If the conditions are fulfilled, one state term is estimated per constellation. Therefore, the number of receiver code biases to be estimated is $(\mathbf{N}^f - 2)\mathbf{N}^c$,
- second frequency δt_{12} , third frequency δt_{13} , and fourth frequency δt_{14} receiver phase biases are only estimated if $N^f \geq 2$, $N^f \geq 3$, or $N^f = 4$, respectively. If the conditions are fulfilled, one state term is estimated per constellation. Therefore, the number of receiver phase biases to be estimated is $(\mathbf{N}^f - 1)\mathbf{N}^c$,
- one slant ionospheric delay per satellite $\bar{I}_1^s$, meaning $\mathbf{N}^s$ ionospheric state terms. There are also as many ambiguities per satellite as there are frequencies (N^f ambiguity state terms per satellite). Since ambiguities are not estimated for the N^c reference satellites, the total number of ambiguity state terms is $\mathbf{N}^f(\mathbf{N}^s - \mathbf{N}^c)$,
- the measurements include one code and one phase measurement per frequency per satellite. Therefore, the total number of measurements is $2\mathbf{N}^s\mathbf{N}^f$.

Therefore, considering N^{meas} measurements available and N^{states} states to be estimated, the number of degrees of freedom M in the uncombined multi-frequency DCM is summarized in Equation (4.9).

$$\begin{aligned}
M &= N^{meas} - N^{states} \\
&= 2N^f N^s - (3 + 1 + N^c + N^c + (N^f - 1)N^c + (N^f - 2)N^c + N^s + N^f(N^s - N^c)) \\
&= N^f N^s - 4 - N^f N^c + N^c - N^s
\end{aligned} \tag{4.9}$$

4.3.2 Redundancy of the classic PPP model

In a classic PPP model, many of the state terms in the previous section are not estimated. For instance, the receiver phase bias state terms are specific to the DCM, and do not need to be estimated. In this case, the uncombined form of the model shown in Equation (2.3) is analyzed. The states estimated in the classic float PPP model are:

- **3** receiver coordinates X, Y, Z
- **1** wet zenith tropospheric delay T_r
- $\mathbf{N}^c$ receiver clocks dt_r
- $(\mathbf{N}^f - 2)\mathbf{N}^c$ receiver code biases $IFB_{r,3}$ and $IFB_{r,4}$

- $\mathbf{N}^s$ first-frequency ionospheric delays I_1^s
- $\mathbf{N}^f \mathbf{N}^s$ ambiguities N_i^s , with $i \in \{1, 2, 3, 4\}$

Therefore, the redundancy in a classic PPP model can be calculated as in Equation (4.10).

$$\begin{aligned}
M &= N^{meas} - N^{states} \\
&= 2N^f N^s - (3 + 1 + N^c + (N^f - 2)N^c + N^s + N^f N^s) \\
&= N^f N^s - 4 - N^f N^c + N^c - N^s
\end{aligned} \tag{4.10}$$

From Equations (4.9) and (4.10), both models appear to have the same degrees of freedom, despite the DCM requiring the choice of one reference satellite per constellation. The equivalence is due to the additional state terms in the DCM appearing as a consequence of not estimating the reference satellites' ambiguities, thus balancing the redundancy equation.

4.3.3 Analysis of PPP redundancy

The degree of freedom equation can be used to understand the effect of the additional frequencies and constellations on PPP models, and subsequently performance. In light of the current GNSS context where many constellations and frequencies are available to users, it is important to understand how these additional measurements affect PPP processing. Figure 4.1 shows the global distribution of the number of visible satellites on average over 24 hours. The figure shows up to almost 50 satellites being seen on the Asian continent on average in a day, with a minimum of over 25 satellites in the rest of the world. This high number of visible satellites ensures better satellite geometry for users around the globe, and therefore better Position Dilution Of Precision (PDOP). This increase in the number of satellites is accompanied with an increase in the number of broadcast frequencies as well, as shown in Table 4.1. The table shows that all constellations have satellites broadcasting on three different frequencies, with GLONASS mainly being dual-frequency, as it is being upgraded with a newer generation of multi-frequency satellites. Some constellations, e.g., Galileo and BeiDou-3 even broadcast on four frequencies, therefore giving importance to the work in this chapter.

In order to better understand the impact of all the additional satellites, constellations, and frequencies, the degree of freedom function is plotted as a function of the three parameters (number of satellites, constellations and frequencies) in Figure 4.2a.

When it comes to the number of satellites, a clear trend is noticeable: as the number of satellites increases, so does the redundancy, regardless of the number of constellations

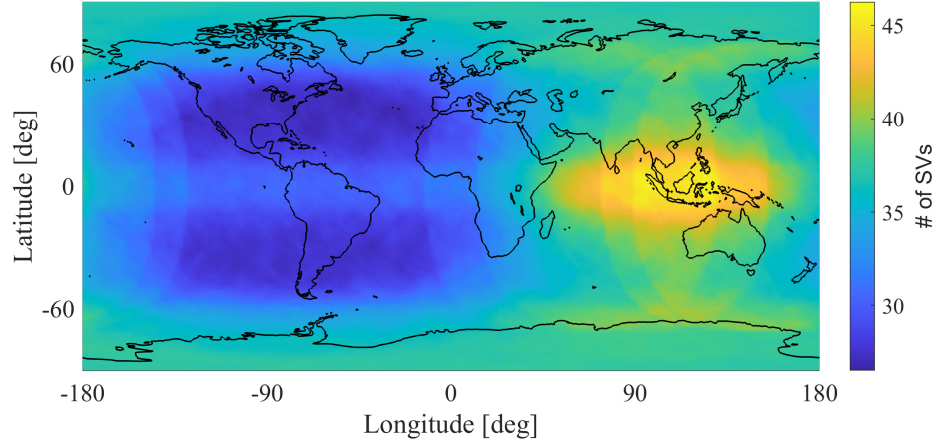
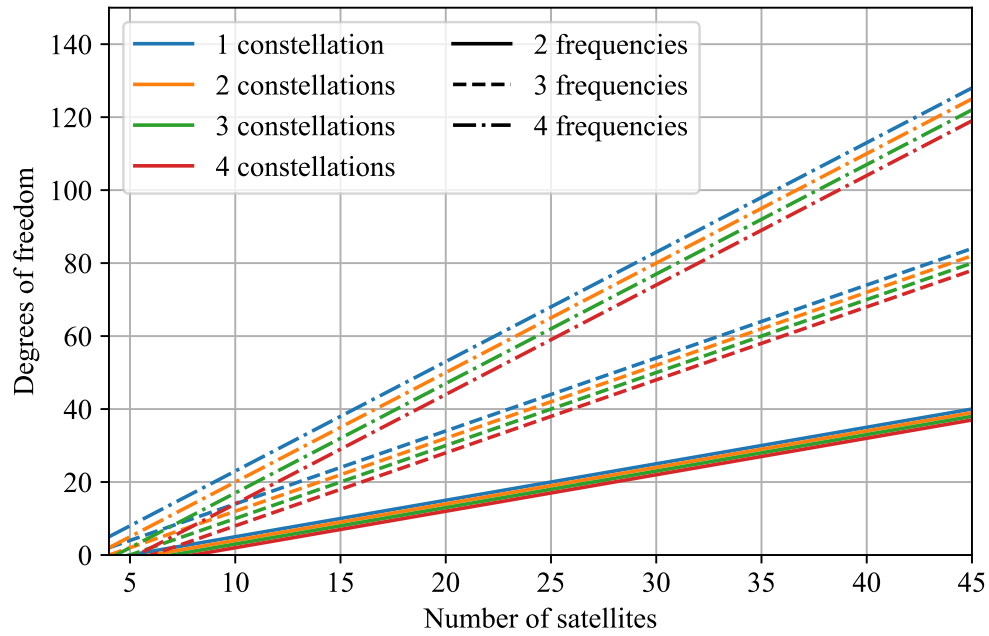


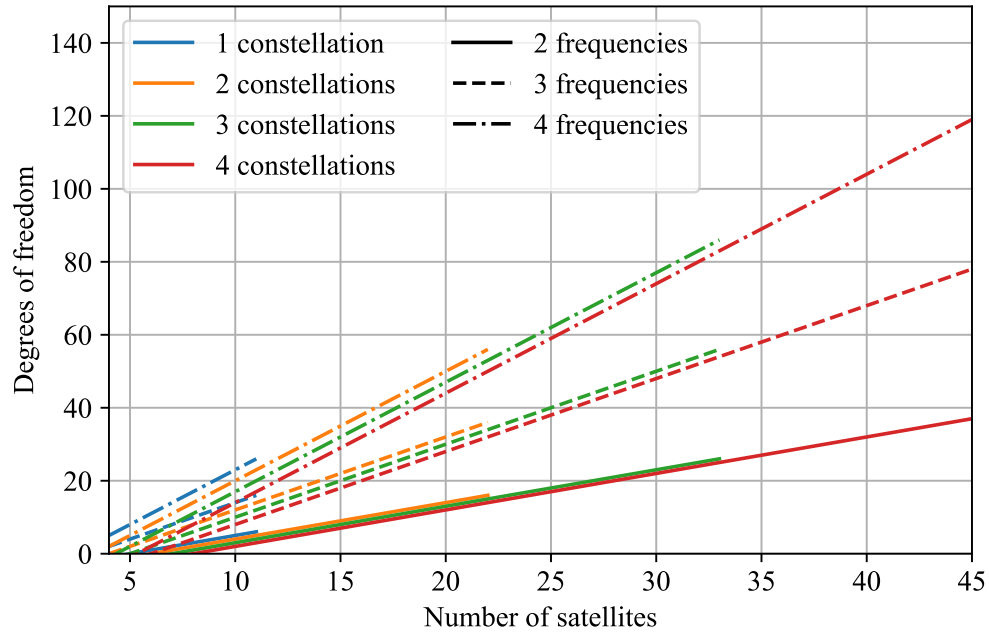
Figure 4.1: Daily average number of visible GPS, GLONASS, Galileo and BeiDou-2/3 satellites on day 305 of year 2021.

Table 4.1: Number of satellites with one, two, three, and four frequencies for each constellation (QZSS Cabinet Office, 2022).

	Freq. 1	Freq. 2	Freq. 3	Freq. 4
GPS	31	31	17	
GLONASS	22	22	2	
Galileo	25	25	25	25
BeiDou-2	15	15	15	
BeiDou-3	29	29	27	27



(a) Redundancy of the PPP model.



(b) More realistic redundancy of the PPP model.

Figure 4.2: Effect of the number of satellites, constellations, and frequencies on the redundancy / degrees of freedom of the PPP model. (b) is a more realistic version of (a), as it takes into account the limited number of satellites observed with a limited number of constellations.

or frequencies. This trend is expected, as additional satellites come with additional measurements to help with the estimation of the state terms, some of which are common to all satellites, e.g., receiver coordinates, clocks and biases. The number of frequencies has a similar effect on the redundancy, as the latter increases with the number of frequencies processed. Furthermore, the effect of the number of frequencies on the redundancy is more noticeable with a higher number of satellites. Indeed, adding more frequencies increases the slope of the redundancy plots as a function of the number of satellites. Therefore, having a higher number of frequencies on a higher number of satellites leads to a higher increase in the redundancy compared to a lower number of satellites. This observation makes sense, as each frequency adds two measurements (pseudorange and carrier-phase), and one additional state to be estimated (ambiguity), therefore increasing the redundancy by one - considering that receiver-related biases are already estimated through the other satellites. As a consequence, adding one frequency when 10 satellites are available leads to a redundancy increase of 10, as opposed to adding one frequency when 40 satellites are available, which leads to a redundancy increase of 40.

The last parameter to be analyzed is the impact of the number of constellations on the redundancy. Looking at a single number of frequencies, it appears that adding more constellations has an adverse effect on the redundancy, as each additional constellation shifts the redundancy graph towards lower values on the y-axis. This shift is a consequence of the additional receiver state terms that need to be estimated whenever a new constellation is added, including the receiver clocks and code and phase biases. These additional state terms need to be estimated and cannot be modelled easily, as their behaviour is not stable and is receiver-dependent (Wang et al., 2020). On the other hand, one must keep in mind that tracking a higher number of satellites requires the use of multiple constellations, as only a limited number of satellites can be tracked with a single constellation due to the relatively limited number of satellites in orbit per constellation. A more realistic graph would be the one showed in Figure 4.2b, where only a limited number of satellites can be processed in a single-constellation solution. In that case, increasing the number of constellations increases the number of satellites that can be used in the processing, thereby increasing the redundancy of the system. Therefore, an ideal solution would process satellites from all constellations on as many frequencies as available, in order to achieve the highest redundancy in the PPP model. Doing so allows for robustness of the solution, and potentially improvements in performance, although ever-increasing the redundancy is not a goal in itself due to the law of diminishing returns.

4.4 Processing and analysis strategies

The model and algorithms described in the previous section are implemented in the in-house multi-constellation, multi-frequency York-PPP engine at the GNSS lab. In order to perform the analysis, a global set of stations is selected and observations from such stations are processed with the York-PPP engine. The map of stations used in the processing is shown in Figure 4.3. A total of 27 IGS stations are selected by making sure that they support three GPS frequencies, four Galileo frequencies, and that they support the recent BeiDou-3. Observation data from these stations are processed over one week between day 305 and day 311 of year 2021. The observation data has a rate of 30 seconds, and the data are split into three hour long datasets that are processed individually in kinematic mode without assuming that they are static. The total number of 3-hour datasets being processed is 1448, which is considered a high enough number to be able to draw conclusions on the performance of each processing strategy. The parameters and strategies of the estimation of various states are summarized in Table 4.2.

The frequencies selected for the processing are:

- GPS: L1 (1575.42 MHz), L2 (1227.60 MHz), and L5 (1176.45 MHz),
- GLONASS: G1 ($1602 + \frac{9}{16}k$ MHz), and G2 ($1246 + \frac{7}{16}k$ MHz),
- Galileo: E1 (1575.42 MHz), E5b (1207.140 MHz), E5a (1176.45 MHz), and E6 (1278.75 MHz),
- BeiDou-2: B1-2 (1561.098 MHz), B2b (1207.140 MHz), and B3 (1268.52 MHz),
- BeiDou-3: B1 (1575.42 MHz), B2a (1176.45 MHz), and B2b (1207.140 MHz).

In order to perform PPP-AR, satellite products are required. In this work, rapid products are used, with satellite orbits and clocks being provided by GeoForschungsZentrum (GFZ) (Deng et al., 2017), and the compatible satellite code and phase biases being provided by Centre National d’Etudes Spatiales (CNES) (Laurichesse and Blot, 2016). The products are provided in RINEX format, with the code and phase biases being available in an OSB format, as such format simplifies the use of satellite biases at the user side, as opposed to differential biases. The processing is performed in simulated real-time. Satellite and receiver antenna corrections are corrected for using the IGS14 ANTEX file (Schmid et al., 2016). For the signals for which antenna corrections are available in the ANTEX file, the corrections are applied. For satellites with missing antenna corrections, corrections from nearby available signals are used. For instance, as GPS L5 antenna corrections are missing from the ANTEX file, GPS L2 antenna corrections are used, assuming that they should be similar to the L5 corrections. A similar issue exists with BeiDou-3, as ANTEX corrections

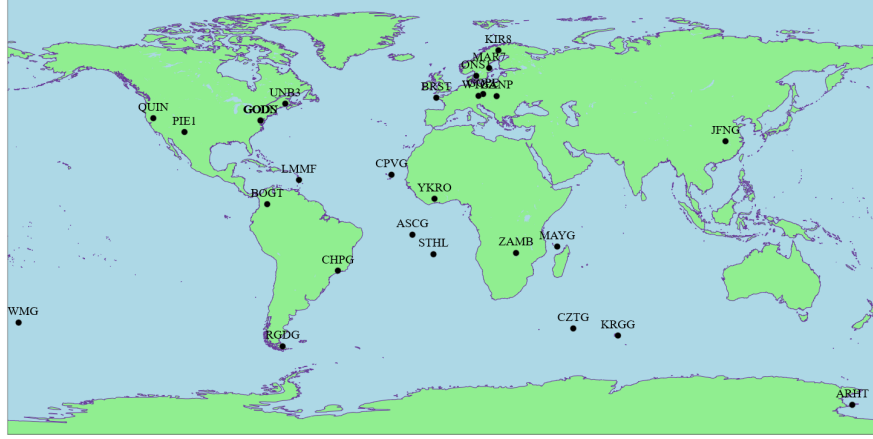


Figure 4.3: Map of the stations used in the processing.

Table 4.2: Processing strategy for the estimated parameters.

Parameter	Strategy
Receiver coordinates	Estimated with process noise equivalent to 100 km/h
Receiver reference coordinates	IGS SINEX positions
$\overline{dt_r}$ and $\overline{\delta t_r}$	Estimated as white noise processes
$\delta t_{12}, \delta t_{13}, \delta t_{14}, IFB_{r,3}$, and $IFB_{r,4}$	Estimated as white noise processes
Tropospheric delay	Dry: GMF model and mapping function (Kouba, 2009). Wet: estimated as a random walk process with process noise of $0.05 \text{ mm}/\sqrt{h}$
Ionospheric delays	Estimated as white noise processes
Ambiguities	Estimated as constants on each continuous arc
Elevation angle cut-off	7°
Weighting strategy	Elevation dependent weighting: $\sigma = \frac{\sigma_{90}}{a+b \sin el}$ with σ_{90} equal to 0.1 m and 0.001 m for the pseudorange and carrier-phase measurements, respectively, and el being the elevation angle. a and b are determined based on a residual and measurement quality analysis and set to 0.15 and 0.85, respectively.

are not available for the BeiDou-3 B1 and B2a signals. B1-2 and B2b corrections are applied in that case, respectively, as the frequencies are relatively close.

In terms of measurement weighting, all pseudorange and carrier-phase measurements from the same satellite, respectively, are given the same weight regardless of frequency. Tuning the measurement weight further depending on measurement noise is intended as future work. Additionally, BeiDou measurements are given half the weight of other constellations. The reason behind this choice is given that BeiDou-3 is a relatively new constellation, and in order to avoid any unwanted behaviour either because of the quality of the products or the signals, giving less weight to the constellation ensures that it does not negatively impact the solution while still benefiting from the additional measurements. This 50% value is empirical based on processing experience. Other constellations are weighted similarly, with GLONASS's residual rejection thresholds being doubled to account for its inter-frequency biases. Future work will include a further investigation and fine-tuning of these weights. In addition to this stochastic modelling, due to the existence of inter-system biases between BeiDou-2 and BeiDou-3, the two generations are treated as two different constellations in the PPP engine, meaning different receiver states as well as reference satellites are chosen for each generation. Furthermore, the BeiDou-2 group delay variations are corrected for using the frequency- and elevation-dependent corrections provided by Wanninger and Beer (2015). Other corrections, such as the phase wind-up, relativistic effect and Earth rotation, are applied following the IERS conventions (Kouba and Mireault, 1998).

Ambiguity resolution is performed by using the Modified Least-squares AMBiguity Decorrelation Adjustment (MLAMBDA) method (Chang et al., 2005). The uncombined float ambiguities on each frequency are fed into the MLAMBDA function together with their covariances. Since the estimated ambiguities are already implicitly single-differenced, they do not contain any non-integer biases and additional differencing is therefore not required. The resulting fixed ambiguities from MLAMBDA are used to update the state terms, and a standard ratio test is used for validating the fixed ambiguities. Ambiguity resolution is performed on all the ambiguities, without selecting a subset, in order to show the impact of the additional frequencies on the success rate of the fixing. Partial ambiguity fixing will be considered as part of future work.

As part of the analysis, an epoch-by-epoch solution is also generated. This solution is similar to the other solutions presented in this chapter (and dissertation so far) in terms of processing strategy, except that the filter is reset every epoch. This way, information from previous epochs is not carried forward to following epochs, making the filter behave like a Single Point Positioning (SPP) solution, but with additional measurement processing enhancements compared to SPP.

In terms of ionospheric activity during the period of processing, Figure 4.4 shows the hourly average Kp and f10.7 indices, which were retrieved from <https://omniweb.gsfc.nasa.gov>.

gov/form/dx1.html. The Kp index characterizes the magnitude of geomagnetic storms and is used to describe the disturbances in the Earth’s magnetic field. Its values range between 0 and 9. The figure shows that, for most days, the index is below 4, showing quiet to unsettled conditions, except for a one-day period, where the Kp index reaches 7 indicating a strong geomagnetic storm taking place. The f10.7 index on the other hand is an indicator of solar activity, and takes values ranging from 50 to above 400. Given that the period of interest is during a period of low solar activity, the f10.7 index is relatively low.

4.5 Position performance analysis

The datasets described in Section 4.4 are processed using the algorithms in Section 4.2, following the strategies discussed in Section 4.4. The first subsection focuses on analyzing the overall performance of the proposed algorithms in a typical PPP analysis. This analysis is done by looking at rms and convergence time metrics and comparing them between different frequency modes and processing modes, as well as by comparing them to the literature. The second subsection focuses on drawing comparisons between the proposed algorithms and RTK in terms of performance. This comparison is done by using stricter comparison thresholds equivalent to those that could be used in RTK, as well as by analyzing the performance of an epoch-by-epoch PPP solution with all the measurement enhancements and determining its performance compared to the expected RTK performance.

4.5.1 Average solution performance

This subsection analyzes the average performance of the flexible multi-constellation multi-frequency PPP-AR solution using 2-, 2/3-, and 2/3/4-frequency measurements. Both float and fixed solutions are analyzed. The datasets described in Section 4.4 are first processed in their entirety without any pre-filtering or outlier removal. Since no outliers were removed in the processing, rather than only assessing the average across all datasets, which would contain outlier results, the average solutions based on three different percentiles are analyzed (100^{th} , 95^{th} and 67^{th}). The results from averaging out positioning errors from all the datasets described in Section 4.4 are summarized in Figure 4.5. Given the research’s goal of achieving centimetre-level positioning near-instantaneously, the initial period of processing is the period of interest with a focus on sub-decimetre accuracies. Thus, the figure only shows the first 20 minutes of processing and up to 20 cm error.

Figure 4.5 shows the average horizontal and vertical error time series from all 1448 datasets at the 100^{th} , 95^{th} (2σ), and 67^{th} (1σ) percentiles. Both float and fixed (with and without ambiguity resolution, respectively) solutions are shown for each frequency combination, although only one float solution is shown regardless of the frequency combination. The reason for only showing the performance of one float solution is that the additional frequen-

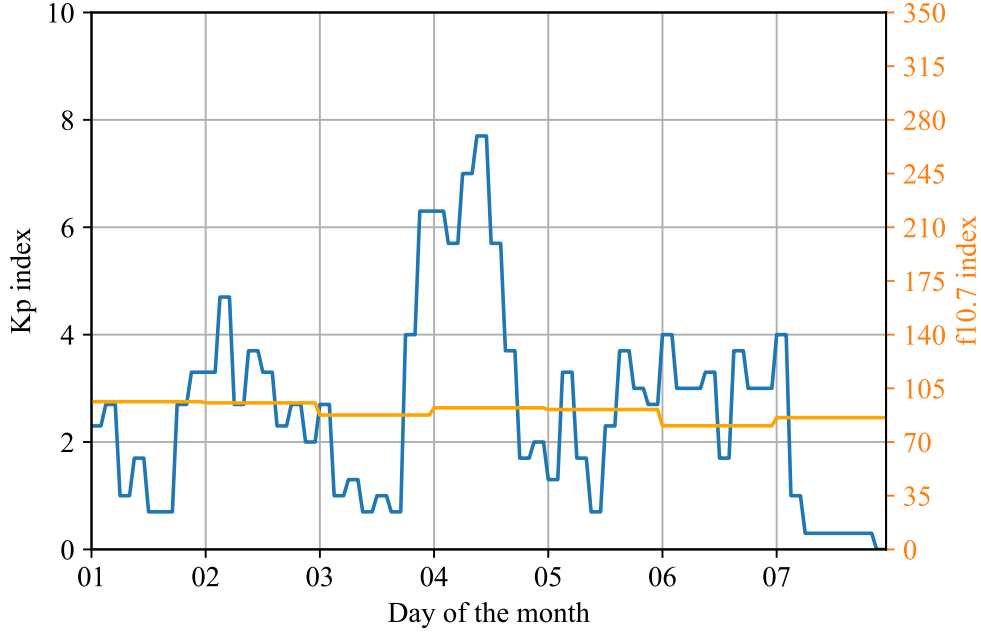


Figure 4.4: Kp and f10.7 indices for the period of processing, on November 2021.

cies do not have an impact on the float solutions, as shown by Naciri and Bisnath (2021a,b); Psychas et al. (2021); Geng et al. (2020), as the additional frequencies do not improve, e.g., the satellite geometry. In contrast, the additional frequencies have a noticeable impact on the fixed solutions at all percentiles due to the fact that adding frequencies leads to correct fixing of ambiguities earlier in the processing. Indeed, adding more frequencies not only reduces convergence time, but it also reduces initial errors, as the initial errors of the dual-, triple, and quadruple-frequency solution (referred to as 'all freq.' in the figure) reach horizontal values as low as 12 cm at the 100th percentile, 8 cm at the 95th percentile, and 1 cm at the 67th percentile, respectively. These initial values are significantly lower compared to the other frequency combinations, as well as the float solution. This reduction in initial errors has been seen in the literature, for example by Li et al. (2020b), which saw improvements up to 42% with a five-frequency solution compared to three- and four-frequency solutions when fixing widelane and extra-widelane ambiguities for Galileo. As a consequence of the decrease in position errors with the addition of frequencies, both convergence time and rms errors are also reduced with the addition of more frequencies, as shown in Figure 4.6.

Figure 4.6 summarizes the statistics for the results shown in Figure 4.5. The convergence time in the figure is defined as the time it takes to reach and stay below 10 cm horizontal error until the dataset's last epoch of processing. First, similar performance can be noticed for all float solutions, whether in terms of convergence time or rms. This behaviour is expected

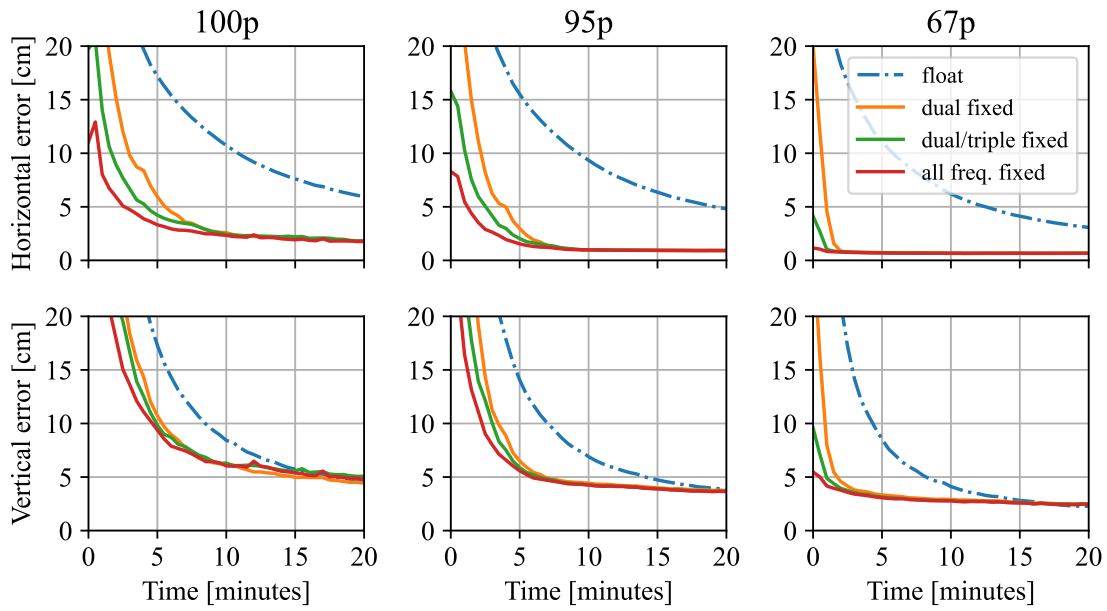
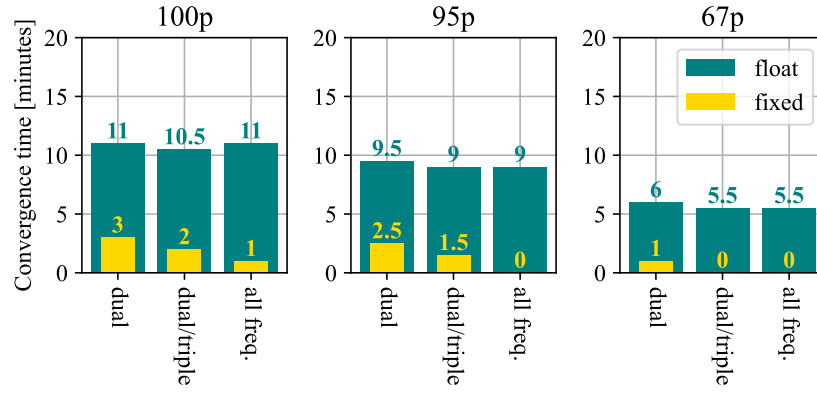
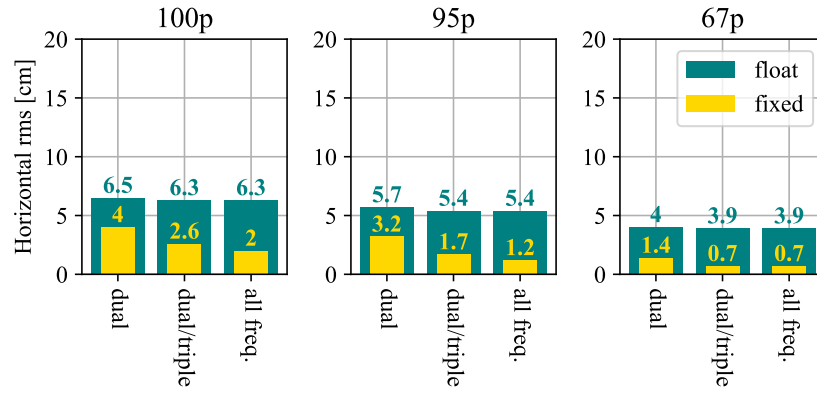


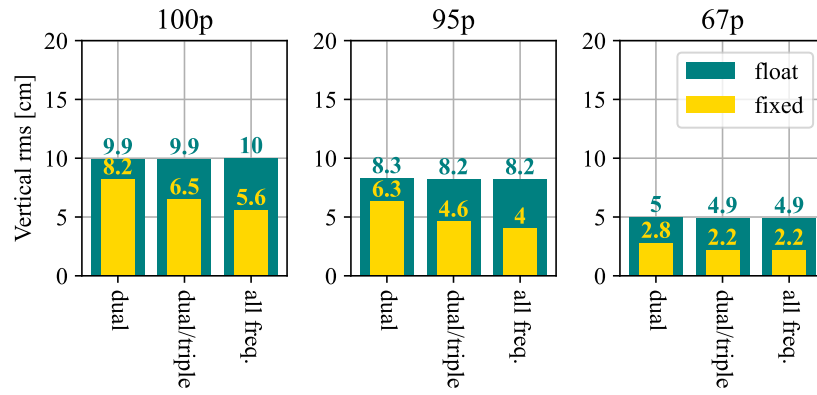
Figure 4.5: Time series of the average horizontal and vertical errors at the 100th, 95th, and 67th percentiles for the dual-frequency, dual- and triple-frequency and all-frequency fixed, and dual-frequency float solutions. The figure focuses on the first 20 minutes of processing and on the decimetre-level errors. The Xth percentile is computed by taking, for each epoch, the average of the X% datasets with the lowest error, with $X \in \{100, 95, 67\}$.



(a) Convergence time.



(b) Horizontal rms.



(c) Vertical rms.

Figure 4.6: Average 100th, 95th, and 67th percentile (a) convergence times to 10 cm horizontal error, (b) horizontal rms, and (c) vertical rms for the dual-, dual- and triple-, and all-frequency solutions, with and without AR. These statistics correspond to the time series in Figure 4.5.

as, as was discussed, the additional frequencies have limited effect on the float solutions, and most of their effect is in improving ambiguity fixing. It can be noticed that fixing ambiguities leads to a 73% improvement in convergence time for the dual-frequency solution at the 100th percentile. This significant decrease in convergence time demonstrates the importance of ambiguity resolution in improving PPP performance and in having PPP perform comparably to RTK. The significant decrease is emphasized when adding more frequencies to the processing, as adding one additional frequency decreases the convergence time by one minute to reach 2 minutes, and adding two frequencies decreases the convergence time even more to reach 1 minute. Note that the measurement rate is 30 seconds, therefore the statistics are in steps of 30 seconds as well. These convergence improvements are reached by looking at the average of all datasets. At both the 95th and 67th percentiles, the all-frequency solution converges instantaneously, with convergence time values of zero, i.e., convergence with the very first epoch of measurements. The improvements from the additional frequencies are attributed to the fact that more ambiguities per satellite are included in the fixing process. Given that the ambiguities from the same satellite are correlated, including more ambiguities provides more information to MLAMBDA, which enables improved ambiguity fixing, and therefore provides improved PPP-AR performance (Geng and Bock, 2013; Naciri and Bisnath, 2021a,b). These results show the possibility of instantaneous few centimetre-level performance with PPP without the use of local augmentation. This possibility has also been shown by Laurichesse and Banville (2018), where dual-frequency GPS and four-frequency Galileo measurements have been used to demonstrate that instantaneous centimetre-level positioning can be achieved using only global products. The authors demonstrated such on a limited sample size using only one receiver in Australia over one hour, and made use of Global Ionospheric Map (GIM) as a priori constraints for the ionosphere. In the current chapter, the study is broadened to more signals and constellations, and it demonstrates the performance for a global distribution of receivers and its consistency across multiple stations.

Similar conclusions can be reached when looking at the rms statistics, in the sense that the rms is comparable between all float solutions, that ambiguity resolution improves the rms, and that adding more frequencies decreases the rms. It is interesting to note that the all-frequency solution has an overall horizontal rms approximately half that of a dual-frequency solution. This significant decrease is attributed to the errors at the initial stages of processing which are decreased with additional frequencies, leading to a smaller overall rms, as well as to a reduced convergence time. As expected, ambiguity resolution has less of an effect on the vertical component than it does on the horizontal component, due to the vertical component being harder to estimate in general as a consequence of the satellite distribution around the antenna.

In order to better understand the performance shown in Figures 4.5 and 4.6, Figure 4.7 highlights the 95th percentile performance for each individual station. Each bar is based on all seven days of processing from each station and is computed for each station as the convergence time of the solution that is generated by taking the epoch-by-epoch average of the 95% lowest horizontal errors. The performance metric of interest here is the convergence time to 10 cm horizontal error.

Figure 4.7 corroborates the observations from the overall average results. Some variation in performance between stations is noticeable when it comes to the float solution, as stations converge within 6.5 to 15 minutes. These results are in accordance with the previous result, as the average float solution regardless of station converges within 9.5 minutes, falling in between the individual station maximum and minimum performances. Fixing ambiguities on two frequencies leads to a noticeable decrease, as most stations converge within less than 5 minutes, and many converge within 2 minutes or less, which is the average convergence time based on all datasets shown in Figure 4.6. Finally, fixing ambiguities on up to four frequencies shows the possibility of instantaneous convergence for many stations, as 63% of stations converge instantaneously, and 89% within one minute or less. This means that only three stations require more than one minute (three epochs) to converge. This promising result is in line with the average results in Figure 4.6, as the average solution converged instantaneously. The dual- and triple-frequency results have been omitted in the figure in order to keep the clarity of the figure. The results from the mixing of dual- and triple-frequency data fall in between the yellow and red bars in the figure, as expected. The conclusion can be drawn that modern PPP-AR can be used for applications requiring sub-decimetre-level horizontal positioning (95%), near-instantaneous positioning/navigation.

4.5.2 Benchmarking against RTK-quality

This subsection performs a more rigorous comparison against RTK-like performance through: 1) a more strict convergence threshold, and 2) the analysis of an epoch-by-epoch solution and the comparison of its performance with RTK-like requirements. Figure 4.8 summarizes the performance for each station by looking at the convergence time to 2.5 cm rather than 10 cm and 67th percentile rather than 95th, as was done for Figure 4.7. The reason behind this choice is to analyze the possibility of achieving the (N)RTK-like requirement of near-instantaneous 2.5 cm horizontal error at 1σ (Bisnath et al., 2013).

Figure 4.8 shows that the instantaneous convergence to 2.5 cm requirement that was set is reached for many stations. Indeed, while all float solutions require at least 16.5 minutes to reach 2.5 cm for each station, both the dual-frequency, and dual-, triple-, and quadruple-frequency (referred to as 'all freq.') fixed solutions see noticeable improvements, as 81% of stations converge instantaneously with all frequencies, compared to only 7% for the

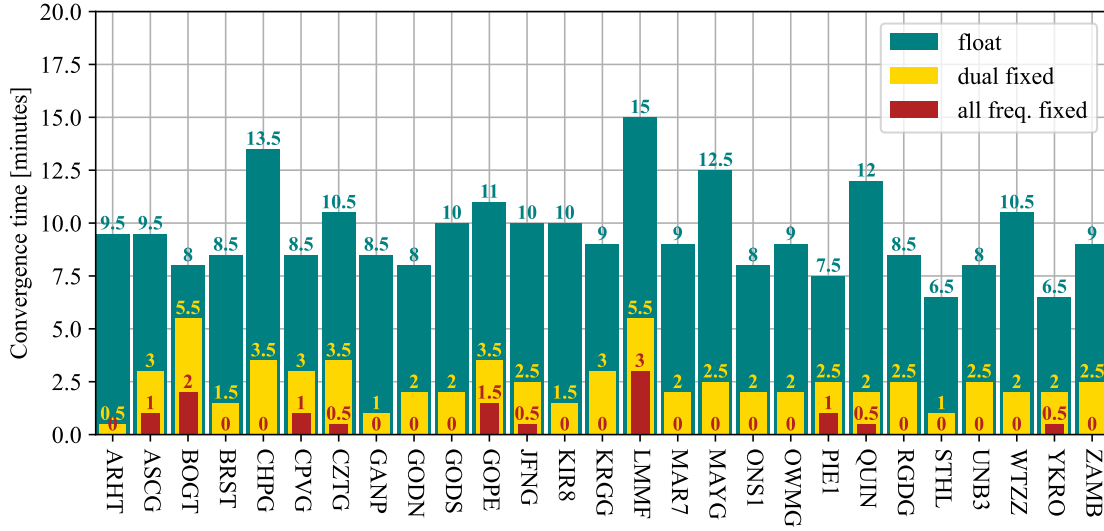


Figure 4.7: 95th percentile average convergence times to 10 cm per station. Each bar is computed by 1) computing the convergence times of all datasets, 2) only keeping the 95% smallest convergence times, and 3) computing the mean of these smallest convergence times.

fixed dual-frequency solutions. Furthermore, 89% of all-frequency stations converge within a minute, compared to 37% for the fixed dual-frequency solutions. These improvements show the importance of the additional frequencies to reach the few centimetre level of errors and maintain that level of accuracy: while fixing ambiguities on two frequencies helps tremendously in achieving centimetre positioning quickly, more frequencies allow for more reliable fixing in the initial stages of the processing, and therefore instantaneous centimetre-level positioning.

In order to further compare multi-constellation, multi-frequency PPP-AR performance to RTK-like performance, the proposed model is tested through an epoch-by-epoch filter. PPP has traditionally used sequential estimation filters, where information from previous epochs is used in following epochs. The reason behind the use of such sequential filters is to introduce the constraint of the ambiguities being constant, and to use previous information and satellite movement to help estimate parameters more accurately. However, given the results shown in Figure 4.8, where instantaneous centimetre-level positioning is achievable, it is interesting to look at the possibility of an epoch-by-epoch solution, where each epoch is treated independently and previous information is not used. Such a solution would rely on the additional constellations and frequencies to fix ambiguities correctly instantaneously to achieve the required accuracies. Similar work was done by Geng and Guo (2020), where EWL and WL ambiguities are fixed for Galileo and BeiDou-3 and single-epoch sub-decimetre positioning is sought. The novelty in this work goes further by reaching for centimetre-

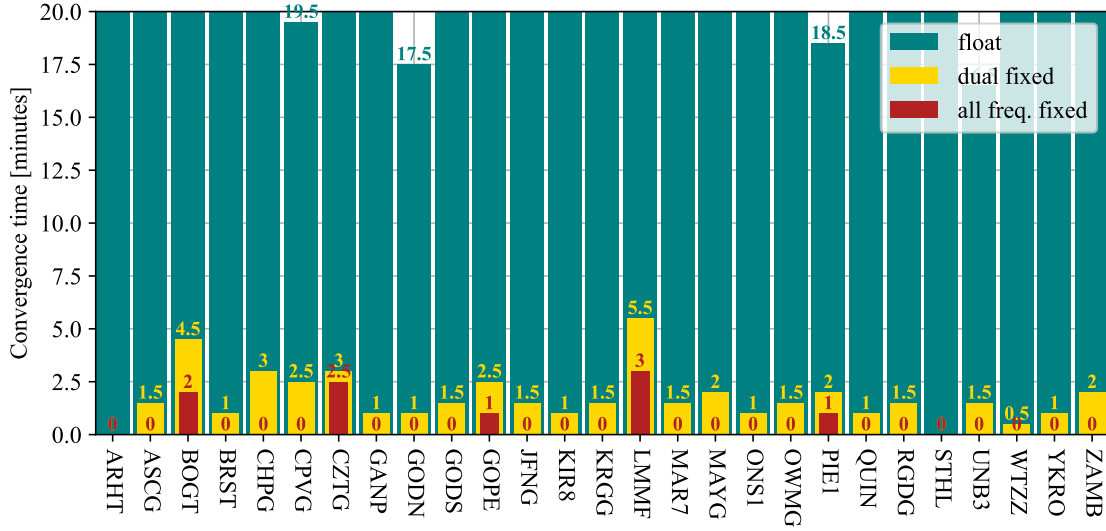
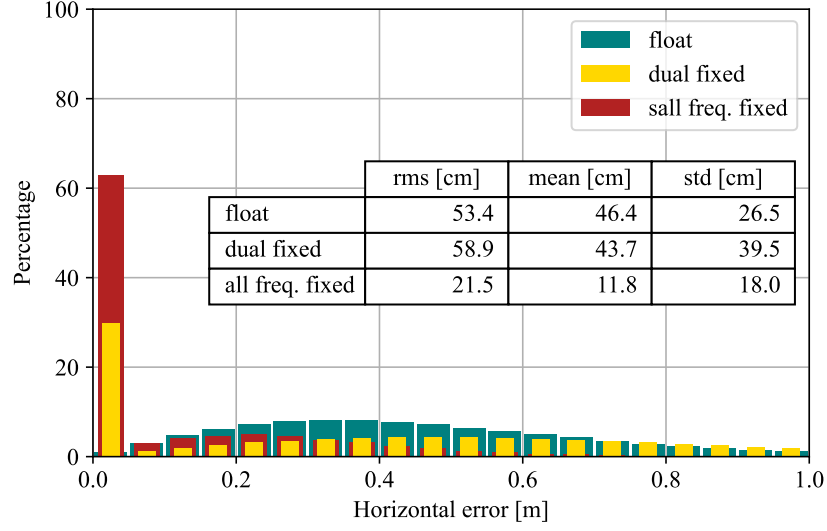


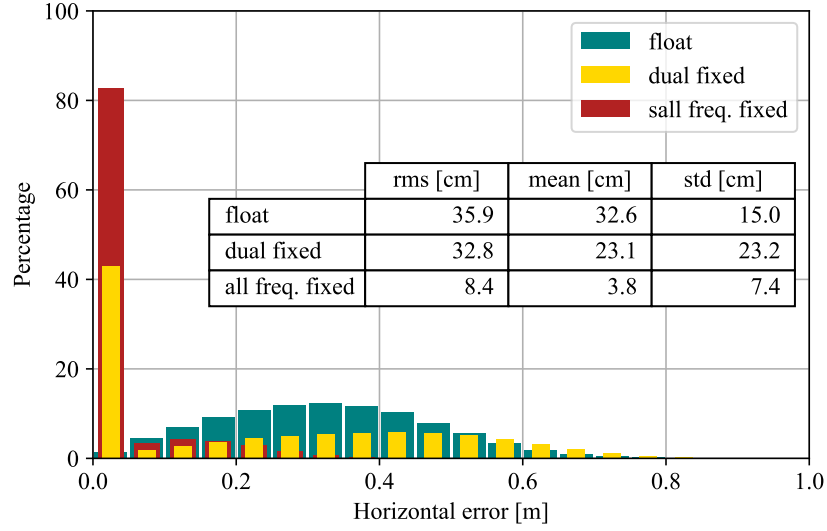
Figure 4.8: 67th percentile average convergence times to 2.5 cm per station. Each bar is computed by 1) computing the convergence times of all datasets, 2) only keeping the 67% shortest convergence times, and 3) computing the mean of these shortest convergence times.

level positioning by fixing all ambiguities (up to four) for GPS, Galileo and BeiDou-2/3. Having such single-epoch precise solutions would mean more robust PPP, as previous faulty epochs would not be used in current epochs in case of, e.g., undetected cycle-slips, or high multipath. The distribution of horizontal errors from such epoch-by-epoch processing is shown in Figure 4.9 at both the 100th and 67th percentiles.

The results from Figure 4.9a show that single-epoch, decimetre-level positioning is a possibility when fixing ambiguities on as many frequencies and constellations as are available. The mean horizontal error for the float dual-frequency results is 46 cm, compared to 44 cm for the fixed dual-frequency results, and 12 cm for the fixed all-frequency results. These mean values, along with the shape of the graphs, show that single-epoch fixing in dual-frequency mode does not lead to significant improvements in performance. In fact, the fixing in dual-frequency mode appears to degrade many of the float solutions, as the means are comparable but the standard deviation of the fixed results is 40 cm, compared to 26 cm for the float result. Therefore, single-epoch processing in dual-frequency mode is not an option to achieve higher precision positioning, as fixing ambiguities in a single epoch is a tough task to perform. However, as seen in the previous subsection, adding more frequencies helps achieve more reliable fixing, which is the primary conclusion from Figure 4.9. Indeed, the mean of the all-frequency, fixed distribution is approximately four times lower than the float and dual-frequency fixed solutions, with a standard deviation of 18 cm, which is also smaller. This improvement is highlighted by the fact that 96% of all epochs are below half



(a) 100th percentile distribution.



(b) 67th percentile distribution.

Figure 4.9: (a) 100th and (b) 67th percentile distributions of the epoch-by-epoch horizontal errors using all datasets. Each bin corresponds to 5 cm. The i th percentile distribution, with $i \in \{100, 67\}$, means that the i % epochs with the lowest horizontal errors are plotted.

a metre but, more importantly, 66% and 61% of all epochs are below a decimetre and 2.5 cm, respectively. This performance is noticeably better than the other solutions and it is attributed to the benefits of the additional frequencies in more reliable fixing.

The results in Figure 4.9a are based on all epochs of processing from all stations. Focusing on the 67th percentile (1σ) leads to the results in Figure 4.9b, where the benefits on the all-frequency solution are clear. Conclusions from Figure 4.9a still hold, as fixing ambiguities on two frequencies still leads to worse performance for many epochs, and the all-frequency solution has lower mean and standard deviation. In this 1σ case, 99.9% of epochs are below half a metre, 86% are below a decimetre, and 81% are below 2.5 cm. These results show PPP being able to achieve RTK-like performance, i.e., centimetre-level epoch-by-epoch positioning, when it comes to an epoch-by-epoch solution. These results are in agreement with and expand on the results shown by Laurichesse and Banville (2018), where centimetre-level solutions were achieved with dual-frequency GPS and quadruple-frequency Galileo in an epoch-by-epoch solution on a sample dataset, and where the prediction that adding more constellations and frequencies will help make that performance a reality for more datasets is made. The prediction is realized in this work, where performance better than 2.5 cm is achieved for 81% of epochs (1σ) in single-epoch processing when using as many as four frequencies and constellations. Such performance is believed to be comparable to typical performance that RTK is capable of achieving, i.e., epoch-by-epoch centimetre-positioning, and a tremendous improvement compared to typical dual-frequency PPP results.

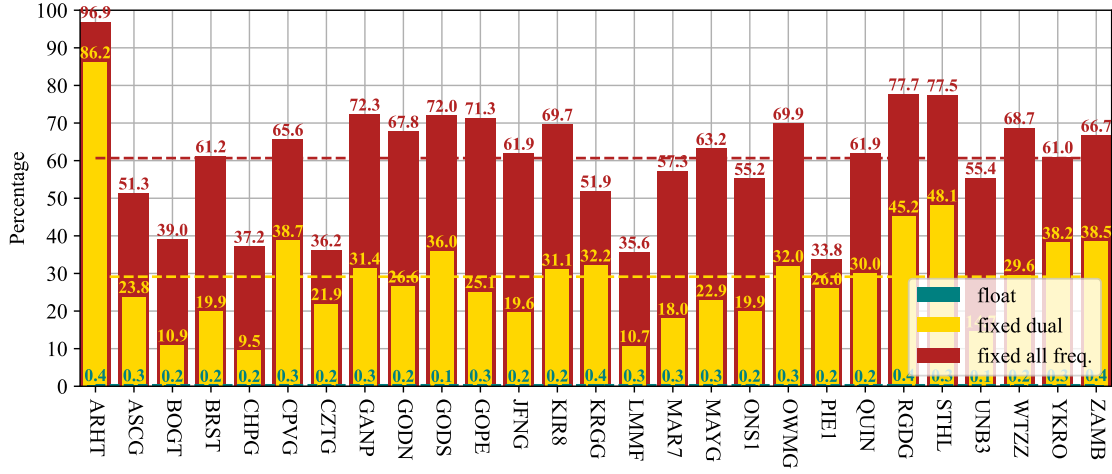
In Figure 4.9a, the all-frequency solution has a spike at the leftmost side of the x-axis, followed by a Gaussian curve with a smaller magnitude. In order to understand this behaviour, and to investigate further, the station-specific percentage of epochs below 2.5 cm is plotted in Figure 4.10. Both the 100th and 67th percentiles are included on different subfigures. Figure 4.10a shows the percentage of epochs from each station that have a horizontal error below 2.5 cm. Each bar is based on one week's worth of station data. The metric of horizontal errors below 2.5 cm in each epoch is equivalent to typical performance expected from RTK, where a high percentage of epochs is expected to reach such accuracies instantaneously. For instance, Bock et al. (2000) showed that 16.8% of epochs fall outside of the 5.7 cm, 4.3 cm, and 38.4 cm outlier bounds in North, East and Up, respectively, on a 37 km baseline, meaning that 83.2% epochs are within the defined intervals. Figure 4.10 shows that it is highly unlikely to reach instantaneous centimetre-level accuracy with float solutions, as only a maximum of 0.4% of epochs has an error below 2.5 cm for all stations. Ambiguity resolution significantly increases the percentage of stations below 2.5 cm in dual-frequency mode, as stations have 29% of epochs below 2.5 cm on average, with a minimum of 9.5% of epochs for station CHPG and a maximum of 86.2% of epochs for station ARHT. The results are improved further when adding all frequencies, as the mean number of epochs

jumps to 61%, with the minimum percentage of converged epochs reached by station PIE1 with 33.8%, and a maximum of 96.9% reached by station ARHT.

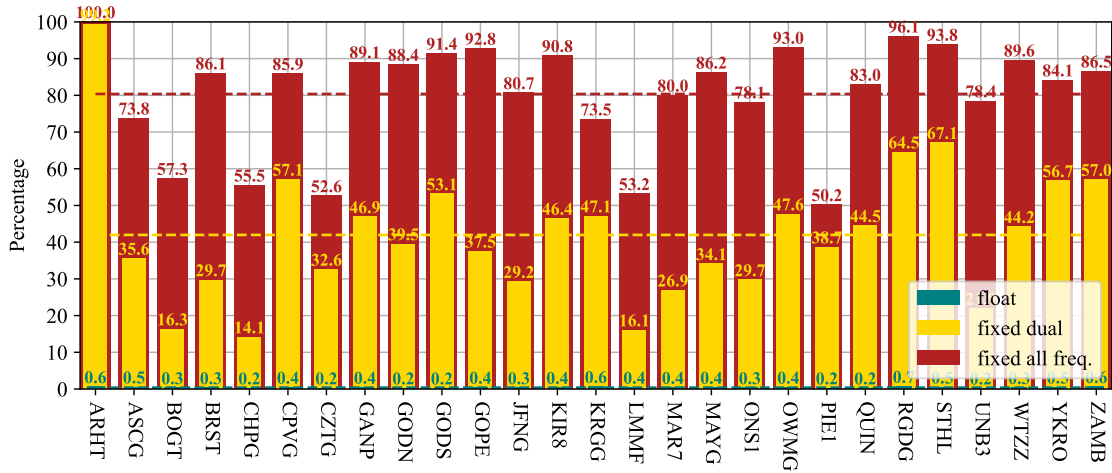
The average number of epochs below 2.5 cm for both dual-frequency and dual-, triple-, and quadruple-frequency results corroborate the results in Figure 4.9a, as approximately 30% of epochs are below 2.5 cm in dual-frequency mode, compared to approximately 60% with all frequencies. Figure 4.10a can therefore help understand the distribution in Figure 4.9a. One noticeable point is the very good performance from station ARHT, for which 97% of epochs are below 2.5 cm. Many of this station's epochs are driving the initial peak in Figure 4.9. However, this station's epochs are not the only driver, as many other stations have over 60% of their epochs below the strict threshold.

Looking at the 67th percentile performance in Figure 4.10b helps gain further insight into the results, as it eliminates some of the worse-performing epochs. The figure shows the impressive performance that is achievable by fixing ambiguities on as many frequencies from as many constellations as are available. The results show that many stations (18 out of 27) are able to have more than 80% of epochs within 2.5 cm error using only a single epoch of data. On average across all stations, 80.4% of epochs are below 2.5 cm, which is in accordance with the results from Figure 4.9b. These results are impressive, as they show the level of single-epoch performance that can be achieved by adding more frequencies, together with fixing ambiguities on all the additional frequencies. These results show that it is possible to achieve instantaneous single-epoch precise positioning. However, one can see that certain stations perform better than others. In order to provide further insight into the difference in performance between stations, Figure 4.11 shows the time series of ambiguity-fixed horizontal errors for three stations with varied performance in Figure 4.10: station ARHT, which is representative of a well-performing station; station GOPE, which is representative of an average-performing station; and station BOGT, which is representative of a bad-performing station. It should be noted that the percentages in Figure 4.11 are based on all epochs and correspond to the 100th percentile percentages, and that all three solutions correspond to the "all-frequency" processing discussed in previous figures. The figure shows that, for stations ARHT and GOPE, more than 90% of epochs have their ambiguities fixed correctly and the solution converging every epoch, despite the epochs being processed independently. Additionally, the same percentages can be seen for both the 10 cm and 2.5 cm thresholds due to the fact that fixing the ambiguities correctly leads to centimetre-level positioning. On the other hand, station BOGT has many incorrect fixes, as only 49.7% of epochs are below one decimetre, while the rest of the epochs are at a few decimetres, which is similar to a single point positioning solution, as information from previous epochs is not used.

Looking at the results in Figures 4.10 and 4.11, two questions can be raised:



(a) 100th percentile performance.



(b) 67th percentile performance.

Figure 4.10: Percentage of epochs with horizontal errors below 2.5 cm per station in the epoch-by-epoch solutions at the (a) 100th and (b) 67th percentiles. Each bar represents the percentage of a station's epochs below 2.5 cm horizontal error over one week. At the 67th percentile, each bar is based on the 67% epochs with the lowest horizontal errors. The horizontal dashed lines represent the average across all stations.

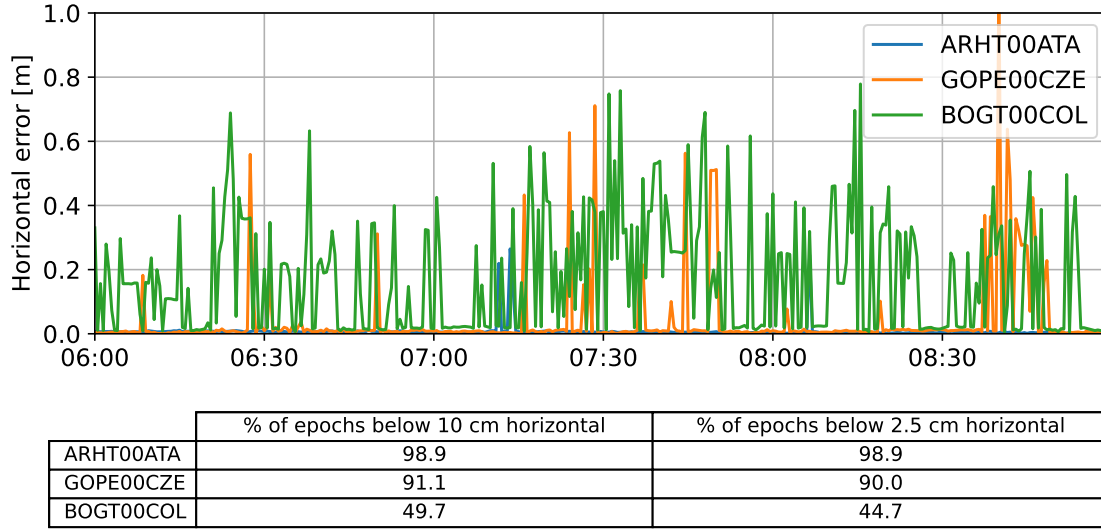


Figure 4.11: Time series of the multi-constellation all-frequency ambiguity-fixed epoch-by-epoch solutions of one well-performing station (ARHT00ATA), one average-performing station (GOPE00CZE), and one bad-performing station (BOGT00COL) on day 305 of year 2021 between 6 AM and 9 AM UTC. The table shows the percentage of epochs (based on all epochs over three hours) which are below 10 cm and 2.5 centimetre errors.

- What else is needed to increase the average number of epochs within 2.5 cm accuracy from 80% to higher percentages?
- Why do some stations behave worse than others?

In the following section, a discussion and elements of responses to the above questions are provided.

4.5.3 Analysis and improvement of single-epoch results

This subsection starts with the exploration of some avenues that could lead to improvements in the epoch-by-epoch solution performance, followed by the analysis of some causes for the discrepancy in performance between stations.

4.5.3.1 One-, two-, and three-epoch PPP

Figure 4.10 shows that, at the 67th percentile, on average, 80% of epochs have an accuracy better than 2.5 cm. While this performance is outstanding and a tremendous improvement compared to other performance, it begs the question of what needs to be done to increase the percentage to values close to 95% and higher. The previous section has been focused on analyzing an epoch-by-epoch solution, where each epoch is processed independently from the

previous ones. However, in some applications, including one or two more epochs of processing could still be acceptable, which is equivalent to one minute of data in a three-epoch solution. Therefore, Figure 4.12 shows the average performance that is achieved by each station when processing every epoch independently (*one epoch*), every two epochs independently (*two epochs*), and every three epochs independently (*three epochs*). The single-epoch solution is the same as the one showed in Figure 4.10b. The reason behind including additional epochs is that the sequential processing can help improve some of the epochs where the position error is higher than 2.5 cm, as information from previous epochs is used to improve the second epoch's and third epoch's positions in a two-epoch and three-epoch solution, respectively.

The figure shows that, as expected, improvements in performance are noticed when including more epochs in the processing. The average percentage of epochs below 2.5 cm error across all stations is 80%, 87%, and 90% for the single-epoch, two-epoch, and three-epoch solutions, respectively. It is important to note that the addition of a second epoch of processing has a more significant impact on worse-performing stations than it is on well-performing stations. For instance, station PIE1 improves from only 50.2% of epochs accurate to 2.5 cm to 67.2%, which is a significant improvement with the inclusion of only one additional epoch of information. Similar levels of improvements are noticeable for all stations with less than 80% of epochs below 2.5 cm error on an epoch-by-epoch solution. On the other hand, stations with more than 80% of epochs below 2.5 cm error on the epoch-by-epoch solution see less improvements with the additional epoch, as is the case, for example, for stations OWMG, ZAMB, and GODS.

With the inclusion of a third epoch, the level of improvement is lower than with the inclusion of the second epoch, as the average across all stations increases only by 3%. This lower impact of a third epoch can be expected due to the law of diminishing returns, as the information provided by each additional epoch would have less and less impact the more epochs are included in the processing. It should also be noted that, in a two- or three-epoch solution, the first epoch is the same between them and the single-epoch solution. Therefore, in a two-epoch solution, only the second epoch would be different, meaning that the solution on half the total number of epochs would be the same as a single-epoch solution. Similarly, in a three-epoch solution, the first two epochs are the same as in a two-epoch solution, meaning that only 33% of the total number of epochs would be different compared to the two-epoch solution. This analysis provides the performance at the station level. In order to assess the overall performance regardless of the station, Figure 4.13 shows the 67th percentile performance based on all epochs for the three solutions: processing every epoch independently, processing every two epochs independently, and processing every three epochs independently.

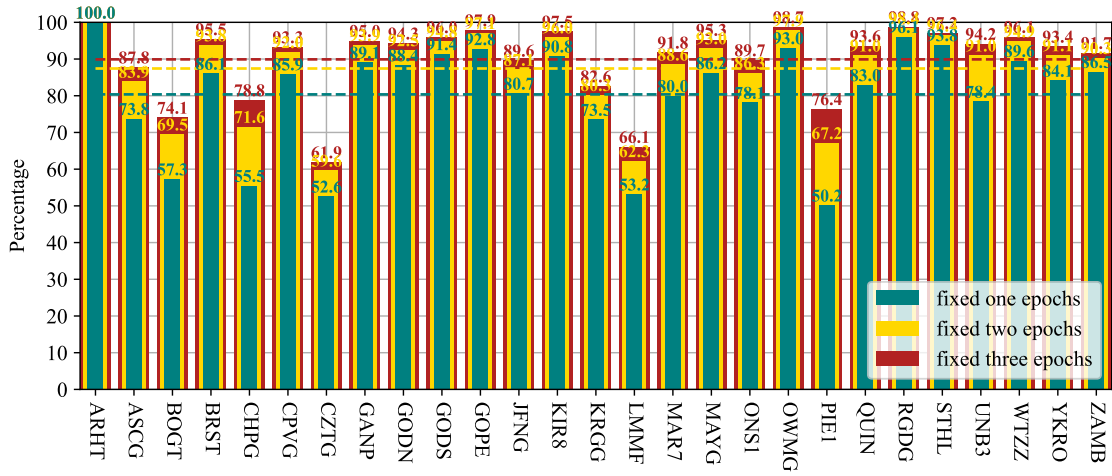


Figure 4.12: Percentage of epochs with horizontal errors below 2.5 cm per station at the 67th percentile for three solutions: processing each epoch, every two epochs, and every three epochs independently. All solutions use all frequencies on all constellations. Each bar represents the percentage of a station’s epochs below 2.5 cm horizontal error over one week. At the 67th percentile, each bar is based on the 67% epochs with the lowest horizontal errors. The horizontal dashed lines represent the average across all stations.

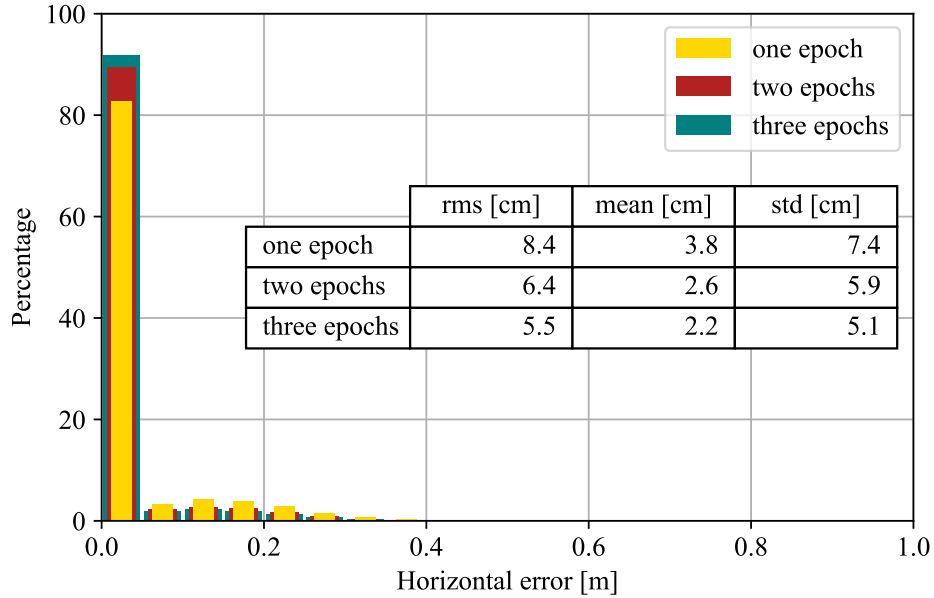


Figure 4.13: 67th percentile distributions of the one-, two-, and three-epoch solutions’ horizontal errors using all datasets. Each bin corresponds to 5 cm. The figure is based on the 67% epochs with the lowest horizontal errors.

Figure 4.13 corroborates the conclusions from Figure 4.12, especially in terms of the law of diminishing returns. Including more epochs in the processing leads to the leftmost peak at 5 cm error to be higher, with the increase being lower with the addition of the third epoch than it is with the second epoch. A difference in the sidelobe can be noticed as well, as the bars with errors > 5 cm decrease with the addition of epochs since the epochs with errors higher than 5 cm are improved and become part of the leftmost bar. This shift is confirmed by both the mean and standard deviation, which decrease with the addition of epochs. Therefore, this analysis shows that including one or two epochs in the processing allows to achieve near-instantaneous precise positioning without any local augmentation and to provide improvements to the single-epoch solutions. Unfortunately, solely including these additional epochs does not lead to percentages close to 95%, meaning that additional measures are required to further improve the solutions.

4.5.3.2 Augmentation with Global Ionospheric Map (GIM)

The purpose of this chapter is to achieve RTK-like performance using only products that would be available to users worldwide, without being constrained to a set location and without requiring local infrastructure / information. In addition to the satellite products used so far (precise satellite orbits, clocks, and code and phase biases), one set of global products that has not been used is Global Ionospheric Map (GIM). These maps consist of ionospheric corrections which are computed based on a global network of stations continuously tracking satellites. These products can either be predicted and disseminated prior to the expected time of their use, or generated based on GNSS measurements, therefore leading to latency in their dissemination in the case of rapid and final products (https://igs.org/products/#ionospheric_products). In this work, in order to assess the effect of such maps on the epoch-by-epoch performance, and whether it can further improve the performance, the predicted maps from CODE are used. The ionospheric corrections are used as pseudo-measurements, meaning that the estimation process is left the same and that the ionospheric corrections are treated as measurements in the estimation process. Given that they are treated as measurements, they need to be given proper weighting in the weight matrix. GIM corrections are known to be precise to approximately 2-10 total electron content units (TECU) with one TECU being equal to 0.163 m on the L1 signal approximately (Zhang et al., 2019b; Ho et al., 1997). In this work, the ionospheric measurements are given weight equivalent to one fourth of the pseudorange weight, with the weighting being elevation dependent, as shown in Table 4.2. Figure 4.14 shows the performance that is achieved by applying the predicted GIM corrections as pseudo-measurements to the epoch-by-epoch solution shown in the previous section.

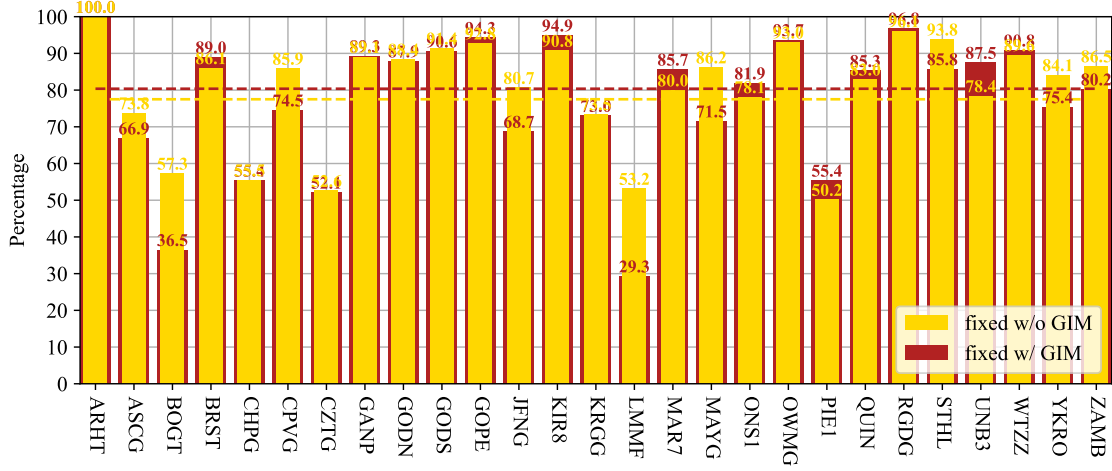


Figure 4.14: Percentage of epochs with a horizontal position error below 2.5 cm error at the 67th percentile with and without the use of Global Ionospheric Map (GIM).

Figure 4.14 shows the performance per station at the 67th percentile. Similarly to before, each bar graph is computed by taking, for each station, the 67% epochs with the lowest horizontal errors, and computing the percentage of those epochs below 2.5 cm horizontal error. The figure shows that, while some stations benefit from the inclusion of these ionospheric corrections, others suffer. For instance, stations such as BRST, KIR8, MAR7, and UNB3 see improvements in performance, while other stations remain relatively the same, such as stations WTZZ, CHPG, and GODS. On the other hand, many stations see degraded performance, for example stations BOGT, LMMF, and MAYG. This difference in performance could be due to the ionospheric corrections not being precise enough. As a consequence, if the ionospheric estimates in the multi-constellation, multi-frequency model without the use of GIM are more accurate than the ionospheric corrections introduced by GIM, it is expected that the position performance would decrease. Conversely, if the ionospheric estimates without GIM are less accurate than the GIM corrections, then performance is expected to be improved. Overall, the conclusion is that using GIM corrections do not lead to overall improvements of the solutions, as the average percentage across all stations decreases with the introduction of GIM. It is interesting to note that the effect is similar between stations in the same region. For example, stations BOGT and LMMF are in the same region, and they are subject to the same level of degradation. And stations PIE1 and QUIN are in the same region, and see the same level of marginal improvement. Similarly, stations BRST, KIR8, MAR7 and ONS1 are all in the same region and see similar levels of improvements. These similarities between stations in the same regions hint that the quality of the ionospheric corrections might not be the same across all regions. Therefore, one could attempt to im-

prove these results using more sophisticated stochastic modelling for GIM measurements to ensure that the weight given to each measurement better correlates with its precision.

4.5.3.3 Discrepancy between stations

From the discussions of Figures 4.10 and 4.11, two questions were raised, with the first question relating to improving the overall performance of the epoch-by-epoch solution, while the second one relates to the difference in performance between stations. Attempts at answering the first question were provided in the previous two subsections. In the current subsection, analysis will be presented to explain the different performance between stations.

First, the difference in performance could be due to a difference in the number of visible satellites and satellite geometry. Indeed, if, due to its location, a station "sees" less satellites than others, its performance is expected to be worse. Similarly, if a station is placed such that satellite signals from one direction are blocked, its satellite geometry (through the PDOP) is expected to be degraded, therefore decreasing performance. In order to assess such effect, the correlation plots between the average number of satellites / PDOP and the epoch-by-epoch performance are plotted for each station in Figure 4.15. Each dot is computed by taking the average across all epochs for each station. The percentages are the same as those in Figure 4.10.

Figure 4.15 shows that all stations see a minimum of 22 satellites on average, with the PDOP being lower than 1 for most stations. These values show good overall coverage and a sufficient number of satellites for PPP processing. It can be noticed that station ARHT for example has the higher number of satellites on average, with just above 36 satellites being processed on average, leading to an average PDOP close to 0.7. These values correlate with the performance shown in Figure 4.10, where ARHT is the best performing station. On the other hand, station CHPG, which is one of the worse performing stations, is also the station with the lowest number of satellites and higher PDOP. However, general correlation between number of satellites / PDOP and epoch-by-epoch position performance is hard to conclude. For instance, for station BOGT is one of the worse performing stations in Figure 4.10, despite having an average PDOP of 0.8. Therefore, while the number of satellites and satellite geometry could explain in part the difference in performance, they are not the sole reason for the discrepancy in performance.

Another possible reason for the discrepancy in performance between stations could be the type of receiver and antenna being used. For instance, a certain antenna type could lead to worse performance if, e.g., the receiver antenna corrections are not precise enough, etc. In order to test the presence or absence of correlation between the epoch-by-epoch performance and the receiver and antenna types, Figure 4.16 shows the same bar graphs

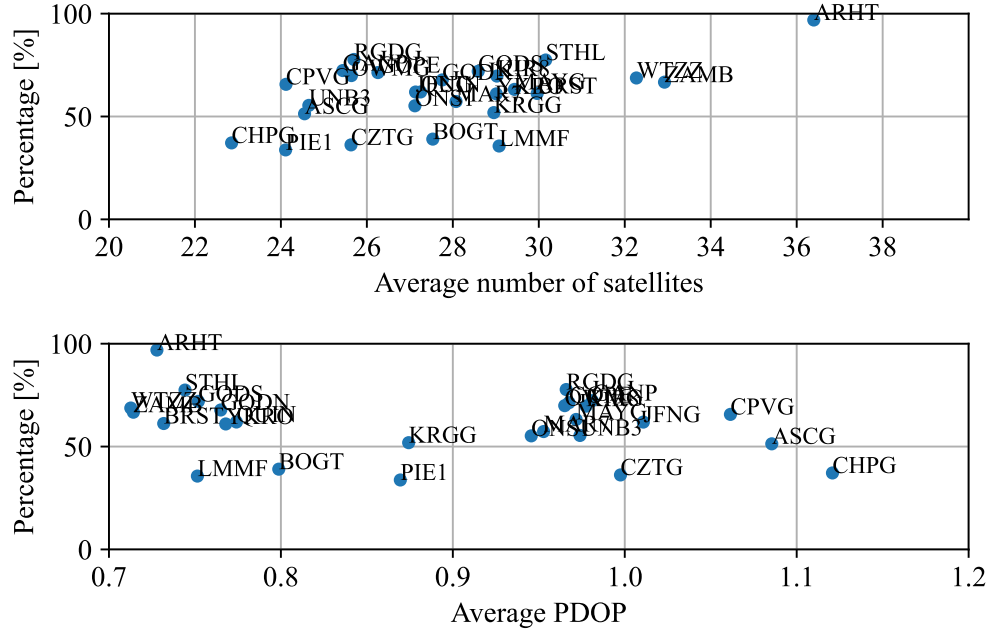
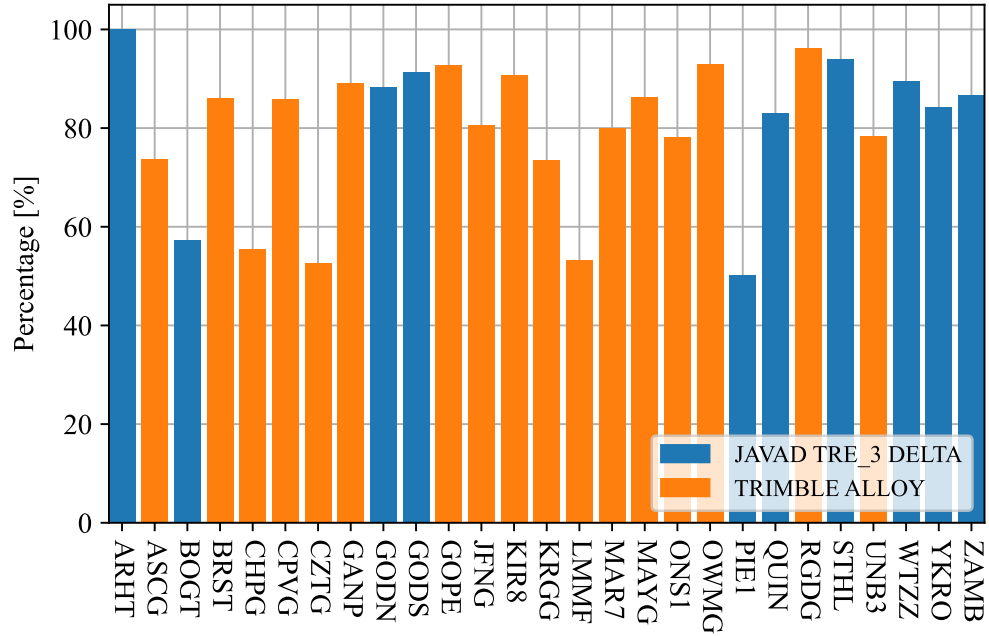


Figure 4.15: Correlation figure between the percentage of epochs below 2.5 cm errors at the 67th percentile in an epoch-by-epoch solution and the average number of satellites (top), and of PDOP (bottom). Each dot is generated based on all data for that station.

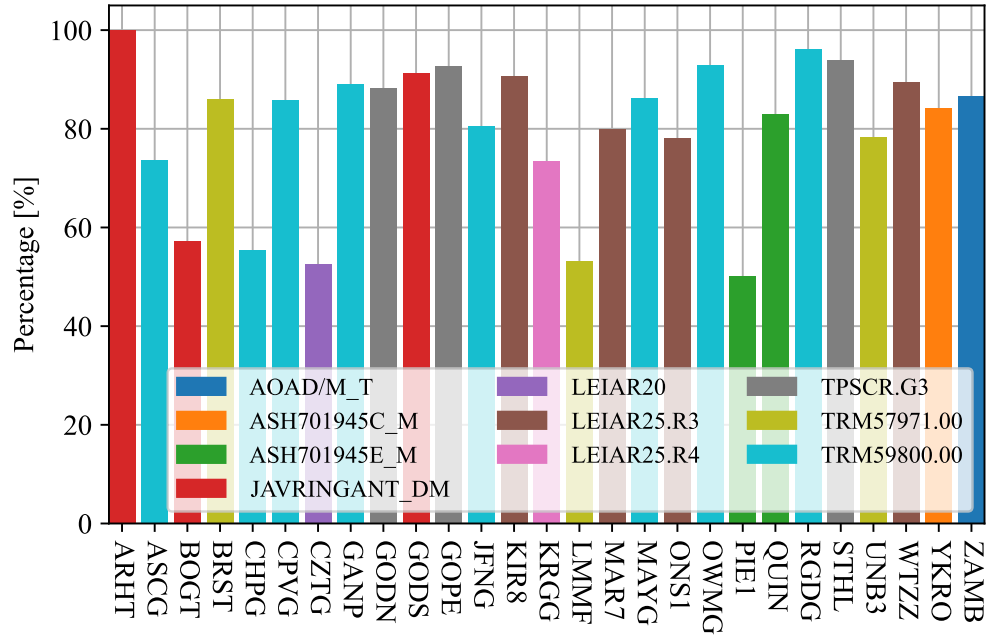
as in Figure 4.10b, except that the bars are colour-coded based on the receiver and the antenna types.

Analyzing the receiver type first, Figure 4.16 shows the absence of correlation between receiver type and the performance, as, for example, station ARHT has the best performance, while having the same receiver as stations BOGT and PIE1 with some of the worse performance. As a consequence, the difference in performance cannot be attributed to the difference in receiver types. Looking at the antenna type, the same conclusion can be held, although more types of antennas are used than receiver types. Looking at station ARHT again, it appears to have the same antenna type as station BOGT, while the latter has a worse performance than the former. Similarly, stations QUIN and PIE1 have the same antenna type, but different levels of performance. Therefore, the difference in performance cannot be attributed to the difference in antenna types either. Consequently, this analysis concludes that the difference in hardware is not the reason for the difference in performance between the stations, extending the mystery further.

One additional analysis that was performed to understand the station-specific performance was to look for correlation between the stations' locations on Earth and their performance. For that purpose, Figure 4.17 was generated by plotting the stations that were used in this work, and colour-coding their markers based on the 67th percentile epoch-by-epoch



(a) Correlation between performance and receiver type.



(b) Comparison between performance and antenna type.

Figure 4.16: Bar graphs of the 67th percentile performance of the epoch-by-epoch solutions. The percentage values are the same as in Figure 4.10b. Figure 4.16a is color-coded based on the receiver type, while Figure 4.16b is color-coded based on the receiver antenna type.

performance shown in Figure 4.10b. It is hard to draw a definite conclusion from Figure 4.17. Stations within the same region tend to have similar performance. For instance, three stations in Latin America/Caribbeans (BOGT, LMMG, and CHPG) have similar performance close to 50%, although the stations are thousands of kilometres away from each other. This performance could be due to, e.g., close proximity to the equator for example. However, stations in Africa around the same latitude have better performance (stations CPVG and YKRO with percentages of 80%). Similarly, stations RGDG, CZTG, and KRGG are on similar latitudes, but have different performance. Stations on similar longitudes have different performances as well. It appears that, while stations in the same region tend to have similar performance (stations in Europe, southwest Africa, Caribbeans, etc), there are many exceptions that lead to thinking that the geographical location of the station is not the sole reason for varied performance (QUIN and PIE1, CZTG and KRGG, etc).

To summarize, more research is needed to understand why some stations, e.g., ARHT, perform better than others, e.g., BOGT, in order to attempt to make the results from most stations similar to station ARHT. Doing so would lead the way to more robust PPP. Attempts were made in this research to understand the different behaviour between stations. For example, the types of receivers and antennas were assessed in order to verify that good or bad performance are not due to the antenna or receiver types, however no correlation was found. Additionally, the locations of the receivers on Earth were analyzed to eliminate better or worse performance being due to a different satellite view. Unfortunately, adjacent stations were found to have different performance, ruling out this possibility. An analysis based on the number of satellites in view and of the PDOP was performed, rendering no direct correlation with the worse performing stations. Other correlations were studied but none were found. Rather than focusing on external factors, one area of improvement is the stochastic model in the processing. For instance, in this chapter, the same level of pseudorange and carrier-phase noise was assumed across all stations. Such assumption might not be realistic, as stations use different receiver and antenna combinations, and they are also located in different environments, which might affect the noise in the measurements. Additionally, the same weight is given to measurements on different frequencies, which may be improved with signal-specific values. Therefore, fine-tuning the stochastic model in general and adapting it to individual stations is expected to further reduce the difference in performance between stations and lead to further consistency of results.

4.6 Summary

For the past few decades, RTK has been the standard for precise positioning because of its ability to achieve centimetre-level positioning instantly for short baselines or within GNSS networks. However, RTK requires the presence of local reference stations with known coor-

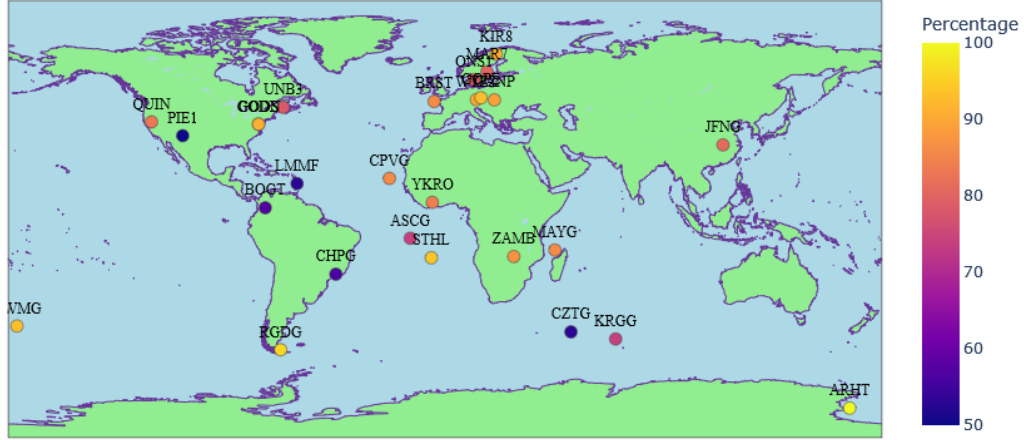


Figure 4.17: Location of the stations, color-coded based on the percentages of epochs below 2.5 cm horizontal error for the 67th percentile epoch-by-epoch multi-GNSS multi-frequency solution, as shown in Figure 4.10b.

dinates, which are not always available, as they are costly to maintain. PPP has superseded RTK when it comes to areas without RTK coverage, as it does not require additional infrastructure other than a sparse worldwide network of receivers. Since PPP only relies on the use of global corrections which can be transmitted either through Internet, satellites, or from the constellations themselves, PPP can be used anywhere in the world where GNSS signals can be tracked. The only caveat is that PPP has been known to require tens of minutes to reach acceptable sub-decimetre accuracies. The work in this research uses the current multi-constellation multi-frequency context to provide as many measurements as available to PPP, which is a measurement-dependent technique. These additional measurements, together with ambiguity resolution, are able to allow PPP to achieve RTK levels of performance without using any local information. To be able to use as many measurements, a novel, flexible, uncombined, quadruple-frequency, decoupled clock model is derived and used to process data from receivers spread around the world.

The results in this chapter show that instantaneous positioning is possible, as the average solution based on 1448 datasets converged and stayed below 10 cm horizontal error instantaneously at 2σ and 1σ . These solutions are computed by taking the epoch-by-epoch average across all datasets after eliminating the 5% and 33% datasets with the highest errors, in order to assess the average performance after eliminating outlier results. This impressive

performance is confirmed by individual stations, as, on average, 63% of stations converge instantaneously, while 89% converge within one minute when using all frequencies from all constellations. These results show that PPP can indeed reach sub-decimetre accuracies instantaneously, provided that ambiguities are fixed on as many frequencies and constellations as are available. These results are even more impressive when tightening the thresholds, as it is found that 81% of stations converge below 2.5 cm instantaneously at 1σ , and that 89% converge below 2.5 cm within one minute at 1σ . However, one of the strengths of RTK is its ability to provide centimetre-level epoch-by-epoch positions for short to medium baselines. Such an epoch-by-epoch solution is generated with the proposed multi-frequency multi-constellation solution, and it is shown that 66% and 61% of all epochs are below a decimetre and 2.5 cm, respectively. This is achieved by processing every epoch of data individually, making these results even more impressive, as only 4% of epochs would converge below a decimetre for a typical all-constellation dual-frequency float solution. Furthermore, it is shown that, on average, individual stations have 60% of their epochs below 2.5 cm in single-epoch processing, with station ARHT even having 96% of its epochs instantaneously converged to 2.5 cm. The result show that the 1σ epoch-by-epoch solutions for 18 stations out of 27 all have at least 80% of their epoch within 2.5 cm, demonstrating the impressive RTK-like performance that is achievable when leveraging many of the constellations and signals currently available to GNSS users.

Such results are drastic improvements in performance, as they show the benefit of leveraging all the constellations in orbit and all the frequencies being broadcast, and they show that PPP can in fact reach centimetre-level accuracy instantaneously, without needing any of the local information or infrastructure. This performance demonstrates that it is possible to achieve centimetre-level positioning anywhere in the world using only a single receiver and a stream of global satellite corrections. Additional work is needed to further understand the difference in performance between stations for epoch-by-epoch processing, and to attempt having all stations perform as well as station ARHT. Some avenues were explored in this chapter, but conclusive answers were not found. Furthermore, only three frequencies were used for BeiDou-3, and an interesting addition would be the inclusion of the two other frequencies being broadcast by the constellation, as well as the fifth Galileo frequency. Further related work also includes analysis of performance using corrections transmitted by Galileo’s High Accuracy Service (HAS), as it paves the way to corrections being transmitted by constellations themselves, allowing users to obtain high precision positions through signals broadcast by constellations alone. The latter constitutes an integral part to the second part of the next chapter.

Chapter 5

Self-serve RTK: mixing global and regional products

This dissertation has focussed thus far on using global precise satellite correction products to generate PPP solutions, and on ways to improve the PPP solutions to reach near-instantaneous centimetre-level positioning without the use of any local information. The purpose from this work is to have PPP perform as well as, or even better than, RTK in areas where RTK is not feasible, such as in remote areas, in the ocean, etc. This chapter and the next open the door to the place that PPP can have in the world in the near-future. This place is described within the context of the tremendous improvements that PPP has been subject to, as demonstrated in Chapter 4. This chapter is split into two parts / chapters, with each part looking at a different aspect of the near future. This first part focusses on *self-serve RTK*, which refers to the use of NRTK corrections in PPP processing, as well as the mixing of both NRTK corrections and PPP corrections from different solutions in a hybrid and improved global solution. The motivation behind this work is explained in Section 5.1.

Conversely, if focussing solely on the global products, the main advantage of PPP is that it is a point positioning technique that only requires, in addition to live satellite signals, satellite corrections, which it receives either through the Internet or commercial satellite signals. The Galileo constellation just introduced a High Accuracy Service (HAS) (EUSPA, 2023), where PPP-like corrections are broadcast from space directly by the constellation itself. This approach makes PPP truly a point positioning technique, as users only need to use the signals broadcast by the GNSS constellations, leading to the work described in the second part of the chapter, described as *RTK from Space*. This service is the first of its kind on a global scale. In this second part of the chapter, an analysis of the early test signals of Galileo HAS is performed. It is expected that, once all frequencies and constellations are added to HAS, comparable performance to the previous chapter should be achieved; though only Galileo and GPS corrections are proposed by the service.

The work in both parts of this chapter has been performed as part of the Galileo Improved Services for Cadastral Augmentation Development On-field Validation (GISCAD-OV) project (GEOWEB S.p.A., 2022), and all the data used in this chapter has been received in the context of the project. The project is a cadastral surveying project resulting from a consortium of 14 members, involving GNSS companies, research centres and universities, surveying organizations, as well as information, technology and communication companies. The project, proposed to the EU Horizon 2020 research and innovation programme, was approved in 2019, and it is focussed on cadastral surveying with a main scope to “design, develop and validate an innovative and cost-effective High Accuracy Service for Cadastral Surveying applications, based on GPS+Galileo High Accuracy Service (HAS) and Precise Point Positioning-Ambiguity Resolution (PPP-AR) quick convergence advanced techniques” (GEOWEB S.p.A., 2022). The GNSS Laboratory at York University is one of the contributors, with a focus on scientific validation of the positioning results. Initial results from the research in this chapter have been published in Naciri et al. (2022).

5.1 Introduction and motivation

While having PPP replace RTK due to its ease of use and improved performance is an attractive thought, reality dictates that such replacement will probably not take place, or that it would take time to happen, due to many reasons. For instance, much of the infrastructure in precise positioning applications is based on (N)RTK processing. Therefore, moving such applications to using PPP would require changes to the infrastructure, which could be costly. Additionally, PPP performance has been traditionally known to degrade with lower-grade hardware or in urban environments. As a consequence, one cannot solely rely on PPP for precise positioning in those areas/environments, as additional research is required to improve PPP performance in those cases. Additionally, the research in this dissertation is focussed on accuracy and convergence time, but certain applications have other priorities, such as integrity and reliability of the solutions. Therefore, rather than having PPP completely replace (N)RTK, the future of precise positioning will most likely consist of a mix of both techniques to provide reliable precise positions to users in all fields of application.

One possibility from such mixing of techniques is to enable the use of local ionospheric information whenever available, which constitutes the *self-serve* part of the chapter. The idea behind this mixing is that users can rely on PPP for positioning. When the user reaches areas where PPP becomes less reliable, or more generally, when the user reaches an area with a local network of NRTK stations, they can make use of corrections from the local network to augment their PPP solution. For example, one application case would be a user driving on a highway where they can rely on PPP for positioning. However, when they

enter the city with its signal obstructions and multipath, they can make use of NRTK corrections to augment their PPP solution. More generally, the method can be used if high positioning performance is required. However, mixing corrections from two different sources, and especially from a global and a regional network is challenging due to the complexity of combining different corrections. The mixing is difficult due to the two correction sources using different models and algorithms, but also because of the difference in size between the networks, as satellite corrections are harder to decorrelate in local networks compared to global networks. While augmenting PPP with regional atmospheric corrections is not a new concept, mixing PPP corrections with regional atmospheric corrections is. Indeed, typically in the literature, the regional atmospheric corrections are generated by a regional network by fixing the satellite orbit, clock, and bias corrections to globally estimated values, and estimating the atmospheric corrections in the network. However, in NRTK, satellite orbits, clocks, and biases are estimated alongside the atmospheric corrections by the regional network, without the global constraint. The absence of the global constraint, coupled with the regional nature of the network, leads to the regional corrections being correlated, and therefore harder to combine with the global products. The mixing of these incompatible global and regional corrections constitutes the novelty in this first part of the chapter, where a proof-of-concept to extract local ionospheric corrections from local NRTK products, which would be compatible with global satellite products is proposed.

While NRTK and PPP are both techniques that appeared and evolved separately, they can both be seen as two facets of the same solution. Indeed, both techniques rely on the use of networks of receivers to estimate SSR corrections, with the latter being sent to users to be used in undifferenced processing to achieve precise positions. However, while in concept similar, both techniques have particularities that make switching from one to the other a not-so-trivial task. The first part of this chapter starts with explaining some of the differences between both techniques and the details that have to be taken into account to be able to process NRTK corrections in PPP mode with a PPP user engine. Once that processing established, the chapter continues with a description of a proof-of-concept on how a PPP solution based on PPP corrections from a global network of stations can be augmented with NRTK corrections from a regional network of stations to enhance PPP performance and achieve NRTK-like performance. Such task is especially difficult given that both sets of corrections are generated by two different service providers, as well as the fact that the networks used to generate the corrections are independent and different in size (one being a regional network and the other being global).

5.2 NRTK corrections for PPP processing

PPP is a technique that saw the light in the 90's as a way to reduce the computational burden from processing multiple receivers simultaneously by computing satellite corrections and fixing them at the user side (Zumberge et al., 1997), but also as a way to eliminate the dependence on local base stations for precise positioning, as is the case for RTK (Kouba and Héroux, 2001). PPP is based on the fact that precise satellite corrections can be estimated from a global network of continuously-operating stations, and that the satellite corrections can be transmitted to users. The latter, only using the satellite corrections and the observation data received from their antenna, can compute their precise position without performing any differencing. NRTK was developed as a way to expand RTK to cover a region, as, rather than having to be within a few tens of kilometres from the base station, users can be in a larger region within the network's boundaries and still be able to receive corrections for their location (Townsend et al., 1999; Talaya and Bosch, 1999). Data from reference stations are sent to a network provider that computes corrections and interpolates them based on the user's location before sending them to the user. This NRTK model evolved to use SSR corrections rather than OSR corrections, as, rather than sending corrections for each signal for the user's location, the NRTK provider directly estimates satellite and atmospheric corrections based on the local/regional network of reference stations and the user's location, and broadcasts them to the user to be applied to their measurements (Wübbena and Willgalis, 2001). Such an approach drastically reduces the bandwidth required to transmit corrections to users, especially since SSR corrections can be broadcast at different rates depending on their behaviour over time.

Given these descriptions of both techniques, the distinction between NRTK and PPP becomes blurry, as both techniques are SSR-based and use a network of stations to estimate satellite corrections before broadcasting them to the user. The main differences between the two techniques therefore reside in the size of the networks used to generate the corrections, as well as the presence/absence of atmospheric corrections. Indeed, due to the smaller size of the NRTK networks and the relative close proximity between reference stations, ionospheric and tropospheric corrections can be precisely interpolated for users within the network's boundaries and broadcast to them, therefore leading to quick convergence of the user solutions (Wübbena et al., 2005; Talbot et al., 2002). In the case of PPP, such precise atmospheric information is impossible to obtain due to the large distances separating reference stations, therefore leading to PPP being a measurement-dependent technique and requiring a convergence period. In order to achieve the goal of self-serve RTK, a first step is to ensure that NRTK corrections can be used in PPP mode. Meaning, taking corrections destined for NRTK processing, can a typical PPP processing tool, such as York-PPP, generate an NRTK-like solution in terms of performance?

For the purpose of this testing, the York-PPP engine has been modified to accept such NRTK corrections. In addition to accommodating for the different format used to transmit these corrections compared to typically used formats in PPP, modifications include ensuring that similar models are used between the network and the user. Indeed, in NRTK, as well as in PPP, the models and conventions used by the user have to be compatible with those used by the network, as small discrepancies can lead to large position errors. In PPP, conventions are set to ensure that users can make use of any correction source and keep the same level of performance without having to tune their engines based on the correction sources (McCarthy et al., 2004). For instance, the reference frame in which the user's coordinates are computed is set by the reference frame that was used during the satellite orbits' estimation. Therefore, in PPP, the convention is to use the IGS14 frame, which is very similar to ITRF 2014 (Reischung et al., 2016). Other conventional models include but are not limited to the solid Earth tides corrections, the satellite antenna corrections, etc. While these conventions are universal in the case of PPP, it is not the case for NRTK, where corrections are generated by regional networks and are mostly destined for commercial use. For instance, in the case of this work, the NRTK corrections use different sign conventions when it comes to the application of satellite corrections, leading to many metres of position error instead of the few decimetres that were expected. Additional discrepancies include a different model being used for the wind-up correction, as well as the non-application of permanent Earth tides - unlike what is usually done for PPP.

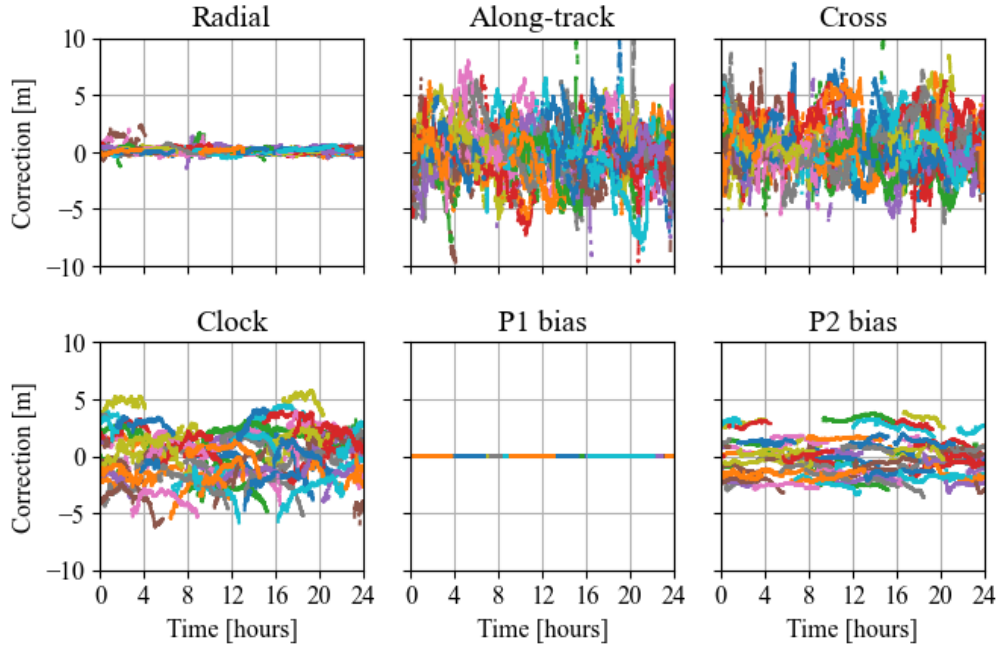
5.2.1 Analysis of NRTK corrections

While solving these issues and ensuring compatibility between the network and the user engine allowed for the correct ingestion of NRTK corrections in the PPP engine, the debugging process was very tedious due to the nature of the corrections, as the size of the network impacts the decorrelation of the corrections, leading to corrections/models having a bigger impact on the user solutions than they would with global PPP corrections. Because of the NRTK networks only covering a relatively small region on Earth (typically a country or a region in a country), they are geometrically small relative to the larger distance of tens of thousands of kilometres that GNSS signals travel from the satellite to the user. This concentrated distribution leads to high correlation between the different satellite and atmospheric states to be estimated by the network (Khodabandeh and Teunissen, 2015; Sato et al., 2022). These correlations are especially true between satellite corrections on one side and atmospheric corrections on the other, as atmospheric delays seep into satellite corrections, leading to a loss of the physical meaning of the corrections. This loss of physical meaning is showcased in Figure 5.1, where SSR satellite corrections are shown from two sources: an NRTK stream where corrections are generated by a regional network in France,

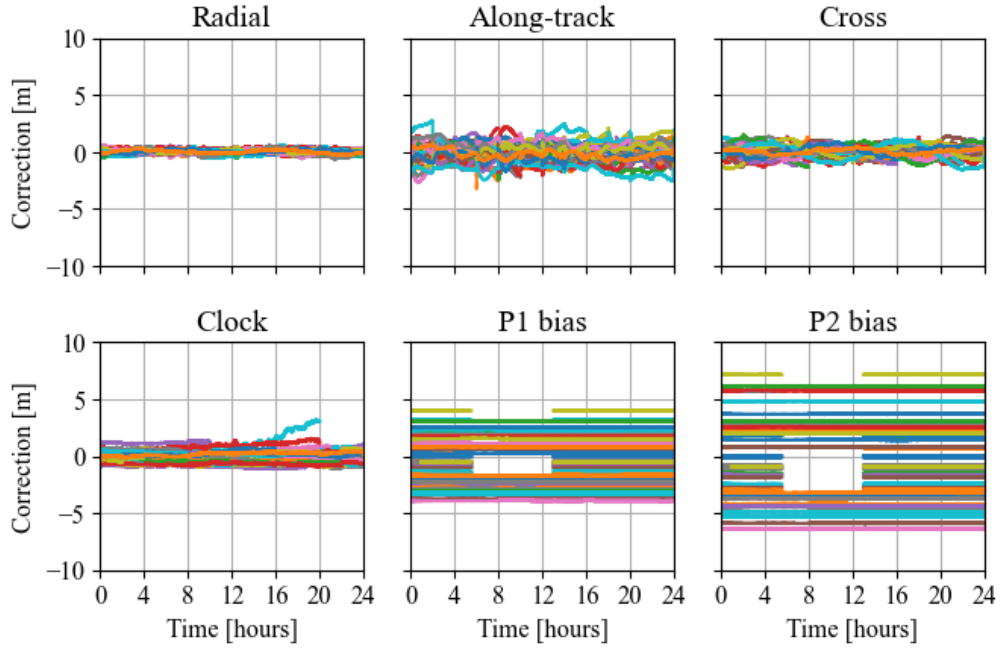
and a global PPP corrections stream. The network of stations used in the generation of the regional corrections can be found in Figure 5.5a as the black triangles, where the distance between reference stations is typically below 75 km. This processing, as well as all the processing in this chapter, is performed on dual-frequency Galileo and GPS measurements due to availability. All processing is performed in simulated real-time in kinematic mode.

The effect of the difficulty to decorrelate corrections, as well as the difference in models can be seen from Figure 5.1. The latter can be noticed from the first-frequency code bias corrections (P1 bias). While the bias is different for each satellite in the global network, it is always equal to zero for the regional solution. This difference is due to a difference in configuration in the network solution and it should not impact user processing. Indeed, the network provider can select a set of constraints in their corrections generation, and setting the first-frequency code bias corrections to zero only means that those code biases would be re-distributed between the satellite clock and the satellite code biases on other frequencies. The effect of the high correlation between the corrections in the regional network can most easily be seen by looking at the trend in, e.g., the clocks. Indeed, the high correlation between corrections tends to translate to atmospheric corrections affecting the satellite-dependent corrections. In the case of regional networks, satellites would rise and set a few times over the course of a day due to the movement of the satellites and due to the network being concentrated in one region on Earth. Therefore, a clear trend would be seen in the atmospheric delays as, as the satellite is rising from the horizon and moving towards its zenith, the delay would decrease, as the signal would have to travel through a thinner layer of the atmosphere. The delay would decrease until the satellite reaches its highest point in the user's sky view, before increasing again while the satellite sets. This trend would not be observed in the satellite-dependent terms (clocks, biases) in a global network thanks to the global distribution of the reference stations, as these corrections are only satellite-dependent and do not depend on the satellite's position in the sky. However, in the case of Figure 5.1a, satellite rise and fall trends can be seen when looking at the satellite clock corrections. This variation between a satellite's rise and its fall are due to the atmospheric corrections affecting these satellite-dependent terms and clearly show the effect of the small size of the network on the estimated corrections. The same trends can be seen in other corrections, though not as easy to identify due to the scale of the figures.

In order to showcase these trends in satellite-dependent corrections, Figure 5.2 shows the correlation between satellite-dependent corrections (orbits, clocks, and code biases) and satellite elevation angle. Given that the corrections are only satellite-dependent, they should not be correlated to the elevation angle; unlike the atmospheric delays, which are higher the lower the satellite elevation angle. In the figure, the correlation between the corrections and elevation angle is clear, as the correlation plots have a funnel shape. The correction values tend to have higher values with lower elevation angles, and lower values with higher elevation



(a) Corrections generated from a regional network.



(b) Corrections generated from a global network.

Figure 5.1: Global and regional satellite radial, along-track, cross-track, clock, and L1/E1 and L2/E5b code bias corrections for GPS and Galileo satellites on day 230 of year 2021. Each color corresponds to a different satellite.

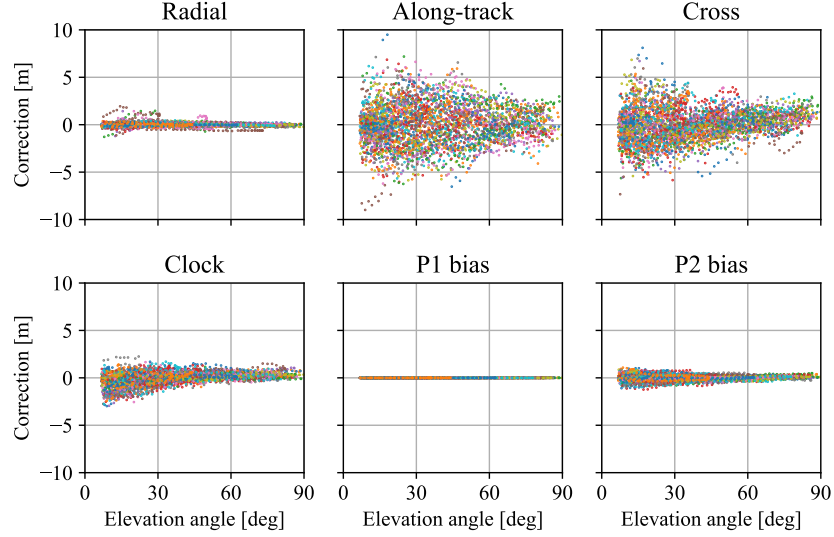


Figure 5.2: Correlation figures between regional corrections and the GPS and Galileo satellites' elevation angles on day 230 of year 2021. Each color corresponds to a different satellite. The mean values for each satellite are removed to highlight trends.

angles. In the case of global corrections, this clear correlation does not exist. Therefore, the figure provides further evidence as to the loss of physical meaning of the corrections when estimated with a regional network of stations.

5.2.2 Effect of corrections on user performance

The high correlation between corrections in the regional network that was shown means that the estimated corrections are not physically meaningful. Therefore, the full range of corrections is expected to be used together without processing subsets of corrections (for example only using satellite orbits and clocks in the processing), as, it was noticed that individual corrections have effects on the user solution which are different from their effect if they were generated from a global network. To quantify some of these effects, Figure 5.3 shows the effect of both satellite code and phase biases on a PPP solution that only uses global corrections, and on a PPP solution that only uses regional corrections. Both sets of corrections are similar to the ones shown in Figure 5.1.

Figure 5.3a shows the performance that is typically expected in PPP when using global corrections: satellite code biases only affect the solution during the initialization and convergence periods, while the satellite phase biases have a minimal impact on the solution. The limited impact of code biases after convergence is mainly due to the fact that such biases tend to be constant/slow-varying over the course of the day. Therefore, as the filter converges, it is able to absorb those uncorrected code biases into the ambiguities and other

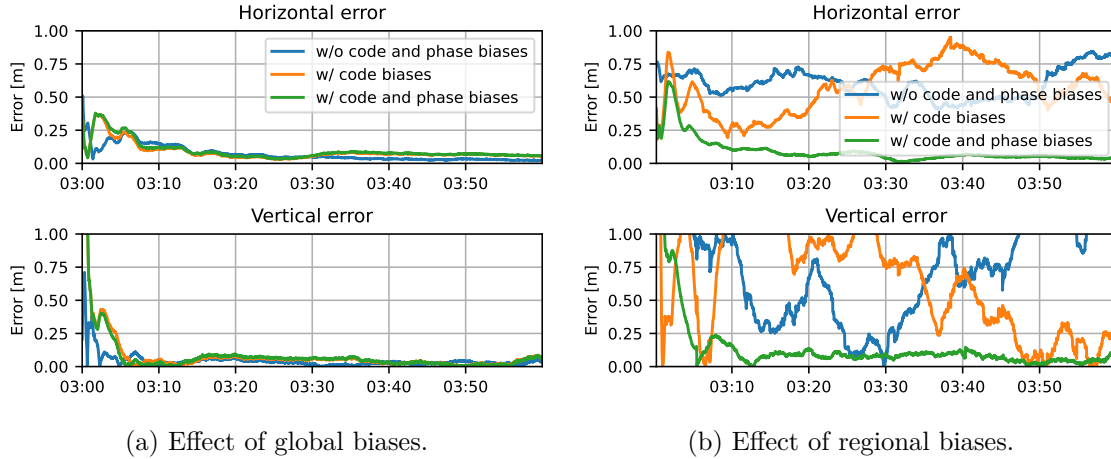


Figure 5.3: Effect of satellite code and phase biases on float PPP solutions using both global and regional products on station VILL in Spain on day 81 of year 2022. The solutions are generated based on dual-frequency observations for GPS and Galileo.

state terms, limiting their effects after convergence. The phase bias on the other hand minimally affects the solution when not performing AR, as phase biases get absorbed by the ambiguities if not corrected for.

Figure 5.3b tells another story when it comes to regional corrections. First, the solution without satellite code nor phase biases appears to not converge over time and it stays around the same range of many decimetres error over the whole duration of the processing. This behaviour is unexpected for a PPP solution, as due to the sequential nature of the filtering being performed, the solution should converge to better accuracies over time. However, due to the satellite orbit and clock corrections losing their physical meaning, the PPP solution using those corrections does not behave as a typical PPP solution. Similarly, including satellite code biases in the processing has an unexpected effect as well, as it has an effect on the solution over the whole duration of the processing, and not simply on the initial epochs of processing. Additionally, the effect appears to be substantial, leading to many decimetres difference with the solution without code biases. This effect from the code biases is much more significant than in the case of global corrections. Similarly, phase biases have a much more significant effect in the regional case than they do in the global one. Indeed, the solution starts off behaving like a typical solution, as a convergence trend is noticed when the satellite phase bias corrections are included. The overall rms decrease from 59.6 cm to 13.3 cm with the inclusion of the phase biases for the horizontal component, and from 78.0 cm to 35.2 cm for the vertical component. This example shows the effect of the loss in physical meaning of the corrections and the importance of including the full range of corrections in the processing.

This solution with satellite code and phase biases does not include the full range of regional corrections, as it does not use the ionospheric and tropospheric corrections. To highlight the effect of such corrections, Figure 5.4 shows the effect of the regional atmospheric corrections, as well as AR, on the regional solution with code and phase biases shown in Figure 5.3b. In the case of ionospheric corrections, they are typically not used as direct corrections, and residual errors are estimated to account for any interpolation errors. However, since the user engine in this case uses a PPP approach, estimating the residual ionospheric error per satellite would be equivalent to estimating the ionospheric delays as unknowns, limiting the corrections' benefits. Therefore, the approach that was adopted here is to use the regional ionospheric corrections as measurements by adding rows to the design matrix. By doing so, the impact of the corrections on the processing can be adjusted using the measurement weight matrix. Given the high precision of the regional ionospheric corrections, they are given similar weight as carrier-phase measurements. When it comes to tropospheric corrections, they are used directly as corrections without estimating any residual errors. This simpler approach could be improved by estimating residual errors as well. However, it was not done as part of this work, as the effect of the residual errors was found to be limited.

First, by focussing on Figure 5.4a, the float solution without ionospheric corrections is the same solution as the one with code and phase biases in Figure 5.3b, and it uses regional satellite orbits, clocks, and code and phase biases. The solution behaves like a typical float PPP solution and takes close to 10 minutes to converge to the sub-decimetre-level. Including ionospheric corrections to the float solution leads to the green solution, where a noticeable decrease in errors is seen for both horizontal and vertical components. With the addition of the precise ionospheric corrections, the rms decrease from 13.3 cm to 3.4 cm horizontally and from 35.2 cm to 8.4 cm vertically. This decrease in error shows the benefit that NRTK has over PPP, as the former can take advantage of precise ionospheric corrections to improve positioning performance. Fixing ambiguities to integers, even without precise atmospheric corrections leads to a reasonably good solution, as the solution converges to the centimetre-level for most of the processing, except for a few jumps in the initial stages of the processing. These jumps are removed once the ionospheric corrections are included (*fixed w/ ionosphere*), and the rms values for that solution are 1.0 cm horizontally and 7.4 cm vertically. Therefore, these results show the importance of both precise ionospheric corrections and AR in reaching instantaneous convergence to the centimetre-level with corrections from a regional network.

However, the rms of the vertical component is still relatively high, and trends can be seen in the times series of that component. These trends are removed once the tropospheric corrections are applied in Figure 5.4b, and the vertical rms is decreased from 7.4 cm to 1.1 cm, showing a significant decrease. This result again shows the importance of using

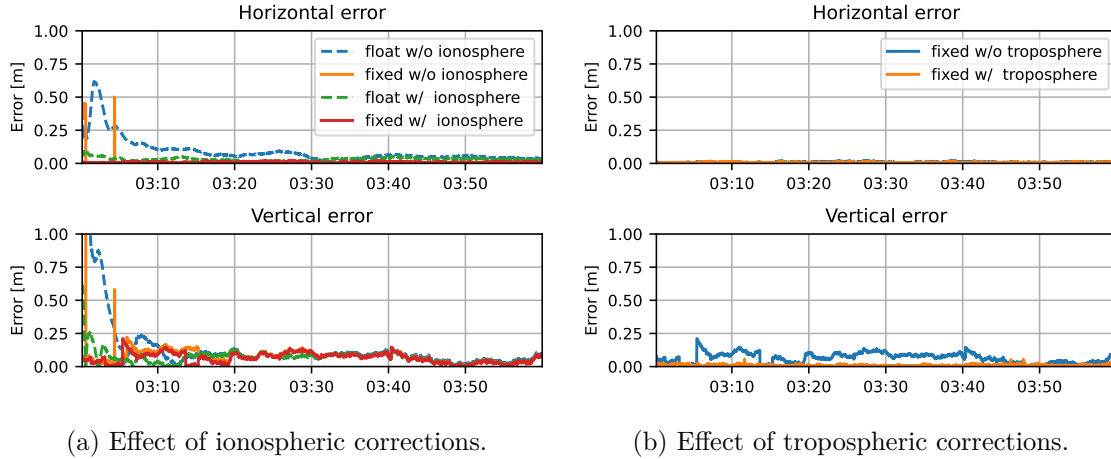


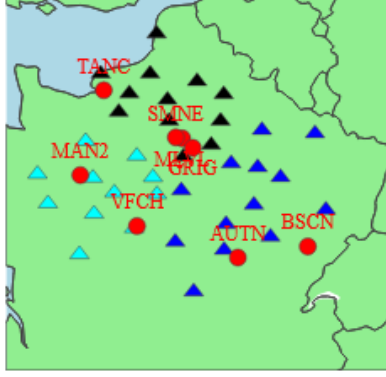
Figure 5.4: Effect of regional atmospheric corrections on float and fixed PPP solutions for station VILL in Spain on day 81 of year 2022. The solutions are generated based on dual-frequency observations for GPS and Galileo.

the full range of corrections in NRTK, as the corrections are highly correlated against themselves, and the model that is used in PPP mode is not completely compatible with the regional satellite and ionospheric corrections. Additionally, since the tropospheric correction mainly affects the vertical component, the incompatibility of the troposphere model with the expected correction mainly affects the vertical component.

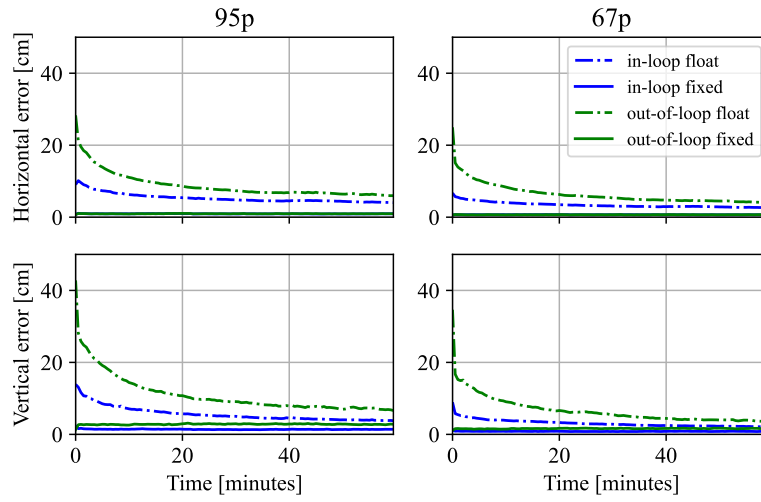
5.2.3 PPP positioning with NRTK corrections

To summarize the performance from the processing of NRTK corrections in PPP processing using the full range of corrections, a larger dataset is used to generate the results in Figure 5.5. For these results, one network in France, comprised of three sub-networks, is used to generate NRTK corrections. Each sub-network is identified by triangles of different colours. As a sanity check, these reference stations are re-processed in PPP user mode to assess their performance, which constitutes the *in-loop* solution. To test the performance of the network for users within the network's boundaries, the stations identified by the red circles are processed as well, constituting the *out-of-loop* solutions. A total of 1,269 and 410 hourly datasets are processed for the *in-loop* and *out-of-loop* solutions, respectively. The solutions use the full range of corrections, i.e., satellite orbits, clocks, and code and phase biases, as well as ionospheric and tropospheric corrections.

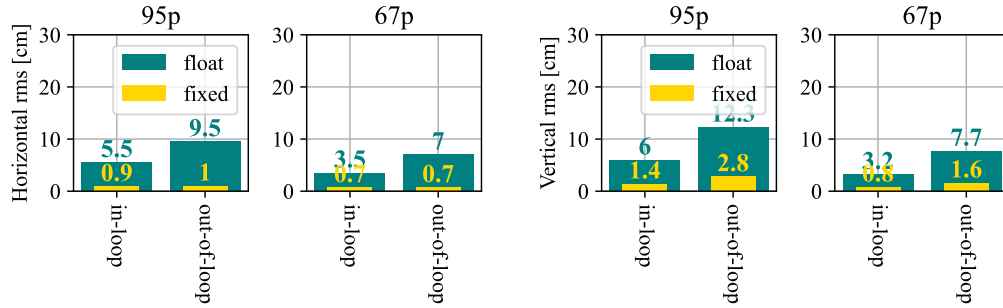
Figure 5.5 shows that the performance that was shown in Figure 5.4 with the use of all NRTK corrections can be reproduced at a large scale. Indeed, both *in-loop* and *out-of-loop* solutions have similar performance, as the float solutions take time to converge, while the fixed solutions converge instantaneously to the centimetre-level. Both *in-loop*



(a) Locations of the receivers.



(b) Time series.



(c) Horizontal rms.

(d) Vertical rms.

Figure 5.5: Average position results for PPP processing with NRTK corrections generated from a network in France. *in-loop* results are based on processing the same stations that were used to generate the corrections (triangles in Figure 5.5a), while *out-of-loop* results are based on processing stations within the NRTK network's boundaries, but not used in the correction generation (red circles in Figure 5.5a).

and *out-of-loop* solutions have an overall rms of 1 cm at the 95th percentile, showing that consistent instantaneous centimetre positioning can be achieved with NRTK corrections in PPP mode. A slight degradation can be noticed in the vertical component for the *out-of-loop* solution, as the overall rms reaches approximately 3 cm compared to 1.4 cm for the *in-loop* solution. This difference in performance could be attributed to the interpolation of the troposphere for the *out-of-loop* solution that could be causing the slightly higher error compared to the *in-loop* solution that uses the same stations for network and user processing. Therefore, the results shown in this subsection demonstrate that, with careful handling of corrections and compatibility of models between network and user, it is possible to achieve NRTK-like performance with regional NRTK corrections, i.e., instantaneous centimetre-level positioning, in a PPP engine.

5.3 Combination of global and regional corrections

Augmenting global PPP solutions with regional atmospheric corrections is not a novel concept, and is usually referred to as PPP-RTK in the literature (Banville et al., 2014; Psychas et al., 2021; Nadarajah et al., 2018). The precise atmospheric corrections in such an approach are usually estimated by, in the regional network, fixing the satellite orbits, clocks, and biases to the global products and estimating and modelling the ionospheric and tropospheric delays within the network’s boundaries. Constraining the satellite products to global products leads to physically-meaningful ionospheric and tropospheric corrections. However, in the case of NRTK corrections, all corrections, including satellite orbits, clocks, and biases, are estimated from the same regional network, leading to estimated corrections which are hard to decorrelate, as was shown in Section 5.2. In that section, it was demonstrated that NRTK corrections can be used in PPP processing to generate solutions at the same level of performance as an NRTK solution. It was demonstrated that, due to the regional scale of the network of reference stations, the estimated corrections are not compatible with globally estimated corrections. The main reason for such incompatibility is the difficulty decorrelating corrections in a regional network, therefore putting a physical limitation to the compatibility between regional and global products. In the following section and for the remainder of this chapter, this discrepancy between regional and global products is studied and the possibility of combining regional and global corrections in a hybrid PPP solution is assessed. A proof-of-concept on how such a combination can be generated is provided through two different approaches.

In Figure 5.1, the difference between satellite-dependent (orbit, clock, and code bias) corrections between global and regional networks was shown. More importantly, the figure demonstrated that atmospheric effects seep into the satellite-dependent corrections due to the difficulty of decorrelating individual corrections caused by the geographically con-

centrated nature of the network. The fact that atmospheric state terms seep into other corrections means that the ionospheric corrections are not physically meaningful either. To highlight the loss of physical meaning from the regional atmospheric corrections, Figure 5.6 compares the ionospheric and tropospheric delays for station JARG in France. The comparison is performed between the atmospheric corrections for the station's location based on a regional NRTK network of stations, and the atmospheric delays based on global PPP processing. The global atmospheric delays are generated by processing the station's observations in PPP mode using the global satellite products. Generating the global atmospheric delays this way mean that a convergence period would be observed. However, its effect on the extraction of the biases is found to be negligible, and the estimated global atmospheric delays are considered to be good gauges of the delays in the remainder of the analysis. For simplicity, the global and regional tropospheric delays are not shown in the figure.

Figure 5.6 clearly shows the differences between the global and regional atmospheric delays. First, focussing on the ionosphere, it appears that the atmospheric delays between the global solution and the regional corrections follow similar trends overall, showing that the regional ionospheric corrections still retain some of their physical nature. However, when comparing absolute values and comparing values between satellites, the two sets of ionosphere delays do not match. To highlight the differences, the bottom left sub-figure shows the difference between the two top sub-figures. It is obvious that there are metre-level differences between the two sets of ionospheric estimates, as between-satellite biases are observed, with an overall rms of 59.1 cm. These metre-level biases and high rms demonstrate that the ionosphere generated from NRTK is not compatible with the PPP solution that uses global products. It is worth noting that the global ionospheric delays are computed based on a PPP solution. Thanks to the convergence of PPP solutions, these estimated delays can be considered close to the "true" delays, as, if not initially, these delays would be close to the "true" delays after a few minutes of processing. As such, between-satellite biases are still observed after many minutes of processing and towards the end of the processing hour, demonstrating that these biases are not an artifact of the estimation process.

Better compatibility appears to exist between the tropospheric delays from the two sources, as the overall rms is 9.1 cm, approximately 6 times lower than the rms of the ionospheric differences. Additionally, the difference in troposphere appears to be more significant initially due to tropospheric convergence from the PPP solution. The period after convergence therefore demonstrates that the tropospheric delays from the NRTK network are compatible with the global PPP solution. One possible explanation for this compatibility is that the tropospheric delays are estimated as zenith delays, which are mapped to each satellite using mapping functions. Given that these zenith delays are common to all satellites in view, and therefore common to all observations, they can be better decorrelated from the rest of the corrections. Similar compatibility between regionally- and globally-extracted

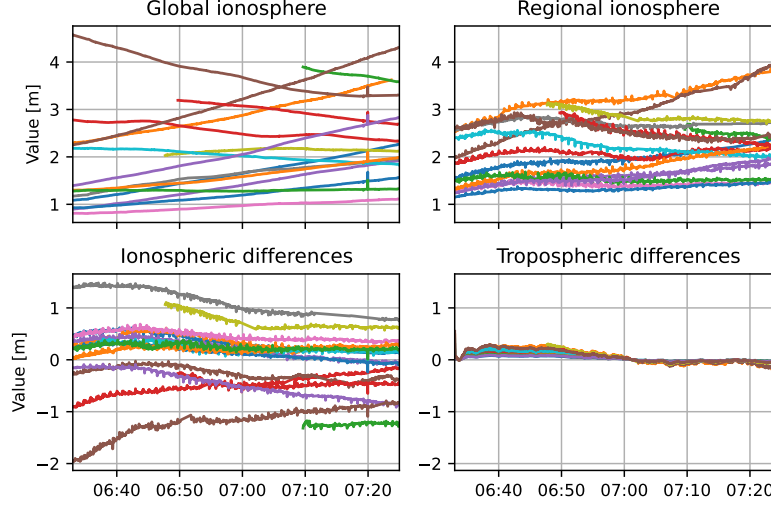


Figure 5.6: Global ionosphere (top left), regional ionosphere (top right), their differences (bottom left), and difference between global and regional troposphere (bottom right) for station JARG in France on day 273 of year 2021. Each colour represents a different satellite, with satellites from Galileo and GPS being shown.

troposphere have been observed in Dousa and Bennitt (2013). The conclusions from Figure 5.6 in terms of ionosphere and troposphere compatibility hold true for different stations from different networks and at different times. For brevity, only one station is shown, but its conclusions can be generalized.

5.3.1 Method description

Figure 5.6 shows that it is difficult to successfully augment a global PPP solution with ionospheric corrections from an independent regional source, as directly using the regional ionospheric corrections would degrade the global PPP solution due to the presence of large between-satellite differences. In order to allow for the mixing of corrections, the discrepancies between the two sets of ionospheric estimates have to be mitigated. In this section, a proof-of-concept is presented where precise ionospheric corrections that are compatible with the global products are generated. The proposed method starts with the computation of global ranges using the receiver coordinates and global satellite orbits, clocks, biases, and ionospheric and tropospheric estimates, as well as regional ranges using receiver coordinates and regional satellite orbits, clocks, biases, and ionospheric and tropospheric corrections. The main principle behind the proposed method is that, computing the differences between the full global and regional OSR ranges should mainly consist of the discrepancies between both sets of products. Therefore, by computing the full ranges and differencing them, one can extract the biases between the two sets of products, which can be used to recover pre-

cise ionospheric corrections that are compatible with the global products. Only ranges on two frequencies are used, as two frequencies provide enough information for bias extraction, without requiring additional frequencies. Consider satellite s and receiver r , the global ranges that are to be used in the method are shown in Equation (5.1).

$$\begin{cases} P_{1_G}^s &= \sqrt{(X_G^s - X_r)^2 + (Y_G^s - Y_r)^2 + (Z_G^s - Z_r)^2} - cdt_G^s + B_{1_G}^s \\ P_{2_G}^s &= \sqrt{(X_G^s - X_r)^2 + (Y_G^s - Y_r)^2 + (Z_G^s - Z_r)^2} - cdt_G^s + B_{2_G}^s \\ P_{1_G,full}^s &= P_{1_G}^s + T_G^s + I_{1_G}^s \\ P_{2_G,full}^s &= P_{2_G}^s + T_G^s + \gamma_2 I_{1_G}^s \end{cases} \quad (5.1)$$

In Equation (5.1), $P_{1_G}^s$ and $P_{2_G}^s$ are the first- and second-frequency OSR ranges using the global satellite orbits, clocks, and code biases without including atmospheric delays, respectively, while $P_{1_G,full}^s$ and $P_{2_G,full}^s$ are the same ranges including ionospheric and tropospheric delays. These atmospheric delays are derived from PPP processing using the global products. In addition to these global ranges, regional ranges as shown in Equation (5.2) are computed as well.

$$\begin{cases} P_{1_R,full}^s &= \sqrt{(X_R^s - X_r)^2 + (Y_R^s - Y_r)^2 + (Z_R^s - Z_r)^2} - cdt_R^s + T_R^s + I_{1_R}^s + B_{1_R}^s \\ P_{2_R,full}^s &= \sqrt{(X_R^s - X_r)^2 + (Y_R^s - Y_r)^2 + (Z_R^s - Z_r)^2} - cdt_R^s + T_R^s + \gamma_2 I_{1_R}^s + B_{2_R}^s \end{cases} \quad (5.2)$$

In Equation (5.2), $P_{1_R,full}^s$ and $P_{2_R,full}^s$ are the first- and second-frequency OSR ranges using the regional satellite orbits, clocks, and code biases, as well as the regional ionospheric and tropospheric corrections. Due to the presence of certain model and convention discrepancies as were described in Section 5.2, both global and regional ranges should include certain correction models, for example the satellite antennas corrections, as global and regional networks might use different satellite reference point conventions, or solid Earth tides corrections. Once the ranges are computed, the difference between the sets of ranges can be computed as shown in Equation (5.3).

$$\begin{cases} d_{1,r}^s &= P_{1_R}^s - P_{1_G,full}^s \\ d_{2,r}^s &= P_{2_R}^s - P_{2_G,full}^s \end{cases} \quad (5.3)$$

In Equation (5.3), $d_{1,r}^s$ and $d_{2,r}^s$ are the differences between ranges and they should only contain ionospheric and tropospheric delays that are compatible with the global products, as well as the discrepancies between regional and global products. The next step consists of extracting ionosphere and troposphere delays from $d_{1,r}^s$ and $d_{2,r}^s$. One way to perform such extraction consists of computing the ionosphere-free combination between $d_{1,r}^s$ and $d_{2,r}^s$ to eliminate the ionospheric delays. However, it was found that doing so introduces undesired

biases to the extracted troposphere, as the discrepancies between products are still present in the extracted troposphere. As an alternative, and given the relative similarity between the regional and global tropospheres in Figure 5.6, it was found that a more reasonable approach consists of subtracting the regional tropospheric delays from $d_{1,r}^s$ and $d_{2,r}^s$. By doing so, the extracted ionospheric delay can be computed as shown in Equation (5.4) as the difference between $d_{1,r}^s$ and the regional troposphere.

$$I_{1,ext}^s = d_{1,r}^s - T_R^s \quad (5.4)$$

Naturally, the extracted ionosphere is still contaminated by the biases induced by the discrepancies between the two sets of corrections. In order to eliminate the biases from the extracted ionospheric delays, the full global and regional ranges, $P_{1,G,full}^s$ and $P_{1,R,full}^s$ are used, as the differences between the two sets of ranges contain the biases caused by the discrepancies between both sets corrections. Once these biases are estimated, they can be directly applied to the extracted biased ionosphere to recover precise ionospheric corrections which are compatible with global satellite corrections. Two approaches to estimate these biases are detailed in the remainder of this chapter: a first approach estimates the biases as the average over time of the OSR double-differences, while the second approach is suitable for real-time processing and estimates the biases in parallel to the user processing. In terms of the tropospheric delays, regional corrections can directly be used, as the latter are found to be compatible with the global corrections. To summarize the method that was developed and explained above, Figure 5.7 shows a diagram of the steps that are followed to extract a set of atmospheric corrections that are compatible with global satellite corrections.

5.3.2 Application to a sample dataset

The purpose of this subsection is to give more details on the method described above. The same dataset for station JARG in France is used over the same time period as is shown in Figure 5.6. To visualize the discrepancies between the products, Figure 5.8 is generated, with the top sub-figure representing the difference between $I_{1,ext}^s$ and the reference global ionosphere. Therefore, the sub-figure only contains the discrepancies between global and regional products. To confirm this is the case and that the extracted ionosphere is the sum of the global ionosphere and the discrepancies, the differences between the full global OSR range $P_{1,G,full}^s$ and the full regional OSR range $P_{1,R,full}^s$ are plotted in the bottom sub-figure. The sub-figure shows that the difference between the full OSR ranges and the extracted ionosphere match and that, if the discrepancies were to be removed from the extracted ionosphere, the latter would become compatible with the global products. The repeated spikes in the figure are due to the regional correction generation.

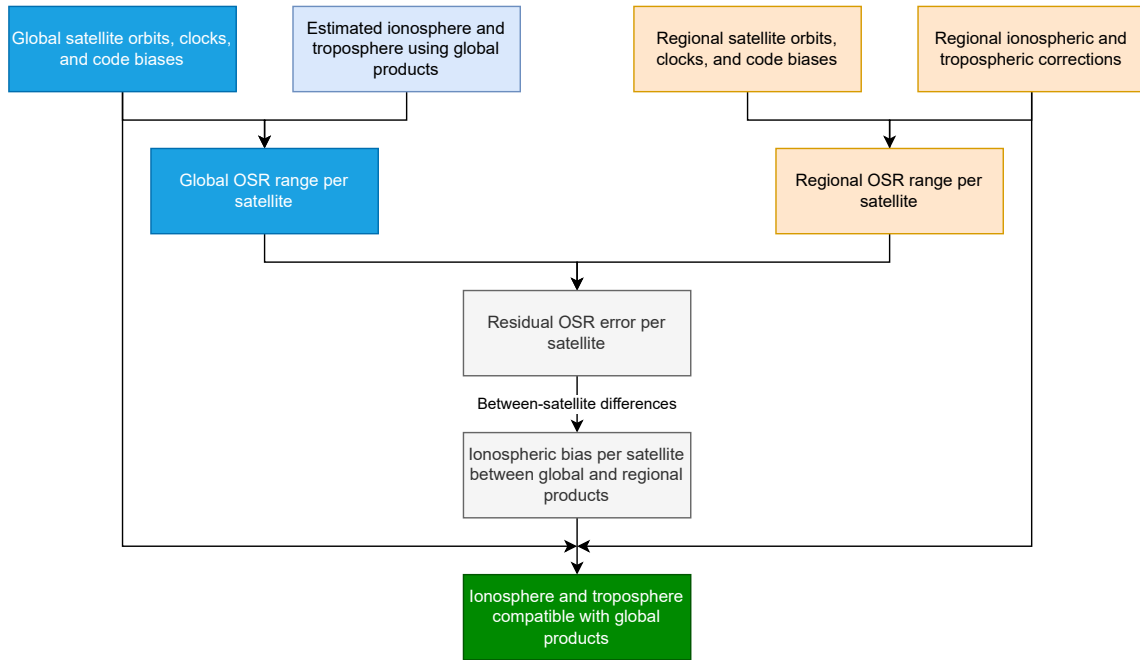


Figure 5.7: Diagram of the strategy used to generate precise atmospheric corrections that are compatible with global products from NRTK corrections.

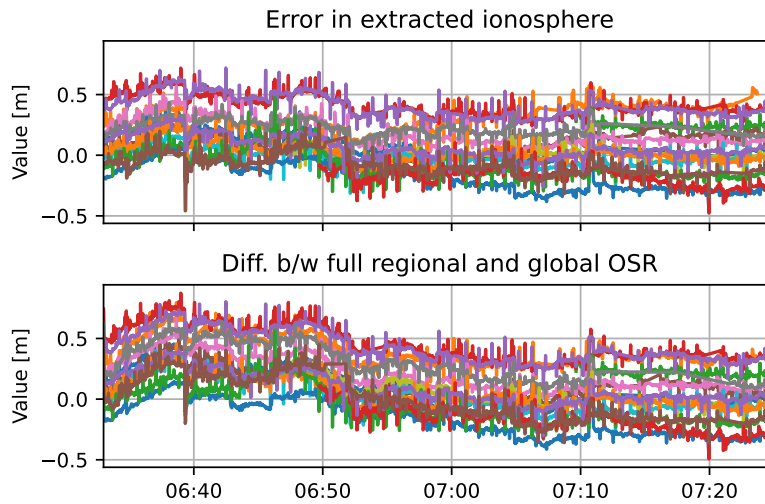


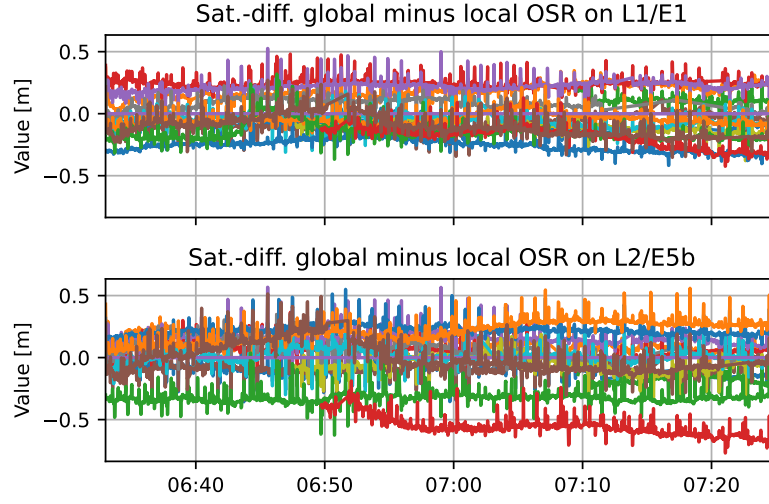
Figure 5.8: Error in the extracted ionospheric delays (top), and difference between estimated global ranges and corrected regional ranges (bottom) for station JARG on day 273 of year 2021.

The difference in full OSR ranges in Figure 5.8 can be seen to contain trends which are common to all satellites. Given that such common trends tend to be absorbed by receiver-dependent terms in the user processing, performing between-satellite differencing is a preferred approach, as it allows the elimination of such trends and only focusses on satellite-specific variations. To do so, one satellite is selected as reference for each constellation and its range differences are subtracted from those of the other satellites. The described procedure leads to between-satellite differences of the differences between full global and regional OSR ranges, referred to as double-differenced OSR ranges. The latter are plotted in Figure 5.9a.

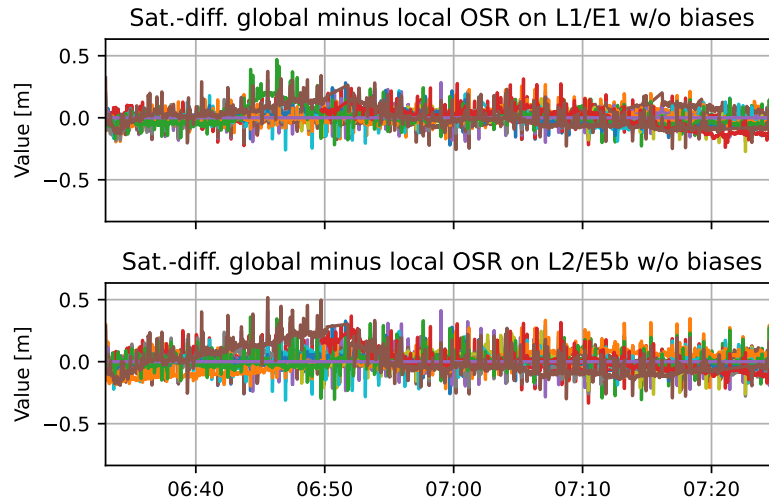
It is clear from Figure 5.9a that differencing between satellites removes the trends seen in Figure 5.8. In addition to the double-differences on L1 for GPS and E1 for Galileo, double-differences are also shown for GPS L2 and Galileo E5b signals for reference. The double-differences on all frequencies show that the biases tend to be constant over an hour and that the between-satellite biases can reach between several decimetres and up to one metre. The rms of the double-differences reach 15.3 cm on L1/E1 and 19.4 cm on L2/E5b. Similarly to Figure 5.8, the double-differences show regular jumps that happen for all satellites throughout the processing. These jumps are artifacts of the regional processing, and remain despite the computation of ranges with all corrections, and despite the double-differencing. In a user solution, these jumps would be absorbed by the residuals and would not impact performance. Therefore, for our purpose, the most harmful behaviour in the figure to a user solution are the large biases between satellites.

The first approach that is analyzed in the current subsection consists of computing the average of the double-difference for each satellite and subtracting it back from the double-differences. In such approach, the averaging requires many epochs to be processed, therefore making it impractical for real-time processing, but suitable for post-processing. Removing the average double-differences transforms Figure 5.9a into Figure 5.9b. Removing the satellite-dependent biases has a significant impact on the double-differences, as the overall rms decrease from 15.3 cm to 5.1 cm on L1/E1, and from 19.4 cm to 6.1 cm on L2/E5b. These decreases in rms are significant and demonstrate that the satellite-dependent biases are the biggest contributors to the big discrepancy between the products. In order to confirm that the estimated satellite-dependent biases correspond to the discrepancies between products, Figure 5.10 shows the between-satellite differences of the differences between extracted ionosphere and global ionosphere with and without the estimated satellite-dependent biases.

In Figure 5.10, the top sub-figure is the between-satellite differences of the errors in the extracted ionosphere that were shown in Figure 5.8. As expected, performing between-satellite differencing removes the common trends between satellites and only leaves out the discrepancies that are specific to each satellite. Applying the biases that were estimated from the double-differences of the full ranges to the extracted ionosphere leads to the bottom sub-



(a) Double-differences.



(b) Double-differences after removing the average biases.

Figure 5.9: Between-satellite, between-source double-differences of the full OSR ranges for station JARG on day 273 of year 2021. Constant satellite-dependent biases are removed from Figure 5.9a in Figure 5.9b.

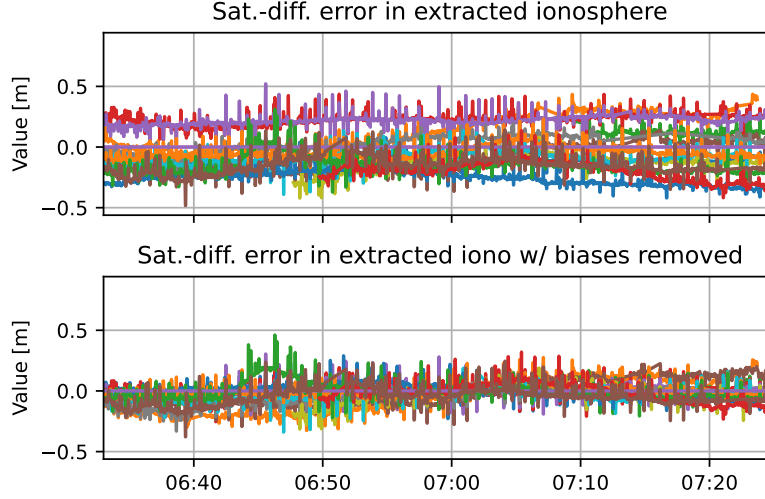


Figure 5.10: Between-satellite differences of the error in the extracted ionospheric delays without (top) and with (bottom) the removal of the between satellite-differences. The latter were estimated as shown in Figure 5.9.

figure in Figure 5.10. It is clear that the estimated biases remove the discrepancy between the extracted and global ionospheric delays, as the overall rms decreases from 15.9 cm to 7.2 cm with the application of the biases. Therefore, this figure shows that the estimated biases can be used to extract precise ionospheric corrections which are compatible with global satellite products.

In order to demonstrate the compatibility of these newly extracted ionospheric corrections with the global satellite corrections, Figure 5.11 shows the PPP performance using the different sets of products. The *regional* solution uses regional orbits, clocks, biases, ionosphere and troposphere. The *global* solution uses global satellite orbit, clock, and bias corrections. The *global w/ extracted iono* solution uses global satellite orbit, clock, and bias corrections, together with the extracted unbiased ionospheric corrections for which the residual errors were shown in Figure 5.10. As expected, the regional solution converges instantaneously and remains at the centimetre-level due to the high quality of the corrections and the presence of precise atmospheric corrections. The rms for the regional solution are 0.5 cm horizontally and 0.9 cm vertically. The global solution behaves as expected, as it takes two minutes to converge and stays at the centimetre-level, with post-convergence rms of 1.5 cm and 1.5 cm for the horizontal and vertical components, respectively. Finally, the global solution with the extracted ionospheric corrections shows that the extracted ionosphere is compatible with the global products. Indeed, adding the extracted ionosphere as pseudo-measurements to improve the estimation process leads to the augmented solution converging instantaneously to the centimetre-level, with rms of 1.5 cm and 1.6 cm hori-

zonally and vertically, respectively. This result shows that the mixing of corrections was successful and that the PPP solution can be augmented with NRTK corrections from an independent source. While the augmentation of PPP solutions with regional atmospheric corrections has been published in the literature (Psychas and Verhagen, 2020; Nadarajah et al., 2018), this presented mixing of corrections is novel. Indeed, the conventional approach in the literature is to constrain the satellite orbits and clocks to globally-estimated values, leading to the regional atmospheric corrections conserving their physical meaning. However, in the context of this mixing, the regional atmospheric corrections are generated alongside satellite orbits, clocks, and biases by and in an NRTK network. Since all corrections are estimated by the same regional network, atmospheric corrections lose their physical meaning, due to the difficulty in decorrelating corrections, adding a complexity that has not been researched and published so far. Potential improvements of this result include further testing through the use of more datasets, or even the use of epoch-by-epoch processing. The work can be further expanded to use the GLONASS and BeiDou constellations rather than only GPS and Galileo.

5.3.3 Adaptation for real-time processing

In the previous subsection, the possibility of mixing corrections from two independent sources, with one being NRTK corrections, was demonstrated. However, in the sample dataset that was used, the same station was used to both estimate the biases and apply them to extract unbiased ionospheric corrections. The approach that was proposed in the previous subsection can therefore not be applied to real-time processing without contributions from the NRTK network provider, as biases are computed as the average of the double-differenced OSR corrections over a certain time period. The method can still be useful though in certain applications if, for instance, the biases are estimated by the NRTK network provider and transmitted to the user who can apply them in real-time, or if the user allows for a "calibration" period where they estimate the biases before they start using them to extract the desired ionospheric delays.

In the current subsection, a different approach is used where, instead of requiring a period where the biases need to be estimated, the latter are estimated in parallel to the user PPP processing. By allowing the estimation of the biases in parallel, the user can therefore start extracting precise ionospheric corrections which are compatible with their global solution as soon as regional corrections are available, and without having to rely on the NRTK to provide the biases for real-time processing. This approach opens the door for the real-time application of the mixing of corrections using only SSR corrections from global and regional network providers. The main idea behind this approach is that, instead of estimating the biases as the average over a long period of time, the biases can be estimated in epoch t

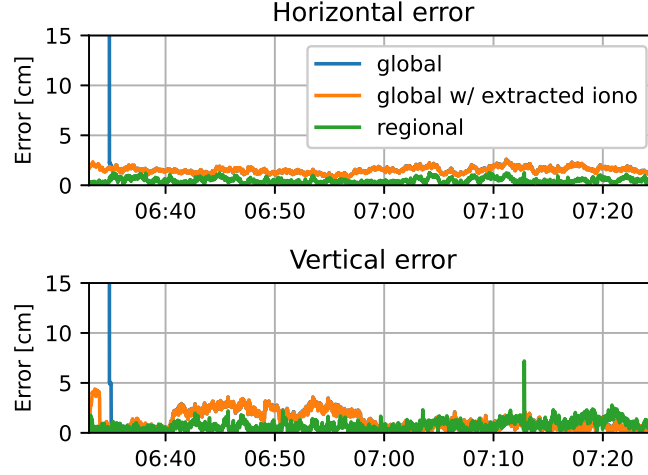


Figure 5.11: Ambiguity-fixed position performance for station JARG on day 273 of year 2021 using regional corrections, global corrections, and global corrections together with the extracted regional ionospheric corrections. All solutions use dual-frequency Galileo and GPS data.

and be used in epoch $t + 1$. Naturally, this approach would only work with the assumption that the biases are slowly varying from one epoch to the next. In the case of a data gap, the latest bias would be used, through such scenario has not been test. Figure 5.12 shows a swimlane diagram of the proposed approach.

The approach described in Figure 5.12 follows the following logic: for the first epoch of processing:

1. Generate PPP solution only using global products (satellite orbits, clocks, and biases),
2. Form global OSR using global products and the estimated ionosphere and troposphere from step 1,
3. Form regional OSR using regional products (satellite orbits, clocks, biases, and ionosphere and troposphere),
4. Compute and save double-differences per satellite . These values correspond to the biases caused by the discrepancy between products, and they will be used for the next epoch's extraction of the ionosphere.

For an epoch t that is not the first epoch of processing:

1. Form global OSR only using satellite products and without any atmospheric corrections, referred to as $P_{1,G}^s$ and $P_{2,G}^s$ in Equation (5.1),

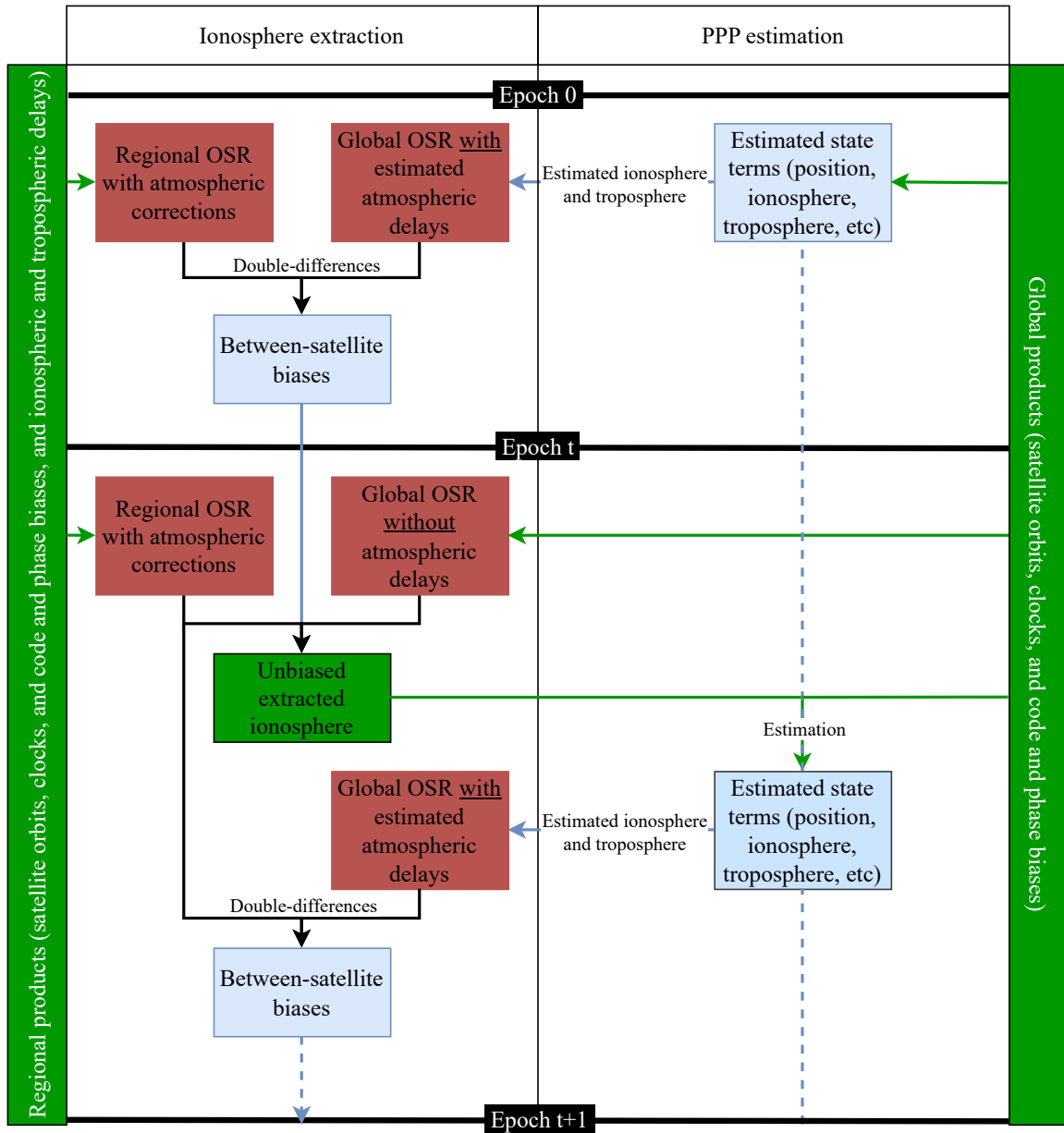


Figure 5.12: Swimlane diagram of the strategy to mix regional and global SSR products in real-time. Green boxes and arrows refer to SSR corrections, light blue boxes and arrows refer to estimates, and red boxes refer to OSR ranges computed with the SSR corrections/estimates.

2. Form regional OSR with atmospheric corrections being applied, referred to as $P_{1,R,full}^s$ and $P_{2,R,full}^s$ in Equation (5.2),
3. Extract the biased ionosphere from the ranges, referred to as $I_{1,ext}^s$ in Equation (5.4),
4. Extract unbiased ionospheric corrections by applying the biases that were estimated in epoch $t - 1$,
5. Generate a PPP solution using the global satellite products and the unbiased ionosphere that was extracted in the previous step,
6. Repeat steps 2 to 4 described in "first epoch" algorithm to estimate the biases to be used in the next epoch.

In this proposed approach, the bias estimation only happens at the end of the epoch and the estimated biases are used in the next epoch's processing. Choosing not to use the current epoch's biases in the PPP estimation is a choice that allows for each epoch to only be processed once, reducing the required computational power, and opens the possibility for real-time mixing of global and regional corrections. In order to compare this approach to the one proposed in the previous section, the former has been applied to the same dataset that was used in section 5.3.2. The comparison of the estimated biases with both approaches is shown in Figure 5.13.

As expected, Figure 5.13 shows that, while the average approach leads to constant biases being estimated, the real-time approach is able to capture the variations in the biases, and therefore represents a more realistic representation of the biases which are due to the discrepancies between global and regional products. The effect of these estimated biases on the extracted ionosphere is shown in Figure 5.14. The figure summarizes the quality of the extracted ionosphere using the real-time approach rather than the average approach that was described in section 5.3.2. The rms statistics for the quantities in the figure are summarized in Table 5.1.

Comparing Figure 5.14a to Figure 5.9b shows that, after the removal of the biases, the real-time approach is able to remove some of the trends that were observed with the average approach. Indeed, given that, in the average approach, the biases are estimated as the average of the double-differences across the duration of the datasets, the estimated biases cannot absorb the time variations in the biases. Therefore, such variations remain in the double-differences even after removing the estimated biases. In the real-time approach, given that the biases are estimated every epoch, they can absorb the variations in the double-differences. However, because the bias estimation is delayed relative to the PPP estimation, as the biases for each epoch are estimated from the previous epoch, the jumps in the double-differences are not eliminated, as they are sudden jumps between epochs. Regardless, the

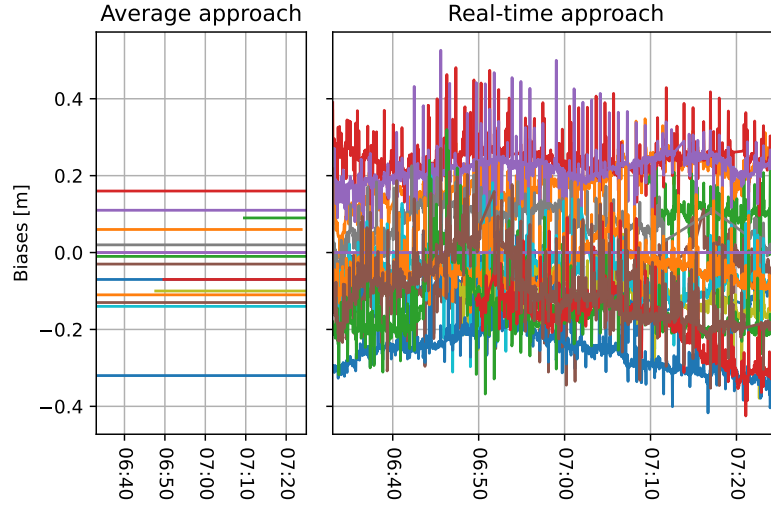
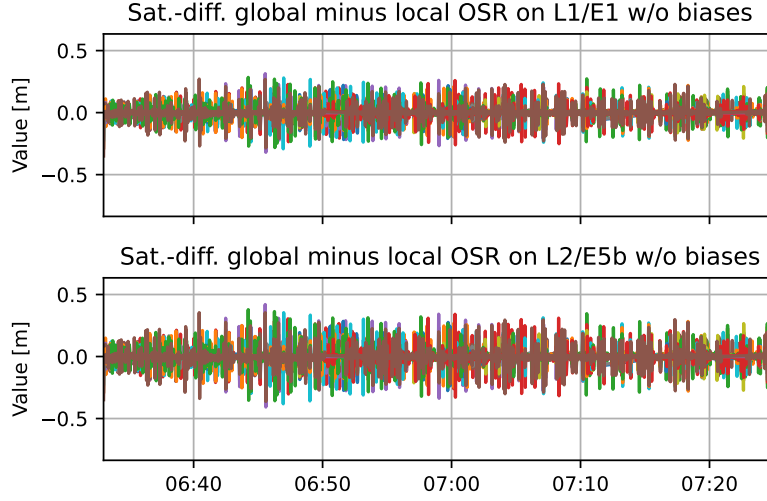


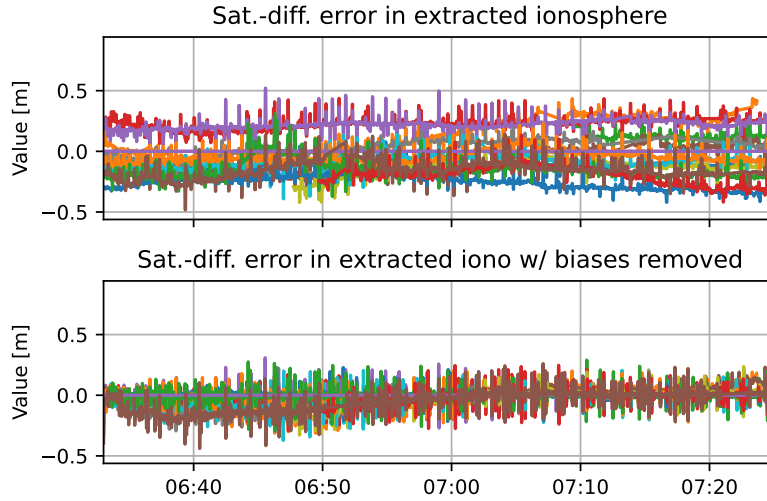
Figure 5.13: Comparison of the estimated biases with the average and real-time approaches for station JARG on day 273 of year 2021.

Table 5.1: rms statistics comparison using both approaches described in sections 5.3.2 and 5.3.3 for the different ranges and extracted biases of interest. These statistics relate to the dataset that was used in the current section for station JARG on day 273 of year 2021.

	Approach	Overall rms [cm]
DD OSR w/ biases on L1/E1		15.2
DD OSR w/o biases on L1/E1	Average approach	5.1
	Real-time approach	2.9
DD OSR w/ biases on L2/E5b		19.4
DD OSR w/o biases on L2/E5b	Average approach	6.1
	Real-time approach	3.7
DD of biased extracted ionosphere		15.9
DD of unbiased extracted ionosphere	Average approach	7.2
	Real-time approach	5.9



(a) OSR double-differences.



(b) Double-differences of extracted ionospheric corrections.

Figure 5.14: Between-satellite, between-source double-differences of the full OSR ranges for station JARG on day 273 of year 2021 after the removal of the biases with the real-time approach (5.14a), and between-satellite difference of the error in the extracted ionosphere with and without biases removal with the real-time approach (5.14b).

real-time approach leads to lower rms of the unbiased double-differences, as the approach leads to overall rms of 2.9 cm and 3.7 cm for L1/E1 and L2/E5b, respectively, as opposed to 5.1 cm and 6.1 cm with the average approach. The same conclusions and observations hold for the extracted ionospheric corrections by comparing Figure 5.14b to Figure 5.10. The real-time approach is able to remove more trends than the average approach, as the rms of the unbiased extracted ionosphere decreases from 7.2 cm with the average approach to 5.9 cm with the real-time approach, keeping in mind that both approaches present a significant improvement compared to the biased ionospheric corrections with their overall rms of 15.9 cm.

The last step of the analysis is to assess the impact of the extracted ionospheric corrections with the real-time approach on the position performance. As was seen in Figure 5.11, the average approach leads to ionospheric estimates that are precise enough that they lead to instantaneous convergence to the centimetre-level of the augmented global solution. Therefore, the improved ionospheric delays that are derived with the real-time approach lead to the same position performance as with the average approach. Despite providing the same performance, the real-time approach described in the current section is still preferred, as it can be applied in real-time while providing more precise ionospheric corrections.

Rather than showing the same performance between both approaches for the JARG dataset, the two approaches described above to derive the discrepancy-caused biases have been applied to a different dataset to assess their performance. The dataset is from station ISLA in France, collected on day 272 of year 2021 between UTC hours 2 and 3, with a worse global performance, as it requires ten minutes to converge to the centimetre-level. The results are summarized in Figure 5.15 and show good performance from the augmented solutions. While the global solution without ionospheric corrections requires ten minutes to converge to the centimetre-level, the two solutions with ionospheric corrections only require a few minutes to converge. This improvement in performance shows that the ionospheric corrections that were derived from the regional corrections are able to contribute to the improvement of the slowly-converging global solution. Additional improvements could be in extending the method to make use of the tropospheric corrections which are expected to improve the performance of the vertical component, which is subject to more variations.

5.4 Summary

In the field of precise positioning, PPP and NRTK tend to be seen as two separate techniques due to their historical evolution and applications. However, as was shown in this chapter, the two techniques are very similar in principle and can be seen as two facets of GNSS augmentation. While PPP uses a global network of reference stations to estimate satellite corrections in the form of precise satellite orbits, clocks, and biases, NRTK relies

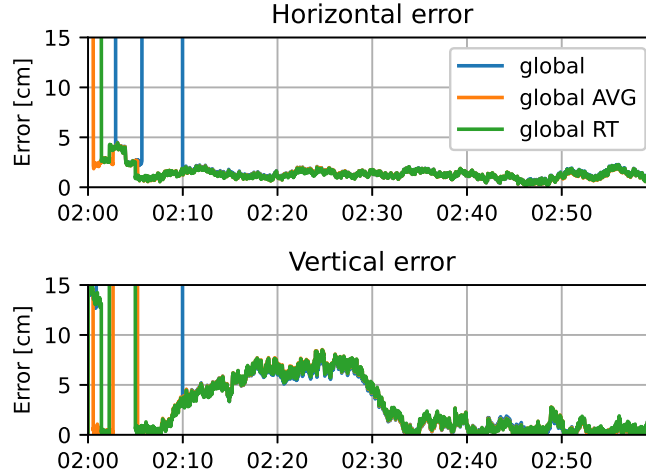


Figure 5.15: Ambiguity-fixed position performance for the global solution without atmospheric augmentation, and with two global solutions that use ionospheric corrections derived using the average and real-time approaches for station ISLA in France on day 272 of year 2021.

on a network of stations typically covering a region or area in a country to estimate SSR corrections. Similarly to PPP, NRTK estimates satellite orbits, clocks, and biases. Additionally, thanks to the small scale of the NRTK network, precise atmospheric corrections can be estimated as well and broadcast to the user, leading to quick convergence to highly precise positions for the users.

In the first part of this chapter, it was demonstrated that NRTK corrections can be used for PPP processing. By first analyzing both sets of SSR corrections, it was shown that SSR corrections estimated from a regional network are harder to decorrelate due to the small scale of the network compared to the user-satellite ranges. This more difficult decorrelation manifests itself, for example, in atmospheric delays seeping into orbit and clock corrections, as the latter are subject to variations depending on the satellite’s elevation angle. As a consequence, the products generated from a regional network are not physically meaningful and are meant to be used together to ensure consistency. It was demonstrated that in using the full range of corrections, NRTK corrections could be used in a PPP processing engine, keeping in mind that conventions and models have to be consistent with the network provider (for example carrier-phase wind-up, solid Earth tides, etc). Using all corrections, including atmospheric corrections, was shown to achieve NRTK-like performance, as instantaneous convergence to the centimetre-level was achieved for stations which are part of the reference stations network, as well as for stations which are not and which are within the network’s boundaries.

The second part of this chapter focussed on the difficult task of mixing corrections between different providers. One potential application of such mixing resides in the possibility to augment a user’s global solution with precise regional atmospheric corrections whenever available. However, rather than combining corrections of the same nature and from the same provider, the proposed method targets agnostic mixing of global PPP corrections and regional NRTK corrections which are generated from independent providers using networks of different scales. This task is especially difficult due to the regional corrections not being physically meaningful. In this chapter, a proof-of-concept was proposed, which is based on forming the full OSR ranges using both sets of corrections, with the assumption that the residual errors would contain the discrepancies between the two sets of products. Two approaches are proposed to estimate the biases caused by these discrepancies: one approach is based on computing the average of the OSR differences over time, while the second approach is based on the estimation of the biases at the end of every epoch and their use to correct for the next epoch’s ionospheric corrections. Both approaches were tested on two sample datasets and showed promising results, as the global solutions were improved with the introduction of the newly estimated ionospheric corrections. The robustness of this proof-of-concept can be further tested by applying it to a larger number of datasets, as well as by analyzing the temporal and spatial variability of the estimated biases.

Chapter 6

RTK from space: Galileo HAS

The RTK and NRTK techniques have been widely accepted as reliable and mature technologies that can achieve centimetre-level positioning accuracy in seconds for geodetic surveys (Gao et al., 1997; Jensen and Cannon, 2000; Townsend et al., 1999). In (N)RTK, users receive Observation State Representation (OSR) messages, which correspond to range corrections from a single reference station or a local reference network to eliminate/reduce most of the GNSS errors. Alternatively, PPP and PPP-RTK (PPP with local atmospheric augmentation) have proven to achieve similar levels of positioning accuracy with State Space Representation (SSR) corrections, e.g., precise satellite orbit and clock products, and a standalone GNSS receiver (Bisnath and Gao, 2009a; Geng et al., 2011; Teunissen et al., 2010; Zumberge et al., 1997), and these corrections are either broadcast through the Internet, or transmitted by commercial satellites.

In addition to the broadcasting of SSR over the Internet and commercial satellites, a recent trend has seen the emergence of the broadcasting of SSR corrections through GNSS constellations directly. The Centimetre-Level Augmentation Service (CLAS) in the Japanese Quasi-Zenith Satellite System (QZSS) was the first satellite centimetre-level augmentation service to become operational (Miya et al., 2016). The service provides QZSS and GPS PPP-RTK corrections over areas with QZSS coverage around Japan. The Chinese BeiDou has a similar regional PPP service called PPP-B2b, where multi-GNSS (only BeiDou and GPS at the moment) PPP corrections are broadcast to users in China and its surrounding areas through BeiDou's GEO satellites over the B2b signal (Liu et al., 2020a). Both QZSS CLAS and BeiDou PPP-B2b provide decimetre-level positioning (Hao et al., 2020; Tao et al., 2021; Xu et al., 2021).

Similarly to these regional services, the European Union is developing the Galileo High Accuracy Service (HAS), which is expected to be the pioneer GNSS in providing PPP orbit, clock, and code and phase bias corrections worldwide for free through its E6 signal (EU-SPA, 2020). At Service Level 1 (SL1), the target performance is to allow for 95% horizontal and vertical positioning accuracy of better than 20 cm and 40 cm, respectively, within 5

minutes, with 99% availability. At Service Level 2 (SL2), the service will be extended to provide atmospheric corrections to users over the European Coverage Area (ECA) - reducing the convergence time target to 100 seconds. Galileo HAS is designed to support the GPS and Galileo constellations on the L1/L5/L2C and E1/E5b/E5a/E6/E5AltBOC signals, respectively. The Galileo High Accuracy Initial Service (SL1 provision only based on the current Galileo stations' network) service became operational on January 24th, 2023 EUSPA (2023), and its maturity can benefit not only geodetic applications such as cadastral surveying, offshore exploration, and civil engineering, but also emerging markets such as Intelligent Transportation Systems (ITS), unmanned vehicles and drones, and augmented reality (EUSPA, 2020).

The EU Agency for the Space Programme (EUSPA), responsible for the Galileo service's provisions, is managing the Galileo HAS service Phase 0, enabling high accuracy testing and experimentation, and has been declared operational on January 24th, 2023. As part of Phase 0 and the HAS testing, Fernandez-Hernandez et al. (2022) analyzed early versions of the test signals, where simulated closed-loop tests were performed using data from September 2020, as well as signal-in-space tests using data from May 2021. The results showed that the SL1 target specifications after convergence could be met under nominal conditions and with good visibility conditions, with the 95% Signal-In-Space Ranging Error (SISRE) being 9.5 cm and 16 cm for Galileo and GPS, respectively. Hauschild et al. (2022) also made use of early Galileo HAS test signals from September 2021 and focussed on precise orbit determination of satellites in Low Earth Orbit (LEO).

In this second part of the chapter, an in-depth analysis of the HAS corrections and their PPP performance from a campaign in summer 2022 is performed, where Galileo and GPS satellite orbit, clock, and code bias corrections were broadcast. The main goals of this section are to comprehensively analyze the broadcasted test HAS corrections' quality by assessing 1) correction availability, 2) satellite orbit and clock corrections and SISRE, 3) satellite code bias corrections, and 4) float PPP solutions using worldwide stations. The PPP performance is compared to the HAS Full Service expected performance (EUSPA, 2020).

First, details about HAS and its application method on the user side are provided, followed by an analysis of the corrections availability. The satellite products are analyzed by comparing the satellite orbits and clocks to reference values from the Center for Orbit Determination in Europe (CODE), as well as analyzing the SISRE; in addition to analyzing the code bias corrections to reference values from the German Aerospace Center DLR. PPP performance is assessed at the 95th and 67th percentiles in both static and kinematic modes and compared to the HAS full service performance specifications (EUSPA, 2020). A simulation of the effect of HAS correction latency on PPP solutions is also performed. Note

that this analysis and research is the first to be published on the topic in the literature, thanks to early access to the Galileo HAS test signals (Naciri et al., 2023).

6.1 Galileo HAS description

The Galileo HAS corrections are transmitted on the E6B signal (1278.75 MHz) within the Galileo C/NAV navigation message. The corrections are transmitted in a format similar to Compact-SSR (CSSR), with the main differences being in the flexibility of message content and update/validity intervals. Indeed, the format used for HAS allows for, i.e., the transmission of orbits and clocks in the same message or at different rates, and a faster clock refresh rate for satellites with less stable clocks.

In this work, the decoding of HAS messages was performed by NovAtel, one of the members of the GISCAD-OV project (GEOWEB S.p.A., 2022). Several NovAtel PwrPak 7 receivers were modified and updated to decode E6B signals in real-time. One receiver is permanently installed within the GISCAD-OV Control Centre in Rome to continuously gather and log HAS messages. The decoded messages are output in a proprietary ASCII format, which can be used to process observation data with HAS corrections.

The Galileo High Accuracy Service is intended to be deployed in two phases: HAS Initial Service (delivering HAS Service Level 1 – SL1 based on the current Galileo stations network) and HAS Full Service (delivering SL1 and SL2 with their target performance) (EUSPA, 2020). The HAS SL1 includes the worldwide transmission, via the Signal-in-Space (SiS) and the Internet, of Galileo and GPS satellite orbits, clocks, and code and phase biases, with specified 95% horizontal and vertical accuracies of 20 cm and 40 cm, respectively. The HAS SiS Interface Control Document (ICD) was published in May 2022, confirming the structure and contents of the HAS Signal (European Union, 2022). SL2 is expected to include atmospheric (troposphere and ionosphere) corrections over the European Coverage Area (ECA), which is expected to improve the targeted convergence time from less than 300 s in SL1 to less than 100 s in SL2 in Europe. Galileo HAS has been in phase 0 in 2021 and 2022.

The HAS corrections are generated using the current Galileo Ground Stations (GSS) network of 14 stations (Fernandez-Hernandez et al., 2022), part of the existing Galileo infrastructure (EUSPA, 2022). Once the corrections are generated, they are transmitted back to the Galileo satellites through 5 up-link stations that are already used to update navigation messages and maintain the constellation in the Galileo infrastructure. The GSS network will be enabling the input data for the HAS Initial Service as well, though more ground stations are expected to be added to achieve the HAS Full Service and related target performance to enable SL1 and SL2.

Considering the scope of the current HAS testing phase as confirmed by EUSPA, the corrections in the HAS SiS and related performance may not be fully representative of the HAS Initial Service and differ from one test session to the next. In this study, results from a test broadcast session in summer 2022 are analyzed, where the transmitted corrections consisted of Galileo and GPS satellite orbits, clocks, and code biases. Phase biases were set as unavailable, though the messages were included in the structure for functional validation.

6.2 Galileo HAS Signal-in-Space

Given the real-time transmission of the corrections and the need to reduce the size of the broadcast messages, the HAS corrections are transmitted in a State Space Representation. The transmitted corrections should be used, along with the broadcast satellite orbits and clocks, to recover the precise satellite orbits and clocks. The transmitted orbital corrections δR^s are in the radial ϵ_R , along-track ϵ_A , and cross-track ϵ_C (RAC) directions and refer to the I/NAV and LNAV ionosphere-free antenna phase centres for Galileo and GPS, respectively (European Union, 2022). The orbit corrections need to be converted to an Earth-Centred Earth-Fixed (ECEF) frame before being added to the broadcast orbits. The conversion from RAC to ECEF frames is done as shown in equation 6.1, where x^s and $\dot{x}^s$ are vectors containing the broadcast satellite position and velocity, respectively. The equation yields the HAS corrections in ECEF. Once the ECEF corrections are recovered, the precise ECEF satellite orbits can be computed by simply adding the ECEF corrections to the broadcast ECEF orbits. The precise satellite orbits will refer to the ionosphere-free antenna phase centre in the Galileo Terrestrial Reference Frame (GTRF). For a triple- or quadruple-frequency user processing HAS in uncombined mode (without forming linear combinations of measurements), it is necessary to ensure that the satellite orbits on each frequency refer to that frequency's antenna phase centre using satellite antenna corrections.

$$\begin{cases} \epsilon_A &= \frac{\dot{x}^s}{|\dot{x}^s|} \\ \epsilon_C &= \frac{x^s \times \dot{x}^s}{|x^s \times \dot{x}^s|} \\ \epsilon_R &= \epsilon_A \times \epsilon_C \\ \delta X^s &= [\epsilon_R, \epsilon_A, \epsilon_C] \delta R^s \end{cases} \quad (6.1)$$

A similar strategy is to be used for the satellite clocks, as the HAS corrections must be added to the broadcast satellite clocks. The reference time for Galileo HAS is the Galileo System Time (GST), and the corrections are to be added to the ionosphere-free broadcast satellite clock. One must ensure that the relativistic effect is corrected for as well, when adding the HAS clock corrections to the broadcast clocks. During the processing, the user must apply code biases to offset between the frequency bias of each signal and the

ionosphere-free combination of the biases that are present in the satellite clock. In general, Observable-specific Signal Biases (OSB) are to be applied to the pseudorange measurements on the corresponding frequencies. The same principle holds for phase biases, though they are not part of this study, as they have not been transmitted as part of the testing campaigns.

Given that navigation messages are updated up to every 10 minutes for Galileo and every 2 hours for GPS, the HAS corrections have to be matched to their corresponding navigation message through an Issue Of Data (IOD) identifier, which is broadcasted along with the HAS corrections. The IOD should be used to find the corresponding IODnav and IODE (IOD Ephemeris)/IODC (IOD Clock) navigation messages for Galileo and GPS, respectively. The IOD range is 0-1023 for Galileo and 0-255 for GPS.

The design of the HAS messages is such that the corrections are constrained to be within certain values. Table 6.1 shows the range of each correction. Phase bias and atmospheric corrections are omitted, as they have not been transmitted as part of the test campaigns. In addition to being constrained to certain ranges, the corrections can take certain values to indicate the status of the correction. For instance, all corrections have a specific value each to indicate that the correction is not available, while the satellite clocks are used to indicate to the user that the satellite's corrections are not to be used. Such indicators provide some indication to the user on the health of a satellite's corrections and allow the rejection of unusable corrections.

During the test campaign analyzed in this work, the broadcast configuration was such that the HAS messages are split into two categories: slow messages, and fast messages. Slow messages contain the orbit and code (and phase) bias corrections and are updated every 50 seconds, while the fast messages contain the satellite clock corrections, updated every 10 seconds. In addition to having to be used with their corresponding navigation messages, HAS corrections have validity intervals, after which they should not be used. As satellite code biases tend to be stable over a few hours (Deng et al., 2021; Villiger et al.,

Table 6.1: Range and specific indicators of the HAS satellite orbit, clock, and code bias corrections. Validity and update interval values are specific to test campaigns. The indicators are specific to the NovAtel decoder format, and binary values for HAS can be found in European Union (2022).

Correction type	Range [m]	Indicators	Validity interval [s]	Update interval [s]
Radial	+/- 10.2375	-10.24: not available	300	50
Along-track	+/- 16.376	-16.384: not available	300	50
Cross-track	+/- 16.376	-16.384: not available	300	50
Delta clocks	+/- 40.95	-40.96: not available 40.95: not to be used	60	10
Code biases	+/- 20.46	-20.48: data not available	3600	50

2019), their validity interval is much longer than orbit and clock corrections. Additionally, since orbit corrections tend to have less variations than clock corrections, they (orbits) have longer validity intervals than clocks. The separation into two slow and fast messages and the validity interval and indicators are specific to the test campaigns, all of which can be changed. For instance, the validity and update intervals of the messages is part of the broadcast information and is not a fixed value.

6.3 Assessment of HAS corrections

In the current section, the HAS corrections broadcast during the summer 2022 campaign are analyzed. Before analyzing the PPP performance, the current section focuses on analyzing the corrections themselves in terms of availability, as well as comparison to other products widely-used in PPP. The section starts with an analysis of the availability of the corrections, followed by analysis with the quality of satellite orbit and clock corrections, before ending with an analysis of the quality of the satellite code bias corrections.

6.3.1 Availability analysis

The following subsection analyzes the availability of HAS corrections during the campaign that took place during summer 2022, with a focus between days 242 and 248 of year 2022. The campaign consisted of Galileo and GPS orbits, clocks, and code biases. The availability analysis only involves orbits and clocks, as code biases were always available during the campaign given their longer validity intervals. As an indicator of the availability of corrections for both constellations, Figure 6.1 shows the status for each satellite on day 247 of year 2022. Three main statuses are identified:

- **Good:** The corrections are good to be processed by the user,
- **Corrections too old:** The latest corrections are past their validity interval,
- **Unusable corrections:** The corrections contain one of the indicators in Table 6.1 to indicate unavailable corrections, or corrections to not be used.

Figure 6.1 is generated by computing the availability every 30 seconds over the course of the day. As the figure shows, the corrections are available throughout the day, with brief periods without corrections for certain satellites, which are more frequent for GPS than for Galileo. Satellites E20 and E22 are missing availability status, due to E22 being set as “not usable” and E20 being set as “not available” by the Galileo constellation. Additionally, some satellites have their HAS corrections set as unusable for the duration of the day (E01, E14, E18, G25 and G28), due to the status of the satellites by their constellation. e.g., E01 has been set as “not available” since DOY 243, 2022 by the Galileo constellation, while E14

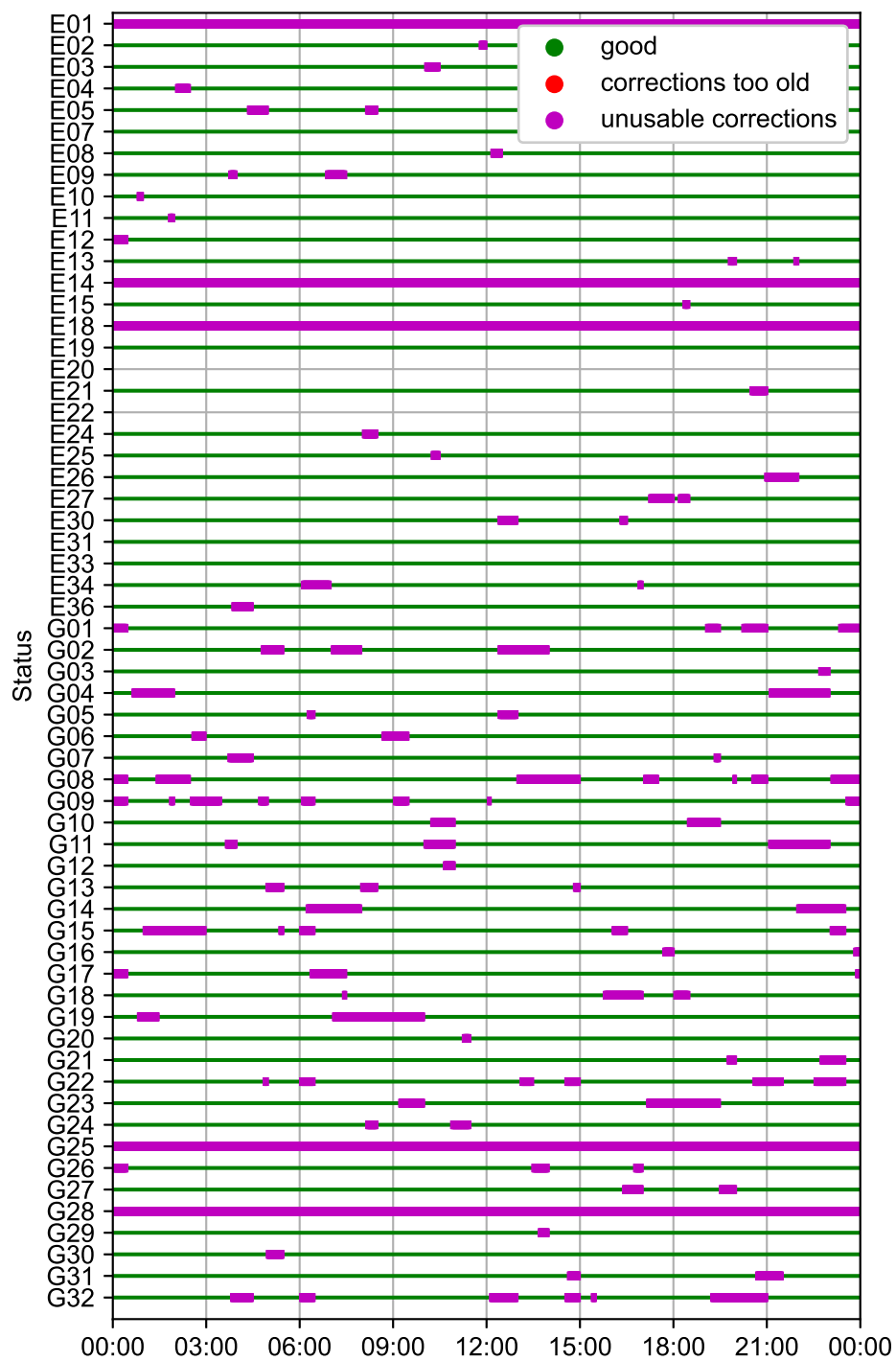


Figure 6.1: Availability of HAS orbit and clock corrections on day 247 of year 2022 for Galileo and GPS satellites.

and E18 are set as “not usable” satellites. PRN 28 is not attributed to any GPS satellite, and G25 has been set as “unusable” by the GPS constellation since DOY 231, 2022. In terms of the intermittent outages, they are due to the corrections being set as unusable in the decoded HAS messages. It is apparent from Figure 6.1 that the outages are more frequent for GPS than Galileo. The overall availability appears to be relatively reasonable, as the intermittent outages for most satellites appear at different times of the day for each satellite. Therefore, a PPP solution would not be reinitialized, as most satellites would be processed continuously.

To quantify correction availability, Figure 6.2 details the availability per satellite over seven days between days 242 and 248 of year 2022. The figure is generated by computing, for each satellite, the percentage of epochs with available orbit and clock corrections. The percentage based on epochs every 30 seconds over one week.

The figure shows that all Galileo satellites, shown in the top sub-figure, with available corrections have corrections available at least 90% of the time, with some satellites having corrections available 100% of the time. GPS satellites, in the bottom sub-figure, have a relatively lower average availability of 90.7%, due to the intermittent outages shown in Figure 6.1, as compared to 96.5% for Galileo. While Galileo has a lowest availability of 92% for satellite E24, GPS suffers from lower minimum availabilities, as satellite G32 has an availability of 72.9%, with many satellites having availabilities between 84% and 90%. These values are in accordance with the time series shown in Figure 6.1, though Figure 6.2 is based on a longer period of seven days rather than one week, as was the case for Figure 6.1. Therefore, the figure shows very good availability for Galileo satellites, while GPS satellites tend to have worse availability. Despite the lower availability, GPS satellites are still expected to improve a Galileo-only solution.

6.3.2 Satellite orbit and clock corrections

Along with the availability of the corrections, analyzing the performance of the products requires comparison with other sources. In the current subsection, the HAS orbit and clock corrections are compared to final orbit and clock products from the Center for Orbit Determination in Europe (CODE). Given that the products from CODE are final products, which are the highest quality of products, they can be used as references to compare against HAS products. The analysis is performed for the summer 2022 test broadcast, which contained both Galileo and GPS corrections. The analysis consists of a direct comparison of the products, as well as analysis of the SISRE.

The direct comparison consists of analyzing the difference between orbit and clock products in the RAC frame. To do so, the difference between both sets of clocks (HAS and CODE) is taken, and the mean value per constellation removed. The reason behind removing the

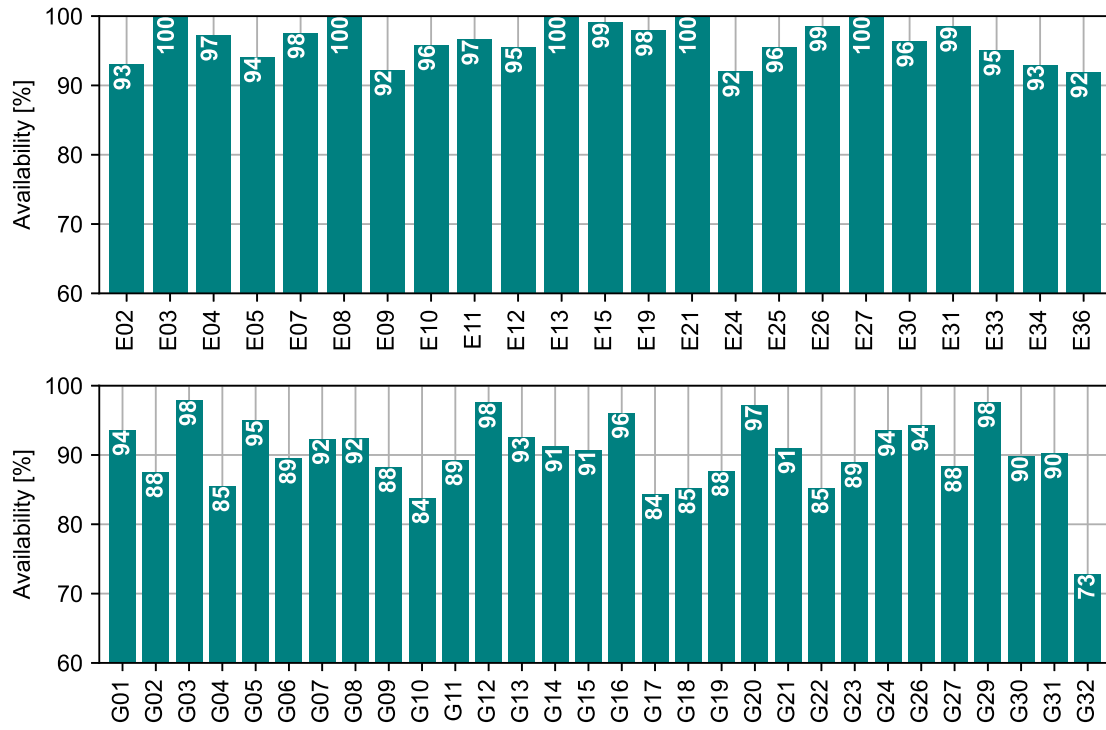


Figure 6.2: Percentage of time with available HAS orbit and clock corrections per Galileo (top) and GPS (bottom) satellite between days 242 and 248 of year 2022.

mean value is that clock products tend to have common variations between satellites from the same constellation, which would not impact the user solution, as the common variations would be absorbed by the receiver clock. These common variations would be different between analysis centres, depending on the models used in the generation of the satellite products. Additionally, one has to keep in mind that HAS clocks refer to the ionosphere-free combination of the LNAV signals (L1 C/A and L2P (Y), or C1C and C2P in RINEX3 convention) for GPS and I/NAV signals (E1/E5b, or C1C and C7Q in RINEX3 convention) for Galileo (European Union, 2022), while the CODE final products refer to the ionosphere-free combination of C1W and C2W (in RINEX3 convention) signals for GPS and C1C and C5Q for Galileo (Villiger et al., 2019). Consequently, alignment between these different conventions is performed using satellite biases to refer both sets of clocks to the same combinations of signals.

When it comes to orbit comparison, the ECEF coordinates from both HAS and CODE are differenced, and the conversion from ECEF to RAC is performed. Given that the transformation matrix from RAC to ECEF δR^s is orthogonal, the conversion from ECEF to RAC can be performed by multiplying by the transpose of the matrix. Consequently, the orbital errors in RAC ($\delta R, \delta A, \delta C$) are computed as shown in equation 6.2. Note that the HAS orbits refer to the ionosphere-free antenna phase centre, while CODE orbits refer to the centre of mass. To counter for the mismatch, an antenna correction is applied to move the CODE orbits to the ionosphere-free antenna phase centre.

$$\begin{bmatrix} \delta R \\ \delta A \\ \delta C \end{bmatrix} = \begin{bmatrix} \epsilon_A & \epsilon_B & \epsilon_C \end{bmatrix}^T \begin{bmatrix} X_{HAS} - X_{CODE} \\ Y_{HAS} - Y_{CODE} \\ Z_{HAS} - Z_{CODE} \end{bmatrix} \quad (6.2)$$

While the direct comparison of products is a good indicator of the quality of the products, it does not necessarily reflect how such products would affect user performance. The SISRE is a closer indicator, as it takes into account for example the projection on the line-of-sight of the different orbit components. The SISRE depends on the user location, though it is possible to compute a global SISRE that represents the average SISRE to be expected when using the products (Montenbruck et al., 2015). In this work, two SISRE are considered: one orbit SISRE that only considers orbit errors, while the total SISRE considers the clock errors as well. Both SISRE formulations used in this work are detailed in equation 6.3.

$$\begin{cases} SISRE_{orb} &= \sqrt{w_R^2 \delta R^2 + w_{AC}^2 (\delta A^2 + \delta C^2)} \\ SISRE_{tot} &= \sqrt{(w_R \delta R - \delta CLK)^2 + w_{AC}^2 (\delta A^2 + \delta C^2)} \end{cases} \quad (6.3)$$

In equation 6.3, $SISRE_{orb}$ and $SISRE_{tot}$ are the orbit and total SISRE, respectively. δR , δA , and δC are the radial, along-track and cross-track errors, while δCLK is the clock error (with the mean removed). w_R and w_{AC} are coefficients that reflect the projection of the individual corrections to the line-of-sight. w_R is taken as 0.98, while w_{AC} is taken as 1/61 for Galileo and 1/49 for GPS (Montenbruck et al., 2015).

Figure 6.3 highlights the orbit errors in the RAC frame, as well as the 3D combination, in addition to the clock errors for day 247 of year 2022. The mean per constellation in the clock errors is removed to eliminate common biases between HAS and CODE. The rms error for the individual components of each constellation are summarized in Table 6.2.

In terms of orbit errors, the rms of the 3D errors are 9.0 cm and 8.6 cm for Galileo and GPS, respectively. These rms values are mostly due to the along- and cross-track components, with the radial component being 3.2 cm and 2.4 cm for Galileo and GPS, respectively. Consequently, the higher along- and cross-track rms are expected to have limited impact on the user performance, as the components will be projected to the line-of-sight between the satellite and the user. Similar to the radial component, the clock component has a direct impact on user performance. The rms of the clock errors are 15.5 cm (0.52 ns) for Galileo and 14.5 cm (0.48 ns) for GPS, showing comparable rms between Galileo and GPS clocks. However, as can be seen in Figure 6.3, two satellites have relatively larger clock errors: satellite G19 with a clock error around 0.4/0.5 m and satellite E12 with a clock error of around 0.6/0.7 m. The reason behind these two larger clock errors is still unknown, but may be associated with the test nature of the HAS broadcast. Removing these two satellites from the computation leads to clock errors of 8.8 cm (0.29 ns) and 11.6 cm (0.39 ns) for Galileo and GPS, respectively.

The comparison with CODE products yields comparable results to those shown by Hauschild et al. (2022) and Fernandez-Hernandez et al. (2022), though the broadcast period and the reference products are different. BeiDou’s PPP-B2b service follows a similar principle as Galileo HAS, in the sense that PPP corrections are being broadcast from the BeiDou satellites directly. PPP-B2b consists of PPP corrections for BeiDou-3 and GPS. Comparing the values in Table 6.2 with already published results for PPP-B2b, it appears that GPS (the common constellation) orbit corrections from Galileo HAS have less error

Table 6.2: rms of the radial, along-track, cross-track, 3D orbit, and clock errors of HAS relative to CODE final products on day 247 of year 2022. The rms are computed taking into account all satellites from each constellation. Values are in cm.

	Radial	Along-track	Cross-track	3D	Clock	Clock excluding G19 and E12
Galileo	3.2	6.6	5.3	9.0	15.5	8.8
GPS	2.4	6.4	5.3	8.6	14.5	11.6

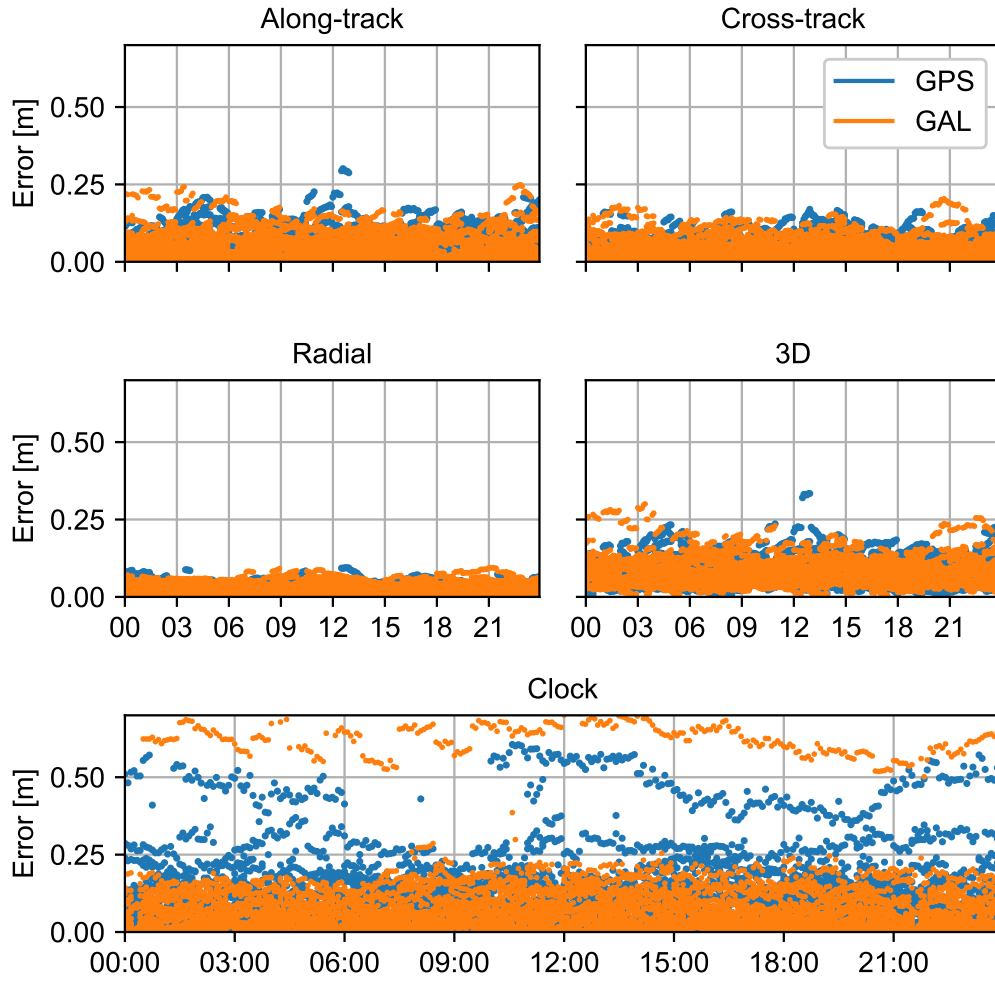


Figure 6.3: Along-track (top left), cross-track (top right), radial (center left), and 3D (center right) orbital errors and clock (bottom) errors of Galileo and GPS HAS corrections relative to final CODE orbits and clocks on day 247 of year 2022.

than in PPP-B2b, though the latter has more accurate clocks (Tao et al., 2021; Nie et al., 2021).

A SISRE analysis is performed in Figure 6.4, where both the SISRE considering only orbit corrections, and considering both orbit and clock corrections are analyzed. The results behave as expected given the results in Figure 6.3: both Galileo and GPS SISRE perform reasonably well, except for G19 and E12, for which the higher clock errors lead to higher total SISRE. The 24-hour rms for Galileo are 3.1 cm and 10.8 cm for the orbit SISRE and total SISRE, respectively, excluding E12. In contrast, the GPS rms are 2.4 cm and 11.6 cm for the orbit SISRE and total SISRE, respectively, excluding G19. These values show comparable quality between Galileo and GPS. Comparable values were obtained by Hauschild et al. (2022) for the Galileo constellation for the September 2021 test broadcast, though GPS SISRE were higher for that campaign compared to this study of the summer 2022 test broadcast. Similar conclusions are made when comparing to results by Fernandez-Hernandez et al. (2022), which focussed on the September 2020 test broadcast. Additionally, for general comparison, these GPS SISRE values appear to be lower than the PPP-B2b corrections by Tao et al. (2021), which showed larger clock variations for GPS. Compared to SISRE from CNES products in the same paper, the HAS GPS orbit SISRE and total SISRE are comparable.

In addition to showing the evolution of SISRE over one day, it is interesting to look at the SISRE per satellite, as presented in Figure 6.5. Each bar in the figure represents the SISRE rms over the whole day on day 247 of year 2022, with both the orbit-only and total SISREs being presented. As expected from Figure 6.5, all satellites from both constellations have reasonable orbit SISRE at the centimetre-level, as well as total SISRE at the decimetre-level. Both constellations behave similarly, with values in the same range. The total SISRE are consistent for all satellites, except for G19 and E12 due to their higher clock errors. Similarly to the above results, these SISRE values are comparable to values by Hauschild et al. (2022) and Fernandez-Hernandez et al. (2022), with the exception of the GPS total SISRE, which are improved in the summer 2022 test broadcast.

6.3.3 Satellite code bias corrections

In addition to the satellite orbit and clock corrections, the last remaining products to be analyzed from the test campaigns are the code biases. The analysis is performed by comparing the HAS test signal code biases with final DCBs from the German Aerospace Center DLR (Montenbruck et al., 2014b). Given that the DLR DCBs are final products, and that they are comparable to the sub-nanosecond level to other DCB products from analysis centres such as CODE and the Chinese Academy of Sciences (CAS) (Liu et al., 2019), they can be taken as the reference to compare against.

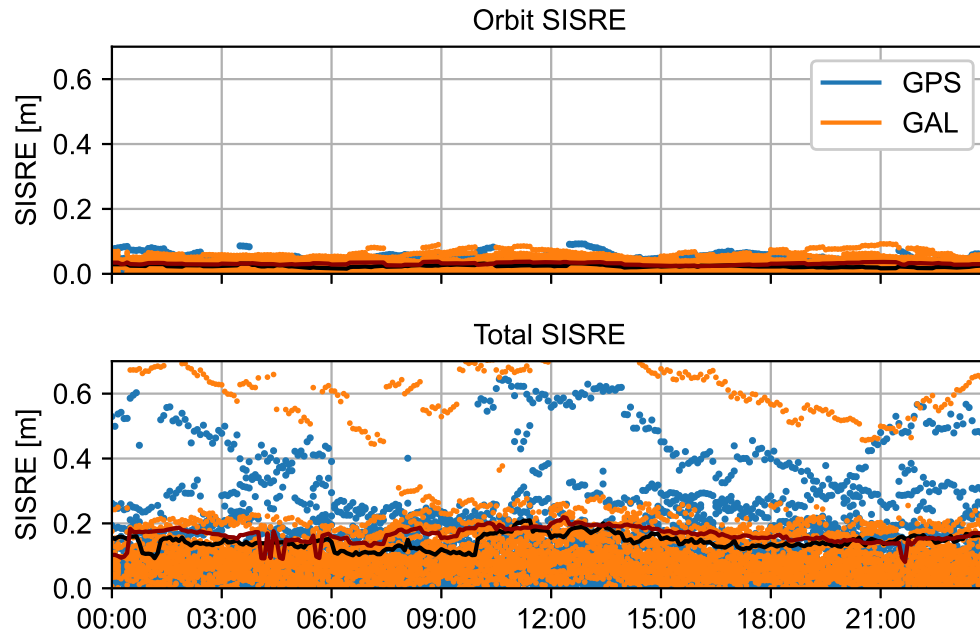


Figure 6.4: Orbital SISRE (top) and total SISRE (bottom) of Galileo and GPS HAS corrections relative to final CODE orbits and clocks on day 247 of year 2022. The horizontal dark red lines represent the Galileo rms, while the black lines represent the GPS rms. Satellites G19 and E12 are excluded from the rms computation.

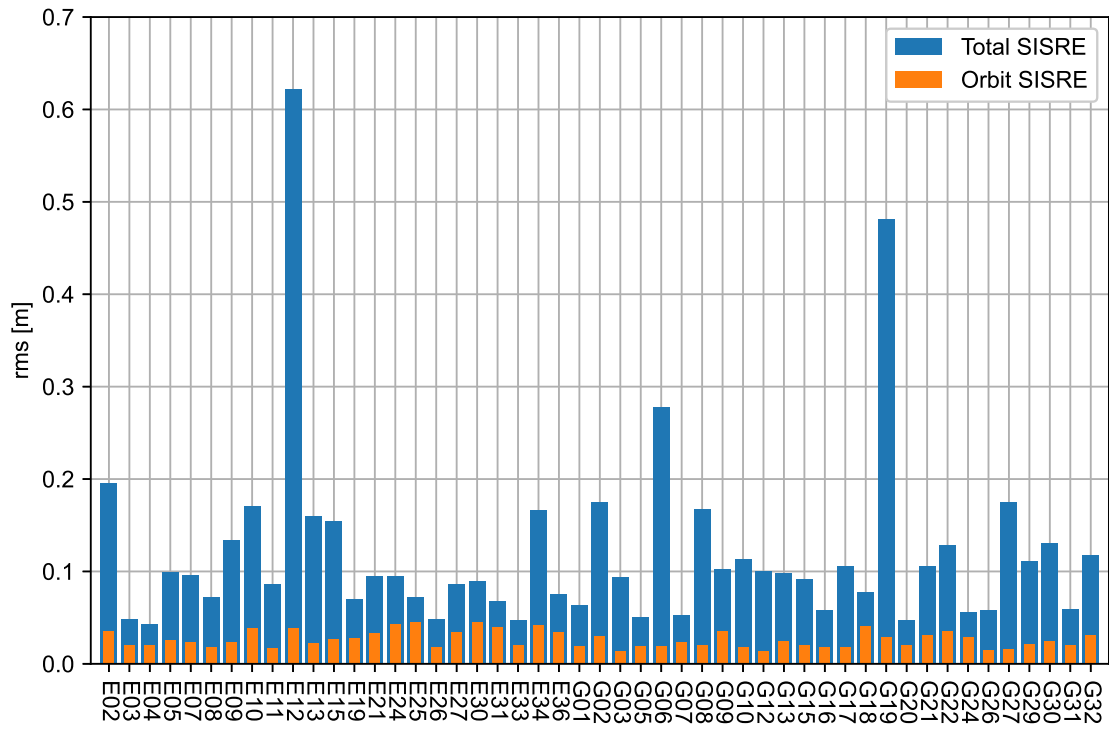


Figure 6.5: Daily average orbital SISRE (orange) and total SISRE (blue) per Galileo and GPS satellite of HAS corrections relative to final CODE orbits and clocks on day 247 of year 2022.

The code bias corrections in the HAS test signal were not computed by the Galileo infrastructure. Instead, they were introduced by configuration based on DCBs from CAS to test and validate the HAS SiS ICD. Therefore, code biases in the test broadcast are not representative of the HAS Initial Service to be declared. Nonetheless, the results obtained during the testing campaign are described. The code biases were in the OSB format, meaning that the corrections are pseudo-absolute (“absolute” because each bias is applied directly to its measurement, and “pseudo” because the biases are, in theory, relative, but the user does not need to know the reference signals) and can be applied directly to the pseudorange measurements. During the summer 2022, the code bias corrections were broadcast for signals E1 C no data (C1C), E5A Q no data (C5Q), E5B Q no data (C7Q), and E6 C no data (C6C), for Galileo, and L1 C/A (C1C), L2C (C2L) and L2P (C2P) for GPS, with the identifiers in parenthesis being the signal codes in RINEX 3 format.

In order to perform the comparison with the reference DCBs, the HAS DCBs are created by differencing OSBs. For instance, the HAS C1C-C5Q DCB is recovered by subtracting the C1C OSB from the C5Q OSB. In total, four DCB combinations are analyzed: C1C-C5Q, C1C-C7Q, and C1C-C6C for Galileo, and C1C-C2L for GPS. The C2P bias is not analyzed due to the absence of DCBs involving the signal. The evolution of the DCB differences for all available satellites over one week in summer 2022 is shown in Figure 6.6. Each data point represents the average over a day, though the code biases tend to be constant over the course of a day. The code biases in the test campaign are constant with adjustments at 2 am UTC. A mean value of -1.81 ns has been removed from the GPS DCB difference, as the constellation-dependent mean would be absorbed by the receiver clock at the user level.

Figure 6.6 shows code bias stability over the week, as the code biases from the test campaign are within less than one ns of the DLR DCBs. The difference in DCB between HAS and the DLR products appears to be at the sub-nanosecond-level, with rms of 0.29 ns, 0.27 ns, and 0.23 ns for C1C-C5Q, C1C-C7Q, and C1C-C6C, respectively. For GPS C1C-C2L, the rms is 0.21 ns. These values are in accordance with typical rms values when comparing DCBs generated from different analysis centres (Villiger et al., 2019; Deng et al., 2021; Li et al., 2017). Galileo satellite E25 experiences higher errors on September 4th, as can be seen from all three DCB plots. These higher errors are due to a jump in all HAS biases for that satellite that lasted for 24 hours, where the jumps reached 1.7 ns, 3.0 ns, 3.1 ns, and 3.1 ns for C1C, C7Q, C5Q, and C6C, respectively.

Figure 6.7 summarizes the mean and standard deviation of DCB differences for each satellite over the course of the same week for all four DCBs. The figure is intended to show the performance per satellite rather than show the evolution over time, as was presented in Figure 6.6. Results show good consistency, as most satellites are within 0.50 ns of the DLR DCBs, except for satellite E25 due to its higher errors on September 4th, as shown in Figure 6.6. The standard deviation of GPS DCBs is lower than that of Galileo, showing

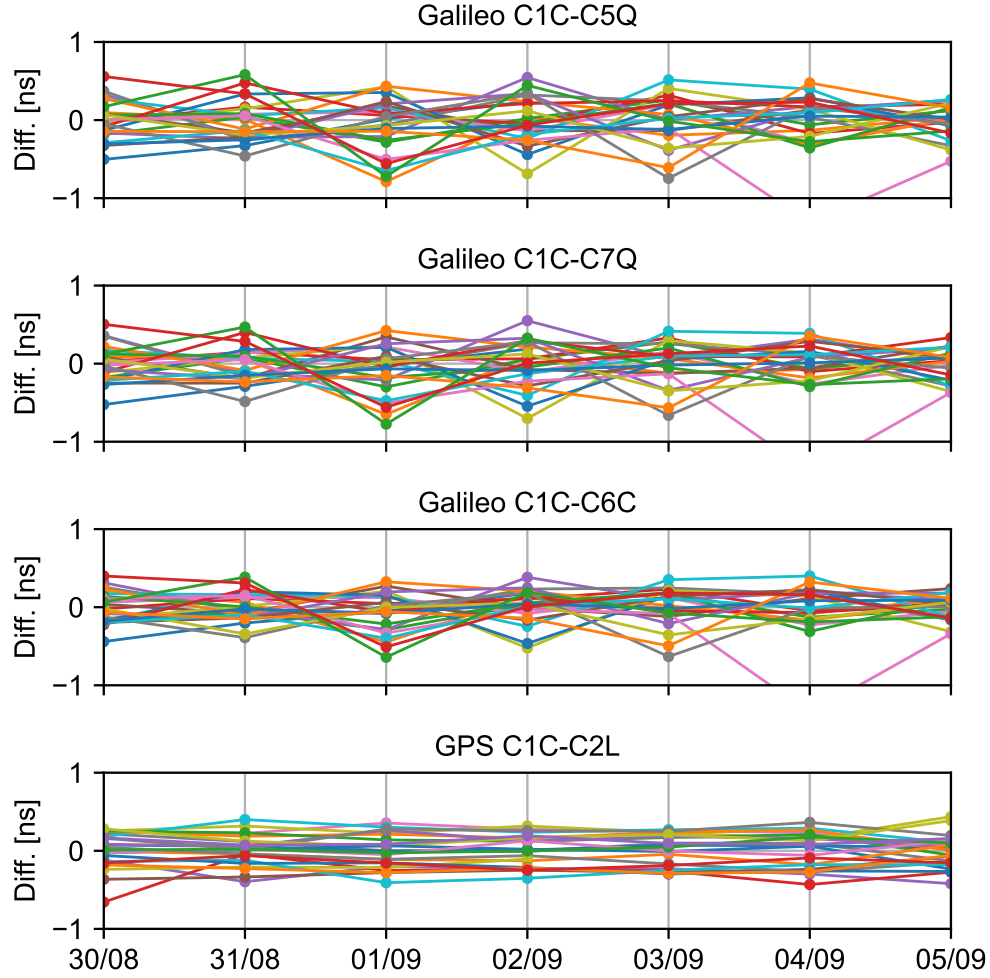


Figure 6.6: Daily average difference between HAS code biases and final DCB products from DLR. The graphs are for the period between days 242 and 248 of year 2022 for Galileo and GPS satellites/signals. Each colour represents a different satellite. The x-axis is in the format “day/month”. On September 4th, satellite E25 goes beyond the y-axis and reaches values of -1.3 ns, -1.4 ns, and -1.4 ns for C1C-C5Q, C1C-C7Q, and C1C-C6C, respectively.

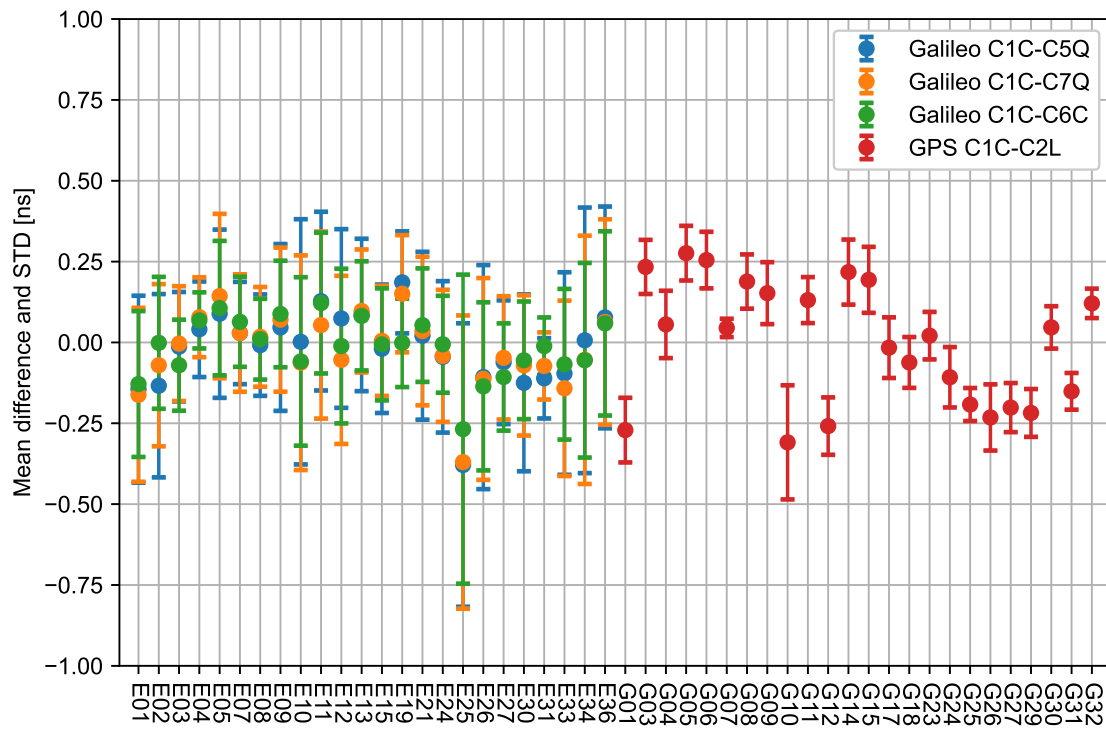


Figure 6.7: Mean difference and standard deviation per Galileo and GPS satellite of HAS code biases relative to final DCB products from DLR. The statistics are based on one week of data, between days 242 and 248 of year 2022.

good stability over several days for GPS DCBs. These values show that clock and DCB corrections from HAS are aligned with IGS-type corrections, as clock and DCB differences are at the sub-nanosecond level.

6.4 PPP performance with Galileo HAS corrections

The performance of Galileo HAS products is analyzed by generating PPP solutions using the HAS corrections. The current section begins with a description of the processing strategy used for the PPP processing, as well as a description of the data being processed. The section continues with PPP results from the summer 2022 test broadcast, before ending with a preliminary analysis of the effect of HAS correction latency on PPP performance.

6.4.1 Processing strategies and data

The PPP processing is performed with the York-PPP engine from the GNSS Lab. The software has been modified to accept HAS corrections as input, as well as any modifications related to the nature of the corrections, i.e., orbit corrections relating to the ionosphere-free antenna phase centre, validity intervals, etc. The PPP tests are performed on a set of 33 global IGS stations. A map of the stations is provided in Figure 6.8. Given that HAS GPS code biases are only available for the C1C, C2L and C2P signals, the stations are selected by ensuring that they track C1C and at least one of the two L2 signals. The tests are performed over a period of one week between days 242 and 248 of year 2022. A total of approximately 1200 independent three-hour long datasets is processed.

The observation data are processed with the processing strategies described in Table 6.3 using a sequential least-squares filter. Both Galileo and GPS measurements are processed using HAS orbits, clocks, and code biases. All data are processed in dual-frequency mode

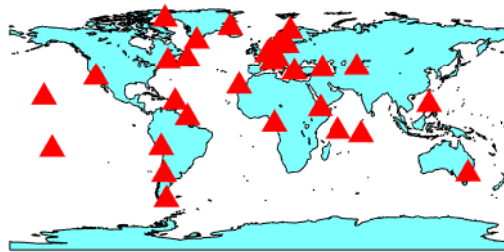


Figure 6.8: Map of the stations used in the PPP processing and HAS analysis.

(E1/E5b for Galileo, and L1/L2 for GPS) and in uncombined processing (using raw single-frequency measurements without forming linear combinations of measurements). The satellite and receiver antenna corrections are applied from the IGS14 ANTEX file (Schmid et al., 2016). All other necessary corrections are applied, including phase wind-up, relativistic effect and Earth rotation following the IERS conventions (Kouba and Mireault, 1998). An elevation angle cut-off of 7 degrees is applied to reject satellites near the horizon from the processing. To recover satellite orbit and clock corrections from HAS corrections, broadcast orbits and clocks are computed using daily merged navigation files from the IGS.

For the analysis, the horizontal and vertical convergence times are defined as the times it takes to reach and settle below 20 cm and 40 horizontal and vertical accuracy, respectively. These definitions are based on the Galileo service specification metrics (EUSPA, 2020). In terms of accuracy, the rms of the position errors is assessed over the whole duration of processing, including the convergence period. Note that the performance specifications are defined post-convergence.

6.4.2 Positioning results

Figure 6.9 illustrates the evolution over time of the average horizontal and vertical errors at the 95th and 67th percentiles for the data being processed in both static and kinematic modes. Each solution in the figure is generated by taking the average of the 95% and 67% lowest position errors on every epoch for each constellation and static/kinematic combination. The figure shows expected general performance: the solutions start at the few-decimetres-level and converge to better accuracies over time; using two constellations provides better performance than a single-constellation solution; and static processing provides better performance than kinematic processing. Additionally, the GPS-only solutions appear to be worse than the Galileo-only solutions. This behaviour is due to the GPS solutions tending to have a lower number of processed satellites – mainly because of limitations in the availability of signals for the code biases, as the average number of processed GPS satellites across all datasets is 5.4, compared to 7.3 for Galileo.

Figure 6.10 summarizes the convergence time statistics to 20 cm horizontal and 40 cm vertical accuracies for the solutions in Figure 6.9. The results show that the convergence time service specification for the HAS full Service of 5 minutes is met at the 67th percentile level for the Galileo-only and combined Galileo and GPS solutions, and nearly met at the 95th percentile. As expected, the GPS-only results take much longer to converge than the Galileo-only solution, though it is worth noting that adding GPS to the Galileo-only solution improves the latter’s performance. In terms of differences between static and kinematic processing results, other than the GPS-only solution that suffers from less satellites being processed, all other solutions provide comparable results between processing modes. These

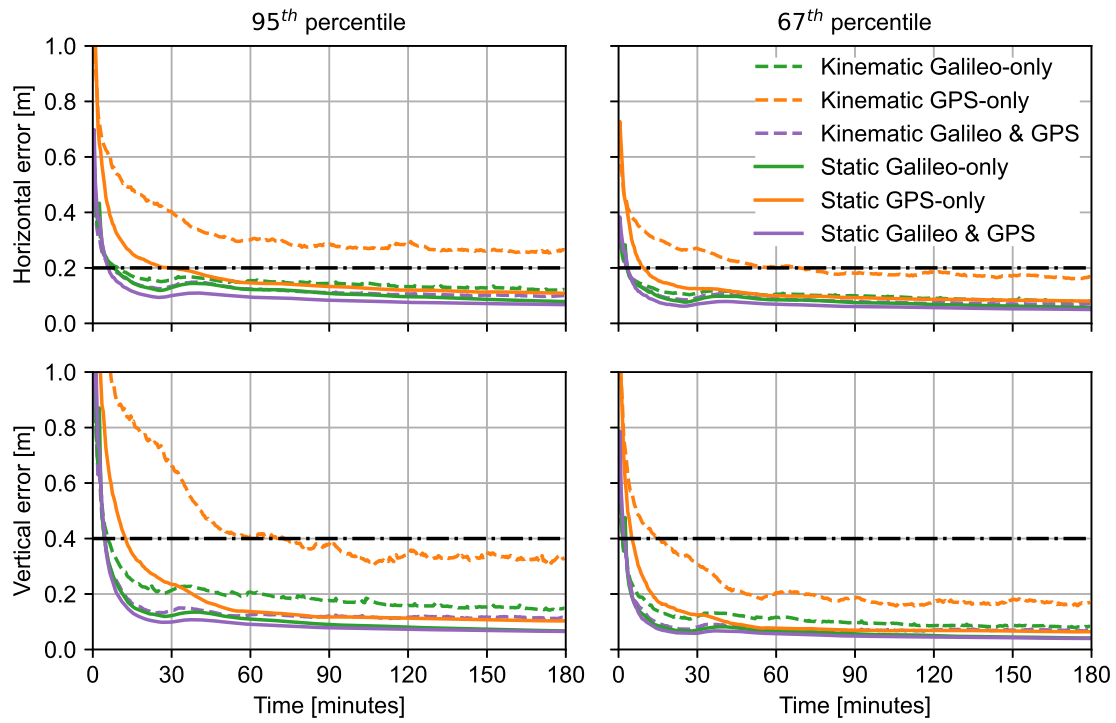


Figure 6.9: Average PPP horizontal (top row) and vertical (bottom row) performance at the 95th (left column) and 67th (right column) percentiles. The solutions include Galileo-only, GPS-only, and Galileo+GPS HAS results in both static and kinematic modes. Horizontal dashed lines represent HAS service specification.

Table 6.3: PPP processing strategy for the HAS processing.

Parameter	Processing strategy
Receiver coordinates	Kinematic mode: estimated with white noise equivalent to 100 km/h Static mode: estimated as constants
Receiver reference coordinates	IGS SINEX positions
Receiver clocks	One per constellation, estimated as white noise
Tropospheric delay	Dry component: GMF model and mapping function (Kouba, 2009) Wet component: Estimated as a random walk process
Ionospheric delays	Estimated as white noise
Ambiguities	Estimated as constants over each continuous arc
Satellite orbits, clocks, and code biases	Galileo HAS
Measurement weighting strategy	Elevation dependent weighting: $\sigma = \frac{\sigma_{90}}{a+(1-a)\sin\lambda}$ with σ_{90} equal to 0.1 m and 0.001 m for the pseudorange and carrier-phase measurements, respectively, based on a residual and measurements quality analysis, and a set to 0.15. λ is the satellite's elevation angle.

statistics are reasonable and are expected to improve prior to the transition to SL1. This statement is especially relevant given that the broadcast session was a test broadcast and that the focus was on achieving the targeted accuracies rather than the targeted convergence time. Recall that the HAS specifications consider the use of both Galileo and GPS, with their code and phase biases. The combined Galileo and GPS solution is close to the convergence time service specification for HAS full service with convergence times of 6 minutes and 7.5 minutes in static and kinematic modes, respectively.

In addition to the convergence time, the solution accuracy is analyzed and summarized in Figure 6.11 in the form of the overall rms over the whole three hours. The figure shows that all solutions meet the Galileo HAS specifications of 20 cm horizontal error and 40 cm vertical error at the 95th percentile, apart from the GPS-only solution. Other than that solution, all Galileo-only and combined Galileo and GPS solutions exceed the specifications. Given the HAS full service service specification, the results processed linked to the HAS test corrections meet the user performance specifications in terms of accuracy. As expected from analyzing Figure 6.9, the Galileo-only solution performs better than the GPS-only solution due to more Galileo satellites being available in general, as well as having a higher correction availability, as was shown in Figures 6.1 and 6.2. Additionally, processing both constellations

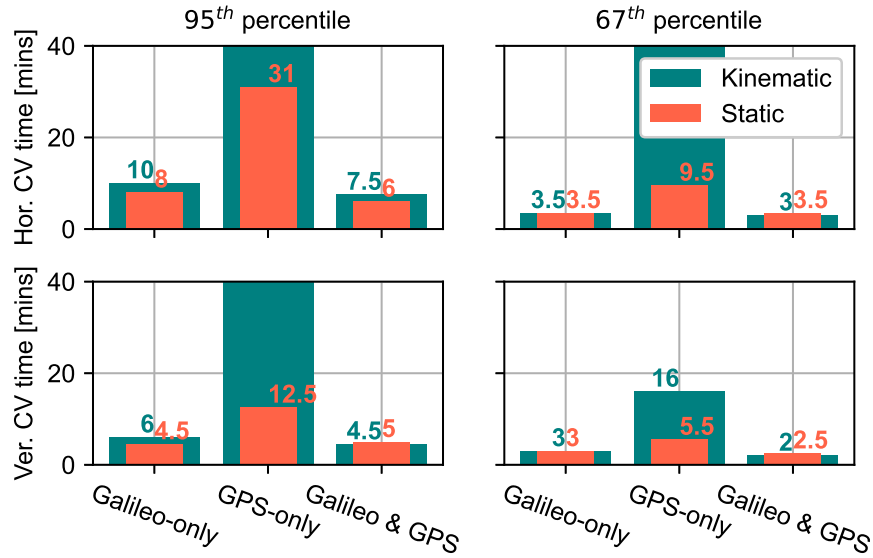


Figure 6.10: Convergence time statistics for Galileo-only, GPS-only and Galileo+GPS HAS solutions in both static and kinematic modes. The statistics correspond to the results in Figure 6.9. Horizontal convergence corresponds to convergence below 20 cm horizontal error, and vertical convergence corresponds to convergence below 40 cm vertical error.

together leads to a solution that is better than the single-constellation solutions, as the number of processed satellites is higher.

6.4.3 Effect of simulated correction latency on HAS position results

In terms of real-time PPP, the performance at the user side depends not only on correction accuracy, but also on the latency of the corrections, which is the difference between the time at which corrections were supposed to be applied and the time they were actually applied. Latency of SSR corrections tends to range from a few seconds to a few tens of seconds (Hadas and Bosy, 2015; Wang et al., 2018). Figure 6.12 shows the effect of Galileo HAS latency on the performance at the 95th percentile from processing all datasets used in the previous figures. The latency is simulated by delaying the application of the HAS corrections by a set number of seconds. The processing is performed in kinematic mode, with both Galileo and GPS corrections being applied.

As expected from Figure 6.12, the solution performance gets worse with higher latencies, as corrections start being applied later than the duration of their validity periods. The performance degradation affects both accuracy and convergence time, as the rms increase from 13.1 cm and 17.6 cm to 16.4 cm and 24.1 cm for the horizontal and vertical components, respectively, when comparing the absence of simulated latency to a latency of 60 seconds.

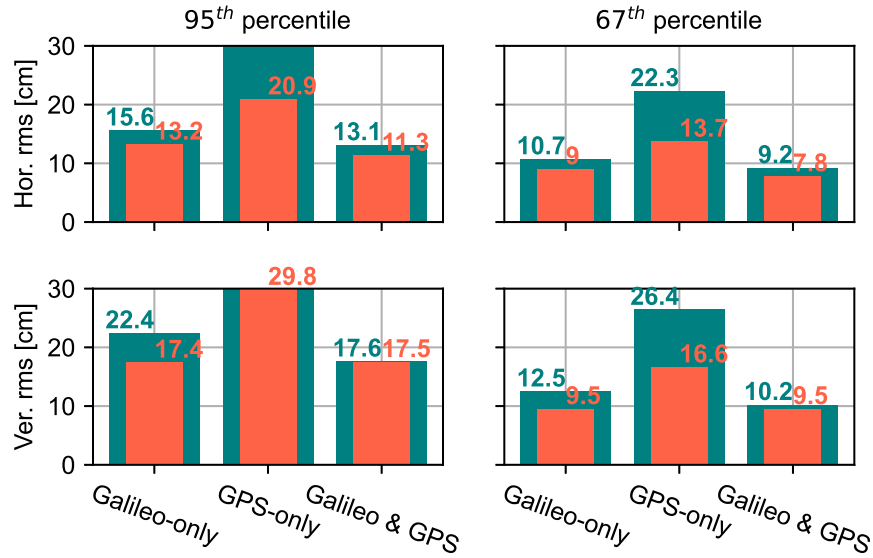


Figure 6.11: rms statistics for Galileo-only, GPS-only and Galileo+GPS HAS solutions in both static and kinematic modes. The rms is computed over the whole three hours of processing. Green and orange bars represent kinematic and static mode results, respectively. The 95th percentile vertical rms are 30.4 cm and 52.9 cm in static and kinematic modes, respectively.

It should be noted though that the HAS service specification of 20 cm horizontal and 40 cm vertical accuracies are still met, even with a latency of 60 seconds. However, the degradation in convergence time brings the solution even further from the service specification of 5 minutes convergence, as the convergence time with a latency of 60 seconds is double the one without introduced latency. Naturally, these results are only indicators of the scale of the latency's impact on performance, as the corrections used here are already affected by the real latency of the corrections. Also, service providers apply latency-reducing countermeasures to minimize this effect on position solutions.

6.5 Summary

Galileo is the first GNSS constellation to provide a global PPP service to users through its HAS service. With the service being expected to become operational in 2022, the GISCAD-OV project members have had early access to Galileo HAS test signals for experimentation purposes. Live Galileo HAS test signals have been broadcast during testing campaigns, where different combinations of corrections have been transmitted and analyzed in an effort to complete an end-to-end experimentation (involving end users) and inject the feedback obtained into the HAS initial service development final stages prior to entry into

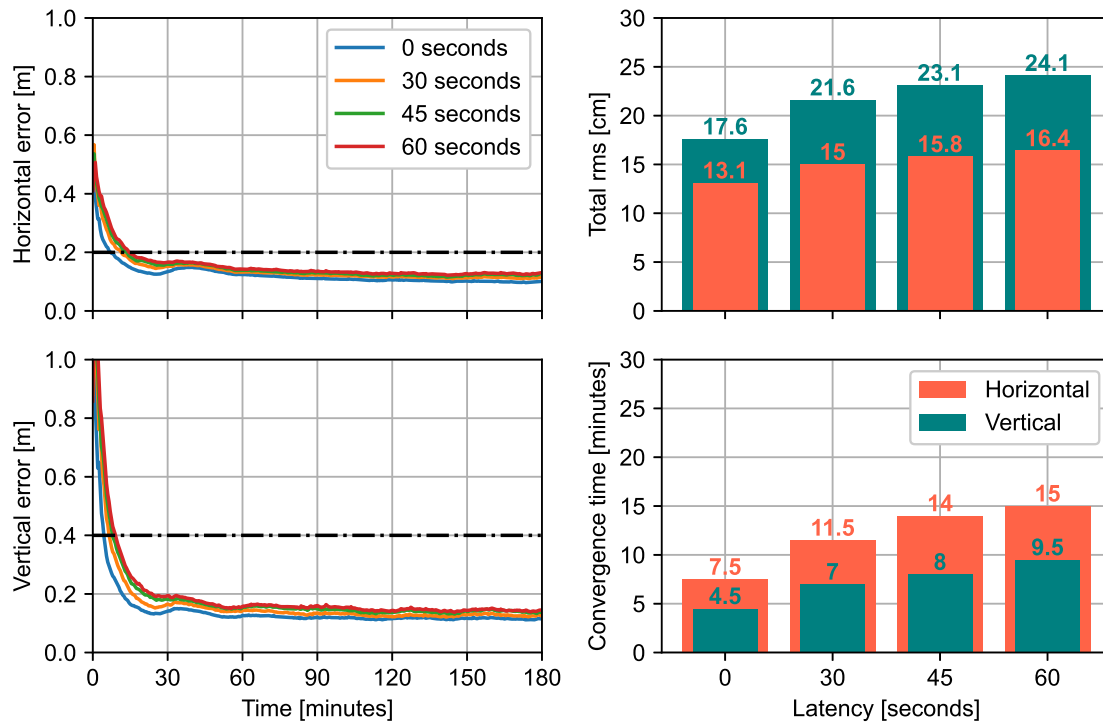


Figure 6.12: Effect of simulated Galileo HAS correction latency on PPP performance at the 95th percentile based on processing all datasets described in the previous section. The data are processed in kinematic mode using Galileo and GPS satellite orbits, clocks and code biases. The horizontal lines represent the service specification.

Service Level 1. The presented study analyzes HAS test corrections and related performance results based on specific set ups for a summer 2022 testing campaign, where Galileo and GPS satellite orbit, clock, and code bias corrections were broadcast.

This study shows good availability of corrections, as the average availability is found to be 90.7% for GPS and 96.5% for Galileo over a period of one week. Comparing the broadcasted test HAS corrections to final products from CODE shows good performance. Excluding two satellites for which the clock errors were found to be too large, the rms for the orbit SISRE are 1.6 cm and 1.4 cm for Galileo and GPS, respectively, while the rms of the total SISRE are 10.6 cm and 11.8 cm for Galileo and GPS, respectively, over a period of 24 hours. In addition to orbit and clock corrections, satellite code bias corrections were analyzed, and they showed good consistency with final code biases from DLR, with rms of 0.29 ns, 0.27 ns, and 0.23 ns for Galileo C1C-C5Q, C1C-C7Q, and C1C-C6C, respectively, and 0.21 ns for GPS C1C-C2L.

PPP processing was performed on global IGS stations using HAS test signals' corrections. The results indicate that the HAS accuracy service specification for full service of horizontal and vertical rms at the 95th percentile of 20 cm and 40 cm, respectively, were met, as the combined Galileo and GPS solution reached rms of 13.1 cm (11.3 cm) and 17.6 cm (17.5 cm) horizontally and vertically, respectively, in kinematic (and static) mode at the 95th percentile, which are well below the targets. The convergence time target of 5 minutes was not met due to the current test campaign not including satellite phase biases, which prevents PPP carrier-phase ambiguity resolution and subsequent improvement in positioning accuracy and reduction in initial convergence time. The combined Galileo and GPS solutions converged below 20 cm horizontal and 40 cm vertical errors within 6 minutes in static mode and 7.5 minutes in kinematic mode at the 95th percentile. These results are not considered indicative of the HAS initial service depicted in the HAS Information note (EUSPA, 2020), although are showing promising results in this specific user scenario. Additionally, simulating latency has shown that a latency of 60 seconds can still lead to results within the HAS accuracy service specification.

Chapter 7

Conclusions and recommendations for future research

Conclusions are drawn based on the work that was described in the earlier chapters of the dissertation. In addition, recommendations for future research that can be performed as a continuation of the work are provided.

7.1 Conclusions

While PPP is an augmentation that was developed a few decades ago, it is still an active research subject. The technique has historically been known for its precise positioning potential, though requiring minutes to tens of minutes of convergence time. Fortunately, thanks to the ever-changing GNSS landscape, there is no shortage of research topics. For instance, research on PPP ranges from improving its performance in general (Banville et al., 2014; Ge et al., 2008; Geng et al., 2020; Laurichesse and Blot, 2016) to integrity assessment (Du et al., 2021; Seepersad and Bisnath, 2013; Juan et al., 2012) to applications to low-cost and ultra-low-cost receivers (Laurichesse et al., 2017; Aggrey et al., 2020; Wang et al., 2021; Vana, 2021) to name a few. In this dissertation, the focus has been on improving PPP performance, with a focus on geodetic hardware, as an attempt to understand the limiting factors behind PPP's performance and attempt to improve the latter. This research does so by reconstructing and expanding a PPP-AR model, the Decoupled Clock Model (DCM), from first principles, as well as the processing of measurements from all four GNSS constellations on as many as four frequencies. By doing so, PPP performance is assessed and compared to RTK-type of performance. Additionally, part of this work is dedicated to using NRTK corrections in PPP processing, as well as a novel proof-of-concept where NRTK corrections are used alongside PPP products to provide a hybrid PPP solution. Finally, the dissertation ends with, for the first time in the published literature, an in-depth analysis

of the Galileo HAS corrections from its early test signals and the PPP performance that is achievable with those early signals.

In order to summarize the performance that was demonstrated in the dissertation, Table 7.1 shows the achieved performance for four of the solutions that were shown in this dissertation. The table shows, for instance, that the proposed four-constellation, four-frequency PPP-AR solution is able to achieve instantaneous centimetre-level positioning, of which many applications can benefit. Examples of potential applications consist of autonomous driving where highly precise positions on a global scale are needed, scientific applications for, e.g., earthquake monitoring, civil engineering applications for, e.g., infrastructure monitoring or cadastral surveying, etc. The fact that the proposed global solution does not require local infrastructure reduces the costs, though a subscription to a global PPP corrections stream could still be required. However, with Galileo HAS providing PPP corrections on a global scale for free, such subscriptions may not be necessary depending on applications and their requirements.

7.1.1 Four-constellation, four-frequency PPP-AR (Chapters 3 and 4)

PPP is a technique that is tremendously benefiting from the modernization and improvements in the GNSS constellations. Indeed, PPP is a measurement-dependent technique that relies on good quality and number of measurements to precisely estimate the user's position. In this part of the dissertation, it is demonstrated that PPP performance depends on the number of measurements available, as PPP performance improves the more constellations, and therefore more satellites, used. However, this approach is proven to have a limit due to the law of diminishing returns, as ever-increasing the number of satellites yields smaller returns the more satellites are available. On the other hand, AR is proven to be a crucial component in improving PPP performance and reducing convergence time, as rather than waiting for them to converge naturally to integer values, carrier-phase ambiguities can be fixed to integers to speed up the convergence process.

While dual-frequency PPP-AR is shown to provide improvements to the convergence time compared to float solution, its performance does still not reach the goal from this dissertation, which is centimetre-level positioning within seconds. As a consequence, another avenue in the form of additional frequencies is explored. To accommodate for the additional frequencies and measurements, a PPP-AR model named the DCM is derived from first principles. The model is derived in a way that it processes raw measurements without forming linear combinations on as many as four frequencies and four constellations. The model can accommodate for the different number of frequencies being broadcast by each constellation / satellite generation, as well as different constellations. Results from processing large numbers of datasets show that adding frequencies does not have an impact on the

Table 7.1: Summary of the main performance demonstrated in the dissertation, and magnitudes of the achieved convergence times, as well as means of achieving the performance.

	Hor. conv. threshold [cm]	Conv. time [min]	Mean of achieving performance
Global float	10	~ 10	Four-constellation processing without ambiguity resolution
Proposed global	2.5	Instantaneous	Four-constellation, four-frequency processing with ambiguity resolution
Mixed global and regional	10	Instantaneous	Regional NRTK ionospheric corrections used to hasten convergence of global dual-constellation, dual-frequency solution
Galileo HAS	20	~ 7.5	Test signals of Galileo HAS using two-constellation dual-frequency orbit, clock, and code bias corrections

float solutions that do not fix the ambiguities, as measurements on the additional frequencies do not strengthen the estimation of the parameters. However, additional frequencies have an impact on ambiguity-fixed solutions. The reason for this impact is that ambiguities are correlated between frequencies, leading to improved fixing with the addition of more frequencies. It is shown that, on average, with a four-constellation, four-frequency solution, the proposed model can achieve instantaneous convergence to the centimetre-level at the 95th percentile. The proposed algorithm is also tested on kinematic data, where a geodetic receiver and antenna were placed on the roof of a car that was driven close to the York University campus. The results show that instantaneous convergence to the centimetre-level is possible for such kinematic tests as well.

7.1.2 Epoch-by-epoch PPP and RTK-like quality (Chapter 4)

Up until recently, single-epoch PPP was a foreign concept, as PPP is known for its long initial convergence time. With the advancements in constellations and user processing techniques, such a concept is becoming more and more possible. It is shown that epoch-by-epoch centimetre-level positioning with PPP is possible using only global PPP products. This performance is made possible thanks to the use of additional constellations, but mainly due to the use of up to four frequencies and the fixing of carrier-phase ambiguities to integers. It is demonstrated that PPP solutions without AR could not perform at the centimetre-level when epochs of data are processed individually; while fixing ambiguities on two frequencies opens the possibility of epoch-by-epoch processing, as approximately 40% of epochs are at the centimetre-level at the 67th percentile, showing the importance of AR for in-

stantaneous centimetre-level single-epoch PPP. It is shown that including two additional frequencies improves the percentage of epochs at the centimetre-level to 80%, demonstrating the importance of the additional frequencies in ensuring correct fixing of ambiguities to their integer values.

The proposed single-epoch solution performance is analyzed based on multiple stations, where 67% of stations are shown to have at least 80% of their epochs at the centimetre-level with a single epoch of data at the 67th percentile. However, differences in performance are noticed between stations, as some of the stations have lower percentages, reaching as low as 50%. While this type of percentage is unprecedented, it also sparks interest in trying to understand what causes the difference in performance per station. A few avenues are explored to analyze the discrepancy in performance in terms of, for example, average number of satellites observed by the station, the type of receiver and antenna used for the station, location on Earth, etc. Unfortunately, clear correlation has not been found. Additionally, in an attempt to increase the percentages of epochs at the centimetre-level even more, two different avenues are explored: one consists of using two or three epochs of data rather than one, as three epochs of data will only require one minute to collect and will still represent a near-instantaneous solution. This avenue shows improvements with the inclusion of more epochs, though the improvements diminish the more epochs are used. The other avenue that is explored is the use of Global Ionospheric Map (GIM) which are shown to not be precise enough to provide significant improvements to user positioning.

7.1.3 NRTK corrections for PPP (Chapter 5)

NRTK and PPP have historically been viewed as two different precise positioning techniques, with NRTK being a variation of RTK that requires multiple base stations surrounding the user, and PPP being a technique that can be used anywhere without any local infrastructure as long as the user has access to satellite correction data. However, both techniques are based on similar principles, as, for both techniques, SSR corrections are estimated from a network of stations and transmitted to the user. Due to the smaller size of the network and shorter distance between stations, NRTK is able to provide highly precise atmospheric corrections, which explains the performance that is typically higher with NRTK compared to PPP.

In Chapter 5, a comparison of SSR corrections generated from a regional and a global network is performed. The comparison shows the effect of the size of the network on the estimated corrections, as NRTK corrections are found to not retain their physical meaning. Indeed, it is determined that, due to the smaller size of the regional network compared to the ranges between stations and satellites, the estimated corrections are harder to decorrelate, as, e.g., residual atmospheric effects are observed on satellite orbit and clock corrections. As

a consequence, it is determined that the effect of (not) including corrections has a different effect on user solutions compared to typical PPP solutions. Nonetheless, it is demonstrated that NRTK-type performance can be achieved in a PPP user engine, as long as the full range of NRTK corrections are used and that models are consistent with the network provider.

Due to the difficulty of decorrelating corrections in NRTK, the ionospheric corrections are found to not be compatible with global PPP corrections, as metre-level discrepancies are found between ionospheric delays from the two sources. As a consequence, augmenting a user solution that uses global PPP corrections with atmospheric corrections from a NRTK network becomes impossible. As a way to allow for such mixing of corrections, a novel proof-of-concept is proposed, where biases due to the discrepancies between the two sets of corrections are estimated. The biases are estimated based on computing OSR ranges using both global and regional corrections and are used to extract ionospheric corrections which are compatible with the global products. Initial testing of the proposed algorithms on sample datasets shows that the extracted ionospheric delays are able to augment the global solution and improve its performance, though more tests and analysis are required.

7.1.4 Galileo HAS and precise SPP (Chapter 6)

Thanks to the global nature of PPP corrections, corrections can be disseminated to users through various means. Typically, corrections are disseminated through offline storage in the case of post-processed applications or through, e.g., the Internet or geostationary satellites in the case of real-time applications. Galileo HAS is a new service that has been declared operational in January 2023, and which transmits PPP corrections for Galileo and GPS satellites on the Galileo E6-B signal. This service is a first of its kind with Galileo being the first constellation to transmit PPP corrections on a global scale. In this dissertation, a first in-depth analysis of the corrections being broadcast by the service in its early stages is performed. This analysis is based on a testing campaign that took place in summer 2022, where satellite orbits, clocks, and code biases are transmitted for Galileo and GPS satellites. The satellite orbits and clocks are computed based on a reduced network of 14 global reference stations, while the code biases are introduced from an external source to test out the transmission chain.

Availability of the corrections is analyzed, where good availability is found for all satellites with some intermittent outages for individual satellites. The availability for Galileo satellites is found to exceed 90%, while the availability for GPS satellites is slightly lower, though still above 80% for all satellites. Results from analyzing the corrections show good performance compared to final reference products, which in turn translates to user performance that meets the Galileo HAS target performance in terms of accuracy. The Galileo HAS target performance consists of user solutions accurate to 20 cm horizontally and 40

cm vertically to be achieved within 5 minutes at service level 1 at the 95th percentile. Results from the analysis show that the target performance is almost achieved in terms of convergence time, as convergence is achieved within 6 minutes rather than 5 minutes. Finally, a simulated latency analysis is performed, where delays are introduced before using the corrections in the processing. Results show that, even with a latency of 60 seconds, the user performance can meet the HAS target accuracy performance. The results from this analysis are promising, as they show that global precise positioning is possible with signals solely being transmitted by the GNSS constellations. Therefore, only using a single GNSS receiver and without any additional information source, users can achieve centimetre-level positioning, which is similar to SPP in principle but much improved in terms of performance.

7.2 Recommendations

In this dissertation, a few avenues are explored to improve PPP performance which demonstrates that RTK-like performance, i.e., instantaneous, centimetre-level positioning, is possible with PPP. It is also demonstrated that NRTK corrections can be used for PPP user processing, and that they can be mixed with global PPP products for a hybrid solution. Finally, analysis of early Galileo HAS signals shows that PPP solutions can be generated using only signals broadcast by GNSS constellations. In the following, recommendations for future work are provided based on the work that has been presented in this dissertation, as well as additional work that can be performed to strengthen PPP and broaden its potential applications.

7.2.1 Five-frequency PPP-AR

The uncombined DCM model that is derived as part of this work allows the processing of measurements on up to four frequencies. Given that the Galileo and BeiDou constellations broadcast on up to five frequencies, the derived DCM can be expanded to accept one additional frequency. Such expansion should be relatively straightforward, as it follows the same logic as the expansion to three or four frequencies. Five-frequency processing has been subject to some research in the literature (Li et al., 2020b,a), which has shown benefits to five-frequency processing, though it is still debatable as to whether an increased number of processed frequencies is more beneficial than appropriate spacing between the processed frequencies (Psychas et al., 2021). The effect of the additional frequency could, for instance, be studied in the context of the epoch-by-epoch processing, as the additional frequency could be the missing piece to achieve consistent epoch-by-epoch centimetre-level solutions.

7.2.2 Partial ambiguity resolution (AR)

In this dissertation, a "brute-force" approach is adopted when it comes to fixing ambiguities: all ambiguities that are available are fixed to integer values (with simple conditions in terms of elevation angle of the satellite, etc.) in order to assess the effect of the additional measurements on the reliability of the ambiguity-fixed solution. However, another approach can be taken where a subset of ambiguities can be fixed rather than all ambiguities. By appropriately selecting this subset, the ambiguity-fixed position solution can be improved compared to the one with all ambiguities being fixed. The approach that fixes a subset of ambiguities is referred to as partial AR in the literature, and it has been subject to much research (Geng and Shi, 2017; Li and Zhang, 2015; Banville et al., 2021). Strengthening the contributions in this dissertation with partial AR could further improve the solutions, especially when it comes to the epoch-by-epoch solutions. Including partial AR could improve the reliability of the solutions, which would be helpful for the integrity and resilience of the solutions for safety-of-life applications. The study of estimators other than MLAMBDA, such as the Best Integer Equivariant (BIE) is also proposed (Odolinski and Teunissen, 2020; Teunissen, 2003).

7.2.3 Ambiguity validation

Another facet of AR that can be improved relates to the validation of the ambiguities. Indeed, after having selected a set of integer ambiguity candidates, prior to updating the state terms, a validation step takes place to verify the quality of the integer candidates. In this work, and typically in PPP literature, simple validation functions are used, such as the ratio of the best and second best sets of candidates (Shi and Gao, 2012; Liu et al., 2017). Such validation functions have been inherited from RTK-AR, and have been found to not be as effective in PPP-AR. Therefore, one potential improvement of this research work involves further studying and adaptation of validation functions for PPP processing.

7.2.4 Dissection and improvement of epoch-by-epoch results

It is demonstrated in this dissertation that epoch-by-epoch, centimetre-level PPP is a possibility, and that it can consistently be achieved only using global PPP products. This achievement is possible thanks to the use of many frequencies on many constellations, coupled with AR. However, it is found that there is a discrepancy in performance between stations, as some stations are found to perform worse than others. Therefore, a follow-up of this work consists of understanding the cause behind the difference in performance between stations. Understanding this causation would allow for further improvements of the solutions. Additionally, it is believed that further improvements of the solutions overall is possible, and that some of the above recommendations such as partial AR or an additional

frequency could help improve the solutions overall. Other potential ways to improve the epoch-by-epoch solutions could involve further tuning of the stochastic models.

7.2.5 Analysis of full HAS service

The European Galileo has a new service where PPP corrections are broadcast to users directly through the constellation's signals. This service means that users can generate PPP solutions only using their GNSS device without relying on any other information source. In this dissertation, analysis of early test signals is performed, where satellite orbits, clocks, and code biases are tested and used to generate PPP solutions. Potential future work includes analysis of the operational signals for which additional corrections in the form of satellite phase biases are expected to be broadcast, opening the door for AR. Additionally, the service will also broadcast ionospheric corrections for the European coverage area, which would be an interesting addition to analyze. The analysis could include the quality of the transmitted ionospheric corrections, and their effect on PPP user performance. Future work includes multi-frequency processing with HAS signals, as this dissertation only makes use of dual-frequency measurements.

7.2.6 Maturity of global and regional mixing proof-of-concept

A proof-of-concept is proposed where global PPP corrections are used together with NRTK corrections generated from a regional network. It is demonstrated that such mixing is possible and that there are different approaches that can be taken for the mixing to be successful. The proof-of-concept is tested on a limited sample size of datasets. As a consequence, future work includes rigorous testing of the proposed method through a wider range of datasets. Performing such extensive testing would allow for maturity of the proposed method, and would allow for its limitations to be found. Additionally, analysis of the spatial and temporal correlations of the estimated biases could be performed to gain a better understanding of the underlying cause of the discrepancies between global and regional products, and of whether such discrepancies can be modelled. Finally, tests can be extended to include more global and regional corrections providers, and to allow for the processing of all GNSS constellations, and up to five frequencies.

7.2.7 DCM for atmospheric modelling

The expanded DCM that is developed as part of this work is focussed on estimating the user position while mitigating the various other state terms. For instance, it is shown that the ionospheric estimates are tainted with receiver biases and, therefore, the estimated delays do not strictly represent the delays caused by the signals travelling through the atmosphere. The proposed formulation is only one of many formulations that are possible for the model.

As such, it could be interesting to modify the model based on the field of application. For instance, if a user is interested in modelling the atmosphere through its estimation with the DCM, the model could be modified in a way that ionospheric estimates are free of biases and can be used to model the ionospheric delays (Rovira-Garcia et al., 2015; Martire et al., 2023).

7.2.8 Application to low-cost hardware

Work in this dissertation is focussed on using geodetic hardware as a way to evaluate the different methods that are proposed and perform various types of analysis. However, most users tend to use mass-market receivers and antennas, which have a lower cost, and which provide measurements which are known to be more noisy (Vana, 2021; Aggrey et al., 2019; Banville et al., 2019). Therefore, the current research can be expanded to perform AR on such low-cost devices. Very limited research has been published in this area due to the difficulty of the task, as fixing ambiguities on such noisy measurements can prove to be a tricky endeavour. As part of the research surrounding this dissertation, analysis is performed where the effect of AR, inertial measurement unit (IMU) integration, and ionospheric constraining on rapid initialization of PPP solutions in various environments with automotive-grade receivers is performed in Naciri et al. (2021b). While the focus of the publication is on quick and reliable sub-metre solutions, the work can be expanded to study the possibility of quick sub-decimetre solutions with AR, as well as reduction of convergence time and improvement of accuracy.

References

- Abdelazeem M, Çelik RN, El-Rabbany A (2016) MGR-DCB: a precise model for multi-constellation GNSS receiver differential code bias. *The Journal of Navigation* 69(4):698–708
- Aggrey J, Bisnath S (2019) Improving GNSS PPP convergence: The case of atmospheric-constrained, multi-GNSS PPP-AR. *Sensors* 19(3):587
- Aggrey J, Bisnath S, Naciri N, Shinghal G, Yang S (2019) Accuracy trend analysis of low-cost GNSS chips: The case of multi-constellation GNSS PPP. In: *Proceedings of the 32nd International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2019)*, pp 3618–3635
- Aggrey J, Bisnath S, Naciri N, Shinghal G, Yang S (2020) Multi-GNSS precise point positioning with next-generation smartphone measurements. *Journal of Spatial Science* 65(1):79–98
- Agnew DC (1997) Nloadf: A program for computing ocean-tide loading. *Journal of geophysical research: solid earth* 102(B3):5109–5110
- Ashby N (2003) Relativity in the global positioning system. *Living Reviews in relativity* 6(1):1–42
- Banville S (2016) GLONASS ionosphere-free ambiguity resolution for precise point positioning. *J Geod* 90(5):487–496, DOI 10.1007/s00190-016-0888-7
- Banville S, Collins P, Zhang W, Langley RB (2014) Global and regional ionospheric corrections for faster PPP convergence. *Navigation: Journal of The Institute of Navigation* 61(2):115–124
- Banville S, Lachapelle G, Ghoddousi-Fard R, Gratton P (2019) Automated processing of low-cost GNSS receiver data. In: *Proceedings of the 32nd International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2019)*, pp 3636–3652
- Banville S, Geng J, Loyer S, Schaer S, Springer T, Strasser S (2020) On the interoperability of IGS products for precise point positioning with ambiguity resolution. *Journal of Geodesy* 94(1):1–15
- Banville S, Hassen E, Lamothe P, Farinaccio J, Donahue B, Mireault Y, Goudarzi MA, Collins P, Ghoddousi-Fard R, Kamali O (2021) Enabling ambiguity resolution in CSRS-PPP. *NAVIGATION: Journal of the Institute of Navigation* 68(2):433–451

- Bertiger W, Desai SD, Haines B, Harvey N, Moore AW, Owen S, Weiss JP (2010) Single receiver phase ambiguity resolution with GPS data. *Journal of Geodesy* 84(5):327–337, DOI <https://doi.org/10.1007/s00190-010-0371-9>
- Bisnath S (2020a) PPP: Perhaps the natural processing mode for precise GNSS PNT. In: 2020 IEEE/ION Position, Location and Navigation Symposium (PLANS), pp 419–425
- Bisnath S (2020b) Relative positioning and real-time kinematic (rtk). *Position, Navigation, and Timing Technologies in the 21st Century: Integrated Satellite Navigation, Sensor Systems, and Civil Applications* 1:481–502
- Bisnath S, Gao Y (2009a) Current state of precise point positioning and future prospects and limitations. *Observing our changing earth*
- Bisnath S, Gao Y (2009b) Precise point positioning a powerful technique with a promising future. *GPS World* 20(4):43–50
- Bisnath S, Saeidi A, Wang JG, Seepersad G (2013) Evaluation of Network RTK Performance and Elements of Certification—A Southern Ontario Case Study. *Geomatica* 67(4):243–251, DOI <https://doi.org/10.5623/cig2013-050>
- Bock Y, Nikolaidis RM, de Jonge PJ, Bevis M (2000) Instantaneous geodetic positioning at medium distances with the Global Positioning System. *Journal of Geophysical Research: Solid Earth* 105(B12):28,223–28,253, DOI <https://doi.org/10.1029/2000JB900268>
- Cai C, Gao Y, Pan L, Zhu J (2015) Precise point positioning with quad-constellations: GPS, BeiDou, GLONASS and Galileo. *Advances in Space Research* 56(1):133–143
- Chang XW, Yang X, Zhou T (2005) MLAMBDA: a modified LAMBDA method for integer least-squares estimation. *Journal of Geodesy* 79(9):552–565, DOI [10.1007/s00190-005-0004-x](https://doi.org/10.1007/s00190-005-0004-x)
- CNSO (2020) BeiDou Constellation Status. URL <http://www.csno-tarc.cn/en/system/constellation>, accessed on 04.08.2020
- Collins P (2008) Isolating and estimating undifferenced GPS integer ambiguities. In: *Proceedings of the 2008 National Technical Meeting of The Institute of Navigation*, pp 720–732
- Collins P, Lahaye F, Heroux P, Bisnath S (2008) Precise point positioning with ambiguity resolution using the decoupled clock model. In: *Proceedings of the 21st international technical meeting of the satellite division of the Institute of Navigation (ION GNSS 2008)*, pp 1315–1322
- Collins P, Bisnath S, Lahaye F, Héroux P (2010) Undifferenced GPS ambiguity resolution using the decoupled clock model and ambiguity datum fixing. *Navigation, Journal of the Institute of Navigation* 57(2):123–135, DOI <https://doi.org/10.1002/j.2161-4296.2010.tb01772.x>

- Collins P, Lahaye F, Bisnath S (2012) External ionospheric constraints for improved PPP-AR initialisation and a generalised local augmentation concept. In: Proceedings of the 25th international technical meeting of the satellite division of the Institute of Navigation (ION GNSS 2012), pp 3055–3065
- Deng Y, Guo F, Ren X, Ma F, Zhang X (2021) Estimation and analysis of multi-GNSS observable-specific code biases. *GPS Solut* 25(3):100, DOI 10.1007/s10291-021-01139-6
- Deng Z, Nischan T, Bradke M (2017) Multi-GNSS Rapid Orbit-, Clock- and EOP-Product Series. GFZ Data Services DOI <https://doi.org/10.5880/GFZ.1.1.2017.002>
- Dousa J, Bennitt GV (2013) Estimation and evaluation of hourly updated global GPS Zenith Total Delays over ten months. *GPS Solutions* 17(4):453–464
- Du Y, Wang J, Rizos C, El-Mowafy A (2021) Vulnerabilities and integrity of Precise Point Positioning for intelligent transport systems: Overview and analysis. *Satellite Navigation* 2(1):1–22
- El-Mowafy A, Deo M, Rizos C (2016) On biases in Precise Point Positioning with multi-constellation and multi-frequency GNSS data. *Measurement science and technology* 27(3):035,102
- Elizabeth H (2020) China launches final Beidou satellite to complete GPS-like navigation system. URL <https://www.space.com/china-launches-final-beidou-navigation-satellite.html>, accessed on 04.08.2020
- Engel PK (1994) The Global Positioning System: Signals, measurements, and performance. *International Journal of Wireless Information Networks* 1(2):83–105
- European Commission (2016) Open Service Ionospheric Correction Algorithm for Galileo Single Frequency Users
- European Union (2022) Galileo High Accuracy Service Signal-In-Space Interface Control Document (HAS SIS ICD), Issue 1.0. URL https://www.gsc-europa.eu/sites/default/files/sites/all/files/Galileo_HAS_SIS_ICD_v1.0.pdf, accessed on 01.09.2022
- EUSPA (2020) Galileo High Accuracy Service (HAS) Info Note. URL https://www.gsc-europa.eu/sites/default/files/sites/all/files/Galileo_HAS_Info_Note.pdf, accessed on 01.09.2022
- EUSPA (2022) System | European GNSS Service Centre. URL <https://www.gsc-europa.eu/galileo/system>, accessed on 22.08.2022
- EUSPA (2023) Galileo High Accuracy Service goes live! URL <https://www.euspa.europa.eu/newsroom/news/galileo-high-accuracy-service-now-operational>, accessed on 30.01.2023
- EUSPA (2023) What is EGNOS? URL <https://www.euspa.europa.eu/european-space/egnos/what-egnos>, accessed on 08.03.2023
- FAA (2023) Satellite Navigation - Wide Area Augmentation System (WAAS) | Federal Aviation Administration. URL https://www.faa.gov/about/office_org/headquarters_offices/ato/service_units/techops/navservices/gnss/waas, accessed on 08.03.2023

- Fan L, Shi C, Li M, Wang C, Zheng F, Jing G, Zhang J (2019) GPS satellite inter-frequency clock bias estimation using triple-frequency raw observations. *Journal of Geodesy* 93(12):2465–2479
- Fernandez-Hernandez I, Chamorro-Moreno A, Cancela-Diaz S, Calle-Calle JD, Zoccarato P, Blonski D, Senni T, de Blas FJ, Hernández C, Simón J, et al. (2022) Galileo high accuracy service: initial definition and performance. *GPS Solutions* 26(3):1–18
- Francis O, Mazzega P (1990) Global charts of ocean tide loading effects. *Journal of Geophysical Research: Oceans* 95(C7):11,411–11,424
- Gao W, Zhao Q, Meng X, Pan S (2021) Performance of single-epoch EWL/WL/NL ambiguity-fixed precise point positioning with regional atmosphere modelling. *Remote Sensing* 13(18):3758
- Gao Y, Li Z, McLellan J (1997) Carrier phase based regional area differential GPS for decimeter-level positioning and navigation. In: *Proceedings of the 10th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GPS 1997)*. Kansas City, MO, pp 1305–1313
- Ge M, Gendt G, Rothacher M, Shi C, Liu J (2008) Resolution of GPS carrier-phase ambiguities in Precise Point Positioning (PPP) with daily observations. *Journal of Geodesy* 82(7):389–399, DOI <https://doi.org/10.1007/s00190-007-0187-4>
- Geisler I (2006) Performance improvement of network RTK positioning. In: *Proceedings of the 2006 National Technical Meeting of The Institute of Navigation*. Monterey, CA, pp 869–880
- Geng J (2011) Rapid integer ambiguity resolution in GPS Precise Point Positioning. PhD thesis, University of Nottingham UK
- Geng J, Bock Y (2013) Triple-frequency GPS Precise Point Positioning with rapid ambiguity resolution. *Journal of Geodesy* 87(5):449–460, DOI 10.1007/s00190-013-0619-2
- Geng J, Guo J (2020) Beyond three frequencies: An extendable model for single-epoch decimeter-level point positioning by exploiting Galileo and BeiDou-3 signals. *Journal of Geodesy* 94(1):1–15
- Geng J, Shi C (2017) Rapid initialization of real-time PPP by resolving undifferenced GPS and GLONASS ambiguities simultaneously. *Journal of Geodesy* 91(4):361–374
- Geng J, Meng X, Dodson AH, Teferle FN (2010) Integer ambiguity resolution in Precise Point Positioning: Method comparison. *Journal of Geodesy* 84(9):569–581, DOI 10.1007/s00190-010-0399-x
- Geng J, Teferle FN, Meng X, Dodson A (2011) Towards PPP-RTK: Ambiguity resolution in real-time Precise Point Positioning. *Advances in space research* 47(10):1664–1673
- Geng J, Shi C, Ge M, Dodson AH, Lou Y, Zhao Q, Liu J (2012) Improving the estimation of fractional-cycle biases for ambiguity resolution in precise point positioning. *Journal of Geodesy* 86(8):579–589, DOI <https://doi.org/10.1007/s00190-011-0537-0>

- Geng J, Guo J, Chang H, Li X (2019) Toward global instantaneous decimeter-level positioning using tightly coupled multi-constellation and multi-frequency GNSS. *Journal of geodesy* 93(7):977–991
- Geng J, Guo J, Meng X, Gao K (2020) Speeding up PPP ambiguity resolution using triple-frequency GPS/BeiDou/Galileo/QZSS data. *Journal of Geodesy* 94(1):6, DOI <https://doi.org/10.1007/s00190-019-01330-1>
- Geng J, Wen Q, Zhang Q, Li G, Zhang K (2022) GNSS observable-specific phase biases for all-frequency PPP ambiguity resolution. *Journal of Geodesy* 96(2):1–18
- GEOWEB SpA (2022) GISCAD-OV. URL <https://giscad-ov.eu/>, accessed on 01.09.2022
- Gong X, Gu S, Lou Y, Zheng F, Yang X, Wang Z, Liu J (2020) Research on empirical correction models of GPS Block IIF and BDS satellite inter-frequency clock bias. *Journal of Geodesy* 94(3):1–11
- Gov G (2015) GPS Standard Positioning Service (SPS) Performance Standard. GPS Gov Website Accessed January 4
- GPS (2021) GPS.gov: Selective Availability. URL <https://www.gps.gov/systems/gps/modernization/sa/>, accessed on 17.01.2022
- GSC (2020) Constellation information | European GNSS Service Centre. URL <https://www.gsc-europa.eu/system-service-status/constellation-information>, accessed on 04.08.2020
- Guo F, Zhang X, Wang J, Ren X (2016) Modeling and assessment of triple-frequency BDS Precise Point Positioning. *Journal of Geodesy* 90(11):1223–1235, DOI 10.1007/s00190-016-0920-y
- Guo J, Geng J (2018) GPS satellite clock determination in case of inter-frequency clock biases for triple-frequency Precise Point Positioning. *J Geod* 92(10):1133–1142, DOI 10.1007/s00190-017-1106-y
- Hadas T, Bosy J (2015) IGS RTS precise orbits and clocks verification and quality degradation over time. *GPS solutions* 19(1):93–105
- Håkansson M, Jensen AB, Horemuz M, Hedling G (2017) Review of code and phase biases in multi-GNSS positioning. *GPS Solutions* 21(3):849–860
- Hao M, Jiao W, Jia X, Tao Q (2020) Precise Point Positioning performance evaluation of QZSS centimeter level augmentation service. In: *China Satellite Navigation Conference*, Springer, pp 78–88
- Hauschild A, Montenbruck O, Steigenberger P, Martini I, Fernandez-Hernandez I (2022) Orbit determination of sentinel-6A using the Galileo high accuracy service test signal. *GPS Solutions* 26(4):1–13
- Hérroux P, Gao Y, Kouba J, Lahaye F, Mireault Y, Collins P, Macleod K, Tétreault P, Chen K (2004) Products and applications for Precise Point Positioning-moving towards real-time. In: *Proceedings of the 17th international technical meeting of the satellite division of The Institute of Navigation (ION GNSS 2004)*, pp 1832–1843

- Hexagon A (2021) RTK From The Sky | Hexagon Autonomy & Positioning. URL <https://hexagonpositioning.com/rtk-from-the-sky>, accessed on 28.01.2022
- Ho C, Wilson B, Mannucci A, Lindqwister U, Yuan D (1997) A comparative study of ionospheric total electron content measurements using global ionospheric maps of GPS, TOPEX radar, and the Bent model. *Radio Science* 32(4):1499–1512
- Hopfield H (1969) Two-quartic tropospheric refractivity profile for correcting satellite data. *Journal of Geophysical research* 74(18):4487–4499
- International GNSS Service (2018) RINEX The Receiver Independent Exchange Format. URL <https://files.igs.org/pub/data/format/rinex304.pdf>, accessed on 01.01.2021
- Janeau G, Adrados C, Girard I (2001) Is it still necessary to use GPS in differential mode since the elimination of selective availability? *Tracking animals with GPS* p 73
- Jensen A, Cannon M (2000) Performance of network RTK using fixed and float ambiguities. In: *Proceedings of the 2000 National Technical Meeting of the Institute of Navigation, Anaheim, CA*, pp 797–805
- Jiao G, Song S, Ge Y, Su K, Liu Y (2019) Assessment of BeiDou-3 and multi-GNSS precise point positioning performance. *Sensors* 19(11):2496
- Jiao G, Song S, Jiao W (2020) Improving BDS-2 and BDS-3 joint precise point positioning with time delay bias estimation. *Meas Sci Technol* 31(2):025,001, DOI 10.1088/1361-6501/ab41cf
- Juan J, Hernández-Pajares M, Sanz J, Ramos-Bosch P, Aragon-Angel A, Orus R, Ochieng W, Feng S, Jofre M, Coutinho P, et al. (2012) Enhanced precise point positioning for GNSS users. *IEEE transactions on geoscience and remote sensing* 50(10):4213–4222
- Kaplan ED, Hegarty C (2017) *Understanding GPS/GNSS: Principles and applications*. Artech house
- Katsigianni G, Loyer S, Perosanz F (2019) PPP and PPP-AR kinematic post-processed performance of GPS-only, Galileo-only and multi-GNSS. *Remote Sensing* 11(21):2477, DOI <https://doi.org/10.3390/rs11212477>, URL <https://www.mdpi.com/2072-4292/11/21/2477>
- Khodabandeh A, Teunissen P (2015) An analytical study of PPP-RTK corrections: Precision, correlation and user-impact. *Journal of Geodesy* 89(11):1109–1132
- Klobuchar JA (1975) A first-order, worldwide, ionospheric, time-delay algorithm, vol 75. Air Force Cambridge Research Laboratories, Air Force Systems Command, United States Air Force
- Kouba J (2009) Testing of global pressure/temperature (GPT) model and global mapping function (GMF) in GPS analyses. *Journal of Geodesy* 83(3):199–208
- Kouba J, Héroux P (2001) Precise point positioning using IGS orbit and clock products. *GPS Solutions* 5(2):12–28

- Kouba J, Mireault Y (1998) [IGSMail-1943] New IGS ERP Format (version 2). URL <https://lists.igs.org/pipermail/igsmail/1998/003315.html>, accessed on 01.01.2021
- Kremer GT, Kalafus RM, Loomis PV, Reynolds JC (1990) The effect of selective availability on differential GPS corrections. *Navigation* 37(1):39–52
- Laurichesse D (2011) The CNES Real-time PPP with undifferenced integer ambiguity resolution demonstrator. 24th International Technical Meeting of the Satellite Division of the Institute of Navigation 2011, ION GNSS 2011 1:654–662
- Laurichesse D, Banville S (2018) Instantaneous centimeter-level multi-frequency precise point positioning: GPS World. URL <https://www.gpsworld.com/innovation-instantaneous-centimeter-level-multi-frequency-precise-point-positioning/>, accessed on 21.01.2022
- Laurichesse D, Blot A (2016) Fast PPP convergence using multi-constellation and triple-frequency ambiguity resolution. 29th International Technical Meeting of the Satellite Division of the Institute of Navigation, ION GNSS+ 2016 3:2082–2088, DOI <https://doi.org/10.33012/2016.14633>
- Laurichesse D, Privat A (2015) An open-source PPP client implementation for the CNES PPP-WIZARD demonstrator. In: Proceedings of the 28th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2015), pp 2780–2789
- Laurichesse D, Mercier F, Berthias JP, Broca P, Cerri L (2009) Integer ambiguity resolution on undifferenced GPS phase measurements and its application to PPP and satellite precise orbit determination. *Navigation, Journal of the Institute of Navigation* 56(2):135–149, DOI <https://doi.org/10.1002/j.2161-4296.2009.tb01750.x>, URL http://www.ion.org/search/view_abstract.cfm?jp=p&idno=7584
- Laurichesse D, Rouch C, Marmet FX, Pascaud M (2017) Smartphone applications for precise point positioning. In: Proceedings of the 30th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2017), pp 171–187
- Li H, Zhou X, Wu B (2013) Fast estimation and analysis of the inter-frequency clock bias for Block IIF satellites. *GPS Solut* 17(3):347–355, DOI 10.1007/s10291-012-0283-7
- Li H, Li B, Xiao G, Wang J, Xu T (2016) Improved method for estimating the inter-frequency satellite clock bias of triple-frequency GPS. *GPS Solut* 20(4):751–760, DOI 10.1007/s10291-015-0486-9
- Li M, Yuan Y, Wang N, Li Z, Li Y, Huo X (2017) Estimation and analysis of Galileo differential code biases. *Journal of Geodesy* 91(3):279–293
- Li P, Zhang X (2015) Precise point positioning with partial ambiguity fixing. *Sensors* 15(6):13,627–13,643
- Li X, Xie W, Huang J, Ma T, Zhang X, Yuan Y (2019) Estimation and analysis of differential code biases for BDS3/BDS2 using iGMAS and MGEX observations. *J Geod* 93(3):419–435, DOI 10.1007/s00190-018-1170-y

- Li X, Li X, Liu G, Yuan Y, Freeshah M, Zhang K, Zhou F (2020a) BDS multi-frequency PPP ambiguity resolution with new B2a/B2b/B2a+ b signals and legacy B1I/B3I signals. *Journal of Geodesy* 94(10):1–15
- Li X, Liu G, Li X, Zhou F, Feng G, Yuan Y, Zhang K (2020b) Galileo PPP rapid ambiguity resolution with five-frequency observations. *GPS Solutions* 24(1):1–13, DOI 10.1007/s10291-019-0930-3
- Liu C, Gao W, Liu T, Wang D, Yao Z, Gao Y, Nie X, Wang W, Li D, Zhang W, et al. (2020a) Design and implementation of a BDS precise point positioning service. *Navigation* 67(4):875–891
- Liu T, Zhang B, Yuan Y, Li Z, Wang N (2019) Multi-gnss triple-frequency differential code bias (dcb) determination with precise point positioning (ppp). *Journal of Geodesy* 93:765–784
- Liu T, Jiang W, Laurichesse D, Chen H, Liu X, Wang J (2020b) Assessing GPS/Galileo real-time precise point positioning with ambiguity resolution based on phase biases from CNES. *Advances in Space Research* 66(4):810–825, DOI <https://doi.org/10.1016/j.asr.2020.04.054>
- Liu Y, Ye S, Song W, Lou Y, Chen D (2017) Integrating gps and bds to shorten the initialization time for ambiguity-fixed ppp. *GPS solutions* 21:333–343
- Lou Y, Zheng F, Gu S, Wang C, Guo H, Feng Y (2016) Multi-GNSS precise point positioning with raw single-frequency and dual-frequency measurement models. *GPS Solutions* 20(4):849–862, DOI 10.1007/s10291-015-0495-8
- Loyer S, Perosanz F, Mercier F, Capdeville H, Marty JC (2012) Zero-difference GPS ambiguity resolution at CNES-CLS IGS Analysis Center. *Journal of Geodesy* 86(11):991–1003, DOI 10.1007/s00190-012-0559-2
- Malys S, Jensen PA (1990) Geodetic point positioning with GPS carrier beat phase data from the CASA UNO experiment. *Geophysical Research Letters* 17(5):651–654
- Martire L, Krishnamoorthy S, Vergados P, Romans LJ, Szilágyi B, Meng X, Anderson JL, Komjáthy A, Bar-Sever YE (2023) The GUARDIAN system-a GNSS upper atmospheric real-time disaster information and alert network. *GPS Solutions* 27(1):1–13
- McCarthy DD, Petit G, Service IERaRS (eds) (2004) *IERS Conventions (2003)*. No. 32 in IERS technical note, Verl. des Bundesamtes für Kartographie und Geodäsie, Frankfurt am Main
- Mervart L, Lukes Z, Rocken C, Iwabuchi T (2008) Precise point positioning with ambiguity resolution in real-time. In: *Proceedings of the 21st international technical meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2008)*. Savannah, GA, pp 397–405
- Miya M, Fujita S, Sato Y, Ota K, Hirokawa R, Takiguchi J (2016) Centimeter level augmentation service (CLAS) in Japanese quasi-zenith satellite system, its user interface,

- detailed design, and plan. In: Proceedings of the 29th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2016), pp 2864–2869
- Montenbruck O, Hugentobler U, Dach R, Steigenberger P, Hauschild A (2012) Apparent clock variations of the Block IIF-1 (SVN62) GPS satellite. *GPS Solut* 16(3):303–313, DOI 10.1007/s10291-011-0232-x
- Montenbruck O, Hauschild A, Steigenberger P (2014a) Differential Code Bias Estimation using Multi-GNSS Observations and Global Ionosphere Maps. *Navigation* 61(3):191–201
- Montenbruck O, Hauschild A, Steigenberger P (2014b) Differential code bias estimation using multi-GNSS observations and global ionosphere maps. *Navigation: Journal of the Institute of Navigation* 61(3):191–201
- Montenbruck O, Steigenberger P, Hauschild A (2015) Broadcast versus precise ephemerides: a multi-GNSS perspective. *GPS Solutions* 19(2):321–333
- Muellerschoen RJ, Bar-Sever YE, Bertiger WI, Stowers DA (2001) Decimeter accuracy: NASA’s global DGPS for high-precision users. *GPS World* 12(1)
- Naciri N (2022) RTK-quality Positioning with Global PPP Corrections. In: Proceedings of the 35th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2022), pp 2546–2562
- Naciri N, Bisnath S (2020) Multi-GNSS Ambiguity Resolution as a Substitute to Obstructed Satellites in Precise Point Positioning Processing. In: Proceedings of the 33rd International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2020), pp 2960–2971
- Naciri N, Bisnath S (2021a) An uncombined triple-frequency user implementation of the decoupled clock model for PPP-AR. *Journal of Geodesy* 95(5):1–17
- Naciri N, Bisnath S (2021b) Approaching Global Instantaneous Precise Positioning with the Dual-and Triple-Frequency Multi-GNSS Decoupled Clock Model. *Remote Sensing* 13(18):3768
- Naciri N, Bisnath S (2023) Rtk-quality positioning with global precise point positioning corrections. *NAVIGATION: Journal of the Institute of Navigation* 70(3)
- Naciri N, Hauschild A, Bisnath S (2021a) Exploring signals on L5/E5a/B2a for dual-frequency GNSS precise point positioning. *Sensors* 21(6):2046
- Naciri N, Vana S, Seepersad G, Bisnath S (2021b) Rapid Position Initialization for Automated Automotive Applications. In: Proceedings of the 34th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2021), pp 2718–2732
- Naciri N, Yi D, Bisnath S, de Blas FJ, Capua R (2022) Validation of a European High Accuracy GNSS Service for Cadastral Surveying Applications. Denver, Colorado, pp 381–396, DOI 10.33012/2022.18458, URL <https://www.ion.org/publications/abstract.cfm?articleID=18458>

- Naciri N, Yi D, Bisnath S, de Blas FJ, Capua R (2023) Assessment of Galileo High Accuracy Service (HAS) test signals and preliminary positioning performance. *GPS solutions* 27(2):73
- Nadarajah N, Khodabandeh A, Wang K, Choudhury M, Teunissen PJ (2018) Multi-GNSS PPP-RTK: from large-to small-scale networks. *Sensors* 18(4):1078
- NAVCEN (2023) GPS Constellation | Navigation Center. URL <https://www.navcen.uscg.gov/gps-constellation>, accessed on 16.04.2023
- Nie Z, Xu X, Wang Z, Du J (2021) Initial assessment of BDS PPP-B2b service: precision of orbit and clock corrections, and PPP performance. *Remote Sensing* 13(11):2050
- Odolinski R, Teunissen PJ (2020) Best integer equivariant estimation: Performance analysis using real data collected by low-cost, single-and dual-frequency, multi-gnss receivers for short-to long-baseline rtk positioning. *Journal of Geodesy* 94(9):91
- Odolinski R, Teunissen P, Odijk D (2015) Combined GPS+ BDS for short to long baseline RTK positioning. *Measurement Science and Technology* 26(4):045,801
- Office of Science and Technology Policy (2000) STATEMENT BY THE PRESIDENT REGARDING THE UNITED STATES' DECISION TO STOP DEGRADING GLOBAL POSITIONING SYSTEM ACCURACY. URL https://clintonwhitehouse3.archives.gov/WH/EOP/OSTP/html/0053_2.html, accessed on 16.04.2023
- Parkinson BW, Enge PK, Spilker J (1996) Differential GPS. *Progress in Astronautics and Aeronautics: Global Positioning System Theory and Applications* 2(164):3–32
- Pengfei C, Wei L, Jinzhong B, Hanjiang W, Yanhui C, Hua W (2011) Performance of precise point positioning (PPP) based on uncombined dual-frequency GPS observables. *Survey Review* 43(322):343–350
- Prange L, Villiger A, Sidorov D, Schaer S, Beutler G, Dach R, Jäggi A (2020) Overview of CODE's MGEX solution with the focus on Galileo. *Advances in Space Research* 66(12):2786–2798, DOI 10.1016/j.asr.2020.04.038
- Psychas D, Verhagen S (2020) Real-time PPP-RTK performance analysis using ionospheric corrections from multi-scale network configurations. *Sensors* 20(11):3012, DOI <https://doi.org/10.3390/s20113012>
- Psychas D, Teunissen PJG, Verhagen S (2021) A Multi-Frequency Galileo PPP-RTK Convergence Analysis with an Emphasis on the Role of Frequency Spacing. *Remote Sensing* 13(16), DOI <https://doi.org/10.3390/rs13163077>
- QZSS Cabinet Office (2022) List of Positioning Satellites | Technical Information | QZSS (Quasi-Zenith Satellite System) - Cabinet Office (Japan). URL <https://qzss.go.jp/en/technical/satellites/index.html>, accessed on 19.01.2022
- Rebischung P, Schmid R, Herring T (2016) [IGSMAIL-7399] Upcoming switch to IGS14/igs14.atx. URL <https://lists.igs.org/pipermail/igsmail/2016/001233.html>, accessed on 01.01.2021

- Rovira-Garcia A, Juan JM, Sanz J, Gonzalez-Casado G (2015) A worldwide ionospheric model for fast precise point positioning. *IEEE Transactions on Geoscience and Remote Sensing* 53(8):4596–4604
- Saastamoinen J (1972) Atmospheric correction for the troposphere and stratosphere in radio ranging satellites. The use of artificial satellites for geodesy 15:247–251
- Sato Y, Motooka N, Fujita S, Hirokawa R, Nakakuki K, Keshin M, Ikari S, Nakasuka Si (2022) Emergency Warning Utilizing Next-Generation PPP-RTK Service. In: *Proceedings of the 35th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2022)*, pp 3720–3730
- Schaer S (2018) SINEX BIAS—Solution (Software/technique) INdependent EXchange Format for GNSS Biases Version 1.00. Bern, Switzerland
- Schmid R, Dach R, Collilieux X, Jäggi A, Schmitz M, Dilssner F (2016) Absolute IGS antenna phase center model igs08. atx: status and potential improvements. *Journal of Geodesy* 90(4):343–364, DOI <https://doi.org/10.1007/s00190-015-0876-3>
- Seepersad G, Bisnath S (2013) Integrity monitoring in precise point positioning. In: *Proceedings of the 26th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2013)*, pp 1164–1175
- Seepersad G, Bisnath S (2017) An assessment of the interoperability of PPP-AR network products. *The Journal of Global Positioning Systems* 15(1):4, DOI <https://doi.org/10.1186/s41445-017-0009-9>
- Seepersad GG (2018) Improving Reliability and Assessing Performance of Global Navigation Satellite System Precise Point Positioning Ambiguity Resolution. PhD thesis, York University, URL <https://yorkspace.library.yorku.ca/xmlui/handle/10315/35570>, accessed on 01.11.2022
- Shi J, Gao Y (2010) Analysis of the integer property of ambiguity and characteristics of code and phase clocks in PPP using a decoupled clock model. *23rd International Technical Meeting of the Satellite Division of the Institute of Navigation 2010, ION GNSS 2010* 3:2553–2564
- Shi J, Gao Y (2012) A fast integer ambiguity resolution method for ppp. In: *Proceedings of the 25th international technical meeting of the satellite division of the institute of navigation (ION GNSS 2012)*, pp 3728–3734
- Shi J, Gao Y (2014a) A comparison of three PPP integer ambiguity resolution methods. *GPS Solutions* 18(4):519–528, DOI [10.1007/s10291-013-0348-2](https://doi.org/10.1007/s10291-013-0348-2)
- Shi J, Gao Y (2014b) A troposphere constraint method to improve PPP ambiguity-resolved height solution. *Journal of Navigation* 67(2):249–262, DOI [10.1017/S0373463313000647](https://doi.org/10.1017/S0373463313000647)
- Su K, Jin S, Jiao G (2022) GNSS carrier phase time-variant observable-specific signal bias (OSB) handling: an absolute bias perspective in multi-frequency PPP. *GPS Solutions* 26(71):71

- Takasu T, Yasuda A (2010) Kalman-filter-based integer ambiguity resolution strategy for long-baseline RTK with ionosphere and troposphere estimation. In: Proceedings of the 23rd international technical meeting of the satellite division of the institute of navigation (ION GNSS 2010), pp 161–171
- Talaya J, Bosch E (1999) CATNET, a permanent GPS network with real-time capabilities. In: Proc. ION GPS-99, 12th Int. Tech. Meeting of The Satellite Division of The US Institute of Navigation, pp 14–17
- Talbot N, Lu G, Allison T, Vollath U (2002) Broadcast network RTK-transmission standards and results. In: Proceedings of the 15th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GPS 2002), pp 2379–2387
- Tao J, Liu J, Hu Z, Zhao Q, Chen G, Ju B (2021) Initial Assessment of the BDS-3 PPP-B2b RTS compared with the CNES RTS. *GPS Solutions* 25(4):1–16
- Tegedor J, Øvstedal O (2014) Triple carrier precise point positioning (PPP) using GPS L5. *Survey Review* 46(337):288–297, DOI 10.1179/1752270613Y.0000000076
- Teunissen P (2003) Theory of integer equivariant estimation with application to GNSS. *Journal of Geodesy* 77(7):402–410
- Teunissen P (2007) Influence of ambiguity precision on the success rate of GNSS integer ambiguity bootstrapping. *Journal of Geodesy* 81(5):351–358, DOI <https://doi.org/10.1007/s00190-006-0111-3>
- Teunissen P, De Jonge P, Tiberius C (1995) The LAMBDA method for fast GPS surveying. In: International Symposium “GPS Technology Applications” Bucharest, Romania
- Teunissen PJ (1993) Least-squares estimation of the integer GPS ambiguities. In: Invited lecture, section IV theory and methodology, IAG general meeting, Beijing, China, pp 1–16
- Teunissen PJ, Khodabandeh A (2015) Review and principles of PPP-RTK methods. *Journal of Geodesy* 89(3):217–240, DOI 10.1007/s00190-014-0771-3
- Teunissen PJ, Odijk D, Zhang B (2010) PPP-RTK: Results of CORS network-based PPP with integer ambiguity resolution. *Journal of Aeronautics, Astronautics and Aviation* 42(4):223–230
- Teunissen PJG (1995) The least-squares ambiguity decorrelation adjustment: a method for fast GPS integer ambiguity estimation. *Journal of Geodesy* 70(1):65–82, DOI 10.1007/BF00863419
- Townsend B, Lachapelle G, Fortes LP, Melgård TE, Nørbech T, Raquet J (1999) New concepts for a carrier phase based GPS positioning using a national reference station network. In: Proceedings of the 1999 National Technical Meeting of The Institute of Navigation. San Diego, CA, pp 319–326
- Uhlemann M, Gendt G, Ramatschi M, Deng Z (2015) GFZ global multi-GNSS network and data processing results. In: IAG 150 years, Springer, pp 673–679

- Van Diggelen F (1997) GPS and GPS+GLONASS RTK. In: Proceedings of the 10th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GPS 1997). Kansas City, MO, pp 139–144
- Vana S (2021) Low-cost, triple-frequency multi-GNSS PPP and MEMS IMU integration for continuous navigation in urban environments. In: Proceedings of the 34th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2021), pp 3234–3249
- Villiger A, Schaer S, Dach R, Prange L, Sušnik A, Jäggi A (2019) Determination of GNSS pseudo-absolute code biases and their long-term combination. *J Geod* 93(9):1487–1500, DOI 10.1007/s00190-019-01262-w
- Wang J, Huang G, Yang Y, Zhang Q, Gao Y, Zhou P (2020) Mitigation of short-term temporal variations of receiver code bias to achieve increased success rate of ambiguity resolution in PPP. *Remote Sensing* 12(5):7–12, DOI <https://doi.org/10.3390/rs12050796>
- Wang L, Li Z, Wang N, Wang Z (2021) Real-time GNSS Precise Point Positioning for low-cost smart devices. *GPS Solutions* 25(2):1–13
- Wang N, Yuan Y, Li Z, Montenbruck O, Tan B (2016) Determination of differential code biases with multi-GNSS observations. *J Geod* 90(3):209–228, DOI 10.1007/s00190-015-0867-4
- Wang Z, Li Z, Wang L, Wang X, Yuan H (2018) Assessment of multiple GNSS real-time SSR products from different analysis centers. *ISPRS international journal of geo-information* 7(3):85
- Wanninger L, Beer S (2015) BeiDou satellite-induced code pseudorange variations: diagnosis and therapy. *GPS Solut* 19(4):639–648, DOI 10.1007/s10291-014-0423-3
- Wu JT, Wu SC, Hajj GA, Bertiger WI, Lichten SM (1992) Effects of antenna orientation on GPS carrier phase. *Astrodynamics* 1991 pp 1647–1660
- Wübbena G, Willgalis S (2001) State space approach for precise real time positioning in GPS reference networks. In: Proceedings Int. Symp. on Kinematic Systems in Geodesy, Geomatics and Navigation (KIS2001). Banff, Canada, pp 5–8
- Wunsch C, Stammer D (1997) Atmospheric loading and the oceanic “inverted barometer” effect. *Reviews of Geophysics* 35(1):79–107
- Wübbena G, Schmitz M, Bagge A (2005) PPP-RTK: precise point positioning using state-space representation in RTK networks. In: Proceedings of the 18th international technical meeting of the satellite division of the Institute of navigation (ION GNSS 2005), pp 2584–2594
- Xiang Y, Gao Y, Li Y (2020) Reducing convergence time of precise point positioning with ionospheric constraints and receiver differential code bias modeling. *Journal of Geodesy* 94(1):1–13

- Xiao G, Li P, Gao Y, Heck B (2019) A unified model for multi-frequency PPP ambiguity resolution and test results with Galileo and BeiDou triple-frequency observations. *Remote Sensing* 11(2):116, DOI <https://doi.org/10.3390/rs11020116>
- Xin S, Geng J, Guo J, Meng X (2020) On the Choice of the Third-Frequency Galileo Signals in Accelerating PPP Ambiguity Resolution in Case of Receiver Antenna Phase Center Errors. *Remote Sensing* 12(8), DOI <https://doi.org/10.3390/rs12081315>
- Xu P, Shi C, Liu J (2012) Integer estimation methods for GPS ambiguity resolution: an applications oriented review and improvement. *Survey Review* 44(324):59–71, DOI 10.1179/1752270611Y.0000000004
- Xu Y, Yang Y, Li J (2021) Performance evaluation of BDS-3 PPP-B2b Precise Point Positioning service. *GPS Solutions* 25(4):1–14
- Zaminpardaz S, Teunissen PJG (2017) Analysis of Galileo IOV + FOC signals and E5 RTK performance. *GPS Solutions* 21(4):1855–1870, DOI 10.1007/s10291-017-0659-9
- Zhang B, Teunissen PJ, Odijk D (2011) A Novel Un-differenced PPP-RTK Concept. *Journal of Navigation* 64(S1):S180–S191, DOI 10.1017/S0373463311000361
- Zhang B, Teunissen PJG, Yuan Y (2017) On the short-term temporal variations of GNSS receiver differential phase biases. *Journal of Geodesy* 91(5):563–572, DOI 10.1007/s00190-016-0983-9, 00000
- Zhang B, Chen Y, Yuan Y (2018) PPP-RTK based on undifferenced and uncombined observations: theoretical and practical aspects. *Journal of Geodesy* 93(7):1011–1024, DOI 10.1007/s00190-018-1220-5
- Zhang B, Teunissen PJG, Yuan Y, Zhang X, Li M (2019a) A modified carrier-to-code leveling method for retrieving ionospheric observables and detecting short-term temporal variability of receiver differential code biases. *J Geod* 93(1):19–28, DOI 10.1007/s00190-018-1135-1
- Zhang L, Yao Y, Peng W, Shan L, He Y, Kong J (2019b) Real-time global ionospheric map and its application in single-frequency positioning. *Sensors* 19(5):1138
- Zhang X, Zhu F, Zhang Y, Mohamed F, Zhou W (2019c) The improvement in integer ambiguity resolution with INS aiding for kinematic Precise Point Positioning. *Journal of Geodesy* 93(7):993–1010
- Zhang Y, Kubo N, Chen J, Chu FY, Wang A, Wang J (2020) Apparent clock and TGD biases between BDS-2 and BDS-3. *GPS Solutions* 24(1):27, DOI <https://doi.org/10.1007/s10291-019-0933-0>
- Zhodzishsky M, Vorobiev M, Khvalkov A, Rapoport L, Ashjaee J (1999) Dual-frequency GPS/GLONASS RTK: experimental results. In: *Proceedings of the 12th International Technical Meeting of the Satellite Division of the Institute of Navigation (ION GPS 1999)*, pp 805–810

- Zhou F, Dong D, Li P, Li X, Schuh H (2019) Influence of stochastic modeling for inter-system biases on multi-GNSS undifferenced and uncombined Precise Point Positioning. *GPS Solutions* 23(3):1–13
- Zumberge J, Heflin M, Jefferson D, Watkins M, Webb F (1997) Precise point positioning for the efficient and robust analysis of GPS data from large networks. *Journal of Geophysical Research: solid earth* 102(B3):5005–5017