

**FULLY DISTRIBUTED EVENT-TRIGGERED
ROBUST COOPERATIVE CONTROL FOR
MULTI-AGENT SYSTEMS**

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Abstract

Cooperative control plays a vital role in multi-agent systems (MASs) such as surveillance, formation flying, and load transportation, where a team of agents can accomplish tasks beyond the capability of a single agent. However, many existing control protocols require global information such as the Laplacian matrix, limiting their scalability and applicability across different topologies. In addition, limited communication and resource constraints pose challenges for practical implementation. This dissertation addresses these issues by developing fully distributed event-triggered cooperative control methods for disturbed MASs.

First, the fully distributed formation control problem for MASs with external disturbances is studied. An adaptive event-triggered sliding-mode control framework is proposed, where the finite-time reachability of the sliding surface is guaranteed without requiring global information. Zeno behavior is excluded, and a dynamic sliding surface is designed to ensure formation performance.

Second, the formation control problem for a leader-follower MAS under unknown perturbations and limited resources is investigated. A fully distributed adaptive dynamic event-triggered protocol is developed using a sliding-mode approach, such that the integral sliding-mode manifold can be reached in finite time. Then, a novel adaptive dynamic triggering strategy is developed to adjust triggering intervals and reduce resource consumption. Using Lyapunov theory and Barbalat lemma, sufficient conditions for asymptotic stability

are established.

Third, an event-based adaptive robust control protocol is introduced for networked Euler–Lagrange systems with a dynamic leader, handling parameter uncertainties, nonlinearities, and unknown perturbations. A nested adaptive sliding-mode estimator compensates for inaccurate states, while a dynamic triggering strategy reduces communication burden. The proposed fully distributed control law guarantees asymptotic convergence.

Fourth, the problem of fully distributed time-varying formation control is studied for MASs with unknown perturbations, input saturation, and switching topologies. An edge-based adaptive event-triggered scheme is designed, where thresholds on each communication edge reduce transmissions without degrading performance. A robust control protocol is developed to ensure asymptotic convergence while Zeno behavior is excluded.

Finally, experiments with multiple quadrotors validate the proposed methods. The results demonstrate consensus achievement, formation maintenance, and robustness in dynamic environments, confirming the practical effectiveness of the developed approaches.

Dedication

To my parents

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Nomenclature

0_N	Column vector with N entries equal to 0
1_N	Column vector with N entries equal to 1
A^{-1}	Inverse of matrix A
A^T	Transpose of matrix A
$A > 0$	Real symmetric positive definite matrix A
$A \otimes B$	The Kronecker product of matrices A and B
$\text{diag}\{\cdot\}$	Diagonal matrix
$D^+f(x_0)$	The right-hand Dini derivative of the continuous function f at x_0 , $D^+f(x_0) := \lim_{x \rightarrow x_0^+} \sup \frac{f(x) - f(x_0)}{x - x_0}$
$\mathbb{R}$	Space of real numbers
$\mathbb{R}_{\geq 0}$	Space of positive real numbers
$\mathbb{R}^{N \times 1}$	Space of real vectors with N entries
$\mathbb{R}^{N \times M}$	Space of $N \times M$ real matrices
$\mathbb{Z}_{\geq 0}$	Space of non-negative integers

I_N	$N \times N$ unit matrix
$\lambda_{\max}(A)$	Maximum eigenvalue of real symmetric matrix A
$\lambda_{\min}(A)$	Minimum eigenvalue of real symmetric matrix A
$\ \cdot\ $	Euclidean norm for vectors or the 2-norm for matrices
$\ \cdot\ _l$	l -norm for vectors or matrices, $l > 0$
$\text{sign}(x)$	Sign function of vector x , $\text{sign}(x) = [\text{sign}(x_1), \dots, \text{sign}(x_N)]^T$

1 Introduction

1.1 Background and Motivation

Inspired by natural phenomena such as ant foraging and bird flock migration, MASs have attracted widespread attention from scholars both domestically and internationally [3, 4]. Through coordination among individual agents, MASs can accomplish tasks that a single agent cannot complete alone [5]. Taking quadrotor formation as an example, a formation system composed of multiple quadrotors offers higher fault tolerance, greater efficiency, and better scalability compared to a single quadrotor [6]. Such systems have played important roles in military surveillance, load transportation and exploration, as shown in Fig. 1.1. Therefore, MASs are gradually becoming emerging complex systems that involve interdisciplinary fields such as control science, mathematics, biology, computer science, and robotics [7, 8].

With continuous advances in computing and communication technologies, cooperative control of MASs has become increasingly prevalent in practical applications. For instance, in intelligent transportation systems, vehicles can form a MAS and cooperate via communication networks to improve traffic efficiency and safety. In industrial automation, robots can collaborate as a MAS to accomplish complex tasks. In environmental monitoring and control, unmanned aerial vehicles (UAVs) can form MASs to jointly monitor and manage

large areas. Currently, there are two main cooperative control approaches for MASs: centralized control and distributed control. Centralized control relies on a central controller to manage the entire system; however, due to the inherent characteristics of MASs, this approach is difficult to implement. In contrast, distributed control better aligns with the MAS features by assigning control tasks to multiple agents, each of which only has access to local information and collaborates with others to achieve global system control. Therefore, distributed control offers advantages such as fault tolerance, scalability, and real-time responsiveness, and has become the mainstream approach in MAS cooperative control. It is worth noting that the term distributed control is sometimes used interchangeably with decentralized control [9], which may cause ambiguity. In decentralized control, agents compute their control inputs using only their own local measurements without real-time information exchange, and the control gains are often predetermined based on certain global parameters.

Agents in a multi-agent system exchange information through communication networks. Since sensors are often battery-powered to ensure mobility and flexibility, energy resources are typically limited. Consequently, achieving the desired control performance under constraints on energy supply is of critical importance in the design of MAS control strategies. However, the use of communication networks introduces several nontrivial challenges. For example, high-frequency periodic sampling can place heavy loads on the network, potentially resulting in congestion. Moreover, limited communication bandwidth often leads to packet losses. These factors can significantly degrade the overall performance of the MAS and may even cause system instability.

To reduce control updates and communication frequency, event-triggered control (ETC) schemes have been introduced into MASs [10–13]. ETC provides an effective alternative to periodic sampling by determining when each agent should transmit its information

over the network. In MASs with event-triggered transmission, an agent sends its state to neighboring agents only when a certain triggering condition is violated. Once triggered, the agent updates its control input and transmits its information to neighboring agents accordingly. However, this static and node-based triggering mechanism will cause to unnecessary resources consumption. Moreover, most existing studies on ETC for networked MASs have assumed ideal communication channels. However, the communication networks among agents may be unreliable due to link failures or new creations. Therefore, the study of event-triggered control for MASs is still open, and many challenges remain to be addressed.

In real-world applications of MASs such as quadrotor formation control, external disturbances such as wind, sensor noise, or actuator imperfections are inevitable and time-varying. External disturbances can significantly degrade the system performance [14, 15], and are very challenging in control system design. If not properly handled, these disturbances can cause oscillations, and even lead to the instability, especially under limited communication and computation resources. To address this issue, many works have proposed robust control schemes that rely on prior knowledge of the upper bounds of disturbances [2, 16]. However, such information is often difficult to obtain in practice. Thus, the control problem of disturbed MASs with unknown upper bounds has attracted increasing attention.

As a specific case of double-integrator nonlinear MASs, the Euler-Lagrange systems can be used to describe a large class of mechanical systems in practice, such as robot manipulators [17] and quadrotors [18]. However, the impact of nonlinearities and parameter uncertainties on system performance is significant, posing considerable challenges in control system design. Nonlinear dynamics can lead to unpredictable system behaviors and increased sensitivity to external disturbances, while parameter uncertainties can com-

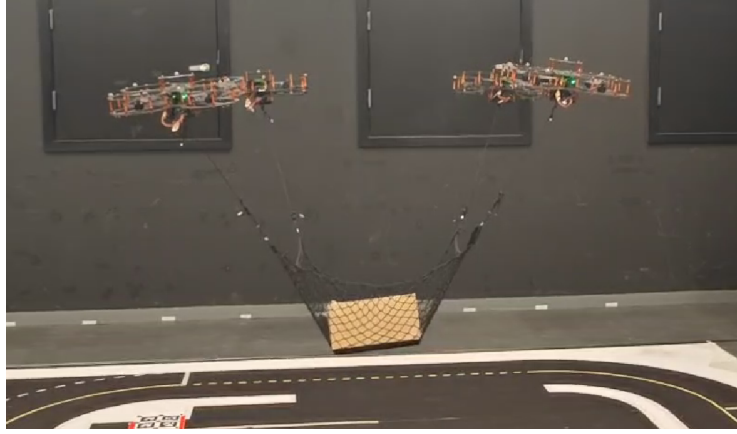


Figure 1.1: Load transportation using quadrotors

promise the accuracy and robustness of control laws. Therefore, in the past two decades, research on EL systems has been continuously carried out and has led to many significant results, which have become an indispensable part of control theory.

Given the widespread presence of unknown perturbations, nonlinearities, and parameter uncertainties, achieving fully distributed robust cooperative control for MASs with limited communication is an urgent and challenging task. However, the study of such systems is not a simple combination of existing results on disturbed nonlinear systems, event-triggered scheme, and robust control with those of cooperative control strategy in multi-agent systems. Instead, it requires more comprehensive research. This study proposes a control framework based on event-triggered mechanism, which addresses the distributed robust cooperative control problem for MASs subject to limited communication without any global information. The proposed approach not only provides more effective control strategies for some problems that have been solved in the existing literature, such as the ETC of disturbed systems, but also offers new ideas and methods for open problems yet to be resolved, such as the fully distributed formation control problem of MASs under unknown perturbations and unreliable communication.

1.2 Literature Review

Researchers have explored various cooperative tasks in MASs, including consensus [19,20], formation control [21,22], and containment control [23,24]. Numerous control architectures have been proposed, with most falling into either centralized or distributed control. In distributed control systems, each agent determines its actions and perceives the environment by relying on local sensors to acquire relative state information from neighboring agents. Agents interact only with their neighbors and do not require access to global information. In contrast, centralized MASs depend on a central controller, whose failure can compromise the entire mission. Distributed control systems, however, offer greater resilience to harsh environments since the malfunction of individual agents does not necessarily impair overall system performance. Additionally, distributed architectures reduce computational load and communication overhead by leveraging local neighbor-to-neighbor interactions.

1.2.1 Consensus Control

The consensus problem, where a group of agents aims to reach a collective agreement, has been extensively studied in the context of MASs. Existing research covers a wide range of topics [25–27], including consensus under both fixed and switching communication topologies, as well as different system dynamics such as first-order integrators and more complex models. In addition, researchers have investigated consensus under various practical limitations, such as limited communication bandwidth, quantization effects, and energy constraints, which are particularly relevant for resource-constrained agents like unmanned aerial vehicles and mobile robots.

1.2.2 Formation Control

Formation, which is a typical control problem of MASs, is to formulate a distributed control protocol for a group of agents to achieve a pre-defined geometry. Some classical methods have been proposed to handle formation control problems for MASs, such as the behavioral strategy [28], the virtual structure method [29], and the leader-follower approach [7]. Several formation control schemes for multiple quadrotors were developed [1, 30, 31], where many potential applications can be found, for instance, persistent surveillance, load transportation and exploration [32–34]. Note that the aforementioned formation control algorithms are all displacement based, in which the geometrical structure of the target formation is defined by inter-agent displacements. Due to the invariance of translational motions, the displacement-based formation control method can realize a translational formation maneuver. Besides, there are two kinds of other formation control approaches: distance-based and bearing-based methods [35–37]. Since the distance constraints are invariant to both translational and rotational motions, some distance-based formation control strategies were developed in [38] and [39] to achieve the translational and rotational formation maneuvers. Then, due to the invariance to translational and scaling motions, some bearing-based formation control schemes, in which target formation is defined by inter-agent bearings, were proposed to realize both translational and scaling formation maneuvers such as [21] and [40]. Recently, some approaches were designed, which define target formations using new types of constant constraints such as barycentric coordinates [41], complex Laplacians [42], and stress matrices [43]. For instance, an affine formation control protocol was developed in [22] based on stress matrices to achieve various formation maneuver behaviors for leader-following MASs. Moreover, to meet the mission requirements, MASs need to change the formation pattern in real time in some application scenarios, such as collision avoidance. Recently, some research results for time-varying formation of MASs have been

obtained [44–46].

Most existing distributed approaches [47–49] rely not only on local information but also on global information such as the eigenvalues of the Laplacian matrix, which characterize the communication topology, and the upper bounds of the leaders’ state derivatives. These global quantities are typically required to determine certain parameters in the control protocols or event-triggering conditions. However, it may be tough to acquire such global information when the network topology is dynamically changing. Thus, the developed schemes in the aforementioned works are not fully distributed. For this reason, some typical fully distributed protocols were proposed for MASs [50–52] by utilizing adaptive mechanisms to tune the coupling gains. However, the existing control methods exhibit limited robustness in the presence of parameter uncertainties and external disturbances.

1.2.3 Robust Control

The parameter uncertainties will affect the control performance of MASs. Motivated by this issue, some research results for cooperative control of MASs have been provided [23,53]. In [23], the distributed containment control law was proposed for mechanical systems with dynamic leaders and parameter uncertainties. The authors of [53] developed an adaptive control protocol for MASs with unknown nonlinear dynamics, without requiring relative velocity measurements. However, the unknown external disturbances were not considered in the aforementioned work. Similarly, the unknown perturbations will degrade the tracking control performance and even induce instability [54]. Consequently, it is necessary to design a robust control algorithm for MASs with disturbances to achieve coordination. Sliding-mode control (SMC) has been widely used in nonlinear systems due to its robustness against unknown dynamics and perturbations [55]. In [14], an event-triggered integral sliding-mode control (ISMC) strategy was proposed to address the distributed finite-time

formation control issue for MASs with perturbations. Note that for traditional SMC, the upper bounds of the perturbations are often required, which is impractical in real-world applications. To overcome this issue, adaptive SMC with a time-varying gain was developed [56]. However, the main issue of this method is that the varying gain increases rapidly due to the large initial error. Moreover, as the disturbance decreases, the varying gain does not reduce, leading to overestimation [57]. Thus, a distributed adaptive time-varying SMC (TVSMC) law was proposed for unknown EL systems, utilizing a time-varying sliding manifold to shorten the reaching phase of SMC [58]. Besides, input saturation limits the magnitude of the control input within a bounded region. Although the saturation nonlinearity usually degrades the system performance and even induces instability, there are only a few studies on cooperative control of MASs with input saturation [26, 59, 60]. In [60], the fully distributed event-based semiglobal consensus of MASs with input saturation was investigated. In [26], the event-based bipartite consensus control protocol was developed for neutrally stable linear systems with actuator saturation. Moreover, in the aforementioned literature, the control systems are assumed to have ideal network communication, enabling continuous communication and controller updates. However, this assumption is unrealistic in practical applications for MASs with constrained energy storage. Therefore, it is necessary to develop an energy-efficient control scheme.

1.2.4 Limited Communication

With the increasing demand for energy conservation, event-triggered control has been widely used in dynamic systems, as it reduces communication congestion caused by continuous interactions among agents while maintaining satisfactory communication efficiency. The concept of event-triggered control for multi-agent systems (MASs) has a rich history, with fundamental contributions dating back more than a decade [10–13]. Authors of [10]

first introduced a distributed event-triggered control scheme that ensures asymptotic consensus without continuous communication, significantly reducing resource usage. Authors of [11] further developed event-based agreement protocols for MASs with sampled-data event detection, establishing rigorous stability conditions. In [12], an iterative event-triggered algorithm was proposed where continuous measurement of the neighbor states is avoided. These pioneering works laid the theoretical foundation for subsequent developments in event-triggered MAS control. In [61], the time-varying formation control problem was studied by utilizing an event-triggered scheme for general linear MASs. In [62], an event-triggered formation tracking control protocol was proposed for multiple mobile robots with a dynamic leader, and the ultimate boundedness was guaranteed via Lyapunov-based stability-analysis methods. In [63], an event-triggered control strategy was developed to solve the formation tracking problem for uncertain EL systems, and the input-output stability was achieved by employing the small-gain theorem. However, in previous static or adaptive event-triggered mechanisms, the triggering thresholds are fixed permanently [64], leading to unnecessary resources consumption. As a result, the dynamic ETC mechanisms were developed in [65–67], where the event-triggered thresholds can be adjusted dynamically over time. In [68], a decentralized dynamic event-triggered neural network control law was developed for nonlinear interconnected large-scale systems subjected to unknown measurement sensitivity and varying control gains. To handle the nonzero-sum games problem for a category of multi-player continuous-time nonlinear systems, authors of [66] proposed a dynamic event-driven adaptive dynamic programming method. However, most existing works only focus on the node-based ETC. When the triggering condition is satisfied, the information of the agent will be transmitted to all its neighbors, which will result in inefficient use of communication resources. Therefore, designing an edge-based triggering mechanism is important. In [69], an asynchronous edge-based event-triggered

communication method was developed for a linear MAS using an adaptive control approach. It is noted that due to link failures or new creations, the interaction topology among quadrotors may be unreliable. This unreliable communication network will make the design of the edge-based triggering scheme more difficult. To the best of the authors' knowledge, no comprehensive study has yet addressed the fully distributed event-triggered cooperative control problem in MASs when confronted with unknown perturbations and unreliable communication.

1.3 Contributions

Among various MASs, multi-quadrotor systems represent a prominent and practical subclass, owing to the high maneuverability and deployment flexibility of quadrotors. These systems are widely used in formation control, cooperative surveillance, and distributed sensing missions. This dissertation makes several contributions to address the distributed cooperative control problem, introduced in Sections 1.1 and 1.2, for a disturbed multi-quadrotor system subject to parameter uncertainties and limited communication without any global information by developing a fully distributed event-triggered robust cooperative control framework.

The contributions of chapter 3 can be summarized as follows:

- Compared with [14, 16, 70], a novel distributed dynamic sliding-mode surface using an adaptive event-triggered approach is proposed. Even with unknown external disturbances, the finite-time reachability of the states of the multi-quadrotor system is guaranteed.
- In comparison with previous results [16, 71, 72], an event-based adaptive SMC algorithm is constructed for the leader-follower multi-quadrotor system to improve the

tracking performance and resource utilization considering unknown external disturbances.

- A novel fully distributed event-triggered formation control (DETFC) framework is formulated without the requirement of global information of the multi-quadrotor system. In comparison with [14,73,74], the proposed framework can guarantee the robust stabilization of the multi-quadrotor system under unknown perturbations, improve the convergence rate, and save resources without degrading the system performance.

The main contributions of Chapter 4 are outlined below:

- A distributed integral sliding manifold (ISM) is proposed, which can guarantee the finite-time reachability and asymptotic convergence of the states of the leader-follower multi-quadrotor system under unknown external disturbances.
- In comparison with previous results [16,71,72], an adaptive SMC algorithm with event-triggered rules is constructed for the multi-quadrotor system, which can efficaciously improve the tracking performance and resource utilization considering system nonlinear coupling and unknown perturbations.
- Under the distributed ISM, a novel adaptive dynamic event-triggered formation control (ADETFC) framework is designed for the multi-quadrotor system with unknown perturbations. In comparison with [1] and [30], the proposed framework with dynamic triggering conditions can guarantee the robust stabilization of the multi-quadrotor system, and decrease the mechanical wear of the actuator caused by continuous information transmission without degrading the system performance.

A summary of the contributions made in Chapter 5 is given below:

- A novel adaptive dynamic triggering scheme is proposed by using the estimated information to reduce unnecessary resources consumption. Different from the static

and adaptive event-triggered strategies [2, 61, 64, 75], the introduction of dynamic variables can dynamically adjust the inter-event interval, further reducing the sampling frequency without degrading tracking performance. In comparison with existing adaptive dynamic triggering strategies [15, 76], the proposed scheme eliminates the need for relative velocity measurements, reducing computational complexity and enhancing feasibility, thereby making it suitable for a broader range of practical applications.

- Unlike [61, 76, 77], which require accurate state information of the leader for controller design, this study removes that dependency by introducing a dual-layer nested adaptive sliding-mode estimator. This estimator reconstructs the dynamic leader's actual states despite measurement inaccuracies. Based on this estimation, a fully distributed adaptive dynamic event-triggered control protocol is designed, eliminating the need for any global information. Compared to existing fully distributed ETC schemes [2, 15], the proposed approach avoids the continuous communication and the uninterrupted updating of the controller, thereby enhancing the utilization of both communication and computation resources.
- It is the first attempt to propose a time-varying adaptive event-based SMC scheme utilizing a time-varying sliding manifold to address the effects of uncertain parameters and unknown perturbations. Compared to the existing SMC algorithms [56, 57], the effect of large initial errors on the control signals has been reduced, leading to the improved control accuracy. Unlike the TVSMC strategy [78], prior knowledge of the upper bounds of the external disturbances is not required. Furthermore, in contrast to the event-based robust controllers in [2, 79], the proposed scheme ensures robust asymptotic stabilization for multiple networked nonlinear EL systems.

Chapter 6 introduces the following significant advancements:

- An adaptive dynamic event-triggered scheme is constructed for edges such that the communication burden can be decreased significantly. Compared with the node-based triggering mechanisms in [15, 61, 80–82], the designed scheme only uses the relative states of the two quadrotors that constitute the edge, and the information transmission between the two quadrotors is asynchronous. The point-to-point communication feature can reduce the communication resources consumption. Different from [69, 83, 84], dynamic variables and adaptive triggering parameters are introduced in each edge, which can dynamically regulate the triggering intervals and further reduce the communication frequency as well as avoid Zeno behavior without degrading the tracking performance.
- Different from [15, 85–87], this study considers the unreliable communication networks, and the corresponding triggering condition associated with each edge is designed by utilizing the adaptive dynamic triggering function and the switching signal. The proposed edge-based ETC protocol is applicable to any switching graphs that meet the jointly connected condition, including fixed graphs as a special case.
- A novel event-based adaptive robust control algorithm is developed to guarantee the time-varying formation tracking of multiple quadrotors under input saturation and external perturbations without knowing the upper bounds. This is different from our previous work [2] and the work in [88], where the knowledge of the upper bounds of perturbations is required. Compared with [15, 20, 25, 61, 80–82, 89, 90], the design of the adaptive robust controller only uses local information, which contributes to the fully distributed fashion of the event-based control scheme.
- In comparison with the existing fully distributed ETC schemes [60, 73, 74, 91], the

asymptotic stability can be guaranteed by utilizing the proposed adaptive robust control scheme for the multi-quadrotor system under unknown perturbations and input saturation. Besides, the design of the controller does not use a global time variable or a common synchronous sampling period, and thus the clock synchronization among quadrotors is not required. Different from [60], the proposed fully distributed ETC protocol is independent of the number of agents. Therefore, it can avoid the ineffectiveness problem of the event-based controller resulted from growing number of agents.

1.4 Dissertation Organization

In view of the current research status of cooperative control for MASs, this dissertation systematically develops a fully distributed event-triggered control design method, building upon previous studies. It conducts in-depth research on the distributed cooperative control problem of multi-quadrotor systems with unknown perturbations and limited communication. This dissertation not only provides new solutions to existing problems, but also presents effective approaches to issues that have not yet been thoroughly studied in the literature.

This dissertation is organized into six chapters in addition to the Introduction. Chapter 2 mainly introduces essential technical terms and foundational concepts, forming the basis for the subsequent chapters of the thesis.

Chapter 3 studies the fully distributed formation control problem of multi-quadrotor systems with external disturbances. By utilizing an adaptive sliding-mode control approach, a fully distributed event-triggered formation control framework is developed.

Chapter 4 investigates the dynamic event-triggered formation control problem for leader-follower multi-quadrotor systems subject to unknown perturbations and limited resources.

The unknown upper bounds of perturbations are considered, and the asymptotic stability of the closed-loop control system is analyzed.

Chapter 5 studies some key challenges such as parameter uncertainties, unknown perturbations, inherent nonlinearities, and limited resources in multiple networked Euler-Lagrange systems with a dynamic leader. Furthermore, the inaccurate measurement of system states and the effect of large initial errors on the varying gain are considered.

Chapter 6 studies the fully distributed event-based time-varying formation control problem for multiple quadrotors with unknown perturbations, input saturation and unreliable communication networks. Different from the node-based triggering mechanism, online triggering parameters and dynamic thresholds with updating laws associated with each edge are developed.

Finally, the conclusions and future work are presented in Chapter 7.

2 Preliminaries

2.1 Graph Theory

Let $\mathcal{G} = 0 \cup \bar{\mathcal{G}}$ represent the communication topology of a leader-follower MAS, where node 0 is the virtual leader and $\bar{\mathcal{G}} = (\mathcal{V}, \mathcal{E})$ denotes the subgraph among N followers. $\mathcal{V} = \{1, 2, \dots, N\}$ represents the node set, and $\mathcal{E} = \{(i, j), i, j \in \mathcal{V}\} \subseteq \mathcal{V} \times \mathcal{V}$ stands for the edge set. Denote the adjacency matrix of $\bar{\mathcal{G}}$ by $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$, in which $a_{ij} = 1$ if and only if $(i, j) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. Note that there are no self-edges (i, i) , i.e. $(i, i) \notin \mathcal{E}$ for $\forall i \in \mathcal{V}$. For a directed graph, if $(i, j) \in \mathcal{E}$, then there exists a directed edge from node j to node i , and node j is called a neighbor of node i . If node 0 can reach any other nodes through a directed path, the graph is said to have a directed spanning tree with node 0 being the root node. Moreover, in an undirected graph, if $(i, j) \in \mathcal{E}$, then $(j, i) \in \mathcal{E}$ as well, which implies that information flows bidirectionally between nodes i and j . If an undirected path exists in an undirected graph between any two distinct nodes, the graph is connected. $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N \times N}$ represents the Laplacian matrix of $\bar{\mathcal{G}}$, with $l_{ii} = \sum_{j=1}^N a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$. The diagonal matrix $\mathcal{A}_0 = \text{diag}(a_{10}, \dots, a_{N0})$ is defined by $a_{i0} = 1$ if the agent i can receive the information from the leader; otherwise $a_{i0} = 0$.

2.2 System Modeling

To validate the proposed control methods, the quadrotor systems are used to conduct experiments. The inner/outer loop architecture [6] is implemented to handle the coordination problem for the multi-quadrotor system. The trajectory is tracked and the pre-defined formation pattern is maintained in the outer loop. The attitudes are stabilized in the inner loop. This research mainly focuses on the outer loop to make the position and velocity converge.

As depicted in [6], the external aerodynamic effects and disturbances are considered, and the outer loop dynamics of the follower quadrotor i is formulated as

$$\begin{aligned}\ddot{x}_i &= -\frac{K_1}{m_i}\dot{x}_i + \frac{T_i}{m_i}(\cos\phi_i \sin\theta_i \cos\psi_i + \sin\phi_i \sin\psi_i) + d_{1i}, \\ \ddot{y}_i &= -\frac{K_2}{m_i}\dot{y}_i + \frac{T_i}{m_i}(\cos\phi_i \sin\theta_i \sin\psi_i - \sin\phi_i \cos\psi_i) + d_{2i}, \\ \ddot{z}_i &= -\frac{K_3}{m_i}\dot{z}_i + \frac{T_i}{m_i}(\cos\phi_i \cos\theta_i) - g + d_{3i},\end{aligned}\tag{2.1}$$

where x_i , y_i and z_i denote the coordinates, respectively. ϕ_i , θ_i and ψ_i are the roll, pitch and yaw angle. d_{1i} , d_{2i} , and $d_{3i} \in \mathbb{R}$ represent the unknown time-varying bounded external disturbances in X , Y and Z , respectively. K_1 , K_2 and $K_3 \in \mathbb{R}_{\geq 0}$ represent the aerodynamic damping coefficient, m_i denotes the mass of the quadrotor, g is the acceleration of the gravity, and T_i stands for the thrust.

Furthermore, the virtual control input is defined as

$$\begin{aligned}u_{xi} &= \frac{T_i}{m_i}(\cos\phi_i \sin\theta_i \cos\psi_i + \sin\phi_i \sin\psi_i), \\ u_{yi} &= \frac{T_i}{m_i}(\cos\phi_i \sin\theta_i \sin\psi_i - \sin\phi_i \cos\psi_i), \\ u_{zi} &= \frac{T_i}{m_i}(\cos\phi_i \cos\theta_i) - g.\end{aligned}\tag{2.2}$$

The thrust of the quadrotor i is calculated as $T_i = m_i\sqrt{u_{xi}^2 + u_{yi}^2 + (u_{zi} + g)^2}$. It is worth noting that the outer-loop dynamics in (2.1)–(2.2) can be expressed in a compact

second-order form. This double-integrator structure is not unique to quadrotors — it is a standard abstraction in the cooperative control literature and can describe a wide range of agent dynamics, such as ground vehicles, fixed-wing UAVs in low-speed flight, underwater vehicles, and spacecraft, provided that the inner-loop control renders the outer-loop dynamics sufficiently fast and accurate. Consequently, although quadrotors are used for experimental validation in this dissertation, the control laws and theoretical results in Chapters 3, 4, and 6 are derived for the general second-order agent model and are applicable to any MAS that admits such an outer-loop representation. In Chapter 5, a different class of dynamics, namely Euler–Lagrange systems, is considered to further demonstrate the applicability of the proposed approach to nonlinear mechanical systems beyond the double-integrator form.

2.3 Experimental System

The experimental system is shown in Fig. 2.1. The system consists of Quanser[®] QDrone quadrotors, characterized by dynamics presented in (2.1), OptiTrack motion capture system with 16 Flex 13 cameras, a workstation, and a router. The unknown perturbations are provided by a Lasko box fan model 3733C and through the ground effects. The workstation calculates the pose of the quadrotors using Motive, the software that comes with the OptiTrack system at 100 Hz. Each QDrone is equipped with four brushless motors, an Intel[®] Aero Compute Board, a 1 kHz IMU sensor, multiple high-resolution cameras, and built-in Wi-Fi. Fig. 2.1 also shows the information flow - the blue dashed lines represent the information received from the workstation or the neighboring quadrotors, and the red dashed lines stand for the data flow sent from each quadrotor.

Remark 1. *The setup operates as a distributed system, with the workstation acting as*

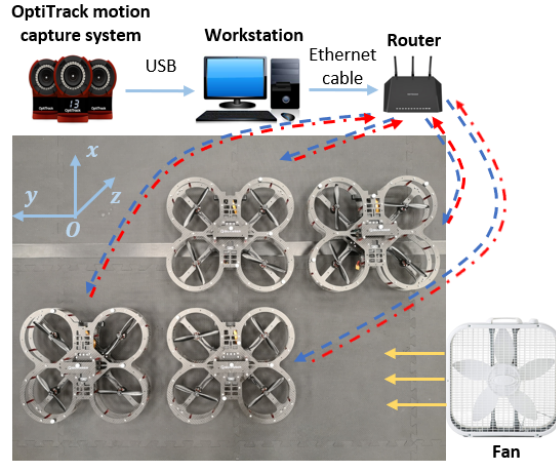


Figure 2.1: Experimental system.

a centralized server that manages each quadrotor's behavior. Given the limited processing power of quadrotors, the workstation handles the control calculations on their behalf. It executes vision processing, state estimation, and control algorithms, and then transmits the appropriate commands to each quadrotor via Wi-Fi. This configuration emulates a distributed control architecture, where the workstation effectively coordinates multiple quadrotors as distributed agents.

3 Fully Distributed Event-Triggered Formation Control

Most existing works about the cooperative control for quadrotors only consider the continuous communication. However, this will lead to unnecessary resources consumption, and the limited bandwidth will make the system suffer from communication burden. Besides, the external disturbances, which are ignored in many previous works, will affect the control performance. Moreover, calculating global information such as the eigenvalues of Laplacian matrix is a complicated task and will take up significant resources to process, especially for large-scale networks. These are all challenges for the control of quadrotors, which are equipped with microprocessors with limited computation/communication capabilities. Therefore, this chapter studies fully distributed formation control problems for multi-quadrotor systems with external disturbances via an event-triggered approach. For leader-follower multi-quadrotor systems, a fully distributed event-triggered formation control (DETFC) framework is proposed by utilizing an adaptive sliding-mode control approach, which does not rely on any global information, such as the network topology. The finite-time reachability of the sliding-mode surface can be guaranteed for quadrotors subject to external disturbances. Meanwhile, the Zeno behavior of the proposed event-triggered protocol is proved to be excluded. A novel dynamic sliding-mode surface is designed to guarantee the formation performance as the quadrotor state trajectories move

on the constructed sliding manifold. Via Lyapunov stability theory, sufficient conditions to ensure the formation results are derived for multi-quadrotor systems. Simulations and experiments are conducted to validate the effectiveness of the proposed control scheme.

3.1 Problem Statement

This section investigates the formation control problem for a multi-quadrotor system which includes N quadrotors with a virtual leader. The communication topology of the system is described by an undirected graph $\bar{\mathcal{G}} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. Let L be the Laplacian matrix of $\bar{\mathcal{G}}$, and the leader-follower matrix is defined as $H = L + \mathcal{A}_0$. The control block diagram is shown in Fig. 3.1.

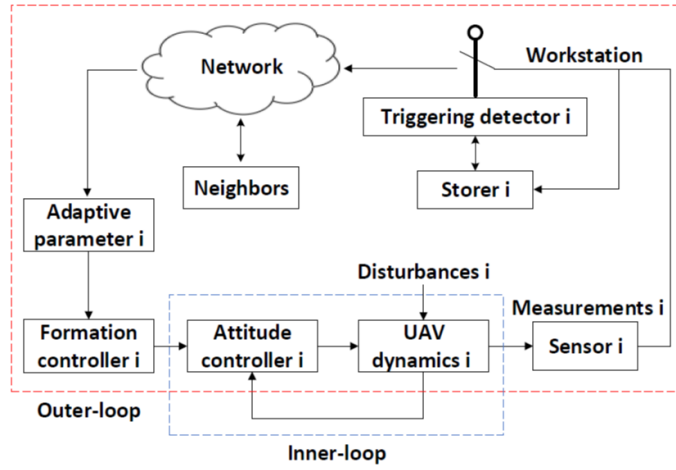


Figure 3.1: Control diagram of the multi-quadrotor system.

Define $u_{pi} = [u_{xi}, u_{yi}, u_{zi}]^T \in \mathbb{R}^{3 \times 1}$, and $f_p = -\text{diag}(K_1/m_i, K_2/m_i, K_3/m_i) \in \mathbb{R}^{3 \times 3}$. Arranging Eqs. (2.1) and (2.2), the subsystem for the quadrotor i is obtained as

$$\begin{aligned} \dot{p}_i &= v_i, \\ \dot{v}_i &= u_{pi} + f_p v_i + d_{pi}, \end{aligned} \tag{3.1}$$

where the position of the quadrotor i is denoted as $p_i = [x_i, y_i, z_i]^T \in \mathbb{R}^{3 \times 1}$, and $v_i =$

$[\dot{x}_i, \dot{y}_i, \dot{z}_i]^T = [v_{xi}, v_{yi}, v_{zi}]^T \in \mathbb{R}^{3 \times 1}$ is the velocity. $d_{pi} = [d_{1i}, d_{2i}, d_{3i}]^T \in \mathbb{R}^{3 \times 1}$ represents the unknown perturbations.

In order to address the fully distributed formation issue for the multi-quadrotor system, for $i \in \{1, 2, \dots, N\}$, two tracking errors are defined as $e_{pi}(t) = p_i(t) - \delta_i - p_0(t)$ and $e_{vi}(t) = v_i(t) - \dot{p}_0(t)$. $p_0 = [x_0, y_0, z_0]^T \in \mathbb{R}^{3 \times 1}$ and $\dot{p}_0 = [\dot{x}_0, \dot{y}_0, \dot{z}_0]^T \in \mathbb{R}^{3 \times 1}$ stand for the position and velocity of the virtual leader, respectively.

In this chapter, the multi-quadrotor system will track the reference position and velocity while maintaining a prescribed formation pattern. Let $\delta_i = [\delta_{xi}, \delta_{yi}, \delta_{zi}]^T \in \mathbb{R}^{3 \times 1}$, $i \in \{1, 2, \dots, N\}$, denote the desired formation relative distance between the quadrotor i and the virtual leader, and the reference trajectory which can be differentiable is provided by the virtual leader. Then, the definition of the formation control for the multi-quadrotor systems is provided as follows.

Definition 1. *The formation control for the multi-quadrotor system (3.1) is achieved in a fully distributed way with tolerances $\varrho_1 > 0$ and $\varrho_2 > 0$, if for any given initial states, there is an event-triggered controller which does not need the eigenvalues of H , such that*

$$\begin{aligned} \lim_{t \rightarrow \infty} \|p_i(t) - \delta_i - p_0(t)\| &\leq \varrho_1, \\ \lim_{t \rightarrow \infty} \|v_i(t) - \dot{p}_0(t)\| &\leq \varrho_2, \quad i = 1, \dots, N. \end{aligned} \tag{3.2}$$

Remark 2. *It is noteworthy to mention that the main objective of this chapter is to propose a fully distributed event-triggered formation control framework (DETFC) for multi-quadrotor systems such that the N followers can achieve a defined formation pattern while tracking the reference commands in the presence of external disturbances and limited resources.*

Remark 3. *This chapter mainly considers the time-invariant formation case, and δ_i is a constant, so as not to overshadow the main work of the proposed fully distributed event-*

based adaptive SMC protocol for multi-quadrotor systems. However, the proposed control strategy can be extended to some other types of maneuver, as done in [40–43].

Before moving on, some necessary assumptions and lemmas are listed to facilitate deriving the main results.

Assumption 1. *The undirected topology $\bar{\mathcal{G}}$ of the multi-quadrotor system is connected. In addition, we assume that at least one follower can receive the leader's information.*

Assumption 2. *The upper bound of the derivative of the acceleration of the virtual leader exists and is known.*

Assumption 3. *It is assumed that the derivative of the external disturbance $\dot{d}_{pi}$ is finite, and the positive constant γ_i is defined such that $\|\dot{d}_{pi}(t)\| \leq \gamma_i$.*

Remark 4. *Assumption 3 is a relaxed condition, which guarantees that the derivative of external disturbances $\dot{d}_{pi}(t)$ are bounded. In practice, this assumption can generally be satisfied.*

Lemma 1 ([92]). *According to Assumption 1, it can be inferred that H is positive definite.*

Lemma 2 ([93]). *The origin of the system is locally finite-time stable for a system described by $\dot{x} = f(x)$ with $f(0) = 0$, if there exist a continuously differentiable positive-definite function $V(x) : D \rightarrow \mathbb{R}$ and $\beta > 0$, $0 < q < 1$, $q \in \mathbb{R}$ such that*

$$\dot{V} + \beta V^q(x) \leq 0, \quad \forall x \in D$$

with settling time depending on initial state $x(0) = x_0$ and satisfies $T(x_0) \leq \frac{V^{1-q}(x_0)}{\beta(1-q)}$.

3.1.1 Design of the Dynamic Sliding-Mode Surface

For $t \in [t_k^i, t_{k+1}^i)$, the sliding-mode manifold for the quadrotor i can be proposed in a fully distributed event-triggered manner as

$$S_i(t) = \sum_{j \in \mathcal{N}_i} a_{ij} (\dot{v}_i - \dot{v}_j) + b_i (\dot{v}_i - \ddot{p}_0) - \left[\sum_{j \in \mathcal{N}_i} a_{ij} (u_{pti} - u_{ptj}) + b_i u_{pti} \right], \quad (3.3)$$

where the control protocol $u_{pti}(t)$ is developed with (3.4). $\mathcal{N}_i = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$ represents the neighbor set of agent i . The compact form of (3.3) is rebuilt as

$$\begin{aligned} S(t) &= H \otimes I_3 (\dot{v} - 1_N \otimes \ddot{p}_0) - H \otimes I_3 u_{pt} \\ &= H \otimes I_3 (u_p + f_p v + d_p - 1_N \otimes \ddot{p}_0 - u_{pt}), \end{aligned}$$

where $S(t) = [S_1^T(t), S_2^T(t), \dots, S_N^T(t)]^T$, $u_p(t) = [u_{p1}^T(t), u_{p2}^T(t), \dots, u_{pN}^T(t)]^T$, $u_{pt}(t) = [u_{pt1}^T(t), u_{pt2}^T(t), \dots, u_{ptN}^T(t)]^T$, $v(t) = [v_1^T(t), v_2^T(t), \dots, v_N^T(t)]^T$, and $d_p(t) = [d_{p1}^T(t), d_{p2}^T(t), \dots, d_{pN}^T(t)]^T$. $\ddot{p}_0 = [\ddot{x}_0, \ddot{y}_0, \ddot{z}_0]^T \in \mathbb{R}^{3 \times 1}$ represents the acceleration of the virtual leader.

3.1.2 Distributed Event-Based Adaptive Sliding-Mode Controller Design

In this part, we firstly define the relative state which can be accessed by the agent i as

$$\begin{aligned} \Lambda_i(t) &= \left\{ \varepsilon_1 \left[\sum_{j \in \mathcal{N}_i} a_{ij} \left[(p_i(t) - p_j(t)) - (\delta_i - \delta_j) \right] \right. \right. \\ &\quad \left. \left. + b_i [p_i(t) - p_0(t) - \delta_i] \right] + \varepsilon_2 \left[\sum_{j \in \mathcal{N}_i} a_{ij} [v_i(t) - v_j(t)] + b_i [v_i(t) - \dot{p}_0(t)] \right] \right\}, \end{aligned}$$

where $\varepsilon_1, \varepsilon_2 > 0$ are the coupling strengths. The fully distributed event-based control protocol is given by

$$\begin{aligned} u_{pti}(t) &= -c_i(t) \Lambda_i(t_k^i), \\ \dot{c}_i(t) &= \frac{1}{\chi} \left(\left(1 - \frac{\eta_1}{2}\right) \Lambda_i^T(t_k^i) \Lambda_i(t_k^i) - \sigma(c_i(t) - \kappa) \right), \end{aligned} \quad (3.4)$$

where $c_i(t)$ denotes the time-varying feedback gain for the proposed controller with $c_i(0) \geq \kappa$. $\eta_1 \in (0, 2)$, $\kappa > 0$ and $\chi > 0$ are constants. $\sigma > 0$ is the tuning parameter. t_k^i with $t_0^i = 0$, $i \in \{1, 2, \dots, N\}$, $k \in \mathbb{Z}_{\geq 0}$ denote the k^{th} sampling instant of $\Lambda_i(t)$.

According to the designed sliding-mode surface (3.3) and the tracking controller (3.4), a novel DETFC protocol is proposed

$$\begin{aligned} u_{pi}(t) &= u_{pti}(t) + u_{psi}(t), \quad t \in [t_k^i, t_{k+1}^i) \\ \dot{u}_{psi}(t) &= -\mathcal{L}_i(t)\text{sign}(S_i(t)), \\ \mathcal{L}_i(t) &= \hat{\gamma}_i(t) + \omega_i + \|f_p\| \|\dot{v}_i(t_k^i)\| + \nu_i, \end{aligned} \quad (3.5)$$

where ν_i is a chosen positive parameter, and ω_i is the tuning scalar. The $\hat{\gamma}_i(t)$ is the estimation of γ_i which is designed as

$$\dot{\hat{\gamma}}_i(t) = \begin{cases} \rho(\|S_i(t)\| + \epsilon_i), & \text{if } \hat{\gamma}_i(t) \leq \gamma_i \\ \rho(\|S_i(t)\| - \epsilon_i), & \text{otherwise} \end{cases} \quad (3.6)$$

where ρ and ϵ_i are positive constants, $\hat{\gamma}_i(0) \geq 0$, and the estimation error is defined as $\tilde{\gamma}_i(t) = \gamma_i - \hat{\gamma}_i(t)$.

Remark 5. According to (3.6), $\hat{\gamma}_i(t)$ will keep increasing before the system states arrive at the sliding manifold $S_i(t)$ in the case of $\hat{\gamma}_i(t) < \gamma_i$, but it will not be larger than γ_i . It is noted that the large enough bounds of γ_i should be estimated to apply the designed event-triggered adaptive sliding-mode controller (3.5) to a practical system. It is reasonable to choose the initial values of the adaptive parameters roughly based on their physical meaning.

For convenience of analysis, define $e_i(t) = \dot{v}_i(t_k^i) - \dot{v}_i(t)$ and $\Phi_i(t) = \Lambda_i(t_k^i) - \Lambda_i(t)$ as the sampling errors for all $t \in [t_k^i, t_{k+1}^i)$.

Then, inspired by [94], the event condition is designed as

$$\begin{aligned} f_i(t) &= \psi_{i1}\Phi_i^T(t)\Phi_i(t) - \psi_{i2}\Lambda_i^T(t_k^i)\Lambda_i(t_k^i) - \alpha e^{-\beta t}, \\ \psi_{i1} &= \zeta c_i(t) + \frac{(2 - \eta_1)(1 - \eta_2)\eta_3}{2\eta_2}, \\ \psi_{i2} &= \frac{(2 - \eta_1)(1 - \eta_3)}{2}, \end{aligned} \quad (3.7)$$

where $\zeta > 0$, $\alpha > 0$, $\beta > 0$, $\eta_2 \in (0, 1)$, and $\eta_3 \in (0, 1)$.

Remark 6. *To improve the tracking performance and handle the chattering problem, a novel event-triggered dynamic sliding manifold in (3.3) is designed. Then, the DETFC framework is proposed to realize the formation for the leader-follower multi-quadrotor system under external disturbances.*

3.2 Main Results

3.2.1 Reachability of Dynamic Sliding Manifold

Theorem 1. *Suppose that the multi-quadrotor system in (3.1) satisfies Assumptions 1-3, then the finite-time reachability of the sliding-mode surface can be guaranteed for quadrotor i by utilizing the proposed DETFC protocol (3.5). The parameter ν_i is chosen as*

$$\nu_i \geq \sup \left\{ \varphi_1 \|\ddot{p}_0(t)\| + \varphi_2 \|f_p\| \|e_i(t)\| \right\}, \quad (3.8)$$

where $\varphi_1 \geq 1$ and $\varphi_2 \geq 1$.

Proof: The Lyapunov function is selected as

$$V(t) = \frac{1}{2} S^T(t) (H \otimes I_3)^{-1} S(t) + \frac{1}{2} \tilde{\gamma}^T(t) \Xi \tilde{\gamma}(t),$$

where $\tilde{\gamma}(t) = [\tilde{\gamma}_1(t), \tilde{\gamma}_2(t), \dots, \tilde{\gamma}_N(t)]^T$, and $\Xi = \text{diag}\{1/\rho\} \in \mathbb{R}^{N \times N}$. The derivative of $V(t)$ can be expressed as

$$\begin{aligned} \dot{V}(t) &= S^T(t) (H \otimes I_3)^{-1} \dot{S}(t) + \tilde{\gamma}^T(t) \Xi \dot{\tilde{\gamma}}(t) \\ &= S^T(t) (\dot{u}_{ps}(t) + f_p \dot{v}(t) + \dot{d}_p(t) - 1_N \otimes \ddot{p}_0(t)) + \tilde{\gamma}^T(t) \Xi \dot{\tilde{\gamma}}(t). \end{aligned}$$

According to the designed DETFC protocol (3.5), the time derivative of $V(t)$ is refor-

culated as

$$\begin{aligned}
\dot{V}(t) &= \sum_{i=1}^N S_i^T(t) \dot{d}_{pi}(t) - \mathcal{L}_i(t) S_i^T(t) \text{sign}(S_i(t)) \\
&\quad + S_i^T(t) f_p \dot{v}_i(t) - S_i^T(t) \ddot{p}_0(t) + \tilde{\gamma}_i(t) \dot{\tilde{\gamma}}_i(t) / \rho \\
&\leq \sum_{i=1}^N S_i^T(t) \dot{d}_{pi}(t) - \mathcal{L}_i(t) \|S_i(t)\| + \|S_i(t)\| \|\ddot{p}_0(t)\| \\
&\quad + \|S_i(t)\| \|f_p\| \|\dot{v}_i(t)\| + \tilde{\gamma}_i(t) \dot{\tilde{\gamma}}_i(t) / \rho.
\end{aligned} \tag{3.9}$$

Based on the definition of the estimation error, considering the designed $\mathcal{L}_i(t)$ in (3.5), the proposed adaptive law $\dot{\gamma}_i(t)$ in (3.6), the selected ν_i in (3.8), and the facts $\|e_i(t)\| + \|\dot{v}_i(t_k^i)\| \geq \|\dot{v}_i(t)\|$, the inequality (3.9) is simplified as

$$\begin{aligned}
\dot{V}(t) &\leq \sum_{i=1}^N S_i^T(t) \dot{d}_{pi}(t) - \omega_i \|S_i(t)\| - \hat{\gamma}_i(t) \|S_i(t)\| - \tilde{\gamma}_i(t) \dot{\hat{\gamma}}_i(t) / \rho \\
&\leq -\rho_1 ((2\lambda_{\min}(H))^{\frac{1}{2}} (\frac{1}{2} S^T(t) (H \otimes I_3)^{-1} S(t))^{\frac{1}{2}} \\
&\quad + (\frac{2}{\lambda_{\max}(\Xi)})^{\frac{1}{2}} (\frac{1}{2} \tilde{\gamma}^T(t) \Xi \tilde{\gamma}(t))^{\frac{1}{2}}) \\
&\leq -\rho_2 V^{\frac{1}{2}},
\end{aligned} \tag{3.10}$$

where $0 < \rho_1 \leq \min\{\omega_i, \epsilon_i\}$ and $0 < \rho_2 \leq \min\{\rho_1 (2\lambda_{\min}(H))^{\frac{1}{2}}, \rho_1 (\frac{2}{\lambda_{\max}(\Xi)})^{\frac{1}{2}}\}$.

Therefore, according to Lemma 2, there exists a time instant $T \leq \frac{2V^{\frac{1}{2}}(x_0)}{\mu_2}$ such that the state trajectory of the multi-quadrotor systems can reach the proposed sliding manifold (3.3) in finite time T . The proof is completed.

Remark 7. It is obtained from [95] that two bounded scalars $M_v > 0$ and $M_{\dot{v}} > 0$ exist, which meet $\|v_q(t)\| \leq M_v$ and $\|\dot{v}_q(t)\| \leq M_{\dot{v}}$ for $q \in 0, 1, \dots, N$. Thus, for $t \in [t_k^i, t_{k+1}^i)$, it is reasonable to choose the appropriate parameter ν_i in (3.5) to satisfy the condition (3.8) based on Assumption 2.

As the states of the multi-quadrotor system reach the developed sliding surface (3.3), it is obtained that $S_i = \dot{S}_i = 0$. Hence, the sliding-mode error dynamics can be expressed

as

$$\dot{e}_p(t) = e_v(t), \quad \dot{e}_v(t) = -C(t)\Lambda(t_k), \quad (3.11)$$

where $\Lambda(t_k) = [\Lambda_1^T(t_k^1), \Lambda_2^T(t_k^2), \dots, \Lambda_N^T(t_k^N)]^T$, $e_p(t) = [e_{p1}^T, e_{p2}^T, \dots, e_{pN}^T]^T$ and $e_v(t) = [e_{v1}^T, e_{v2}^T, \dots, e_{vN}^T]^T$. Let $c(t) = \text{diag}\{c_1(t), \dots, c_N(t)\}$ and $C(t) = c(t) \otimes I_3$.

In this chapter, a fully distributed event-triggered adaptive sliding-mode formation control framework for the outer loop of the multi-quadrotor system is designed, and classical proportional derivative (PD) controllers are developed to stabilize the attitude as in [6]. Once the position and velocity errors arrive in the distributed sliding manifold, the formation problem of the resulting sliding mode dynamics is analyzed with the DETFC protocol.

3.2.2 System Analysis

Theorem 2. *Consider the leader-follower multi-quadrotor system (3.1) with the proposed fully distributed event-triggered adaptive sliding mode protocol (3.5), the states of the resultant sliding mode dynamics (3.11) will achieve the formation under Assumption 1. Moreover, the error dynamics $e_p(t)$ and $e_v(t)$, and the adaptive parameter $c_i(t)$, $i \in \{1, 2, \dots, N\}$, are uniformly ultimately bounded (UUB).*

Proof: According to Lemma 1, the Lyapunov function candidate is chosen as

$$V_1(t) = \frac{1}{2}\xi^T(t)\Pi\xi(t) + \sum_{i=1}^N \frac{\chi}{2} (c_i(t) - \hat{c})^2,$$

where $\xi(t) = [e_p^T(t), e_v^T(t)]^T$, $\Pi = \begin{bmatrix} \mu H^2 & \varepsilon_1 H \\ \varepsilon_1 H & \varepsilon_2 H \end{bmatrix} \otimes I_3$ with $\mu \gg \varepsilon_1$ being a constant, $\hat{c}$ is a positive constant that is to be determined later. Evidently, $V_1(t)$ is positive definite based on [51] and [94]. The time derivative of $V_1(t)$ along the states of the sliding-mode

dynamics is given by

$$\begin{aligned}\dot{V}_1(t) = & \mu e_p^T(t) \left(H^2 \otimes I_3 \right) e_v(t) + \varepsilon_1 e_v^T(t) (H \otimes I_3) e_v(t) \\ & - \Lambda(t) C(t) \Lambda(t_k) + \sum_{i=1}^N \chi \left(c_i(t) - \hat{c} \right) \dot{c}_i(t),\end{aligned}\quad (3.12)$$

where $\Lambda(t) = [\Lambda_1^T(t), \Lambda_2^T(t), \dots, \Lambda_N^T(t)]^T = (H \otimes I_3)(\varepsilon_1 e_p(t) + \varepsilon_2 e_v(t))$.

Then, by utilizing Young's inequality, the time derivative of $V_1(t)$ in (3.12) is rewritten as

$$\begin{aligned}\dot{V}_1(t) \leq & \varepsilon_1 \lambda_{\max}(H) e_v^T(t) e_v(t) + \mu e_p^T(t) \left(H^2 \otimes I_3 \right) e_v(t) \\ & - \left(1 - \frac{\eta_1}{2} \right) \hat{c} \Lambda^T(t_k) \Lambda(t_k) - \sum_{i=1}^N \frac{\sigma}{2} \tilde{c}_i^2(t) \\ & + \sum_{i=1}^N \frac{c_i(t)}{2\eta_1} \Phi_i^T(t) \Phi_i(t) + \varsigma,\end{aligned}\quad (3.13)$$

where $\tilde{c}_i(t) = c_i(t) - \hat{c}$ and $\varsigma = \frac{N\sigma(\hat{c}-\kappa)^2}{2}$. Note that

$$\begin{aligned}- \left(1 - \frac{\eta_1}{2} \right) \hat{c} \Lambda^T(t_k) \Lambda(t_k) \leq & - \left(1 - \frac{\eta_1}{2} \right) (1 - \eta_2) \eta_3 \hat{c} \Lambda^T(t) \Lambda(t) \\ & + \sum_{i=1}^N \left(1 - \frac{\eta_1}{2} \right) \frac{1 - \eta_2}{\eta_2} \eta_3 \hat{c} \Phi_i(t) \Phi_i(t) \\ & - \left(1 - \frac{\eta_1}{2} \right) (1 - \eta_3) \hat{c} \Lambda^T(t_k) \Lambda(t_k),\end{aligned}\quad (3.14)$$

and

$$\begin{aligned}-\Lambda^T(t) \Lambda(t) = & -\varepsilon_1^2 e_p^T(t) \left(H^2 \otimes I_3 \right) e_p(t) - 2\varepsilon_1 \varepsilon_2 e_p^T(t) \left(H^2 \otimes I_3 \right) e_v(t) \\ & - \varepsilon_2^2 e_v^T(t) \left(H^2 \otimes I_3 \right) e_v(t).\end{aligned}\quad (3.15)$$

Choose μ and $\hat{c}$ to satisfy the following conditions:

$$\begin{aligned}\mu &= 2 \left(1 - \frac{\eta_1}{2} \right) (1 - \eta_2) \eta_3 \varepsilon_1 \varepsilon_2 \hat{c}, \\ \hat{c} &\geq \frac{1}{(1 - \eta_1/2) (1 - \eta_2) \eta_3 \varepsilon_1^2 \lambda_{\min}^2(H)}, \\ \hat{c} &\geq \frac{\varepsilon_1 \lambda_{\max}(H) + 1}{(1 - \eta_1/2) (1 - \eta_2) \eta_3 \varepsilon_2^2 \lambda_{\min}^2(H)}.\end{aligned}\quad (3.16)$$

Afterward, we can obtain

$$\begin{aligned}\dot{V}_1(t) \leq & \hat{c} \sum_{i=1}^N \left(\left(\frac{\zeta c_i(t)}{2\eta_1 \zeta \hat{c}} + \frac{(2-\eta_1)(1-\eta_2)\eta_3}{2\eta_2} \right) \Phi_i^T(t) \Phi_i(t) \right. \\ & \left. - \frac{(2-\eta_1)(1-\eta_3)}{2} \Lambda_i^T(t_k^i) \Lambda_i(t_k^i) - \alpha e^{-\beta t} \right) \\ & - \xi^T(t) \xi(t) - \sum_{i=1}^N \frac{\sigma}{2} \tilde{c}_i^2(t) + \hat{c} N \alpha e^{-\beta t} + \varsigma,\end{aligned}\quad (3.17)$$

where $\hat{c} \geq 1/2\eta_1\zeta$. Based on the triggering condition (3.7), $\dot{V}_1(t)$ in (3.17) can be reformulated as

$$\begin{aligned}\dot{V}_1(t) & \leq -\xi^T(t) \xi(t) - \sum_{i=1}^N \frac{\sigma}{2} \tilde{c}_i^2(t) + \hat{c} N \alpha e^{-\beta t} + \varsigma \\ & \leq -\pi V_1(t) + \hat{c} N \alpha e^{-\beta t} + \varsigma,\end{aligned}\quad (3.18)$$

where $\pi = \min \left\{ (2/\lambda_{\min}(\Pi)), (\sigma/\chi) \right\}$. According to the comparison lemma [96], one has

$$V_1(t) \leq \left(V_1(0) - \frac{\varsigma}{\pi} \right) e^{-\pi t} + \frac{\varsigma}{\pi} + \hat{c} N \Delta(t), \quad (3.19)$$

where $\Delta(t)$ is defined as

$$\Delta(t) = \begin{cases} \alpha t e^{-\pi t}, & \text{if } \beta = \pi \\ \frac{\alpha}{\pi - \beta} (e^{-\beta t} - e^{-\pi t}). & \text{if } \beta \neq \pi \end{cases}$$

Thus, it is verified that $\xi(t), \tilde{c}_1(t), \dots, \tilde{c}_N(t)$ are UUB and converge to a bounded set

$$\mathcal{S} \triangleq \left\{ \xi(t), \tilde{c}_1(t), \dots, \tilde{c}_N(t) : V_1(t) \leq \frac{\varsigma}{\pi} \right\}.$$

Therefore, the formation of the multi-quadrotor system (3.1) is realized according to Definition 1. This completes the proof.

Remark 8. It is observed from the proof of Theorem 2 that $e_p(t)$, $e_v(t)$ and $c_i(t)$ will converge to a bounded set in terms of $V_1(t)$ as $\left\{ V_1(t) : 0 < V_1(t) \leq \frac{\varsigma}{\pi} \right\}$, which can be

viewed as an estimation of the attraction region and the error tolerance of $e_p(t)$, $e_v(t)$ and $c_i(t)$. From the definition of ς , one has that the bound of tracking errors can be reduced by selecting a relatively small σ .

3.2.3 Admissibility of Triggering Condition

The monotone increasing sequence which represents the triggering instant is defined as $\{t_k^i\}_{k=0}^\infty$, and we set $t_0^i = 0$ without loss of generality. To avoid the Zeno behavior, the inter-event interval $t_{k+1}^i - t_k^i$ has to be positive for $k \in \mathbb{Z}_{\geq 0}$. The main result is proposed as follows.

Theorem 3. *Considering the multi-quadrotor system (3.1), the minimal inter-event interval is positive and the Zeno phenomenon is eliminated by utilizing the proposed DETFC protocol (3.5) and the triggering condition (3.7).*

Proof: For all $t \in [t_k^i, t_{k+1}^i)$, the right-hand Dini derivative of $\|\Phi_i(t)\|$ is represented as

$$D^+ \|\Phi_i(t)\| = \frac{d}{dt} \sqrt{\Phi_i(t) \Phi_i(t)} = \frac{\Phi_i^T(t) \dot{\Phi}_i(t)}{\sqrt{\Phi_i^T(t) \Phi_i(t)}} \leq \|\dot{\Phi}_i(t)\|, \quad (3.20)$$

and we have

$$\begin{aligned} \dot{\Phi}_i(t) = & -\varepsilon_1 \left(\sum_{j \in \mathcal{N}_i} a_{ij} (e_{vi}(t) - e_{vj}(t)) + b_i e_{vi}(t) \right) \\ & - \varepsilon_2 \left(\sum_{j \in \mathcal{N}_i} a_{ij} (\dot{e}_{vi}(t) - \dot{e}_{vj}(t)) + b_i \dot{e}_{vi}(t) \right). \end{aligned} \quad (3.21)$$

According to Assumptions 2 and 3, for convenience of analysis, assume

$$\begin{aligned} \Upsilon_k^i = & \max_{t \in [t_k^i, t_{k+1}^i]} \left\| \frac{\varepsilon_1}{\varepsilon_2} \Lambda_i(t_k^i) - \frac{\varepsilon_1^2}{\varepsilon_2} \left(\sum_{j \in \mathcal{N}_i} a_{ij} (e_{pi}(t) - e_{pj}(t)) + b_i e_{pi}(t) \right) \right. \\ & \left. + \varepsilon_2 \left(\sum_{j \in \mathcal{N}_i} a_{ij} (\dot{e}_{vi}(t) - \dot{e}_{vj}(t)) + b_i \dot{e}_{vi}(t) \right) \right\|. \end{aligned} \quad (3.22)$$

Therefore, one has

$$D^+ \|\Phi_i(t)\| \leq \frac{\varepsilon_1}{\varepsilon_2} \|\Phi_i(t)\| + \Upsilon_k^i, \quad \forall t \in [t_k^i, t_{k+1}^i]. \quad (3.23)$$

Based on the comparison lemma [96], we obtain

$$\|\Phi_i(t)\| \leq \frac{\varepsilon_2}{\varepsilon_1} \Upsilon_k^i \left(e^{\frac{\varepsilon_1}{\varepsilon_2}(t-t_k^i)} - 1 \right). \quad (3.24)$$

According to the triggering condition (3.7), it is noticed that the sample error $\|\Phi_i(t)\|$ is bounded by

$$\|\Phi_i(t)\| \leq \Gamma_k^i(t), \quad \forall t \in [t_k^i, t_{k+1}^i] \quad (3.25)$$

where $\Gamma_k^i(t) = \sqrt{\left(\frac{\psi_{i2}}{\psi_{i1}}\right) \Lambda_i^T(t_k^i) \Lambda_i(t_k^i) + \left(\frac{\alpha}{\psi_{i1}}\right) e^{-\beta t}}$. Then, at $t = t_{k+1}^i$, there holds $\Phi_i(t_{k+1}^i) > \sqrt{\left(\frac{\alpha}{\psi_{i1}}\right) e^{-\beta t_{k+1}^i}}$, which together with (24) shows

$$t_{k+1}^i - t_k^i \geq \frac{\varepsilon_2}{\varepsilon_1} \ln \left(\frac{\varepsilon_1 \sqrt{\left(\frac{\alpha}{\psi_{i1}}\right) e^{-\beta t_{k+1}^i}}}{\varepsilon_2 \Upsilon_k^i} + 1 \right). \quad (3.26)$$

Due to the existence of the term $e^{-\beta t_{k+1}^i}$, the minimal inter-event interval $t_{k+1}^i - t_k^i > 0$ is guaranteed for any finite horizon. Thus, the Zeno phenomenon can be effectively eliminated during the formation control process for the multi-quadrotor system. This completes the proof.

3.3 Simulation Results

In this section, simulations are presented to verify the validity of the proposed DETFC scheme for the leader-follower multi-quadrotor system with external disturbances. Fig. 3.2 shows the undirected communication topology, where node 0 represents the virtual leader, and 1-4 are the followers. The multi-quadrotor system is modeled as (3.1) with $m_i = 1.121$

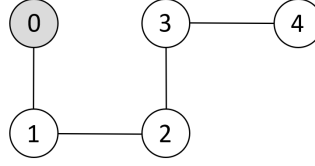


Figure 3.2: Topology of the communication graph.

kg, $g = 9.8 \text{ m/s}^2$ and $K_1 = K_2 = K_3 = 0.01 \text{ N}\cdot\text{s/m}$. The external disturbances are expressed as $d_i = 0.2 \sin(t) \text{ m/s}^2$.

The formation pattern is represented as $\delta_1 = [0, -1, 0]^T \text{ m}$, $\delta_2 = [1, -1, 0]^T \text{ m}$, $\delta_3 = [1, 0, 0]^T \text{ m}$ and $\delta_4 = [2, 0, 0]^T \text{ m}$. The reference trajectory provided by the virtual leader is given as $p_0 = [0.2t, 0.2t, 0.2t]^T \text{ m}$ and $v_0 = [0.2, 0.2, 0.2]^T \text{ m/s}$. In addition, the parameters φ_1 and φ_2 are chosen as $\varphi_1 = \varphi_2 = 1$. The bound of the disturbance derivative γ_i is set to 0.1. For the designed DETFC protocol (3.5) and the triggering function (3.7), we set $\varepsilon_1 = \varepsilon_2 = 10$, $\chi = 4.1$, $\sigma = 0.2$, $\kappa = 1$, $\eta_1 = 0.2$, $\eta_2 = \eta_3 = 0.1$, $\xi = 0.5$, $\alpha = 0.1$ and $\beta = 0.01$. The parameters ω_i , ν_i , ρ and ϵ_i are chosen as $\omega_i = 0.2$, $\nu_i = 1.8$, $\rho = 0.1$ and $\epsilon_i = 45$. The initial conditions for the following quadrotors are set as $p_1(0) = [0.8, 1.9, 0.9]^T \text{ m}$, $p_2(0) = [2.2, 1.8, 1]^T \text{ m}$, $p_3(0) = [2.3, 3.1, 1.2]^T \text{ m}$, $p_4(0) = [3.1, 3, 1.1]^T \text{ m}$ and $v_i(0) = 0_{3 \times 1} \text{ m/s}$. The initial value for the time-varying gain of the controller (3.4) is given as $c_i(0) = 1$, $i \in \{1, 2, \dots, N\}$.

The formation performance for the multi-quadrotor system with external disturbances in three dimensions is depicted in Fig. 3.3. Fig. 3.4 shows the triggering instants and intervals. The event numbers for following quadrotors are 970, 1421, 2452 and 1246, respectively, during the simulation time $[0, 20] \text{ s}$. Compared with the time-triggered method which has 2.2×10^4 sampling instants in total for each quadrotor, the proposed DETFC algorithm only needs about 6.9% of the resources in average to complete the control tasks. The formation errors are demonstrated in Figs. 3.5(a) and (b). Moreover, comparisons

Table 3.1: Comparisons of Simulation Results

Performance indexes	e_p (m)	e_v (m/s)	T_s (s)
DETFC	0.012; 0.005; 0.0055	0.009; 0.003; 0.005	5
Controller in [1]	0.011; 0.008; 0.007	0.009; 0.01; 0.006	7

with the results obtained using the formation control schemes proposed in [1] are conducted so as to validate the efficiency of the developed DETFC framework. The same parameters for quadrotor and external disturbances are implemented in the simulation, and the performance are shown in Figs 3.5(c) and (d). Table 3.1 compares the results using the proposed controller and the controller in [1] in terms of the final position (e_p) and velocity (e_v) tracking errors in X , Y and Z , and the settling time (T_s) in average for four quadrotors. It is obtained from Figs. 3.3-3.5 and Table 3.1 that the proposed control scheme can guarantee both the convergence rate and the formation performance of the multi-quadrotor system under external disturbances, as well as avoid the Zeno behavior. Moreover, the developed DETFC framework is more efficient than the existing methods for multi-quadrotor systems to achieve the formation with less computational resources.

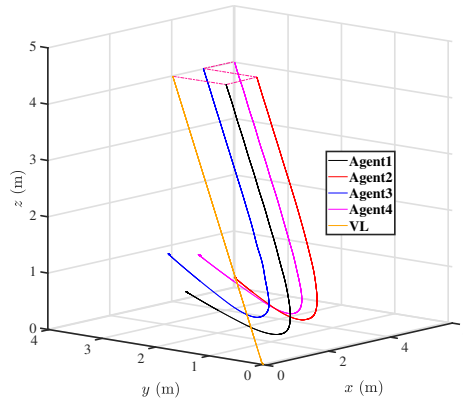


Figure 3.3: Formation trajectories.

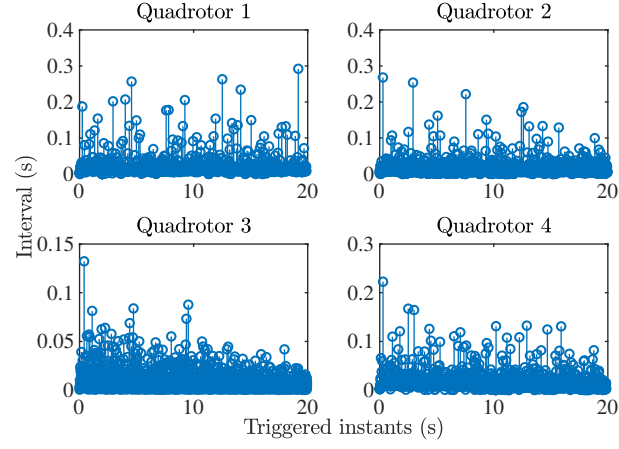


Figure 3.4: Triggered instants and intervals of the following quadrotors.

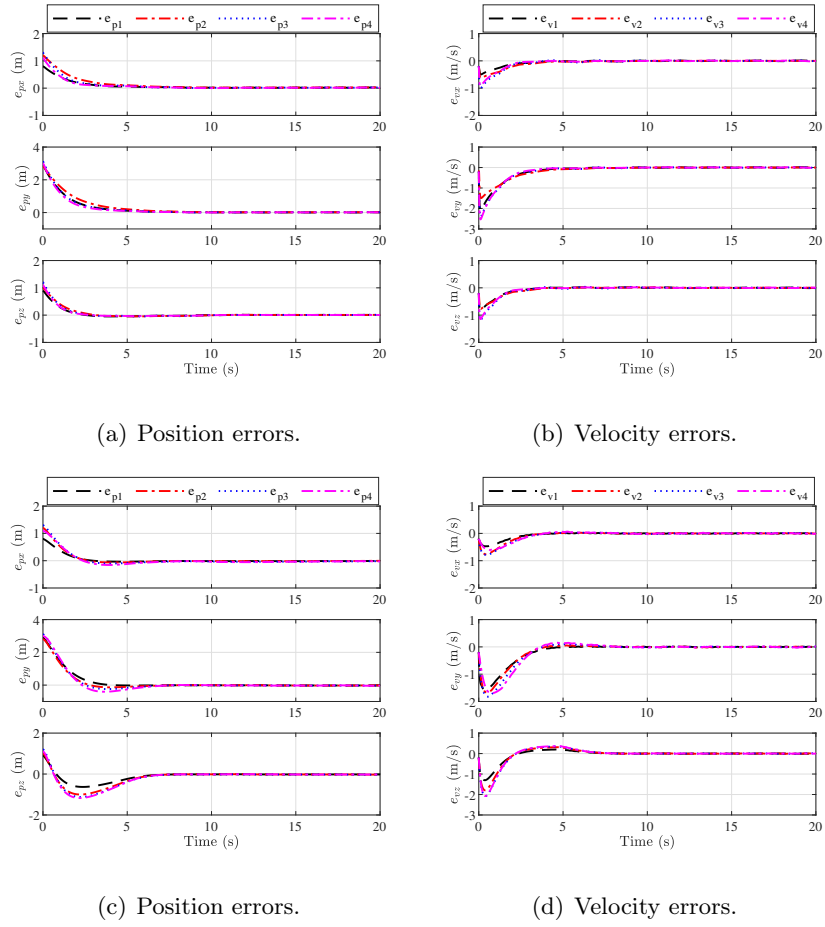


Figure 3.5: Simulation results of tracking errors: proposed controller (a, b); controller in [1] (c, d).

3.4 Experimental Results

In this section, hardware experiments are conducted to verify the effectiveness of the proposed controller using the experimental system shown in Fig. 3.6. The unknown perturbations are provided through the ground effects.

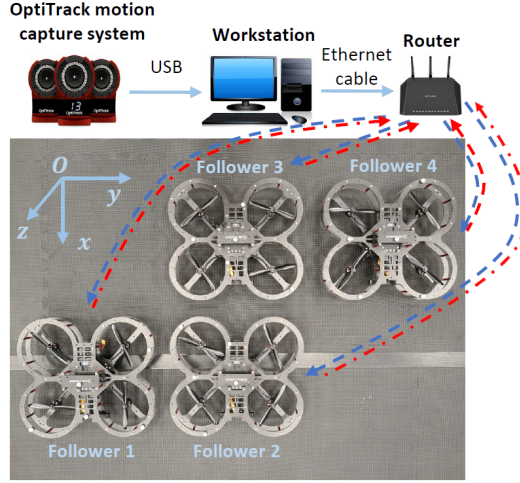


Figure 3.6: Experimental multi-quadrotor system.

The communication graph and parameters are set the same as Sec. 3.3, and the formation geometry is chosen as $\delta_1 = [1, 0, 0]^T$ m, $\delta_2 = [1, 1, 0]^T$ m, $\delta_3 = [0, 1, 0]^T$ m and $\delta_4 = [0, 2, 0]^T$ m. The desired trajectories are chosen as $p_0 = [-1.2+0.12t, -1.2+0.12t, 0.16t]^T$ m and $v_0 = [0.12, 0.12, 0.16]^T$ m/s, and the initial values of the following quadrotors are chosen as $p_1 = [0, -1.2, 0.6]^T$ m, $p_2 = [0, 0, 0.6]^T$ m, $p_3 = [-1.2, 0, 0.6]^T$ m, $p_4 = [-1.2, 1, 0.6]^T$ m and $v_i(0) = 0_{3 \times 1}$ m/s, $i \in \{1, 2, \dots, N\}$.

The quadrotors first took off and hovered at the initial position, then started to track the trajectory and the formation pattern at $t = 0$ s. The experimental tracking results are shown in Fig. 3.7. It can be found that the formation geometry is maintained while tracking the desired trajectory with a satisfactory performance, which validates the ef-

Table 3.2: Comparisons of Experimental Results

Performance indexes	e_p (m)	e_v (m/s)	T_s (s)
DETFC	0.08; 0.07; 0.04	0.05; 0.065; 0.045	2.5
Controller in [1]	0.11; 0.15; 0.11	0.075; 0.115; 0.095	3.1

fectiveness of the proposed DETFC algorithm. For comparison, the formation controller in [1] is implemented, and the results are shown in Fig. 3.8. The comparison of the experimental results is shown in Table 3.2. It is concluded from Figs. 3.7-3.8 and Table 3.2 that the proposed fully distributed formation control framework in this chapter has better convergence rate and tracking performance than the prior method in [1] for multi-quadrotor systems. Moreover, the triggering numbers for following quadrotors are 4711, 4342, 5054 and 5681, respectively, during the experimental period (10 s). Compared to the time-triggered method that has 10^4 sampling instants in total for each quadrotor, the proposed DETFC scheme can effectively save resources.

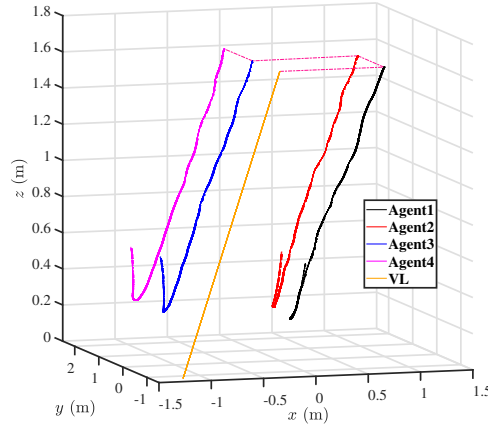
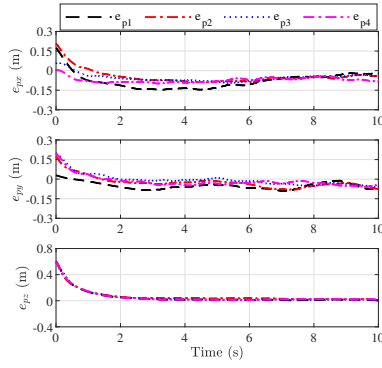
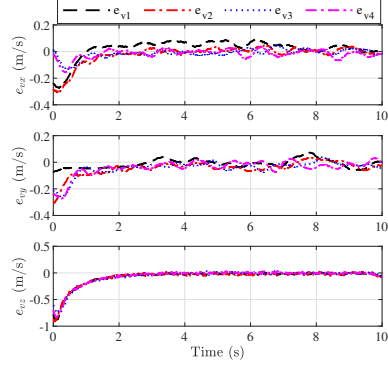


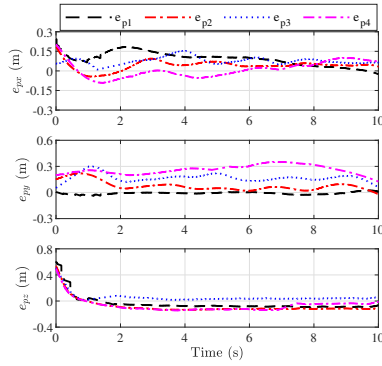
Figure 3.7: Experimental results: formation trajectories.



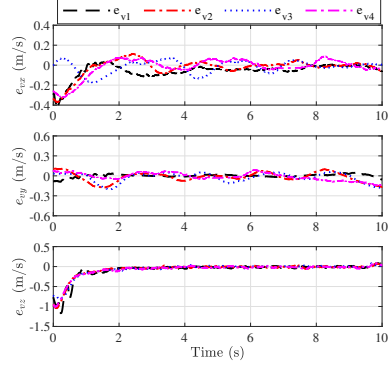
(a) Position errors



(b) Velocity errors



(c) Position errors



(d) Velocity errors

Figure 3.8: Experimental results of tracking errors: proposed controller (a, b); controller in [1] (c, d).

3.5 Summary

In this chapter, the formation control for the leader-follower multi-quadrotor system under limited communication and external disturbances was studied. A distributed dynamic sliding manifold was designed via the event-triggered strategy, and the chattering phenomenon of the controller was improved. In addition, a novel fully distributed event-triggered formation control framework was proposed based on the adaptive SMC algorithm to handle

external disturbances. According to the developed control scheme, Zeno phenomenon was eliminated, and the UUB stability was achieved. The validity of the proposed DETFC scheme has been verified by means of simulations and experiments. Compared to the time-triggered formation controller in [1], the scheme proposed in this chapter does not rely on any global information, reduces unnecessary energy usage, and improves the convergence time and tracking performance.

Although the proposed controller in this chapter can handle external disturbances, the knowledge of the upper bounds of perturbations is required, which is not easy to be obtained in practice. Besides, the developed static triggering scheme only can reduce the communication frequency, but the problem of the frequent update of the controller is not considered. Thus, further works will improve the robustness and the tracking performance for quadrotors under constrained computational and communication capacities.

4 Fully Distributed Adaptive Dynamic Event-Triggered Control

As mentioned in the summary of the preceding chapter, obtaining the upper bounds of external disturbances is difficult. While continuous information transmission will lead to unnecessary resource consumption, the frequent response of the actuator to control commands will make the system suffer from the wear of the actuator. Therefore, this chapter studies formation control problems for leader-follower multi-quadrotor systems subject to unknown perturbations and limited resources via an event-triggered mechanism. A distributed adaptive dynamic event-triggered formation control (ADETFC) protocol is designed by utilizing a sliding-mode control approach, such that the integral sliding-mode manifold can be reached in finite time for the states of quadrotors with unknown external disturbances. A distributed integral sliding-mode surface is proposed to guarantee the formation tracking performance as the state trajectories of multi-quadrotor systems move on the constructed sliding manifold. Then, a novel adaptive dynamic triggering strategy is developed to adjust the triggering interval dynamically, and thus further reduces the unnecessary resource consumption. Via the Lyapunov stability theory and Barbalat lemma, sufficient conditions to ensure the asymptotic stability are derived for leader-follower multi-quadrotor systems. Simulations and experiments to validate the effectiveness of the proposed control scheme are conducted.

4.1 Problem Statement

This chapter studies a leader-follower multi-quadrotor system which includes N quadrotors with a virtual leader. Let $\bar{\mathcal{G}} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ represent the communication topology of the system. The Laplacian matrix of $\bar{\mathcal{G}}$ is denoted as L , and the leader-follower matrix is defined as $H = L + \mathcal{A}_0$. The control block diagram is shown in Fig. 4.1.

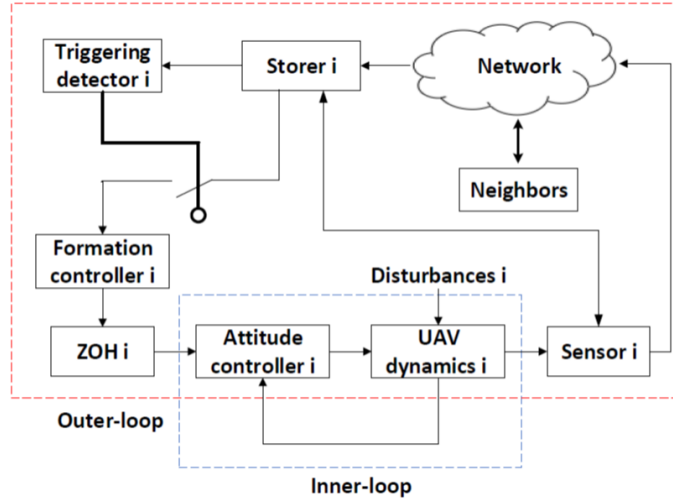


Figure 4.1: Control diagram of the multi-quadrotor system.

Define $u_i = [u_{xi}, u_{yi}, u_{zi}]^T \in \mathbb{R}^{3 \times 1}$, and $f_p = -\text{diag}(K_1/m_i, K_2/m_i, K_3/m_i) \in \mathbb{R}^{3 \times 3}$.

From Eqs. (2.1) and (2.2), the outer loop subsystem for the quadrotor i is obtained as

$$\begin{aligned} \dot{p}_i &= v_i, \\ \dot{v}_i &= u_i + f_p v_i + d_{pi}, \end{aligned} \tag{4.1}$$

where the position of the quadrotor i is denoted as $p_i = [x_i, y_i, z_i]^T \in \mathbb{R}^{3 \times 1}$, and $v_i = [\dot{x}_i, \dot{y}_i, \dot{z}_i]^T = [v_{xi}, v_{yi}, v_{zi}]^T \in \mathbb{R}^{3 \times 1}$ is the velocity. $d_{pi} = [d_{1i}, d_{2i}, d_{3i}]^T \in \mathbb{R}^{3 \times 1}$ is the external disturbances. Let $\delta_i = [\delta_{xi}, \delta_{yi}, \delta_{zi}]^T \in \mathbb{R}^{3 \times 1}$, $i \in \{1, 2, \dots, N\}$, denote the desired formation relative distance between the quadrotor i and the virtual leader. Then, the definition of the formation control for the multi-quadrotor systems is provided as follows.

Definition 2. *The formation control of the multi-quadrotor system is achieved if for any given initial states, the following requirements is satisfied*

$$\begin{cases} \lim_{t \rightarrow \infty} (p_i(t) - p_0(t)) \rightarrow \delta_i, \\ \lim_{t \rightarrow \infty} (v_i(t) - \dot{p}_0(t)) \rightarrow 0, \quad i = 1, \dots, N, \end{cases} \quad (4.2)$$

where $p_0(t) = [x_0, y_0, z_0]^T \in \mathbb{R}^{3 \times 1}$ and $\dot{p}_0(t) = [\dot{x}_0, \dot{y}_0, \dot{z}_0]^T \in \mathbb{R}^{3 \times 1}$ represent the position and velocity of the leader, respectively.

Remark 9. *It should be noted that the main goal of this chapter is to propose a distributed adaptive dynamic event-triggered formation control (ADETFC) framework by utilizing an ISM-based sliding-mode control approach such that the leader-follower multi-quadrotor system can achieve a defined formation pattern while tracking the reference commands in the presence of unknown perturbations and limited resources.*

Some necessary assumptions and lemmas are listed as follows.

Assumption 4. *The undirected topology $\bar{\mathcal{G}}$ of the multi-quadrotor system is connected. In addition, we assume that at least one follower can receive the leader's information.*

Assumption 5. *The upper bound of the acceleration of the virtual leader exists and is known.*

Assumption 6. *It is assumed that the external disturbance d_{pi} is finite, and the unknown positive constant γ_i is defined such that $\|d_{pi}(t)\| \leq \gamma_i$.*

Lemma 3 ([92]). *According to Assumption 4, it can be inferred that the matrix H is positive definite.*

4.1.1 Design of the Integral Sliding-Mode Surface

The distributed integral sliding-mode surface is designed as

$$\begin{aligned}
S_i(t) = & \sum_{j=1}^N a_{ij} (v_i(t) - v_j(t)) + b_i (v_i(t) - \dot{p}_0(t)) \\
& - \sum_{j=1}^N a_{ij} (v_i(0) - v_j(0)) + b_i (v_i(0) - \dot{p}_0(0)) \\
& - \int_0^t \left[\sum_{j=1}^N a_{ij} (\eta_i(\tau) - \eta_j(\tau)) + b_i \eta_i(\tau) \right] d\tau,
\end{aligned} \tag{4.3}$$

where the function $\eta_i(\tau)$ for $i = 1, \dots, N$ is proposed as

$$\begin{aligned}
\eta_i(\tau) = & - \left[\sum_{j=1}^N a_{ij} \operatorname{sig} \left(p_i - p_j - (\delta_i - \delta_j) \right)^{\alpha_1} + b_i \operatorname{sig} (p_i - \delta_i - p_0)^{\alpha_1} \right] \\
& - \left[\sum_{j=1}^N a_{ij} \operatorname{sig} (v_i - v_j)^{\alpha_2} + b_i \operatorname{sig} (v_i - \dot{p}_0)^{\alpha_2} \right],
\end{aligned}$$

where $\alpha_1 > 1$ and $\alpha_2 = \frac{2\alpha_1}{\alpha_1+1}$. Note that for a vector $\varpi = [\varpi_1, \dots, \varpi_N]^T \in \mathbb{R}^{N \times 1}$, $\operatorname{sig}(\varpi)^v = [\operatorname{sign}(\varpi_1) |\varpi_1|^v, \operatorname{sign}(\varpi_2) |\varpi_2|^v, \dots, \operatorname{sign}(\varpi_N) |\varpi_N|^v]^T$. In order to handle the distributed formation tracking issue for the multi-quadrotor system, for $i \in \{1, 2, \dots, N\}$, two tracking errors are defined as $e_{pi}(t) = p_i(t) - \delta_i - p_0(t)$ and $e_{vi}(t) = v_i(t) - \dot{p}_0(t)$. Then, the compact form of (4.3) is rebuilt as

$$S(t) = (H \otimes I_3) (e_v(t) - e_v(0)) - \int_0^t (H \otimes I_3) \eta(\tau) d\tau,$$

where $S(t) = [S_1^T(t), S_2^T(t), \dots, S_N^T(t)]^T$, $e_v(t) = [e_{v1}^T(t), e_{v2}^T(t), \dots, e_{vN}^T(t)]^T$, and $\eta(t) = [\eta_1^T(t), \eta_2^T(t), \dots, \eta_N^T(t)]^T$.

4.1.2 Distributed Event-Based Sliding-Mode Controller Design

According to the developed ISM (4.3), the distributed ADETFC protocol is proposed as

$$\begin{aligned}
u_i(t) = & - \left[\sum_{j \in \mathcal{N}_i} a_{ij} \operatorname{sig} \left(p_i \left(t_k^i \right) - p_j \left(t_k^j \right) - (\delta_i - \delta_j) \right)^{\alpha_1} \right. \\
& \left. + b_i \operatorname{sig} \left(p_i \left(t_k^i \right) - p_0 \left(t_k^i \right) - \delta_i \right)^{\alpha_1} \right] \\
& - \left[\sum_{j \in \mathcal{N}_i} a_{ij} \operatorname{sig} \left(v_i \left(t_k^i \right) - v_j \left(t_k^j \right) \right)^{\alpha_2} + b_i \operatorname{sig} \left(v_i \left(t_k^i \right) - \dot{p}_0 \left(t_k^i \right) \right)^{\alpha_2} \right] \\
& - \left(k_{1i} + \left| \hat{\gamma}_i \left(t_k^i \right) \right| + \mathcal{L} \right) \operatorname{sign} \left(S_i \left(t_k^i, t_k^j \right) \right), \tag{4.4}
\end{aligned}$$

where $k_{1i} > 0$ is a tuning scalar. $\mathcal{L}$ is a positive constant and mainly used to compensate for the external aerodynamic effects. t_k^i with $t_0^i = 0$, $i \in \{1, 2, \dots, N\}$, $k \in \mathbb{Z}_{\geq 0}$ denotes the k th sampling instant of agent i , and t_k^j denotes the last update time of the j -th agent's neighbors. $\mathcal{N}_i = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$ denotes the neighbor set of agent i . $\hat{\gamma}_i$ is the estimation of γ_i , and the adaptive law of unknown perturbations is designed as

$$\dot{\hat{\gamma}}_i(t) = \rho \left(\|S_i(t)\| + \epsilon_i \right), \tag{4.5}$$

where ρ and ϵ_i are positive constants and $\hat{\gamma}_i(0) \geq 0$. The estimation error is defined as $\tilde{\gamma}_i(t) = \gamma_i - \hat{\gamma}_i(t)$. For $t \in [t_k^i, t_{k+1}^i)$, the control protocol can be reformulated as

$$u_i(t) = \eta_i \left(t_k^i, t_k^j \right) - \left(k_{1i} + \left| \hat{\gamma}_i \left(t_k^i \right) \right| + \mathcal{L} \right) \operatorname{sign} \left(S_i \left(t_k^i, t_k^j \right) \right).$$

According to the output of the controller, a novel measurement error for the multi-quadrotor system is defined as

$$\begin{aligned}
e_i(t) = & \eta_i \left(t_k^i, t_k^j \right) - \left(k_{1i} + \left| \hat{\gamma}_i \left(t_k^i \right) \right| + \mathcal{L} \right) \operatorname{sign} \left(S_i \left(t_k^i, t_k^j \right) \right) \\
& - \left[\eta_i(t) - \left(k_{1i} + \left| \hat{\gamma}_i(t) \right| + \mathcal{L} \right) \operatorname{sign} \left(S_i(t) \right) \right]. \tag{4.6}
\end{aligned}$$

Then, an adaptive event-triggering function is developed as

$$\mathcal{H}_i(e_i(t), S_i(t)) = \iota_i \left[\|e_i(t)\|^2 - k_{1i} \|S_i(t)\| - k_{2i} \|S_i(t)\|^2 \right], \quad (4.7)$$

where ι_i and k_{2i} are positive scalars.

The triggering time sequence is determined with a dynamic threshold by the condition

$$t_{k+1}^i = \inf_{l > t_k^i} \left\{ l : \mathcal{H}_i(e_i(t), S_i(t)) > \chi_i(t), \forall t \in [t_k^i, l) \right\}, \quad (4.8)$$

where $\chi_i(t)$ is a dynamic variable, which updates based on

$$\dot{\chi}_i(t) = -\varepsilon_i \chi_i^{\frac{1}{2}}(t) - \frac{\mu_i}{\iota_i} \mathcal{H}_i(e_i(t), S_i(t)) \quad (4.9)$$

with $\varepsilon_i > 0$, $0 < \mu_i < 1$ and $\chi_i(0) > 0$.

Therefore, we can get $\dot{\chi}_i(t) \geq \left(-\varepsilon_i \chi_i^{-\frac{1}{2}}(t) - \frac{\mu_i}{\iota_i}\right) \chi_i(t)$, and $\dot{\chi}_i(t) \geq h_i(t) \chi_i(t)$, where $h_i(t) = -\varepsilon_i \chi_i^{-\frac{1}{2}}(t) - \frac{\mu_i}{\iota_i}$. Thus, we have $\chi_i(t) \geq \chi_i(0) e^{\int_0^t h_i(\tau) d\tau} > 0, \forall t \geq 0$.

Remark 10. *The dynamic variable $\chi_i(t)$ is the key factor for adjusting the threshold dynamically in the triggering function (4.8). When $\chi_i(t) \equiv 0$, the triggering function becomes a static event-triggered mechanism. It is worth noting that the introduction of the variable $\chi_i(t)$ in (4.8) can produce larger triggering intervals and reduce triggering times. In addition, the parameter ι_i in (4.7) also affects the triggering frequency. The smaller the value of ι_i is chosen, the slower the triggering frequency will be.*

Remark 11. *The introduced dynamic variable $\chi_i(t)$ plays an essential role in the exclusion of Zeno behavior. When $\chi_i(t) \equiv 0$, it is difficult to prove that the triggering sequence avoids Zeno behavior. To solve this problem, as done in [73] and [97], the common strategy is to degrade the system performance from the asymptotic stable to the bounded stable. In order to exclude Zeno behavior without sacrificing the system performance in this chapter, the design of the triggering function and the updating law of the dynamic variable can be found in (4.7)-(4.9), and the initial condition needs to satisfy $\chi_i(0) > 0$.*

4.2 Main Results

4.2.1 Reachability of Integral Sliding Manifold

Theorem 4. *Under the designed distributed event-based adaptive sliding-mode control protocol (4.4), and the dynamic triggering condition (4.8), the finite-time reachability of the ISM for the leader-follower multi-quadrotor system in (4.1) can be guaranteed with the Assumptions 4-6. $\mathcal{L}$ is selected as*

$$\mathcal{L} \geq \sup \left\{ \varphi_1 \|\ddot{p}_0(t)\| + \varphi_2 \|f_p\| \left\| v_i(t_k^i) \right\| + \varphi_3 \|f_p\| \|\tilde{\varepsilon}_i(t)\| \right\}, \quad (4.10)$$

where $\varphi_1 \geq 1$, $\varphi_2 \geq 1$, $\varphi_3 \geq 1$, and $\tilde{\varepsilon}_i(t) = v_i(t_k^i) - v_i(t)$.

Proof: Select the Lyapunov function as

$$V_1(t) = \frac{1}{2} S^T(t) (H \otimes I_3)^{-1} S(t) + \frac{1}{2} \tilde{\gamma}^T(t) \Xi \tilde{\gamma}(t),$$

where $\tilde{\gamma}(t) = [\tilde{\gamma}_1(t), \tilde{\gamma}_2(t), \dots, \tilde{\gamma}_N(t)]^T$, and $\Xi = \text{diag}\{1/\rho\} \in \mathbb{R}^{N \times N}$. The derivative of $V(t)$ can be expressed as

$$\begin{aligned} \dot{V}_1(t) &= \sum_{i=1}^N S_i^T(t) (\dot{v}_i(t) - \ddot{p}_0(t) - \eta_i(t)) - \sum_{i=1}^N \left(\tilde{\gamma}_i(t) (\|S_i(t)\| + \epsilon_i) \right) \\ &= \sum_{i=1}^N S_i^T(t) \left(e_i(t) - (k_{1i} + |\hat{\gamma}_i(t)| + \mathcal{L}) \text{sign}(S_i(t)) \right. \\ &\quad \left. + f_p v_i(t) + d_{pi}(t) - \ddot{p}_0(t) \right) - \sum_{i=1}^N \tilde{\gamma}_i(t) (\|S_i(t)\| + \epsilon_i). \end{aligned}$$

One has that $-(|\hat{\gamma}_i(t)| + \tilde{\gamma}_i(t) - \gamma_i) \leq 0$. Therefore, $\dot{V}_1(t)$ can be rewritten as

$$\dot{V}_1(t) \leq \sum_{i=1}^N [\|S_i(t)\| \|e_i(t)\| - k_{1i} \|S_i(t)\| - \epsilon_i \tilde{\gamma}_i(t)].$$

Then, another Lyapunov function is selected with the following form:

$$V_2(t) = V_1(t) + \sum_{i=1}^N \chi_i(t).$$

Afterward, the Young's inequality is implemented, and for any $\pi > 0$, the derivative of $V_2(t)$ can be written as

$$\begin{aligned}
\dot{V}_2(t) &\leq \sum_{i=1}^N \left[\frac{1}{\pi} \|S_i(t)\|^2 + \pi \|e_i(t)\|^2 - k_{1i} \|S_i(t)\| \right] \\
&\quad + \sum_{i=1}^N \left(-\varepsilon_i \chi_i^{\frac{1}{2}}(t) - \mu_i \left(\|e_i(t)\|^2 - k_{1i} \|S_i(t)\| - k_{2i} \|S_i(t)\|^2 \right) \right) - \sum_{i=1}^N \epsilon_i \tilde{\gamma}_i(t) \\
&\leq \sum_{i=1}^N \left[(\pi - \mu_i) \|e_i(t)\|^2 - \varepsilon_i \chi_i^{\frac{1}{2}}(t) - (1 - \mu_i) k_{1i} \right. \\
&\quad \left. \times \|S_i(t)\| + \left(\frac{1}{\pi} - \mu_i k_{2i} \right) \|S_i(t)\|^2 \right] - \sum_{i=1}^N \epsilon_i \tilde{\gamma}_i(t).
\end{aligned}$$

By appropriately choosing the parameters such that $\frac{1}{k_{2i}\mu_i} \leq \pi \leq \mu_i$, one has

$$\begin{aligned}
\dot{V}_2(t) &\leq \sum_{i=1}^N \left[- (1 - \mu_i) k_{1i} \|S_i(t)\| - \epsilon_i \tilde{\gamma}_i(t) - \varepsilon_i \chi_i^{\frac{1}{2}}(t) \right] \\
&\leq -\rho_1 ((2\lambda_{\min}(H))^{\frac{1}{2}} (\frac{1}{2} S^T(t)(H \otimes I_3)^{-1} S(t))^{\frac{1}{2}} \\
&\quad + (\frac{2}{\lambda_{\max}(\Xi)})^{\frac{1}{2}} (\frac{1}{2} \tilde{\gamma}^T(t) \Xi \tilde{\gamma}(t))^{\frac{1}{2}} + \sum_{i=1}^N \chi_i^{\frac{1}{2}}(t)) \\
&\leq -\rho_2 V_2^{\frac{1}{2}}(t). \tag{4.11}
\end{aligned}$$

where $\rho_1 = \min((1 - \mu_i) k_{1i}, \epsilon_i, \varepsilon_i) > 0$, and $\rho_2 = \min\left(\rho_1 (2\lambda_{\min}(H))^{\frac{1}{2}}, \rho_1 \left(\frac{2}{\lambda_{\max}(\Xi)}\right)^{\frac{1}{2}}, \rho_1\right) > 0$. Thus, based on Lemma 2, there exists a time instant $T \leq \frac{2V_2^{\frac{1}{2}}(x(0))}{\rho_2}$ such that the state trajectory of the multi-quadrotor systems can reach the proposed distributed ISM (4.3) in finite time T under unknown perturbations. The proof is completed.

Remark 12. From [95], it is obtained that there exist two positive bounded scalars $M_v > 0$ and $M_{\dot{v}} > 0$, and $\|v_q(t)\| \leq M_v$ and $\|\dot{v}_q(t)\| \leq M_{\dot{v}}$ are constructed for $q \in 0, 1, \dots, N$. Thus, for all $t \in [t_k^i, t_{k+1}^i)$, it is practical to choose proper constant $\mathcal{L}$ in (4.4) to satisfy the condition (4.10). Moreover, the external aerodynamic effects for quadrotors are very small, and thus many of the existing works ignore this term in the quadrotor model, for

example [3, 98]. In our work, Eq. (4.10) will become $\mathcal{L} \geq \sup \{ \varphi_1 \|\ddot{p}_0(t)\| \}$ after removing the external aerodynamic effects, and the proof is similar to the proof of Theorem 4, and thus is omitted here.

When the states of the leader-follower multi-quadrotor system remain on the developed sliding manifold, one has $\dot{S} = 0$ based on the SMC theory [99], and the sliding-mode dynamics are formulated as follows:

$$\dot{e}_{pi}(t) = e_{vi}(t), \quad \dot{e}_{vi}(t) = \eta_i(t). \quad (4.12)$$

Remark 13. In this chapter, classical proportional derivative (PD) controllers are designed to stabilize the attitude as in [6] and [3] so as not to overshadow the main work of the proposed distributed event-triggered formation control framework (4.4) for the outer loop of the multi-quadrotor system.

4.2.2 System Analysis

In this section, the leader-follower formation control problem for the sliding-mode dynamics (4.12) is analyzed based on the distributed adaptive dynamic event-triggered integral sliding-mode control in (4.4).

Theorem 5. Consider the leader-follower multi-quadrotor system (4.1) with the designed distributed ADETFC protocol, the formation tracking is achieved for the sliding-mode dynamics in (4.12) under Assumption 4.

According to [100], the Lyapunov function candidate is chosen as

$$V(t) = W_1(t) + W_2(t) + W_3(t),$$

where

$$\begin{aligned} W_1(t) &= \frac{1}{2} \sum_{i=1}^N e_{vi}^2, \\ W_2(t) &= \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \alpha_{ij} \int_0^{e_{pi}-e_{pj}} \text{sig}(t)^{\alpha_1} dt, \\ W_3(t) &= \sum_{i=1}^N b_i \int_0^{e_{pi}} \text{sig}(t)^{\alpha_1} dt. \end{aligned}$$

According to the sliding-mode dynamics (4.12), the derivative of $V(t)$ is represented as

$$\begin{aligned} \dot{V}(t) &= - \sum_{i=1}^N e_{vi} \left[\sum_{i=1}^N a_{ij} \text{sig}(e_{pi} - e_{pj})^{\alpha_1} + b_i \text{sig}(e_{pi})^{\alpha_1} \right. \\ &\quad \left. + \sum_{j=1}^N a_{ij} \text{sig}(e_{vi} - e_{vj})^{\alpha_2} + b_i \text{sig}(e_{vi})^{\alpha_2} \right] \\ &\quad + \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N a_{ij} \text{sig}(e_{pi} - e_{pj})^{\alpha_1} (e_{vi} - e_{vj}) + \sum_{i=1}^N b_i \text{sig}(e_{pi})^{\alpha_1} e_{vi} \\ &= - \sum_{i=1}^N \left[\frac{1}{2} \sum_{j=1}^N a_{ij} \text{sig}(e_{vi} - e_{vj})^{\alpha_2} (e_{vi} - e_{vj}) + b_i e_{vi} \text{sig}(e_{vi})^{\alpha_2} \right] \leq 0. \end{aligned} \quad (4.13)$$

Hence, one has that $\lim_{t \rightarrow +\infty} \int_0^t \dot{V}(t) dt$ exists and is limited. Based on (4.13), we can have $V(t) \leq V(0)$, and thus the states e_{pi} and e_{vi} are bounded. Moreover, according to the sliding-mode dynamics (4.12), $\dot{e}_{vi}$ is bounded. Therefore, it is inferred that $\dot{V}(t)$ is uniformly continuous. Next, based on Barbalat's lemma [101], one has $\lim_{t \rightarrow +\infty} \frac{1}{2} \sum_{j=1}^N a_{ij} \text{sig}(e_{vi} - e_{vj})^{\alpha_2} (e_{vi} - e_{vj}) + b_i e_{vi} \text{sig}(e_{vi})^{\alpha_2} = 0$, and we can get $\lim_{t \rightarrow +\infty} e_{vi} = 0$. Then, it can be inferred from the sliding-mode dynamics (4.12) that $\lim_{t \rightarrow +\infty} \sum_{j=1}^N a_{ij} \text{sig}(e_{pi} - e_{pj})^{\alpha_1} + b_i \text{sig}(e_{pi})^{\alpha_1} = 0$. Furthermore, one has

$$\begin{aligned} &\lim_{t \rightarrow +\infty} \sum_{i=1}^N e_{pi} \left[\sum_{j=1}^N a_{ij} \text{sig}(e_{pi} - e_{pj})^{\alpha_1} + b_i \text{sig}(e_{pi})^{\alpha_1} \right] \\ &= \lim_{t \rightarrow +\infty} \sum_{i=1}^N \left[\frac{1}{2} \sum_{j=1}^N a_{ij} \text{sig}(e_{pi} - e_{pj})^{\alpha_1} (e_{pi} - e_{pj}) + b_i e_{pi} \text{sig}(e_{pi})^{\alpha_1} \right] = 0. \end{aligned}$$

Hence, it is concluded that $\lim_{t \rightarrow +\infty} e_{pi} = 0$. Consequently, the formation of the leader-follower multi-quadrotor system is achieved based on Definition 2. The proof is completed.

4.2.3 Minimal Interevent Interval Computation

The difference value τ_k^i between any two uninterrupted triggering moments is defined as $\tau_k^i = t_{k+1}^i - t_k^i$ for all $k \in \mathbb{Z}_{\geq 0}$. To avoid the Zeno behavior, τ_k^i has to be positive.

Theorem 6. *Considering the multi-quadrotor system (4.1), the minimal inter-event interval is positive and the Zeno phenomenon is eliminated by utilizing the designed ADETFC protocol (4.4) and the dynamic triggering condition (4.8).*

Proof: For $t \in [t_k^i, t_{k+1}^i)$, in consideration of the dynamic triggering condition (4.8), one has

$$\begin{aligned} D^+ (\|e_i(t)\|) &\leq \|D^+ e_i(t)\| \\ &\leq \|D^+ \eta_i(t)\| + \|D^+ (k_{1i} + |\hat{\gamma}_i(t)| + \mathcal{L}) \text{sign}(S_i(t))\|. \end{aligned} \quad (4.14)$$

For the first term in (4.14), we have

$$\begin{aligned} \|D^+ \eta_i(t)\| &\leq \alpha_1 \left(\sum_{j=1}^N a_{ij} \|p_i - p_j - (\delta_i - \delta_j)\|^{\alpha_1-1} \right) \left(\sum_{j=1}^N a_{ij} \|v_i - v_j\| \right) \\ &\quad + \alpha_1 b_i \|p_i - p_0 - \delta_i\|^{\alpha_1-1} \|v_i - \dot{p}_0\| \\ &\quad + \alpha_2 \left(\sum_{j=1}^N a_{ij} \|v_i - v_j\|^{\alpha_2-1} \right) \left(\sum_{j=1}^N a_{ij} (\|u_i - u_j\| + \|f_p v_i - f_p v_j\|) + 2N\gamma_{\max} \right) \\ &\quad + \alpha_2 b_i \|v_i - \dot{p}_0\|^{\alpha_2-1} (\|u_i\| + \|f_p\| \|v_i\| + \|\ddot{p}_0\| + \gamma_{\max}) \end{aligned} \quad (4.15)$$

where $\gamma_{\max} = \max_i \{\gamma_i\} > 0$ is a bounded constant. In this section, the $\text{sign}(\cdot)$ function is

replaced by the $\tanh(\cdot)$ function [102], and then the second term in (4.14) is formulated as

$$\begin{aligned}
& \|D^+ (k_{1i} + |\hat{\gamma}_i(t)| + \mathcal{L}) \operatorname{sign} (S_i(t)) \| \\
& \leq \left\| D^+ |\hat{\gamma}_i(t)| \tanh (\beta S_i(t)) \right\| + (k_{1i} + |\hat{\gamma}_i(t)| + \mathcal{L}) \left\| 1 - \tanh^2 (\beta S_i(t)) \right\| \left\| \beta \dot{S}_i(t) \right\| \\
& \leq \rho (\|S_i(t)\| + \epsilon_i) + \beta (k_{1i} + |\hat{\gamma}_i(t)| + \mathcal{L}) \left(\sum_{j=1}^N a_{ij} (\|u_i - u_j\| + \|f_p v_i - f_p v_j\|) + 2N\gamma_{\max} \right. \\
& \quad \left. + \left\| \sum_{j=1}^N a_{ij} (\eta_i - \eta_j) \right\| + b_i (\|\eta_i\| + \|u_i\| + \|f_p v_i\| + \gamma_{\max} + \|\ddot{p}_0\|) \right) \quad (4.16)
\end{aligned}$$

where $\beta \gg 1$. Moreover, one has

$$\begin{aligned}
& \|e_i(t)\| \leq (t - t_k) \sigma_i, \\
& \sigma_i = \alpha_1 \left(\sum_{j=1}^N a_{ij} \|p_i - p_j - (\delta_i - \delta_j)\|^{\alpha_1 - 1} \right) \left(\sum_{j=1}^N a_{ij} \|v_i - v_j\| \right) \\
& \quad + \alpha_1 b_i \|p_i - p_0 - \delta_i\|^{\alpha_1 - 1} \|v_i - \dot{p}_0\| \\
& \quad + \alpha_2 \left(\sum_{j=1}^N a_{ij} \|v_i - v_j\|^{\alpha_2 - 1} \right) \left(\sum_{j=1}^N a_{ij} (\|u_i - u_j\| + \|f_p v_i - f_p v_j\|) + 2N\gamma_{\max} \right) \\
& \quad + \alpha_2 b_i \|v_i - \dot{p}_0\|^{\alpha_2 - 1} (\|u_i\| + \|f_p\| \|v_i\| + \|\ddot{p}_0\| + \gamma_{\max}) \\
& \quad + \rho (\|S_i(t)\| + \epsilon_i) + \beta (k_{1i} + |\hat{\gamma}_i(t)| + \mathcal{L}) \left(\sum_{j=1}^N a_{ij} (\|u_i - u_j\| + \|f_p v_i - f_p v_j\|) \right. \\
& \quad \left. + 2N\gamma_{\max} + \left\| \sum_{j=1}^N a_{ij} (\eta_i - \eta_j) \right\| + b_i (\|\eta_i\| + \|u_i\| + \|f_p v_i\| + \gamma_{\max} + \|\ddot{p}_0\|) \right) \quad (4.17)
\end{aligned}$$

with $\|e_i(0)\| = 0$. When $t = t_{k+1}^i$, based on (4.7) and (4.8), one has $\|e_i(t_{k+1}^i)\| > \sqrt{\frac{\chi_i(t_{k+1}^i)}{\iota_i}}$, which together with (4.9) shows

$$\sqrt{\frac{\chi_i(0) e^{\int_0^{t_{k+1}^i} h_i(\tau) d\tau}}{\iota_i}} < \sqrt{\frac{\chi_i(t_{k+1}^i)}{\iota_i}} < \|e_i(t_{k+1}^i)\| \leq \tau_k^i \sigma_i.$$

The above inequality implies that $\tau_k^i > \frac{\sqrt{\frac{1}{\iota_i} \chi_i(0) e^{\int_0^{t_k^i+1} h_i(\tau) d\tau}}}{\sigma_i} > 0$ for any finite horizon, which signifies that Zeno behavior can be effectively avoided. The proof is completed.

4.3 Numerical Simulations

In this section, the validity of the proposed distributed ADETFC algorithm for the leader-follower multi-quadrotor system is presented by simulation. The undirected communication topology is shown in Fig. 4.2, where node 0 stands for the virtual leader and 1-3 are the followers. The multi-quadrotor system is modelled as (4.1) with $m_i = 1.121$ kg, $g = 9.8$ m/s² and $K_1 = K_2 = K_3 = 0.01$ N·s/m. The external disturbances are expressed as $d_i = 0.2 \sin(t)$ m/s².

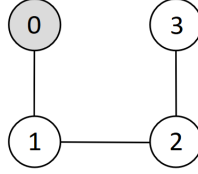


Figure 4.2: Topology of the communication graph.

The reference trajectory provided by the leader is $p_0 = [5 \sin(0.5t), 5 \cos(0.5t), 0.25t]^T$ m and $\dot{p}_0 = [2.5 \cos(0.5t), -2.5 \sin(0.5t), 0.25]^T$ m/s. The formation pattern is given as $\delta_1 = [0, -1, 0]^T$ m, $\delta_2 = [1, -1, 0]^T$ m and $\delta_3 = [1, 0, 0]^T$ m. In addition, the parameters φ_1, φ_2 and φ_3 are selected as $\varphi_1 = \varphi_2 = \varphi_3 = 1$. For the proposed ADETFC algorithm (4.4) and the triggering function (4.8), we set $\alpha_1 = 1.5$, $\alpha_2 = 1.2$, $\rho = 0.1$, $\epsilon_i = 0.1$, $k_{1i} = 10.5$, $k_{2i} = 10$, $\mathcal{L} = 3$, $\iota_i = 0.01$, $\varepsilon_i = 0.1$, and $\mu_i = 0.4$. The initial conditions for the following quadrotors are set as $p_1(0) = [1, 0, 1.25]^T$ m, $p_2(0) = [3, 2, 1.4]^T$ m, $p_3(0) = [1, 2.5, 1.3]^T$ m, and $v_i(0) = 0_{3 \times 1}$ m/s. The initial values for the dynamic variable

χ_i and estimates $\hat{\gamma}_i$ are given as $\chi_i(0) = 3$ and $\hat{\gamma}_i(0) = 0$, respectively, $i \in \{1, 2, \dots, N\}$.

In order to verify the effectiveness and the advantage of the proposed ADETFC strategy, comparisons with different triggering mechanisms and formation control protocols are conducted. Firstly, the following three different triggering mechanisms are studied:

- (i) *Periodic time-triggered mechanism (PTTM)*. The sampled-data packet on quadrotor i is used to update the controller in a periodic fashion. This case is realized by setting $k_{1i} = 0$, $k_{2i} = 0$ and $\chi_i(t) = 0$ in (4.7) and (4.8).
- (ii) *Static event-triggered mechanism (SETM)*. The controller of quadrotor i is updated in an event-triggered manner, and the triggering threshold is fixed permanently. This case can be achieved by setting $\chi_i(t) = 0$ in (4.7).
- (iii) *Dynamic event-triggered mechanism (DETM)*. The event for quadrotor i is triggered in response to a dynamic threshold parameter $\chi_i(t)$. The parameters of triggering functions (4.7)-(4.7) are set as the initialization.

During the simulation time $[0, 20]$ s, there are a total of 1500 sampled-data packets. Fig. 4.3 shows the event release instants for each quadrotor under dynamic and static event-triggered methods, respectively. Table 4.1 provides a quantitatively comparison of how many times the control commands are updated on each quadrotor under different triggering mechanisms.

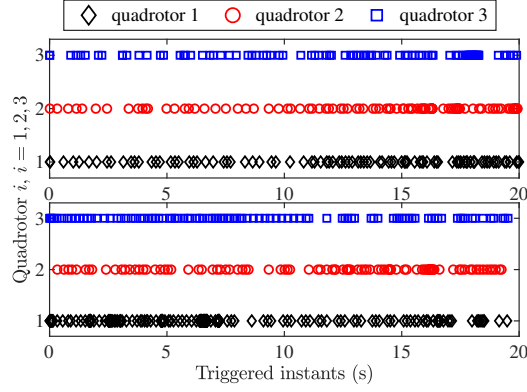


Figure 4.3: Top: triggering instants under dynamic ETC method. Bottom: triggering instants under static ETC method.

Table 4.1: Event Release Numbers of Simulation Results

Quadrotor No.	1	2	3
PTTM	1500	1500	1500
SETM	172	111	116
DETM	90	93	81

Then, the following three different formation control protocols are studied:

- (i) *ADETFC*. The proposed adaptive SMC strategy (4.4) with the designed dynamic event-triggered condition is implemented to achieve the formation for the multi-quadrotor system under external perturbations.
- (ii) *Adaptive static event-triggered formation control (ASETFC)*. In this case, the developed adaptive SMC protocol with the static event-triggered function is applied to perform the formation task for the multi-quadrotor system under external disturbances.
- (iii) *Controller proposed in [1]*. The formation control scheme proposed in [1] is implemented for the multi-quadrotor system, and same parameters for quadrotors and

external disturbances are set.

The formation tracking performance of the leader-follower multi-quadrotor system with external disturbances in three dimensions under ADETFC are demonstrated in Fig. 4.4. The formation tracking errors under ADETFC are depicted in Figs. 4.5(a) and (b). Moreover, comparisons are conducted by using ASETFC and the formation control scheme proposed in [1], and the performances are shown in Figs. 4.5(c) and (d), and Figs. 4.5(e) and (f), respectively. In a meaningful way to discuss the influences of different control algorithms for each quadrotor, some performance comparisons, such as mean squared error (MSE), mean absolute error (MAE), and steady-state error (SSE), are provided in Table 4.2.

Based on the aforementioned simulation results, one can obtain the following.

- (i) The DETM has larger triggering intervals and can always lead to the least sampling frequency than both the PTTM and the SETM, thus having more potential to reduce resource consumption. In comparison with the PTTM which requires 100% sampled-data packets, the proposed DETM only needs about 5.9% in average for each quadrotor to complete the formation tasks, while the SETM needs about 8.9%.
- (ii) The designed ADETFC strategy achieves a better formation performance than the controller developed in [1] in terms of MSE, MAE, and SSE for the multi-quadrotor system with external disturbances, which validates the effectiveness of the control protocol proposed in this chapter. Moreover, the ADETFC with less sampled-data packets achieves similar formation performance as the ASETFC, which verifies the advantages of reducing unnecessary triggers. In the steady state, compared with the ADETFC, the larger SSE under the ASETFC may arise from not triggering some data packets, which contains useful information to achieve the desired formation.

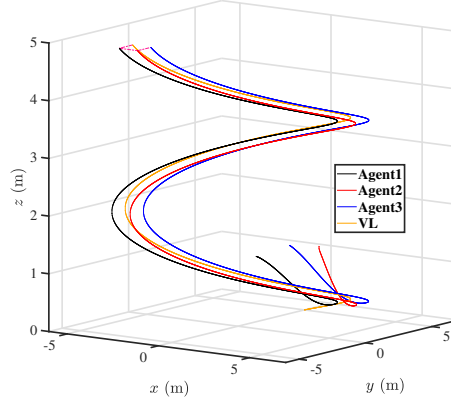
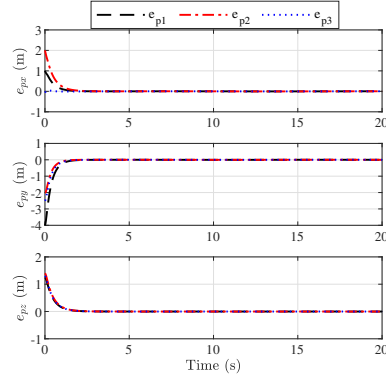


Figure 4.4: Simulated formation trajectories under ADETFC.

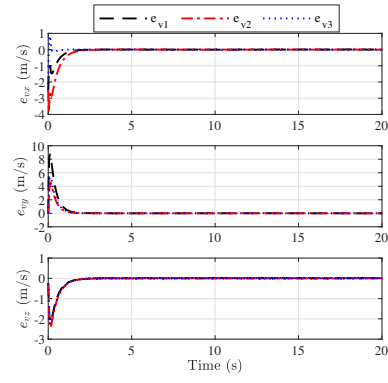
Table 4.2: Tracking Comparisons of Simulation Results

	ADETFC		ASETFC		Controller in [1]	
	Position (m)	Velocity (m/s)	Position (m)	Velocity (m/s)	Position (m)	Velocity (m/s)
MSE of quadrotor 1	0.2656	0.9299	0.338	0.8176	1.8369	1.7136
MSE of quadrotor 2	0.16	0.7145	0.2548	0.6785	1.0218	1.0517
MSE of quadrotor 3	0.1193	0.6229	0.1942	0.4799	0.8358	0.7628
MAE of quadrotor 1	0.1765	0.3986	0.2701	0.4614	0.9972	0.9301
MAE of quadrotor 2	0.1639	0.3483	0.2458	0.4019	0.8839	0.8206
MAE of quadrotor 3	0.1111	0.2591	0.1683	0.2994	0.6668	0.6233
SSE of quadrotor 1	0.0062	0.0061	0.0067	0.0072	0.102	0.0615
SSE of quadrotor 2	0.0072	0.0081	0.0075	0.0085	0.1112	0.0727
SSE of quadrotor 3	0.0059	0.0066	0.0073	0.0071	0.1007	0.0588

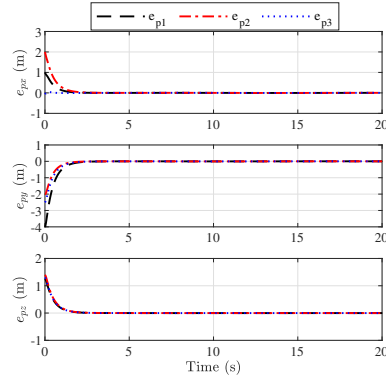
Therefore, it can be concluded that the proposed distributed adaptive dynamic event-triggered SMC strategy can not only guarantee the formation tracking performance, but improve the resource utilization and reduce the actuator wear effectively.



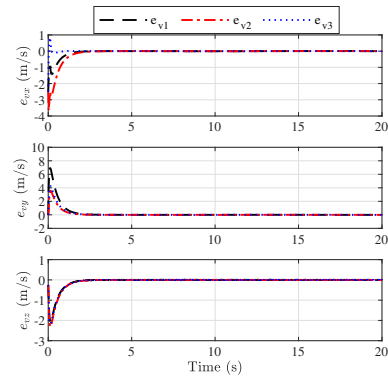
(a) Position errors.



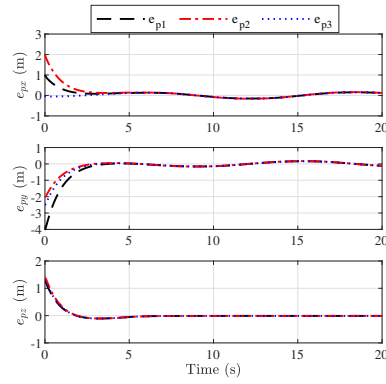
(b) Velocity errors.



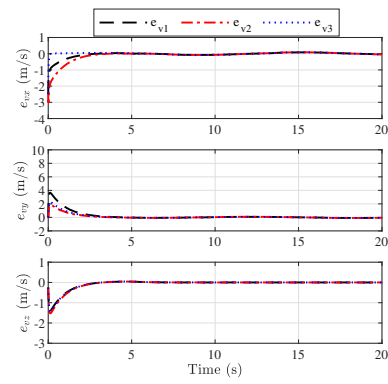
(c) Position errors.



(d) Velocity errors.



(e) Position errors.



(f) Velocity errors.

Figure 4.5: Simulation results of tracking errors: ADETFC (a, b); ASETFC (c, d); controller in [1] (e, f).

4.4 Experimental Studies

In this section, hardware experiments are conducted to validate the advantages and effectiveness of the proposed control law using the experimental system shown in Fig. 4.6.

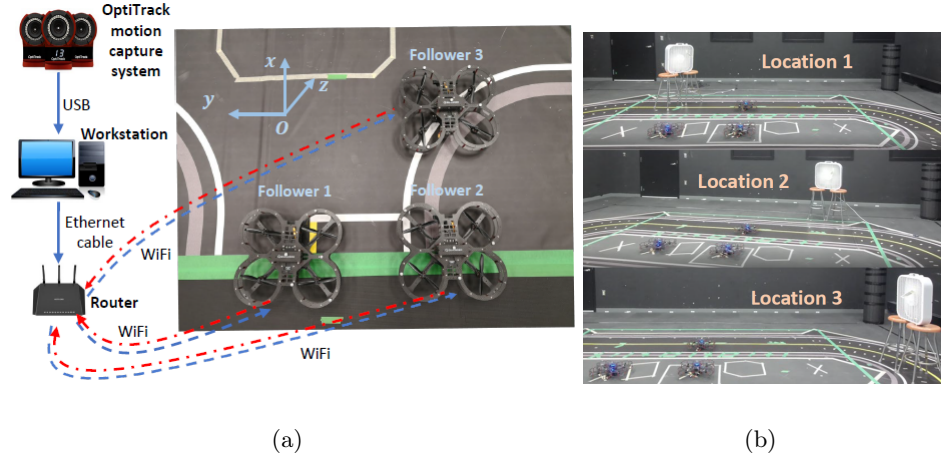


Figure 4.6: Experimental system: multi-quadrotor system (a); setup of fan (b).

The same communication graph and parameters are set as in Sec. 4.3, and the formation geometry is selected as $\delta_1 = [-0.9, 0, 0]^T$ m, $\delta_2 = [-0.9, -0.9, 0]^T$ m, and $\delta_3 = [0, -0.9, 0]^T$ m. The desired trajectories are chosen as $p_0 = [0.9 \sin(0.5t) + 0.9, 0.9 \cos(0.5t), 0.08t + 0.6]^T$ m and $\dot{p}_0 = [0.45 \cos(0.5t), -0.45 \sin(0.5t), 0.08]^T$ m/s. However, due to the limitation of the flight space, we set $x_0 = 1.5$ m when $x_0 > 1.5$ m. The initial values of the following quadrotors are set as $p_1(0) = [0, 1, 0.6]^T$ m, $p_2(0) = [0, 0, 0.6]^T$ m, $p_3(0) = [1, 0, 0.6]^T$ m, and $v_i(0) = 0_{3 \times 1}$ m/s, $i \in \{1, 2, \dots, N\}$. Quadrotors firstly took off and hovered at the initial position, then started to track the trajectory and the formation pattern at $t = 0$ s. The experimental video can be found at <https://youtu.be/QaTFEETZo3o>.

To verify the capability of our proposed controller to handle unknown perturbations, sufficient experiments are conducted with different unknown perturbations, triggering

mechanisms and formation control protocols. Firstly, the ADETFC protocol is tested for the multi-quadrotor system under various unknown perturbations by placing the fan at three different locations, as shown in Fig. 4.6(b), and choosing three different fan speeds: L1, L2 and L3. The position and velocity tracking performance with different fan locations and speeds are shown in Table 4.3 and Table 4.4, respectively. It can be observed that the tracking performances of the followers with the proposed ADETFC protocol are all satisfied even with different disturbance locations and levels. In terms of MSE, MAE, and SSE, the maximum position (or velocity) tracking errors under perturbations are less than 6.5 cm (or 5.5 cm/s), 10.5 cm (or 10 cm/s), and 3.7 cm (or 6.5 cm/s), respectively, which are acceptable and satisfied. Therefore, it can be concluded that the designed ADETFC strategy can handle the unknown perturbation very well.

Table 4.3: Position Tracking Comparisons of Experimental Results for Different Unknown Perturbations

	Location 1			Location 2			Location 3		
	L1	L2	L3	L1	L2	L3	L1	L2	L3
MSE of quadrotor 1 (m)	0.028	0.0636	0.0306	0.054	0.0366	0.027	0.0583	0.0254	0.0524
MSE of quadrotor 2 (m)	0.0146	0.0629	0.0532	0.0501	0.0103	0.0149	0.0123	0.0296	0.0226
MSE of quadrotor 3 (m)	0.0262	0.0199	0.0328	0.0448	0.0112	0.0208	0.0198	0.0113	0.0158
MAE of quadrotor 1 (m)	0.0736	0.1023	0.0716	0.0917	0.0788	0.0691	0.0979	0.0668	0.0941
MAE of quadrotor 2 (m)	0.0468	0.1033	0.0913	0.0708	0.0408	0.0516	0.0457	0.0785	0.0562
MAE of quadrotor 3 (m)	0.0649	0.0524	0.0739	0.0613	0.0446	0.0565	0.0566	0.0408	0.0544
SSE of quadrotor 1 (m)	0.0265	0.027	0.037	0.0225	0.0283	0.0236	0.0305	0.026	0.0315
SSE of quadrotor 2 (m)	0.0283	0.0351	0.0291	0.03	0.0252	0.0264	0.0305	0.0335	0.0346
SSE of quadrotor 3 (m)	0.0272	0.017	0.0298	0.0259	0.0144	0.021	0.028	0.0367	0.0355

Table 4.4: Velocity Tracking Comparisons of Experimental Results for Different Unknown Perturbations

	Location 1			Location 2			Location 3		
	L1	L2	L3	L1	L2	L3	L1	L2	L3
MSE of quadrotor 1 (m/s)	0.0111	0.0359	0.0432	0.006	0.007	0.02	0.0182	0.0199	0.028
MSE of quadrotor 2 (m/s)	0.0411	0.0464	0.0312	0.0283	0.0188	0.0479	0.0252	0.027	0.0234
MSE of quadrotor 3 (m/s)	0.0414	0.044	0.0518	0.0333	0.023	0.0414	0.0521	0.0141	0.0377
MAE of quadrotor 1 (m/s)	0.0435	0.0772	0.084	0.0305	0.035	0.0519	0.0569	0.0573	0.0612
MAE of quadrotor 2 (m/s)	0.08	0.0971	0.0744	0.0904	0.059	0.0911	0.0664	0.0688	0.06
MAE of quadrotor 3 (m/s)	0.0779	0.0832	0.092	0.0933	0.0627	0.0856	0.0921	0.0485	0.072
SSE of quadrotor 1 (m/s)	0.0446	0.0367	0.036	0.04	0.0381	0.0216	0.065	0.0277	0.0461
SSE of quadrotor 2 (m/s)	0.0321	0.0309	0.0061	0.0627	0.0604	0.0453	0.063	0.0415	0.06
SSE of quadrotor 3 (m/s)	0.0252	0.0605	0.029	0.0338	0.0607	0.0409	0.0085	0.0629	0.0633

Then, three different control strategies under perturbations with same location (location 3) and same fan speed (L3) are studied. The experimental formation performance in three dimensions under ADETFC is shown in Fig. 4.7, and the tracking errors are depicted in Figs. 4.8(a) and (b). It can be found that the formation geometry is maintained while tracking the desired trajectory with a satisfactory performance. For comparisons, the ASETFC strategy and the controller in [1] are implemented, and the results are shown in Figs. 4.8(c) and (d), and Figs. 4.8(e) and (f), respectively. The tracking performance under different control strategies are shown in Table 4.5, where it can be observed that the proposed ADETFC framework in this chapter has the better tracking performance for the multi-quadrotor system. Moreover, the comparison of event release numbers on each follower under different triggering mechanism is shown in Table 4.6, where it can be found that the proposed DETM requires the least data packets during the experimental period (15 s). Based on experimental results, it can be concluded that the designed ADETFC strategy can not only reduce the resource consumption and the actuator wear, but improve the tracking performance of the multi-quadrotor system with unknown perturbations, and thus the advantage and the effectiveness of the developed ADETFC are validated.

Table 4.5: Tracking Comparisons of Experimental Results for Different Control Strategies

	ADETFC		ASETFC		Controller in [1]	
	Position (m)	Velocity (m/s)	Position (m)	Velocity (m/s)	Position (m)	Velocity (m/s)
MSE of quadrotor 1	0.0353	0.0162	0.0492	0.051	0.423	0.0373
MSE of quadrotor 2	0.034	0.056	0.1021	0.0394	0.134	0.0922
MSE of quadrotor 3	0.0952	0.0287	0.0585	0.0891	0.1363	0.1233
MAE of quadrotor 1	0.0728	0.0557	0.0863	0.0864	0.2825	0.0766
MAE of quadrotor 2	0.0719	0.0927	0.1304	0.0798	0.1614	0.1222
MAE of quadrotor 3	0.1256	0.069	0.0996	0.1176	0.1557	0.14
SSE of quadrotor 1	0.033	0.0308	0.0397	0.0376	0.0622	0.0742
SSE of quadrotor 2	0.0315	0.0632	0.0373	0.0431	0.0534	0.0607
SSE of quadrotor 3	0.0298	0.0351	0.0354	0.0512	0.0588	0.082

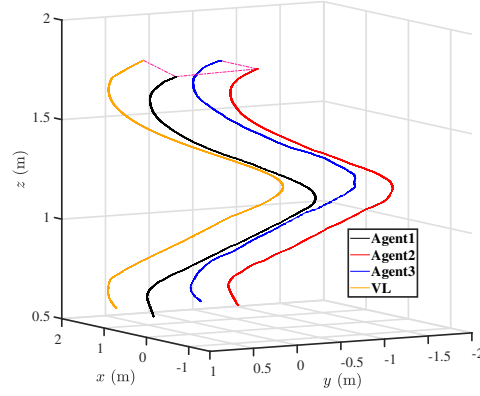
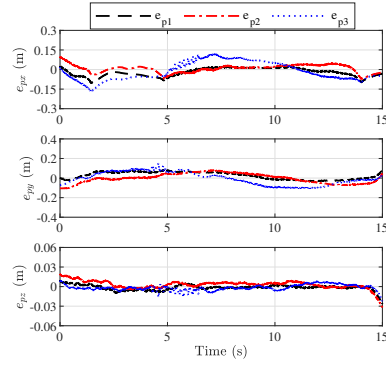


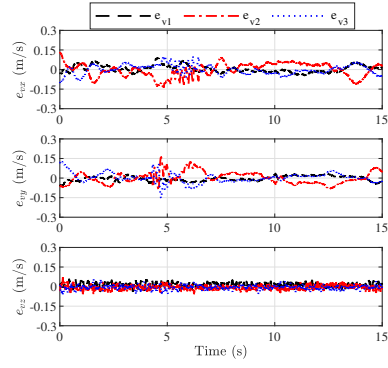
Figure 4.7: Experimental formation trajectories under ADETFC.

Table 4.6: Event Release Numbers of Experimental Results

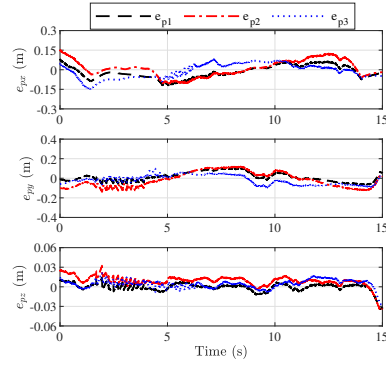
Quadrotor No.	1	2	3
Time-triggered Control	1500	1500	1500
Static ETC	605	577	652
Dynamic ETC	392	420	498



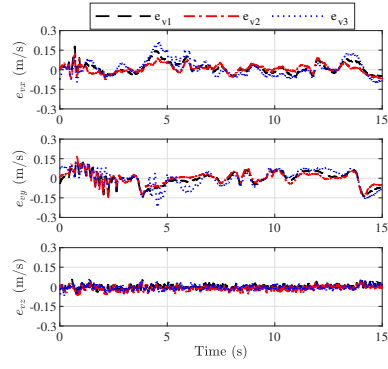
(a) Position errors.



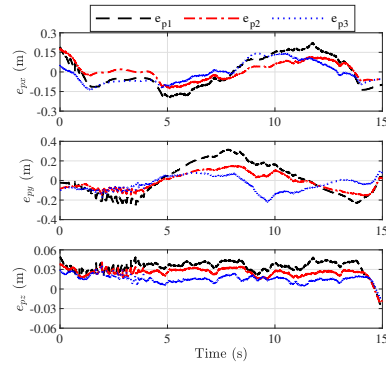
(b) Velocity errors.



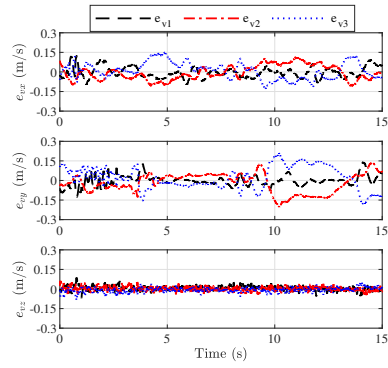
(c) Position errors.



(d) Velocity errors.



(e) Position errors.



(f) Velocity errors.

Figure 4.8: Experimental results of tracking errors: ADETFC (a, b); ASETFC (c, d); controller in [1] (e, f).

4.5 Summary

In this chapter, the formation control problem of the leader–follower multi-quadrotor system subject to unknown external disturbances and limited resources has been studied. A new distributed ISM has been devised such that asymptotic convergence can be guaranteed for the states of the sliding-mode dynamics. In addition, a novel distributed adaptive dynamic event-triggered formation control framework has been proposed by utilizing the ISM-based SMC strategy to cope with unknown perturbations and limited resources. The actuator wear and the unnecessary energy consumption have been efficaciously decreased by the developed dynamic triggering condition. Moreover, the triggering interval can be adjusted dynamically, and the Zeno behavior has been ruled out efficiently. The validity of the proposed distributed ISM-based ADETFC algorithm has been verified by means of simulations and experiments. Compared to the static ETC protocol and the time-triggered control scheme in [1], the developed fully distributed dynamic ETC scheme achieves better tracking performance with less sampling frequency for quadrotors under unknown perturbations.

However, it is noted that the nonlinearities and parameter uncertainties of quadrotors, which will significantly affect the system performance, are not considered in this chapter. Besides, in this chapter, it is assumed that the state measurements of the leader are accurate, which is unrealistic in practical applications. Moreover, obtaining relative velocity measurements among neighboring agents is generally difficult in practice. Thus, future work will focus on designing an event-based robust control law for quadrotors under inaccurate measurements without relative velocity measurements.

5 Adaptive Dynamic Event-Based Robust Control

The leader's information is important for the controller design of leader-follower MASs, but the measurement is often noisy in practice. Some existing works designed the observers to estimate the leader's states by using the upper bounds of the leaders' state derivatives, which are global information and thus they are not fully distributed control. Besides, as a remarkable fact, obtaining relative velocity measurements among neighboring agents is generally more challenging than obtaining relative position measurements in practice. Consequently, this chapter develops an event-based adaptive robust control scheme for multiple networked Euler-Lagrange (EL) systems with a dynamic leader, addressing some key challenges such as parameter uncertainties, unknown perturbations, inherent nonlinearities, and limited resources, for the practical applications of networked robotics and autonomous systems. To reduce the communication network burden and the computational resources consumption, an adaptive dynamic triggering strategy is developed. Additionally, to estimate the inaccurate states, a nested adaptive sliding-mode estimator is proposed. Then, a fully distributed adaptive dynamic event-based time-varying sliding-mode control strategy is developed based on the designed triggering scheme and estimator, without requiring any global information. This strategy reduces the effect of large initial errors on the varying gain during adaptation, and compensates for the influences of inher-

ent nonlinearities, unknown external perturbations, and parameter uncertainties, making it feasible for practical implementation. Moreover, Lyapunov stability theory is used to guarantee the asymptotic convergence of the closed-loop networked systems. Finally, hardware experiments are conducted using multiple quadrotors to validate the effectiveness of the proposed control scheme in multi-agent coordination tasks.

5.1 Problem Statement

This chapter studies multiple networked EL systems including a leader denoted by 0 and N followers. A directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is presented to describe the communication graph, where $\mathcal{V} = \{\nu_0, \nu_1, \dots, \nu_N\}$. Let L represent the Laplacian matrix. Suppose that the leader has no neighbors. This means that the information from any of the agents will not be received by the leader. Then, the Laplacian matrix L is divided into

$$L = \begin{bmatrix} 0 & \mathbf{0}_N^T \\ L_2 & L_1 \end{bmatrix} \quad (5.1)$$

with $L_1 \in \mathbb{R}^{N \times N}$ and $L_2 \in \mathbb{R}^{N \times 1}$.

The dynamics of systems are formulated as

$$M_i(q_i) \ddot{q}_i + C_i(q_i, \dot{q}_i) \dot{q}_i + g_i(q_i) = \tau_i + d_i, \quad i = 1, \dots, N, \quad (5.2)$$

where $q_i, \dot{q}_i, \ddot{q}_i \in \mathbb{R}^{p \times 1}$ represent the vectors of generalized position, velocity, and acceleration. $M_i(q_i) \in \mathbb{R}^{p \times p}$ represents the inertia matrix, which is symmetric positive definite. $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{p \times p}$ denotes the Coriolis and centrifugal matrix, while $g_i(q_i) \in \mathbb{R}^{p \times 1}$ signifies the vector of gravitational torque. τ_i denotes the control input, and d_i is the perturbation. Suppose that an unknown constant ϕ_i exists satisfying $\|d_i\|_1 \leq \phi_i$. Then, some reasonable properties are provided for multiple networked EL systems as

Property 1. $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew-symmetric.

Property 2. For any vectors $x, y \in \mathbb{R}^{p \times 1}$, there is a regressor $Y_i(q_i, \dot{q}_i, x, y)$ such that $M_i(q_i) \ddot{q}_i + C_i(q_i, \dot{q}_i) \dot{q}_i + g_i(q_i) = Y_i(q_i, \dot{q}_i, x, y) \theta_i$ with the unknown vector θ_i representing the parameter uncertainties.

Property 3. For all $q_i \in \mathbb{R}^p$, positive scalars ϑ_M , ϑ_C , and ϑ_g exist satisfying $\|M_i(q_i)\|_2 \leq \vartheta_M$, $\|C_i(q_i, \dot{q}_i)\|_2 \leq \vartheta_C \|q_i\|_2 \|\dot{q}_i\|_2$, and $\|g_i(q_i)\|_2 \leq \vartheta_g$.

This chapter attempts to handle the fully distributed event-based consensus tracking control issue for networked EL systems with a dynamic leader over directed graph. The position tracking error of agent i , $i = 1, \dots, N$, is defined as $\tilde{q}_i \triangleq q_i - q_0$, while the velocity tracking error is $\dot{\tilde{q}}_i \triangleq \dot{q}_i - \dot{q}_0$, where q_0 and $\dot{q}_0 \in \mathbb{R}^p$ are the leader's position and velocity, respectively. By designing an adaptive dynamic event-triggered robust control scheme, the communication and computational resources consumption can be decreased, and the tracking errors $\tilde{q}_i$ and $\dot{\tilde{q}}_i$ will converge to zero as $t \rightarrow +\infty$.

Assumption 7. A directed path to each follower exists in the topology $\mathcal{G}$ for the leader. Moreover, the graph among the followers is bidirectional.

Assumption 8. The leader's velocity $\dot{q}_0$, acceleration $\ddot{q}_0$, and the third-order derivative of generalized position $\dddot{q}_0$ are bounded, where the boundaries are unavailable for followers.

Lemma 4 ([103]). If Assumption 7 holds, one has that L_1 is symmetric positive definite.

It is noted that for a function $f(t)$, $f(t) \in \mathbb{L}_p$ if $\left(\int_0^\infty |f(\tau)|^p d\tau\right)^{1/p}$ exists for $p \in [1, \infty)$, and $f(t) \in \mathbb{L}_\infty$ if $\sup_{t \geq 0} |f(t)|$ exists.

Lemma 5 ([104]). For a differentiable function $g(t)$, if it satisfies $g(t), \dot{g}(t) \in \mathbb{L}_\infty$, and $g(t) \in \mathbb{L}_p$ for $p \in [1, \infty)$, we can obtain $g(t) \rightarrow 0$ as $t \rightarrow \infty$.

Lemma 6 ([96]). For a continuously differentiable function $g(x, u, t)$, which is globally Lipschitz in (x, u) , uniformly in t , consider the system $\dot{x} = g(x, u, t)$. If there is a globally

exponentially stable equilibrium point at the origin for the unforced system $\dot{x} = g(x, 0, t)$, the original system is input-to-state stable (ISS).

Firstly, two auxiliary variables are defined as

$$\hat{q}_{ri} \triangleq \hat{v}_i - \eta \sum_{j=0}^N a_{ij} (q_i - q_j) - f_i \pi_i, \quad (5.3)$$

$$\hat{\ddot{q}}_{ri} \triangleq \hat{a}_i - \eta \sum_{j=0}^N a_{ij} (\dot{q}_i - \dot{q}_j) - \dot{f}_i \pi_i, \quad (5.4)$$

where $\hat{v}_i$ and $\hat{a}_i$ are the follower i 's estimations of the velocity and acceleration of the dynamic leader. $f_i \triangleq e^{-\kappa_i t}$ with $\kappa_i > 0$ and $\pi_i \triangleq -\dot{q}_i(0) + \hat{v}_i(0) - \eta \sum_{j=0}^N a_{ij} (q_i(0) - q_j(0))$.

Then, the sliding manifold is developed as

$$S_i(t) \triangleq \dot{q}_i - \hat{q}_{ri} \triangleq \dot{q}_i - \hat{v}_i + \eta \sum_{j=0}^N a_{ij} (q_i - q_j) + f_i \pi_i. \quad (5.5)$$

Next, define $\xi_{1i} \triangleq \sum_{j=1}^N a_{ij} (\hat{v}_i - \hat{v}_j) + a_{i0} (\hat{v}_i - \dot{q}_0)$ and $\xi_{2i} \triangleq \sum_{j=1}^N a_{ij} (\hat{a}_i - \hat{a}_j) + a_{i0} (\hat{a}_i - \ddot{q}_0)$. Considering the imprecise measurement of the system states, the following estimators are proposed as

$$\dot{\hat{v}}_i = -\hat{\beta}_{1i} \text{sign}(\xi_{1i}), \quad \dot{\hat{\beta}}_{1i} = \epsilon_{1i} \|\xi_{1i}\|_1, \quad (5.6)$$

$$\dot{\hat{a}}_i = -\hat{\beta}_{2i} \text{sign}(\xi_{2i}), \quad \dot{\hat{\beta}}_{2i} = \epsilon_{2i} \|\xi_{2i}\|_1, \quad (5.7)$$

where ϵ_{1i} and ϵ_{2i} are positive scalars.

The event detector is added to reduce the communication burden and the controller update frequency between the sensor and the controller. The triggering instant is defined as the monotone increasing sequence $\{t_k^i\}_{k=0}^\infty$ with $t_0^i = 0$, where k is non-negative integer. For $t \in [t_k^i, t_{k+1}^i)$, the control law is designed as

$$\begin{aligned} \tau_i &= -K_i S_i(t_k^i) + Y_i(t_k^i) \hat{\theta}_i(t_k^i) - \hat{\phi}_i(t_k^i) \text{sign}\left(S_i(t_k^i)\right), \\ \dot{\hat{\theta}}_i &= -\Lambda_i Y_i(t) S_i(t), \\ \dot{\hat{\phi}}_i &= \epsilon_i \|S_i(t)\|_1, \end{aligned} \quad (5.8)$$

where $K_i > 0$ is the gain of the controller, $\Lambda_i > 0$, and $\epsilon_i > 0$. According to the developed sliding manifold and controller, some measurement errors are defined as

$$\begin{aligned} e_i &= K_i S_i \left(t_k^i \right) - K_i S_i(t), \\ \varepsilon_i &= Y_i \left(t_k^i \right) \hat{\theta}_i \left(t_k^i \right) - Y_i(t) \hat{\theta}_i(t), \\ \sigma_i &= \hat{\phi}_i \left(t_k^i \right) \text{sign} \left(S_i \left(t_k^i \right) \right) - \hat{\phi}_i(t) \text{sign} \left(S_i(t) \right). \end{aligned} \quad (5.9)$$

Remark 14. *It can be seen that the varying gain $\hat{\phi}_i$ is related to the norm of $S_i(t)$, and it may increase rapidly due to a large $\|S_i(0)\|_1$, which could lead to large control signals. In such cases, the system is more susceptible to saturation, which will degrade the tracking performance. TVSMC has been employed to remove the reaching phase of the SMC by introducing a time-varying sliding manifold instead of a time-invariant one [78, 105]. With this approach, the sliding surface passes through the system's initial state at the beginning of the motion and moves toward a predefined time-invariant sliding manifold via rotating and/or shifting. Thus, the initial trajectory of the closed-loop system only includes the sliding phase, effectively eliminating the reaching phase and mitigating the influence of initial errors on control performance. However, TVSMC requires prior knowledge of the upper bound of external disturbances, which is not feasible in practice. Hence, a novel adaptive TVSMC scheme is proposed in (5.8) for EL systems by means of the time-varying sliding manifold (5.5), where the adaptive gain $\hat{\phi}_i$ is induced by the external disturbances. It relaxes the assumption that the upper bound of the external disturbances is known. Although the reaching phase cannot be eliminated by applying the adaptive TVSMC, it can be significantly shortened. The effect of initial errors on gain adaptation in the reaching phase will be greatly reduced, ensuring better control performance.*

Then, the triggering function is developed as

$$\Delta_i = \psi_i \left(\|\varepsilon_i(t)\|_2^2 + \|e_i(t)\|_2^2 + \|\sigma_i(t)\|_2^2 - \hbar_i \|S_i\|_2^2 \right), \quad (5.10)$$

where ψ_i and $\hbar_i$ are positive scalars. To determine the triggering instants, the dynamic event-triggered (ET) condition is formulated as

$$t_{k+1}^i = \inf_{l > t_k^i} \left\{ l : \Delta_i(t) > \delta_i(t), \forall t \in [t_k^i, l) \right\}, \quad (5.11)$$

where the dynamic variable $\delta_i(t)$ is updated based on

$$\dot{\delta}_i(t) = -\chi_{1i}\delta_i(t) - \frac{\chi_{2i}}{\psi_i}\Delta_i(t) \quad (5.12)$$

with $\chi_{1i} > 0$, $\chi_{2i} > 0$, and $\delta_i(0) > 0$. Based on (5.10)-(5.12), one has that $\dot{\delta}_i(t) \geq -\left(\chi_{1i} + \frac{\chi_{2i}}{\psi_i}\right)\delta_i(t)$. Thus, we can derive that $\delta_i(t) \geq \delta_i(0)e^{-(\chi_{1i} + \frac{\chi_{2i}}{\psi_i})t} > 0$.

For convenience of analysis, we define $\tilde{\beta}_{1i} = \beta_{1i} - \hat{\beta}_{1i}$, $\tilde{\beta}_{2i} = \beta_{2i} - \hat{\beta}_{2i}$, $\tilde{v}_i = \hat{v}_i - \dot{q}_0$, $\tilde{a}_i = \hat{a}_i - \ddot{q}_0$, $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, and $\tilde{\phi}_i = \phi_i - \hat{\phi}_i$. Based on Property 2, we have

$$M_i(q_i)\ddot{\hat{q}}_{ri} + C_i(q_i, \dot{q}_i)\dot{\hat{q}}_{ri} + g_i(q_i) = Y_i(q_i, \dot{q}_i, \ddot{\hat{q}}_{ri}, \dot{\hat{q}}_{ri})\theta_i. \quad (5.13)$$

Substituting (5.5), (5.8), and (5.13) into (5.2), one has that

$$\begin{aligned} M(q)\dot{S}(t) = & -C(q, \dot{q})S(t) - KS(t) - Y(t)\tilde{\theta}(t) - e(t) \\ & + \varepsilon(t) - \sigma(t) + d(t) - \hat{\phi}(t)\text{sign}(S(t)), \end{aligned} \quad (5.14)$$

where $M(q) = \text{diag}\{M_1(q_1), M_2(q_2), \dots, M_N(q_N)\}$, $S(t) = [S_1^T(t), S_2^T(t), \dots, S_N^T(t)]^T$, $C(q, \dot{q}) = \text{diag}\{C_1(q_1, \dot{q}_1), C_2(q_2, \dot{q}_2), \dots, C_N(q_N, \dot{q}_N)\}$, $K = \text{diag}\{K_1, K_2, \dots, K_N\} \otimes I_p$, $Y(t) = \text{diag}\{Y_1(t), Y_2(t), \dots, Y_N(t)\}$, $e(t) = [e_1^T(t), e_2^T(t), \dots, e_N^T(t)]^T$, $\tilde{\theta}(t) = [\tilde{\theta}_1^T(t), \tilde{\theta}_2^T(t), \dots, \tilde{\theta}_N^T(t)]^T$, $\sigma(t) = [\sigma_1^T(t), \sigma_2^T(t), \dots, \sigma_N^T(t)]^T$, $d(t) = [d_1^T(t), d_2^T(t), \dots, d_N^T(t)]^T$, and $\hat{\phi}(t) = \text{diag}\{\hat{\phi}_1(t), \hat{\phi}_2(t), \dots, \hat{\phi}_N(t)\} \otimes I_p$.

Remark 15. The fully distributed adaptive dynamic triggering mechanism is derived in (5.10)-(5.12) without the relative velocity measurements. To dynamically adjust the threshold of the triggering function (5.10), the dynamic variable $\delta_i(t)$ is introduced with the update law (5.12). If (5.10), which includes the three measurement errors defined in (5.9),

is greater than the dynamic variable at the time instant t_{k+1}^i , the information collected by the sensor is transmitted to the controller via a communication network, and the developed event-based adaptive TVSMC protocol in (5.8) is updated for each agent. Notably, the dynamic auxiliary variable $\delta_i(t)$ in (5.12) helps extend inter-event intervals and reduce triggering times by tuning parameters ψ_i , $\bar{h}_i$, χ_{1i} and χ_{2i} .

Remark 16. In general, for EL systems to achieve coordination, each agent requires both position and velocity information from its neighbors. However, obtaining relative velocity measurements is challenging in practice. To address this problem, an alternative approach is to utilize the dynamic leader's information for cooperative control. However, since the leader's information serves as global information for nodes that are not directly connected to the leader, it becomes inaccessible to them. Therefore, we designed adaptive sliding-mode estimators for each follower to estimate the dynamic leader's information. From the designed controller scheme (5.5)-(5.8) and the ET strategy (5.10)-(5.12), it can be seen that only local information is required, ensuring the fully distributed nature of the event-based control scheme.

5.2 Main Results

5.2.1 System Analysis

Next, the tracking control issue is studied for the system with the proposed controller.

Theorem 7. Under the developed adaptive dynamic ETC protocol (5.8) with the time-varying adaptive sliding-mode estimator (5.6)-(5.7), if Assumptions 7 and 8 hold, the tracking errors of the networked EL systems with parameter uncertainties, unknown perturbations, and inaccurate measurements can achieve the asymptotic convergence.

Proof: For all $t \in [t_k^i, t_{k+1}^i)$, the Lyapunov function candidate is selected as

$$\begin{aligned} V_1(t) = & \frac{1}{2} S^T(t) M(q) S(t) + \frac{1}{2} \tilde{\theta}^T(t) \Lambda^{-1} \tilde{\theta}(t) \\ & + \frac{1}{2} \tilde{v}(t) (L_1 \otimes I_p) \tilde{v}(t) + \frac{1}{2} \tilde{a}(t) (L_1 \otimes I_p) \tilde{a}(t) \\ & + \sum_{i=1}^N \frac{1}{2\epsilon_{1i}} \tilde{\beta}_{1i}^2(t) + \sum_{i=1}^N \frac{1}{2\epsilon_{2i}} \tilde{\beta}_{2i}^2(t) + \sum_{i=1}^N \frac{1}{2\epsilon_i} \tilde{\phi}_i^2(t), \end{aligned} \quad (5.15)$$

where $\tilde{v}(t) = [\tilde{v}_1^T(t), \tilde{v}_2^T(t), \dots, \tilde{v}_N^T(t)]^T$, $\tilde{a}(t) = [\tilde{a}_1^T(t), \tilde{a}_2^T(t), \dots, \tilde{a}_N^T(t)]^T$, and $\Lambda = \text{diag}\{\Lambda_1, \Lambda_2, \dots, \Lambda_N\}$.

The time-derivative of $V_1(t)$ is

$$\begin{aligned} \dot{V}_1(t) = & \frac{1}{2} S^T(t) \dot{M}(q) S(t) + S^T(t) M(q) \dot{S}(t) - \tilde{\theta}^T(t) \Lambda^{-1} \dot{\tilde{\theta}}(t) \\ & + \tilde{v}^T(t) (L_1 \otimes I_p) \dot{\tilde{v}}(t) + \tilde{a}^T(t) (L_1 \otimes I_p) \dot{\tilde{a}}(t) \\ & - \sum_{i=1}^N \frac{1}{\epsilon_{1i}} \tilde{\beta}_{1i}(t) \dot{\tilde{\beta}}_{1i}(t) - \sum_{i=1}^N \frac{1}{\epsilon_{2i}} \tilde{\beta}_{2i}(t) \dot{\tilde{\beta}}_{2i}(t) - \sum_{i=1}^N \frac{1}{\epsilon_i} \tilde{\phi}_i(t) \dot{\tilde{\phi}}_i(t). \end{aligned} \quad (5.16)$$

Following (5.6)-(5.7), we can get that

$$\dot{\tilde{v}} = \dot{\tilde{v}} - \ddot{q}_0 = -\hat{\beta}_{1i} \text{sign} \left[(L_1 \otimes I_p) \tilde{v} \right] - \ddot{q}_0, \quad (5.17)$$

$$\dot{\tilde{a}} = \dot{\tilde{a}} - \ddot{q}_0 = -\hat{\beta}_{2i} \text{sign} \left[(L_1 \otimes I_p) \tilde{a} \right] - \ddot{q}_0. \quad (5.18)$$

From Assumption 2, we can obtain that $\ddot{q}_0$ and $\ddot{q}_0$ are bounded. Suppose that $\|\ddot{q}_0\| < \mu_1$ and $\|\ddot{q}_0\| < \mu_2$, where μ_1 and μ_2 are positive constants. Then, combining (5.6)-(5.8), (5.17), and (5.18), (5.16) is reformulated as

$$\begin{aligned} \dot{V}_1(t) = & \frac{1}{2} S^T(t) \dot{M}(q) S(t) + S^T(t) [-C(q, \dot{q}) S(t) - K S(t) - e(t) - \sigma(t)] \\ & + \tilde{\theta}^T(t) Y(t) S(t) + S^T(t) [-Y(t) \tilde{\theta}(t) - \hat{\phi}(t) \text{sign}(S(t)) + d(t) + \varepsilon(t)] \\ & - \sum_{i=1}^N \hat{\beta}_{1i} \|\xi_{1i}\|_1 - \sum_{i=1}^N \hat{\beta}_{2i} \|\xi_{2i}\|_1 - \tilde{v}^T(t) (L_1 \otimes I_p) \ddot{q}_0 - \tilde{a}^T(t) (L_1 \otimes I_p) \ddot{q}_0 \\ & + \sum_{i=1}^N \left[(\hat{\beta}_{1i} - \beta_{1i}) \|\xi_{1i}\|_1 + (\hat{\beta}_{2i} - \beta_{2i}) \|\xi_{2i}\|_1 \right] + \sum_{i=1}^N (\hat{\phi}_i - \phi_i) \|S_i(t)\|_1. \end{aligned} \quad (5.19)$$

Based on Property 1, it can be derived that

$$\begin{aligned}
\dot{V}_1(t) \leq & - \sum_{i=1}^N K_i S_i^T(t) S_i(t) + \sum_{i=1}^N S_i^T(t) (\varepsilon_i(t) - e_i(t) - \sigma_i(t)) \\
& - \sum_{i=1}^N (\beta_{1i} \|\xi_{1i}\|_1 + \beta_{2i} \|\xi_{2i}\|_1) + \mu_1 \left\| (L_1 \otimes I_p) \tilde{v}(t) \right\|_1 \\
& + \mu_2 \left\| (L_1 \otimes I_p) \tilde{a}(t) \right\|_1 - \sum_{i=1}^N \hat{\phi}_i(t) \|S_i(t)\|_1 \\
& + S^T(t) d(t) + \sum_{i=1}^N (\hat{\phi}_i(t) - \phi_i) \|S_i(t)\|_1.
\end{aligned} \tag{5.20}$$

Since $\sum_{i=1}^N \|\xi_{1i}\|_1 = \left\| (L_1 \otimes I_p) \tilde{v} \right\|_1$ and $\sum_{i=1}^N \|\xi_{2i}\|_1 = \left\| (L_1 \otimes I_p) \tilde{a} \right\|_1$, one has that

$$\begin{aligned}
\dot{V}_1(t) \leq & - \sum_{i=1}^N K_i \|S_i(t)\|_2^2 + \sum_{i=1}^N S_i^T(t) \left(\|\varepsilon_i(t)\|_2 + \|e_i(t)\|_2 + \|\sigma_i(t)\|_2 \right) \\
& - (\beta_1 - \mu_1) \left\| (L_1 \otimes I_p) \tilde{v} \right\|_1 - (\beta_2 - \mu_2) \left\| (L_1 \otimes I_p) \tilde{a} \right\|_1,
\end{aligned} \tag{5.21}$$

where $\beta_1 = \min_i \{\beta_{1i}\}$ and $\beta_2 = \min_i \{\beta_{2i}\}$.

Afterward, we select another Lyapunov function as

$$V(t) = V_1(t) + \sum_{i=1}^N \delta_i(t). \tag{5.22}$$

By using Young's inequality, for any $\pi_1 > 0, \pi_2 > 0$ and $\pi_3 > 0$, one has that

$$\begin{aligned}
\dot{V}(t) = & \dot{V}_1(t) + \sum_{i=1}^N \dot{\delta}_i(t) \\
= & \dot{V}_1(t) - \sum_{i=1}^N \chi_{1i} \delta_i(t) - \sum_{i=1}^N \chi_{2i} \left[\|\varepsilon_i(t)\|_2^2 + \|e_i(t)\|_2^2 + \|\sigma_i(t)\|_2^2 - \hbar_i \|S_i(t)\|_2^2 \right] \\
\leq & - \sum_{i=1}^N \left(K_i - \chi_{2i} \hbar_i - \frac{1}{\pi_1} - \frac{1}{\pi_2} - \frac{1}{\pi_3} \right) \|S_i(t)\|_2^2 \\
& - (\beta_1 - \mu_1) \left\| (L_1 \otimes I_p) \tilde{v} \right\|_1 - (\beta_2 - \mu_2) \left\| (L_1 \otimes I_p) \tilde{a} \right\|_1,
\end{aligned} \tag{5.23}$$

where $\chi_{2i} > \max \{\pi_1, \pi_2, \pi_3\}$.

By choosing $K_i - \chi_{2i}\hbar_i - \frac{1}{\pi_1} - \frac{1}{\pi_2} - \frac{1}{\pi_3} > 0$, $\beta_1 > \mu_1$, $\beta_2 > \mu_2$, we can have $\dot{V}(t) \leq 0$. Since $V(t_k^i) \geq 0$ and $\dot{V}(t) \leq 0$, it is found that $V(t)$ is bounded, and hence one has $S, \tilde{\theta}, \hat{\phi}_i \in \mathbb{L}_\infty$, $\tilde{v}, \tilde{a} \in \mathbb{L}_\infty$, and $\hat{\beta}_{1i}, \hat{\beta}_{2i} \in \mathbb{L}_\infty$.

Next, the vector form of the sliding function is

$$\dot{\tilde{q}} = -\eta (L_1 \otimes I_p) \tilde{q} + S + \tilde{v} - f^T \pi. \quad (5.24)$$

Since all eigenvalues of L_1 are positive, one has that (5.24) is globally exponentially stable at the origin when $S + \tilde{v} = 0$. Based on Lemma 6, we can infer that it is ISS.

Since $S \in \mathbb{L}_\infty$ and $\hat{v} \in \mathbb{L}_\infty$, we can obtain $\tilde{q} \in \mathbb{L}_\infty$ and $\dot{\tilde{q}} \in \mathbb{L}_\infty$. Because $\dot{q}_0, \ddot{q}_0 \in \mathbb{L}_\infty$, one has $\hat{v}, \hat{a} \in \mathbb{L}_\infty$. Next, we can find from (5.3)-(5.4) that $\hat{q}_{ri}, \hat{\ddot{q}}_{ri} \in \mathbb{L}_\infty$. Furthermore, since the external disturbance is bounded and $Y_i(t) \in \mathbb{L}_\infty$, one has $\dot{S} \in \mathbb{L}_\infty$. Besides, we have $\dot{\tilde{v}}, \dot{\tilde{a}} \in \mathbb{L}_\infty$, since $\hat{\beta}_{1i}, \hat{\beta}_{2i} \in \mathbb{L}_\infty$ in (5.17) and (5.18). Following from $V(t_k^i) \geq 0$ and $\dot{V}(t) \leq 0$, we can obtain that $V(\infty)$ is bounded, and thus one has that

$$\begin{aligned} & \int_{t_k^i}^{+\infty} \sum_{i=1}^N \left(K_i - \chi_{2i}\hbar_i - \frac{1}{\pi_1} - \frac{1}{\pi_2} - \frac{1}{\pi_3} \right) \|S_i(t)\|_2^2 d\tau \\ & + \int_{t_k^i}^{+\infty} (\beta_1 - \mu_1) \left\| (L_1 \otimes I_p) \tilde{v} \right\|_1 d\tau + \int_{t_k^i}^{+\infty} (\beta_2 - \mu_2) \left\| (L_1 \otimes L_p) \tilde{a} \right\|_1 d\tau \\ & \leq V(t_k^i) - V(\infty), \end{aligned} \quad (5.25)$$

which means that $S_i \in \mathbb{L}_2 \cap \mathbb{L}_\infty$, $\tilde{v} \in \mathbb{L}_1 \cap \mathbb{L}_\infty$, and $\tilde{a} \in \mathbb{L}_1 \cap \mathbb{L}_\infty$. Because $\dot{S}, \dot{\tilde{v}}$ and $\dot{\tilde{a}}$ are bounded, we can infer from Lemma 5 that $S, \tilde{v}$ and $\tilde{a}$ asymptotically converge to zero. Based on the fact that (5.24) is ISS in relation to S and $\tilde{q}$, it is concluded that $\tilde{q}_i \rightarrow 0, \dot{\tilde{q}}_i \rightarrow 0$ as $t \rightarrow +\infty$, indicating that $q_i \rightarrow q_0$ and $\dot{q}_i \rightarrow \dot{q}_0$.

5.2.2 Admissibility of Triggering Condition

The inter-event interval $t_{k+1}^i - t_k^i$ has to be positive to rule out the Zeno behavior. The proof of Zeno-freeness is provided below.

Theorem 8. *Considering multiple networked Euler-Lagrange systems (5.2), by using the developed adaptive ET scheme (5.10)-(5.12) and the adaptive dynamic event-based robust control protocol (5.8), the minimal inter-event interval is positive, which can achieve the Zeno-freeness.*

Proof: In this section, the $\tanh(\cdot)$ function is used to substitute the $\text{sign}(\cdot)$ function as in [15]. For $t \in [t_k^i, t_{k+1}^i)$, we can have

$$\|\dot{e}_i(t)\|_2 = K_i \|\dot{S}_i(t)\|_2 \leq K_i \|\ddot{q}_i - \hat{\ddot{q}}_{ri}\|_2, \quad (5.26)$$

$$\|\dot{\varepsilon}_i(t)\|_2 = \left\| \dot{Y}_i(t)\hat{\theta}_i(t) + Y_i(t)\dot{\hat{\theta}}_i(t) \right\|_2, \quad (5.27)$$

$$\begin{aligned} \|\dot{\sigma}_i(t)\|_2 &\leq \left\| \dot{\hat{\phi}}_i \tanh(\beta S_i(t)) \right\|_2 + \beta \|\hat{\phi}_i \dot{S}_i(t)\|_2 \\ &\leq \epsilon_i \|S_i(t)\|_1 + \beta \left\| \hat{\phi}_i(t) (\ddot{q}_i - \hat{\ddot{q}}_{ri}) \right\|_2, \end{aligned} \quad (5.28)$$

where $\beta \gg 1$. It follows that

$$\begin{aligned} \|\dot{e}_i(t)\|_2 + \|\dot{\varepsilon}_i(t)\|_2 + \|\dot{\sigma}_i(t)\|_2 &\leq \left\| \dot{Y}_i(t)\hat{\theta}_i(t) + Y_i(t)\dot{\hat{\theta}}_i(t) \right\|_2 + \epsilon_i \|S_i(t)\|_1 \\ &\quad + \left(K_i + \beta \|\hat{\phi}_i(t)\|_2 \right) \|\ddot{q}_i - \hat{\ddot{q}}_{ri}\|_2. \end{aligned} \quad (5.29)$$

Note that all signals in (5.29) are bounded since $Y_i(t)$, $\dot{Y}_i(t)$, $S_i(t)$, $\hat{\phi}_i(t)$, $\hat{\ddot{q}}_{ri}$ and $\hat{\ddot{q}}_{ri}$ are bounded. Moreover, by defining $\Phi_i = \left\| \dot{Y}_i(t)\hat{\theta}_i(t) + Y_i(t)\dot{\hat{\theta}}_i(t) \right\|_2 + \epsilon_i \|S_i(t)\|_1 + \left(K_i + \beta \|\hat{\phi}_i(t)\|_2 \right) \|\ddot{q}_i - \hat{\ddot{q}}_{ri}\|_2$, we can obtain that

$$\|\dot{e}_i(t)\|_2 + \|\dot{\varepsilon}_i(t)\|_2 + \|\dot{\sigma}_i(t)\|_2 \leq \Phi_i. \quad (5.30)$$

Integrating (5.30) from t_k^i to t on both sides, one has that

$$\|e_i(t)\|_2 + \|\varepsilon_i(t)\|_2 + \|\sigma_i(t)\|_2 \leq \Phi_i(t - t_k^i). \quad (5.31)$$

Based on the triggering condition (5.11), when $t = t_{k+1}^i$, we can derive that

$$\frac{\delta_i(t_{k+1}^i)}{\psi_i} + \hbar \left\| S_i(t_{k+1}^i) \right\|_2^2 \leq \left(\|e_i(t)\|_2 + \|\varepsilon_i(t)\|_2 + \|\sigma_i(t)\|_2 \right)^2. \quad (5.32)$$

As a result, the triggering interval satisfies

$$t_{k+1}^i - t_k^i > \sqrt{\frac{\delta_i(t_{k+1}^i)}{\psi_i}} \geq \sqrt{\frac{\delta_i(0)e^{-(\chi_{1i} + \frac{\chi_{2i}}{\psi_i})t_{k+1}^i}}{\psi_i}} > 0,$$

which signifies that the inter-event interval is positive. Thus, for any finite time interval, only limited triggering instants exist, obviating the Zeno phenomenon.

5.3 Experimental Studies

To validate the developed fully distributed event-based control scheme, experiments are conducted using the networked multi-quadrotor system. The experimental system is shown in Fig. 5.1(a). To provide perturbations to quadrotors, a Lasko box fan with three speeds (S1, S2 and S3) is used.

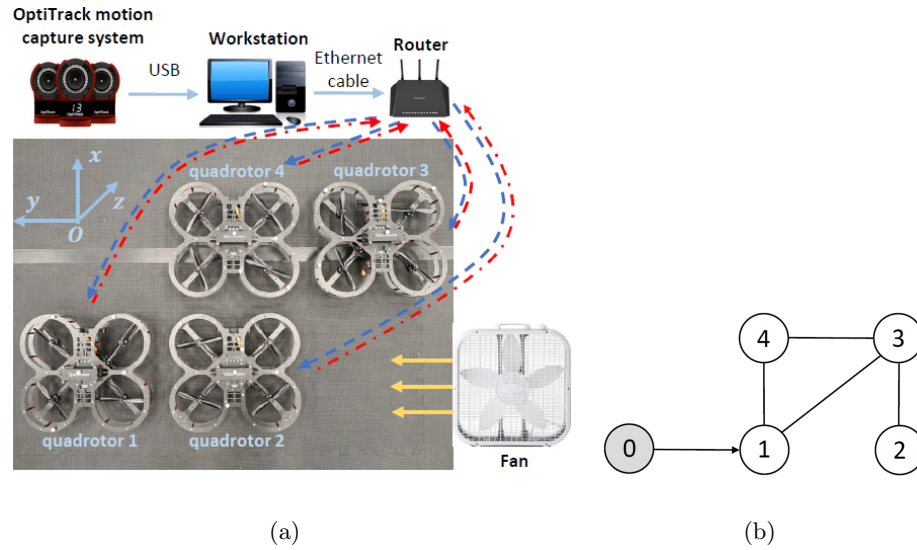


Figure 5.1: Experimental system: (a) multi-quadrotor system; (b) topology of the communication graph.

In this chapter, the inner/outer loop architecture [2] is used to model the quadrotor. Note that the outer loop control is realized by using the designed control protocol, while

the proportional derivative (PD) control scheme is applied to stabilize the inner loop as in [2]. Denote $\psi_i = -\text{diag}(Q_1/m_i, Q_2/m_i, Q_3/m_i) \in \mathbb{R}^{3 \times 3}$, where m_i is the mass, and Q_1 , Q_2 and $Q_3 \in \mathbb{R}_{\geq 0}$ are the damping coefficients. From [2, 15], the subsystem of quadrotor i is modeled as:

$$\begin{aligned}\dot{p}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= \psi_i v_i(t) + u_i(t) + d_{pi}(t),\end{aligned}\tag{5.33}$$

where $p_i = [\tilde{x}_i, \tilde{y}_i, \tilde{z}_i]^T \in \mathbb{R}^{3 \times 1}$ is the position and $v_i = [\dot{\tilde{x}}_i, \dot{\tilde{y}}_i, \dot{\tilde{z}}_i]^T \in \mathbb{R}^{3 \times 1}$ is the velocity. $d_{pi} = [d_{1i}, d_{2i}, d_{3i}]^T \in \mathbb{R}^{3 \times 1}$ represents the unknown perturbations. $u_i = [u_{xi}, u_{yi}, u_{zi}]^T \in \mathbb{R}^{3 \times 1}$. To protect the quadrotors, a safe relative position between the i th follower quadrotor and the leader should be set, which will be a formation tracking control issue. The formation relative distance is denoted as $\varphi_{pi} \in \mathbb{R}^{3 \times 1}$. Then, the objective of the formation control problem is to achieve $p_i(t) - p_0(t) = \varphi_{pi}$ and $v_i(t) - v_0(t) = 0$ as $t \rightarrow +\infty$, where $p_0(t)$ and $v_0(t)$ are the position and velocity of the leader, respectively. To convert the formation problem into the consensus problem, the position and velocity tracking errors are denoted as $e_{pi}(t) = (p_i(t) - \varphi_{pi}) - p_0(t)$ and $e_{vi}(t) = v_i(t) - v_0(t)$, respectively, and then the system of each follower is in the form of (5.33) with the states $[(p_i(t) - \varphi_{pi})^T, v_i^T(t)]^T$.

The topology graph is shown as Fig. 5.1(b), in which the leader is 0 while 1, 2, 3, 4 are following quadrotors. The system is modelled with $m_i = 1.121$ kg, $g = 9.8$ m/s² and $Q_q = 0.01$ N·s/m, $q = 1, 2, 3$. The desired reference command given by the leader is expressed as $p_0 = [0.9 \sin(0.5t), 0.9 \cos(0.5t) + 1.4, 0.06t + 0.7]^T$ m and $v_0 = [0.45 \cos(0.5t), -0.45 \sin(0.5t), 0.06]^T$ m/s, and the formation pattern for three quadrotors are selected as $\varphi_{p1} = [0, -0.7, 0]^T$ m, $\varphi_{p2} = [0, -1.4, 0]^T$ m, $\varphi_{p3} = [0.7, -1.4, 0]^T$ m, and $\varphi_{p4} = [0.7, -0.7, 0]^T$ m, respectively. The initial values of quadrotors are set as $p_1(0) = [0, 1, 0.7]^T$ m, $p_2(0) = [0, 0, 0.7]^T$ m, $p_3(0) = [1, 0, 0.7]^T$ m, $p_4(0) = [1, 1, 0.7]^T$ m, and $v_i(0) = 0_{3 \times 1}$ m/s. For $i = 1, \dots, 4$, we set $\kappa_i = 0.01$ and $\eta = 1$ in (5.5). The param-

eters of the estimator (5.6)-(5.7) are chosen as $\epsilon_{1i} = \epsilon_{2i} = 0.01$ and $\hat{\beta}_{1i}(0) = \hat{\beta}_{2i}(0) = 0.1$. For the designed control law (5.8) and the triggering scheme (5.10)-(5.12), we select $\Lambda_i = 0.1$, $\epsilon_i = 0.02$, $K_i = 4.5$, $\psi_i = 0.3$, $\hbar_i = 0.1$, $\chi_{1i} = 0.1$, $\chi_{2i} = 0.9$, $\hat{\theta}_i(0) = 0$, $\hat{\phi}_i(0) = 0.1$ and $\delta_i(0) = 2$.

To validate the advantages of the designed controller, comparisons are conducted for multiple quadrotors under perturbations (S3). Firstly, different triggering schemes are investigated:

- (i) *Adaptive dynamic event-triggered scheme (ADETS)*. An adaptive ET mechanism with a dynamic triggering threshold variable $\delta_i(t)$ is designed for quadrotor i . The parameters of the triggering scheme (5.10)-(5.12) are set as the initialization.
- (ii) *Static event-triggered scheme (SETS)*. An ET strategy for quadrotor i is implemented corresponding to a fixed triggering threshold. This case is achieved by setting $\delta_i(t)$ as a positive constant in (5.11), e.g., $\delta_i(t) = 0.01$.
- (iii) *Adaptive event-triggered scheme (AETS)*. An adaptive triggering strategy with a fixed triggering threshold proposed in [2] is used for the data transmission and the controller update of the quadrotor i .

Fig. 5.2 shows the triggering numbers of four quadrotors during the time $[0, 15]$ s under different ET laws, respectively, where 1,500 data packets are sampled in total for each quadrotor. It can be observed from Fig. 5.2 that the ADETS has the least sampling instants and thus leads to larger triggering intervals in comparison with the SETS and the AETS. In comparison with the SETS and the AETS, which has 209 and 224 sampling instants (13.9 % and 14.9 % resources utilization) for each quadrotor in average, respectively, the ADETS only needs 10.95 % of the resources to realize the control task. Hence, the developed ADETS can further reduce the resources consumption. Fig. 5.3 shows the event release

instants for each quadrotor under ADETS and SETS, respectively. In addition, the event-based control commands of four quadrotors under ADETS are demonstrated in Fig. 5.4, where it can be found that the designed ADETS can regulate the control frequency.

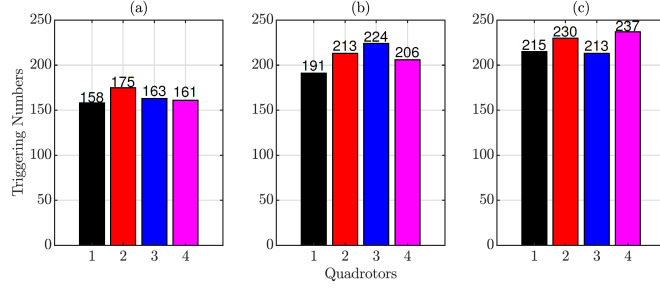


Figure 5.2: Triggered numbers : ADETS (a); SETS (b); AETS (c).

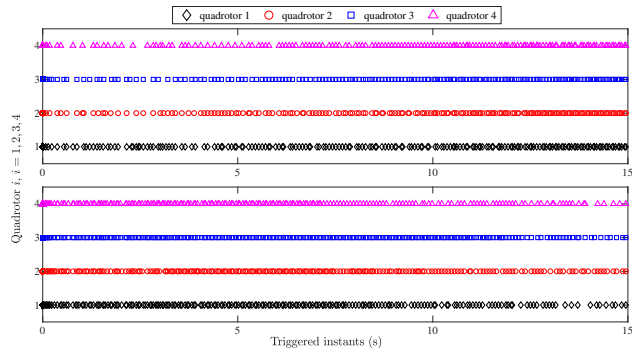


Figure 5.3: Triggered instants : ADETS (Top); SETS (Bottom).

Then, the following fully distributed event-based robust control schemes are studied:

- (i) *Adaptive dynamic ETC*. The designed adaptive TVSMC scheme (5.8) with the proposed dynamic ET strategy (5.10)-(5.12) is applied to realize the robust formation tracking for multiple quadrotors.
- (ii) *Static ETC*. The developed robust control protocol law (5.8) with the static ET scheme is implemented to achieve the formation task for multiple quadrotors.

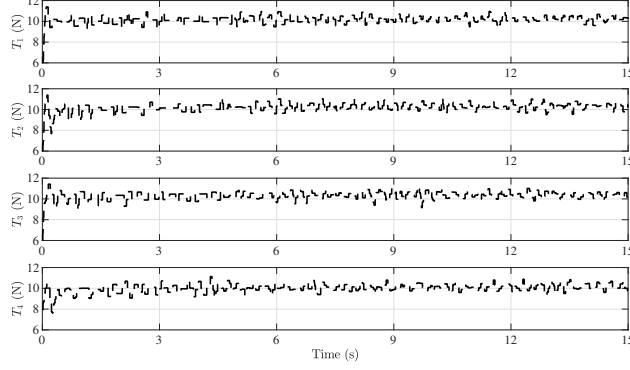


Figure 5.4: Thrusts of quadrotors.

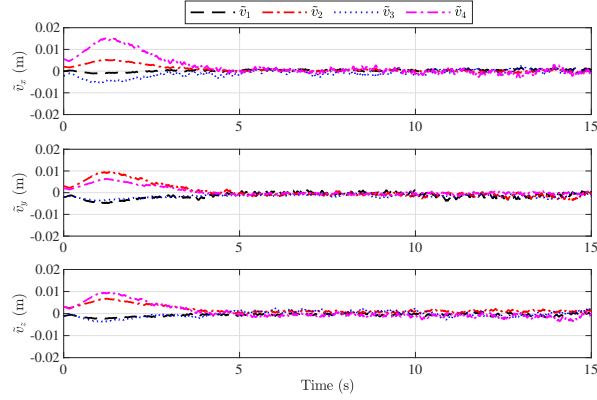


Figure 5.5: Estimation errors of the velocity.

(iii) *Control law proposed in [2]*. The event-based adaptive control protocol designed in [2] is implemented for multiple quadrotors, and same parameters for controllers are set.

We set $t=0$ s when the followers began to track the reference and the desired geometry. Note that the external perturbations are provided by the fan with the highest speed (S3). To obtain the dynamic leader's velocity and acceleration in a fully distributed manner, the designed estimators are implemented. The estimation errors of velocity and acceleration for each quadrotor are demonstrated in Fig. 5.5 and Fig. 5.6, respectively. It is noted that the dynamic leader's velocity in z axes is a constant, thus the estimation of its acceleration

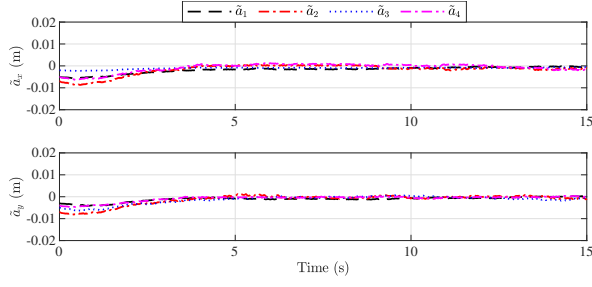


Figure 5.6: Estimation errors of the acceleration.

Table 5.1: Estimation Errors of Velocity and Acceleration

	$\tilde{v}_i$ (cm/s)	$\tilde{a}_i$ (cm/s ²)
MAE of quadrotor 1	0.1844	0.1279
MAE of quadrotor 2	0.3462	0.2148
MAE of quadrotor 3	0.2137	0.1516
MAE of quadrotor 4	0.4764	0.1795

is not required. In terms of the mean absolute error (MAE), the quantitative estimation results for each quadrotor are provided in Table 5.1, where it can be found that the average velocity and acceleration estimation errors are 0.35 cm/s and 0.17 cm/s², respectively. It can be concluded from Fig. 5.5, Fig. 5.6 and Table 5.1 that the performance of the designed estimators meets expectations, providing the potential to achieve high tracking control accuracy.

With the estimated results, the fully distributed formation tracking control is realized, and the formation performance of multiple quadrotors under the developed robust adaptive dynamic ETC scheme is provided in Fig. 5.7. The corresponding formation errors are shown in Figs. 5.8(a) and (d). For comparisons, the robust static ETC protocol and the control scheme in [2] are applied, and the performance is demonstrated in Figs. 5.8(b) and (e), and Figs. 5.8(c) and (f), respectively. As shown in Figs. 5.2, 5.3, and 5.8, the designed robust adaptive dynamic ETC protocol achieves comparable formation tracking performance for multiple quadrotors, even with a lower sampling frequency. This is due

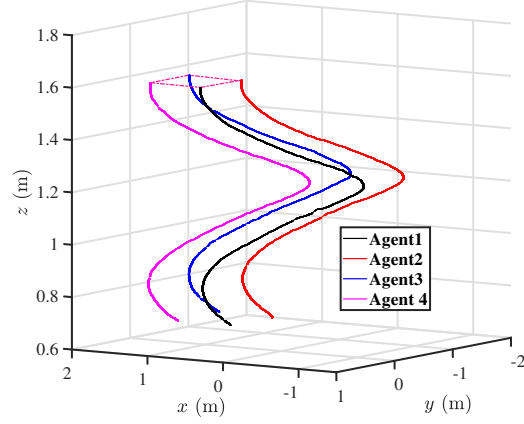
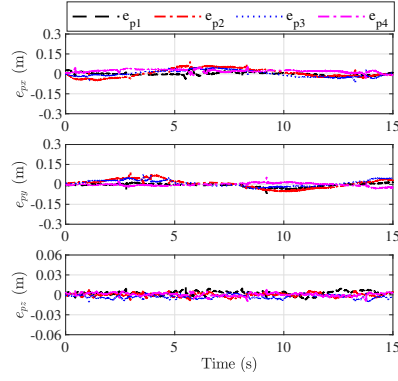


Figure 5.7: Formation trajectories of the following quadrotors.

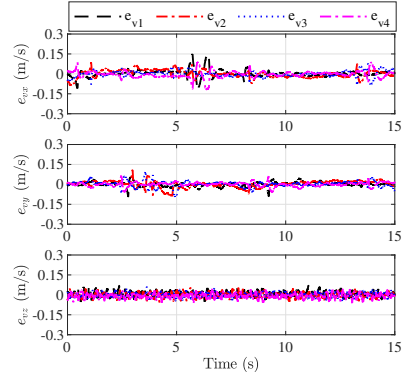
to its higher triggering frequency in the steady-state, where the triggered data contains useful information for maintaining the desired formation. Therefore, the advantages of further reducing resources consumption without degrading the tracking performance of the proposed dynamic ETC law have been demonstrated.

Table 5.2: Tracking Comparisons for Different Control Schemes

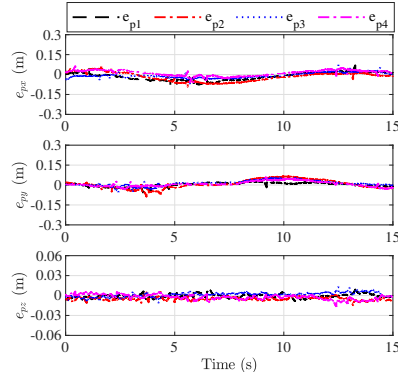
	Dynamic ETC		Static ETC		Controller in [2]	
	Pos. (m)	Vel. (m/s)	Pos. (m)	Vel. (m/s)	Pos. (m)	Vel. (m/s)
MSE of quadrotor 1	0.0056	0.015	0.0112	0.0147	0.0676	0.0564
MSE of quadrotor 2	0.0252	0.0181	0.0244	0.0201	0.0691	0.0588
MSE of quadrotor 3	0.0146	0.0134	0.0193	0.0214	0.0479	0.0294
MSE of quadrotor 4	0.0073	0.0171	0.0106	0.0193	0.0493	0.0284
MAE of quadrotor 1	0.0422	0.0635	0.0502	0.0619	0.1206	0.1123
MAE of quadrotor 2	0.0874	0.0792	0.0848	0.0811	0.1217	0.1205
MAE of quadrotor 3	0.0603	0.0585	0.0794	0.0826	0.1007	0.0897
MAE of quadrotor 4	0.0453	0.0702	0.0491	0.0803	0.1044	0.089



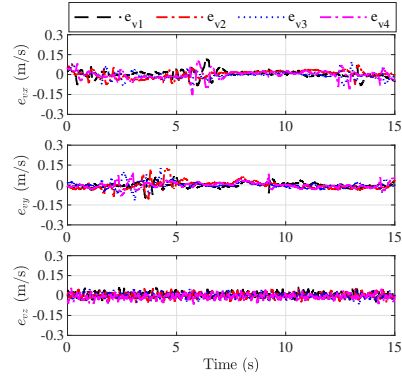
(a) Position errors



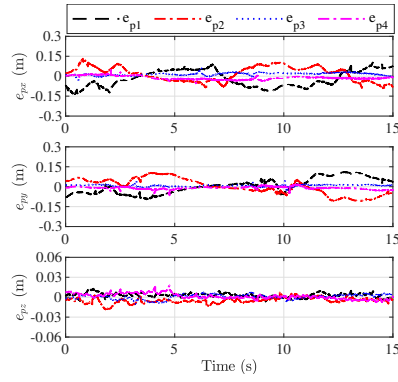
(b) Velocity errors



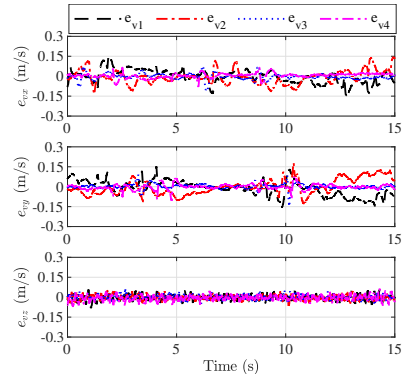
(c) Position errors



(d) Velocity errors



(e) Position errors



(f) Velocity errors

Figure 5.8: Formation tracking errors: adaptive dynamic ETC (a, d); static ETC (b, e); event-based control law in [2] (c, f).

By utilizing the mean squared error (MSE) and the mean absolute error (MAE), the quantitative comparison results for each quadrotor under different control schemes are provided in Table 5.2. From Figs. 5.8(a) and (d), Figs. 5.8(b) and (e), and Table 5.2, it can be found that the position and velocity of each quadrotor can reach and keep the specified responses, which indicates that the proposed fully distributed ETC law can achieve the desired formation of the multi-quadrotor system. Compared with the static ETC, where the average position tracking errors for each quadrotor are 1.64 cm (MSE) and 6.59 cm (MAE), they are 1.32 cm and 5.88 cm, respectively, under the adaptive dynamic ETC. The tracking errors have been improved by 19.5% (MSE) and 10.8% (MAE). In addition, the control strategy in [2] can also achieve the formation in the steady state as shown in Figs. 5.8(c) and (f), but it has worse performance in handling the effects of the imprecise state measurements, uncertainties, and external disturbances during the dynamical process. In terms of MSE and MAE, the average position tracking errors for each drone are 5.85 cm and 11.18 cm. Note that compared with the controller in [2], the tracking errors under the adaptive dynamic ETC with less sampling frequency have been improved by 77.4% (MSE) and 47.4% (MAE). Thus, it can be concluded that the robust formation can be guaranteed via the designed fully distributed robust adaptive dynamic ETC protocol, such that the formation task of multiple quadrotors with limited resources under uncertainties and external perturbations can be achieved with satisfactory performance.

To further validate the robustness of the designed sliding-mode estimators and the adaptive dynamic ETC protocol, perturbations at three fan speeds are tested: S1, S2 and S3. The estimation results for different fan speeds are presented in Table 5.3, while Table 5.4 shows the formation tracking errors of each quadrotor. It can be observed from Tables 5.3 and 5.4 that the estimation and tracking errors did not increase as external disturbances increased. The tracking performance of the quadrotors under various perturbation levels,

Table 5.3: Estimation Comparisons for Different Perturbations

	S1		S2		S3	
	$\tilde{v}_i$ (cm/s)	$\tilde{a}_i$ (cm/s ²)	$\tilde{v}_i$ (cm/s)	$\tilde{a}_i$ (cm/s ²)	$\tilde{v}_i$ (cm/s)	$\tilde{a}_i$ (cm/s ²)
MAE of quadrotor 1	0.2004	0.1568	0.2672	0.1389	0.1844	0.1279
MAE of quadrotor 2	0.3436	0.1859	0.3274	0.2091	0.3462	0.2148
MAE of quadrotor 3	0.3031	0.195	0.4379	0.2368	0.2137	0.1516
MAE of quadrotor 4	0.4066	0.1739	0.3227	0.1934	0.4764	0.1795

Table 5.4: Tracking Comparisons for Different Perturbations

	S1		S2		S3	
	Pos. (m)	Vel. (m/s)	Pos. (m)	Vel. (m/s)	Pos. (m)	Vel. (m/s)
MSE of quadrotor 1	0.0108	0.0094	0.0142	0.0136	0.0056	0.015
MSE of quadrotor 2	0.0117	0.0139	0.0199	0.0166	0.0252	0.0181
MSE of quadrotor 3	0.0152	0.0114	0.0121	0.0153	0.0146	0.0134
MSE of quadrotor 4	0.0091	0.0089	0.0128	0.0168	0.0073	0.0171
MAE of quadrotor 1	0.0488	0.0475	0.0614	0.0597	0.0422	0.0635
MAE of quadrotor 2	0.0515	0.0607	0.0804	0.0687	0.0874	0.0792
MAE of quadrotor 3	0.0634	0.0506	0.0533	0.0641	0.0603	0.0585
MAE of quadrotor 4	0.0471	0.0462	0.054	0.0694	0.0453	0.0702

using the derived estimator-based control law, is satisfactory. Therefore, the robustness of the estimators and the perturbation rejection capability of the controller have been demonstrated. The experimental video can be accessed via <https://youtu.be/Vp6CAFwi4R8>.

5.4 Summary

This chapter studied the fully distributed consensus tracking control issue for multiple networked EL systems considering noisy state measurements, unknown perturbations, parameter uncertainties, and limited resources. A nested adaptive sliding-mode estimator was derived to estimate the imprecise system states. In addition, a fully distributed dynamic ET scheme was proposed to reduce the resources consumption, without requiring relative velocity measurements. The inter-event interval can be regulated, and the Zeno behavior has been eliminated effectively. According to the developed estimator and the triggering scheme, a fully distributed adaptive dynamic event-based TVSMC scheme was designed

for EL systems. This controller does not need the upper bounds of perturbations and any global information. Asymptotic convergence of the closed-loop system was also guaranteed. Experiments were conducted using four quadrotors to validate the effectiveness of the developed robust event-based controller. As compared to the static ETC scheme and the event-based controller in [2], the proposed control protocol can achieve better control performance with less resources consumption for quadrotors under uncertainties, perturbations and inaccurate measurements.

However, the graph topology of the multi-quadrotor system in this chapter is assumed to be fixed, and only the time-invariant formation control is realized. To make it suitable for more general scenarios, one of the future plan is to extend the method to handle the time-varying formation tracking problem for multiple quadrotors with input saturation and unreliable communication networks. Besides, although the developed dynamic triggering scheme can reduce the sampling frequency, this node-based event-triggered mechanism will lead to unnecessary resources consumption. Thus, designing an edge-based triggering method for quadrotors will also be the future work.

6 Fully Distributed Edge-Based Dynamic Event-Triggered Control

The event-triggered control scheme is widely used in MASs because it can decrease the inter-agent interaction frequency while preserving satisfactory communication efficiency. However, most existing works only focus on the node-based ETC. When the triggering condition is satisfied, the information of the agent will be transmitted to all its neighbors. As stated in the summary of the last chapter, this will cause to unnecessary resources consumption. Besides, the unreliable communication networks will make the design of edge-based triggering scheme more difficult. Moreover, the magnitude of the control input of quadrotors is limited within a bounded region. The saturation nonlinearity usually degrades the system performance and even induces instability. Therefore, this chapter investigates the fully distributed event-based time-varying formation control problem for multiple quadrotors with unknown perturbations, input saturation and switching topologies. A novel adaptive dynamic event-triggered scheme is first formulated to alleviate the communication burden and reduce the resources consumption. Meanwhile, online triggering parameters and dynamic thresholds with updating laws associated with each edge are introduced so that the control protocol can be developed in a fully distributed way and the unnecessary communication can be further reduced for the leader-follower multi-quadrotor system with switching topologies without sacrificing the tracking performance. Then, a

fully distributed robust formation control protocol using the low gain feedback technique is developed to guarantee the time-varying formation of multiple quadrotors subject to unknown perturbations and input saturation without requiring global information. Furthermore, sufficient conditions are derived to ensure the asymptotic convergence of the formation error and Zeno-freeness. Hardware experiments are conducted to verify the efficiency of the designed controller.

6.1 Problem Statement

Consider a multi-quadrotor system including N quadrotors and a leader. Let $\mathcal{G} = 0 \cup \bar{\mathcal{G}}$ represent the communication topology, $\bar{\mathcal{G}} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ denotes the subgraph among N quadrotors. The Laplacian matrix is represented by $\mathcal{L}$, and the leader-follower matrix is defined as $\mathcal{H} = \mathcal{L} + \mathcal{A}_0$.

Lemma 7 ([106]). *If the undirected graph $\mathcal{G}$ is connected, the matrix $\mathcal{H}$ is positive definite.*

The switching topology is represented by $\mathcal{G}_{\sigma(t)} = 0 \cup (\mathcal{V}, \mathcal{E}_{\sigma(t)}, \mathcal{A}_{\sigma(t)})$, where $\sigma(t) : [0, +\infty) \rightarrow \mathcal{P}$ is the switching signal. $\mathcal{E}_{\sigma(t)}$ represents the set of all edges, and $\mathcal{A}_{\sigma(t)} = [a_{ij}^{\sigma(t)}] \in \mathbb{R}^{N \times N}$ is the adjacency matrix. $\mathcal{L}_{\sigma(t)} = [l_{ij}^{\sigma(t)}] \in \mathbb{R}^{N \times N}$ stands for the Laplacian matrix. $\mathcal{H}_{\sigma(t)}$ is the leader-follower matrix. Consider an infinite sequence of bounded, nonoverlapping and continuous time intervals $[t_s, t_{s+1})$, where $s \in \mathbb{N}$ and $t_0 = 0$. Let $t_{s+1} - t_s \leq T_d$ with T_d being some positive constant. During each interval $[t_s, t_{s+1})$, there are finite nonoverlapping and continuous subintervals

$$\left[t_s^0, t_s^1 \right), \left[t_s^1, t_s^2 \right), \dots, \left[t_s^{\mathcal{W}_s-1}, t_s^{\mathcal{W}_s} \right), t_s = t_s^0, t_{s+1} = t_s^{\mathcal{W}_s}$$

satisfying $t_s^{j+1} - t_s^j \geq \bar{\tau}$, $j = 0, 1, \dots, \mathcal{W}_s - 1$, where $\bar{\tau}$ is the dwell time, and $\bar{\tau} > 0$. The switching topology $\mathcal{G}_{\sigma(t)}$ is fixed during each time subinterval $[t_s^j, t_s^{j+1})$. Then, let $\bigcup_{t \in [t_s, t_{s+1})} \mathcal{G}_{\sigma(t)}$ represent a union graph across each time interval $[t_s, t_{s+1})$.

Assumption 9. The communication topology $\mathcal{G}_{\sigma(t)}$ is jointly connected, i.e., $\bigcup_{t \in [t_s, t_{s+1})} \mathcal{G}_{\sigma(t)}$ is connected.

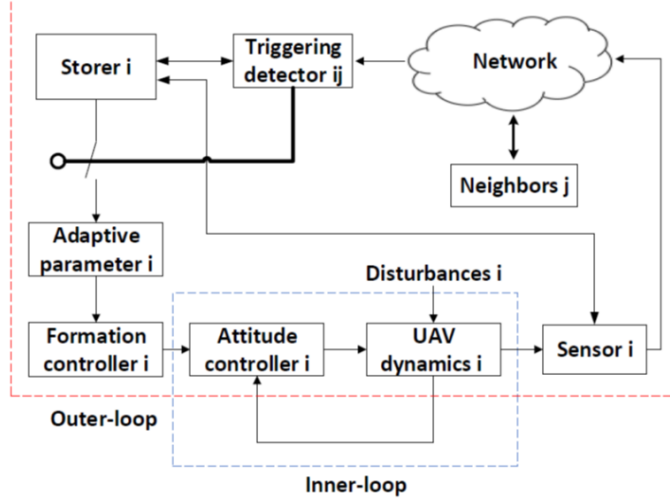


Figure 6.1: Control system diagram.

This chapter mainly focuses on the time-varying formation problem. The control block diagram is shown in Fig. 6.1. Denote $f_p = -\text{diag}(K_1/m_i, K_2/m_i, K_3/m_i) \in \mathbb{R}^{3 \times 3}$. From Eqs. (2.1) and (2.2), the subsystem of quadrotor i is modeled as

$$\begin{aligned} \dot{p}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= f_p v_i(t) + \text{sat}_\Delta(u_i(t)) + d_{pi}(t), \end{aligned} \quad (6.1)$$

where $p_i = [x_i, y_i, z_i]^T \in \mathbb{R}^{3 \times 1}$ is the position and $v_i = [\dot{x}_i, \dot{y}_i, \dot{z}_i]^T \in \mathbb{R}^{3 \times 1}$ is the velocity. $d_{pi} = [d_{1i}, d_{2i}, d_{3i}]^T \in \mathbb{R}^{3 \times 1}$ represents the unknown perturbations. $u_i = [u_{xi}, u_{yi}, u_{zi}]^T \in \mathbb{R}^{3 \times 1}$. $\text{sat}_\Delta(u_i) = [\text{sat}_\Delta(u_{xi}), \text{sat}_\Delta(u_{yi}), \text{sat}_\Delta(u_{zi})]^T \in \mathbb{R}^{3 \times 1}$ denotes the saturation function, and $\text{sat}_\Delta(u_{im}) = \text{sign}(u_{im}) \min\{|u_{im}|, \Delta\}$ for all $m \in \{x, y, z\}$, where $\Delta > 0$. Define $\theta_i(t) = [p_i(t), v_i(t)]^T$, and thus we can have

$$\dot{\theta}_i(t) = A\theta_i(t) + B\text{sat}_\Delta(u_i(t)) + Bd_{pi}(t), \quad (6.2)$$

where A and B are known system matrices with appropriate dimensions.

The model of the leader can be built as $\dot{\theta}_0(t) = A\theta_0(t)$. To solve the fully distributed event-based formation control problem for the multi-quadrotor system, we define $\tilde{\theta}_{i0} = e^{A(t-t_k^{i0})}\theta_i(t_k^{i0})$ and $\tilde{\theta}_{ij} = e^{A(t-t_k^{ij})}\theta_i(t_k^{ij})$, where t_k^{i0} and t_k^{ij} , with $t_0^{i0} = t_0^{ij} = 0$, $k \in \mathbb{Z}_{\geq 0}$, denote the k th triggering instant associated with the edge $(i, 0)$ and the edge (i, j) , respectively. Then, define $e_{i0} = \tilde{\theta}_{i0} - \theta_i$, $e_{ij} = \tilde{\theta}_{ij} - \theta_i$, $\tilde{\psi}_{i0} = \tilde{\theta}_{i0} - h_i - \theta_0$, and $\tilde{\psi}_{ij} = (\tilde{\theta}_{ij} - h_i) - (\tilde{\theta}_{ji} - h_j)$.

The system is required to realize a time-varying formation using the proposed controller in this chapter. Define $h_i(t) = [h_{ip}(t), h_{iv}(t)]^T \in \mathbb{R}^{6 \times 1}$, where $h_{ip}(t)$ and $h_{iv}(t)$ are the formation relative distance and velocity between the quadrotor i and the virtual leader, respectively. The reference trajectory is provided by the virtual leader. The formation tracking for quadrotors is defined as follows.

Definition 3. For any priori given bounded sets $\chi \in \mathbb{R}^{6 \times 1}$ and $\chi_0 \in \mathbb{R}^{6 \times 1}$, the semiglobal time-varying formation tracking can be guaranteed for the dynamics (6.2), if starting from any $\theta_i(0) \in \chi$ and $\theta_0(0) \in \chi_0$,

$$\lim_{t \rightarrow \infty} \|\theta_i(t) - h_i(t) - \theta_0(t)\| = 0, \quad i = 1, \dots, N. \quad (6.3)$$

Remark 17. This chapter attempts to design a fully distributed robust edge-based adaptive dynamic ETC algorithm for multiple quadrotors with switching topologies such that the time-varying formation tracking can be realized for quadrotors under unknown external disturbances, input saturation and limited bandwidth.

Assumption 10 ([107]). The pair (A, B) is asymptotically null controllable with bounded controls (ANCBC). That is, (A, B) is stabilizable and all eigenvalues of A are located in the closed left-half complex plane.

Assumption 11. Suppose that the perturbation d_{pi} is bounded, and an unknown positive constant γ exists satisfying $\|d_{pi}(t)\| \leq \gamma$.

Lemma 8 ([107]). *Suppose Assumption 10 holds. There is an unique matrix $P(\varepsilon) > 0$ for any $\varepsilon \in (0, 1]$, which is the solution of the following algebraic Riccati equation (ARE):*

$$A^T P(\varepsilon) + P(\varepsilon)A - P(\varepsilon)B^T B P(\varepsilon) + \varepsilon I_n = 0. \quad (6.4)$$

Furthermore, $\lim_{\varepsilon \rightarrow 0} P(\varepsilon) = 0$.

To overcome the limitation of the bandwidth and handle the input saturation nonlinearity and unknown perturbations for multiple quadrotors with switching topologies, this section develops a fully distributed edge-based dynamic event-triggered control (EBDETC) protocol to achieve the time-varying formation as

$$u_i = K \sum_{j=0}^N c_{ij} a_{ij}^{\sigma(t)} \tilde{\psi}_{ij} + \sum_{j=0}^N c_{ij} a_{ij}^{\sigma(t)} \mu_{ij} \left(K \tilde{\psi}_{ij} \right) + \pi h_i \quad (6.5)$$

with

$$\dot{c}_{i0} = \kappa_{i0} a_{i0}^{\sigma(t)} \left(\tilde{\psi}_{i0}^T \Gamma \tilde{\psi}_{i0} + 2 \left\| K \tilde{\psi}_{i0} \right\| \right), \quad (6.6a)$$

$$\dot{c}_{ij} = \kappa_{ij} a_{ij}^{\sigma(t)} \left(\tilde{\psi}_{ij}^T \Gamma \tilde{\psi}_{ij} + 2 \left\| K \tilde{\psi}_{ij} \right\| \right), \quad (6.6b)$$

where c_{i0} and c_{ij} are the adaptive coupling weights. $c_{i0}(0) \in \chi_{c_0}$ with the bounded set $\chi_{c_0} \in \mathbb{R}_{\geq 0}$, while $c_{ij}(0) = c_{ji}(0) \in \chi_c$ with the bounded set $\chi_c \in \mathbb{R}_{\geq 0}$. $\kappa_{ij} = \kappa_{ji}$ and κ_{i0} are positive constants. Note that the control response of the system can be faster by choosing larger κ_{i0} and κ_{ij} values in (6.6). Besides, $K = -B^T P(\varepsilon)$ and π are control gain matrices, and $\|\pi h_i\| \leq \bar{\pi}$ holds for a positive constant $\bar{\pi}$. $P(\varepsilon)$ is the solution of the equation (6.4). $\Gamma = P(\varepsilon)BB^T P(\varepsilon)$ is a constant matrix. The nonlinear function $\mu_{ij}(\cdot)$ is defined as

$$\mu_{ij}(\varsigma) = \begin{cases} \frac{\varsigma}{\|\varsigma\|}, & \text{if } \|\varsigma\| \neq 0 \\ 0, & \text{if } \|\varsigma\| = 0 \end{cases} \quad (6.7)$$

Then, define $\xi_i \triangleq \theta_i - h_i - \theta_0$, $i = 1, \dots, N$, as the formation error and thus one has

$$\begin{aligned} \dot{\xi}_i = & A\xi_i + B \text{sat}_{\Delta} \left[K \sum_{j=0}^N c_{ij} a_{ij}^{\sigma(t)} \tilde{\psi}_{ij} + \sum_{j=0}^N c_{ij} a_{ij}^{\sigma(t)} \mu_{ij} \left(K \tilde{\psi}_{ij} \right) + \pi h_i \right] \\ & + B d_{pi} + A h_i - \dot{h}_i. \end{aligned} \quad (6.8)$$

It is noted that the semiglobal time-varying formation can be realized for multiple quadrotors if ξ asymptotically converges to 0 with a bounded initial value, where $\xi = [\xi_1^T, \dots, \xi_N^T]^T$.

The triggering functions are developed as

$$\mathcal{H}_{i0}(t) = \omega_{i0} \left[\left(\frac{1}{2} + 2\iota c_{i0} \right) e_{i0}^T \Gamma e_{i0} + (1 + 2\iota c_{i0}) \|K e_{i0}\| - \frac{1}{4} \tilde{\psi}_{i0}^T \Gamma \tilde{\psi}_{i0} \right], \quad (6.9a)$$

$$\mathcal{H}_{ij}(t) = \omega_{ij} \left[(1 + 2\iota c_{ij}) e_{ij}^T \Gamma e_{ij} + (2 + 2\iota c_{ij}) \|K e_{ij}\| - \frac{1}{4} \tilde{\psi}_{ij}^T \Gamma \tilde{\psi}_{ij} \right], \quad (6.9b)$$

where ι is a positive constant. ω_{i0} and ω_{ij} are positive scalars. The adaptive dynamic triggering mechanisms for determining triggering instants of edge $(i, 0)$ and edge (i, j) are defined as

$$t_{k+1}^{i0} \triangleq \inf \left\{ t - t_k^{i0} > 0 \mid \mathcal{H}_{i0}(t) \geq \beta_{i0}(t) \text{ or } a_{i0}^{\sigma(t)} \neq a_{i0}^{\sigma(t_k^{i0})} \right\}, \quad (6.10a)$$

$$t_{k+1}^{ij} \triangleq \inf \left\{ t - t_k^{ij} > 0 \mid \mathcal{H}_{ij}(t) \geq \beta_{ij}(t) \text{ or } a_{ij}^{\sigma(t)} \neq a_{ij}^{\sigma(t_k^{ij})} \right\}, \quad (6.10b)$$

where β_{i0} and β_{ij} are dynamic thresholds, and the updating laws are designed as

$$\dot{\beta}_{i0}(t) = -\phi_{i0} \beta_{i0}(t) - \frac{\varrho_{i0}}{\omega_{i0}} \left(a_{i0}^{\sigma(t)} \mathcal{H}_{i0}(t) \right) \quad (6.11a)$$

$$\dot{\beta}_{ij}(t) = -\phi_{ij} \beta_{ij}(t) - \frac{\varrho_{ij}}{\omega_{ij}} \left(a_{ij}^{\sigma(t)} \mathcal{H}_{ij}(t) \right) \quad (6.11b)$$

with $\beta_{i0}(0) > 0$, $\beta_{ij}(0) > 0$, $\phi_{i0} > 0$, $\phi_{ij} > 0$, $\varrho_{i0} \geq 1$ and $\varrho_{ij} \geq 1$. Therefore, one has $\dot{\beta}_{i0}(t) \geq -\left(\phi_{i0} + a_{i0}^{\sigma(t)} \frac{\varrho_{i0}}{\omega_{i0}}\right) \beta_{i0}(t)$ and $\dot{\beta}_{ij}(t) \geq -\left(\phi_{ij} + a_{ij}^{\sigma(t)} \frac{\varrho_{ij}}{\omega_{ij}}\right) \beta_{ij}(t)$. Thus, $\forall t \geq 0$, we have $\beta_{i0}(t) \geq \beta_{i0}(0) e^{-\left(\phi_{i0} + a_{i0}^{\sigma(t)} \frac{\varrho_{i0}}{\omega_{i0}}\right)t} > 0$ and $\beta_{ij}(t) \geq \beta_{ij}(0) e^{-\left(\phi_{ij} + a_{ij}^{\sigma(t)} \frac{\varrho_{ij}}{\omega_{ij}}\right)t} > 0$. Note that by selecting smaller values for the parameters ι , ω_{i0} , ω_{ij} , ϕ_{i0} , ϕ_{ij} , ϱ_{i0} and ϱ_{ij} in (6.9)-(6.11), the triggering frequency can be reduced. Moreover, when the triggering instants $t_{k+1}^{ij} = t_{k+1}^{ji}$, which are determined by $\mathcal{H}_{ij}(t) \geq \beta_{ij}(t)$ or $\mathcal{H}_{ji}(t) \geq \beta_{ji}(t)$ or $a_{ij}^{\sigma(t)} \neq a_{ij}^{\sigma(t_k^{ij})}$, it will be a synchronous EBDETC.

Remark 18. By utilizing the node-based triggering mechanism proposed in [20, 25, 89], the agent i will transmit its information to all neighbors at its triggering instants. Different from [20, 25, 89], the edge-based triggering mechanism is developed in this chapter, where agent i only needs to transmit its information to agent j when the edge (i, j) is triggered. Intuitively, the communication frequency and resources consumption can be decreased by utilizing the proposed asynchronous edge-based dynamic triggering strategy (6.9)-(6.11). Note that in comparison with the synchronous edge-event-triggered strategy, where two agents need to transmit their state information to each other synchronously once the connected edge is triggered, the proposed asynchronous triggering method can further reduce the communication frequency.

6.2 Main Results

6.2.1 Stability Analysis

The stability criterion is provided to guarantee the formation for multiple quadrotors with the developed EBDETC scheme.

Lemma 9 (Cauchy's Convergence Criterion [108]). *The sequence $V(t_i), i = 0, 1, 2, \dots$ converges if and only if for any $\Omega > 0$, there is a positive number $\mathcal{W}_\Omega$ such that $\forall s > \mathcal{W}_\Omega$*

$$|V(t_{s+1}) - V(t_s)| < \Omega \quad \text{or} \quad \left| \int_{t_s}^{t_{s+1}} \dot{V}(t) dt \right| < \Omega. \quad (6.12)$$

Theorem 9. *For the multi-quadrotor system (6.1) with input saturation and unknown perturbations on switching topologies satisfying Assumptions 10-11, the time-varying formation tracking can be achieved under the proposed fully distributed EBDETC protocol (6.5) by selecting $h_i(t)$, if there exists a constant gain matrix π such that $\lim_{t \rightarrow \infty} \left((A + B\pi) h_i - \dot{h}_i \right) = 0, i = 1, 2, \dots, N$, is satisfied.*

Proof: The following positive-definite Lyapunov candidate is chosen as

$$\begin{aligned}
V = & \frac{1}{2} \xi_i^T P(\varepsilon) \xi_i + \sum_{i=1}^N \frac{(c_{i0} - \hat{c})^2}{4\kappa_{i0}} + \sum_{i=1}^N \sum_{j=1}^N \frac{(c_{ij} - \hat{c})^2}{8\kappa_{ij}} \\
& + \sum_{i=1}^N \frac{\hat{c}}{2} \beta_{i0} + \sum_{i=1}^N \sum_{j=1}^N \frac{\hat{c}}{2} \beta_{ij},
\end{aligned} \tag{6.13}$$

where $\hat{c}$ is a positive constant. Moreover, for $t \in [t_k^{ij}, t_{k+1}^{ij})$, $\tilde{\psi}_{ij}$ is continuous, and thus it is bounded in $[t_k^{ij}, t]$. Together with the fact that $\lim_{\varepsilon \rightarrow 0} P(\varepsilon) = 0$, it deduces that $\lim_{\varepsilon \rightarrow 0} \kappa_{ij} a_{ij}^{\sigma(t)} \left(\tilde{\psi}_{ij}^T P(\varepsilon) B B^T P(\varepsilon) \tilde{\psi}_{ij} + 2 \left\| B^T P(\varepsilon) \tilde{\psi}_{ij} \right\| \right) = 0$. Therefore, based on (6.6b), it is not hard to obtain that $\lim_{\varepsilon \rightarrow 0} c_{ij}$ is bounded. Similarly, we can have that $\lim_{\varepsilon \rightarrow 0} c_{i0}$ is bounded.

Denote $\bar{\lambda} > 0$ as a constant scalar such that

$$\bar{\lambda} \geq \sup_{\varepsilon \in (0,1], \theta_0(0) \in \chi_0, \theta_i(0) \in \chi, c_{i0}(0) \in \chi_{c0}, c_{ij}(0) \in \chi_c, i=1, \dots, N} V(0).$$

Such a $\bar{\lambda}$ exists since $\chi_0, \chi, \chi_{c0}, \chi_c, \beta_{ij}, \lim_{\varepsilon \rightarrow 0} c_{i0}$ and $\lim_{\varepsilon \rightarrow 0} c_{ij}$ are bounded. Define $L_V(\bar{\lambda}) := \{V \leq \bar{\lambda}, i = 1, \dots, N\}$. Because $L_V(\bar{\lambda})$, $\lim_{\varepsilon \rightarrow 0} c_{i0}$ and $\lim_{\varepsilon \rightarrow 0} c_{ij}$ are bounded, and $\lim_{\varepsilon \rightarrow 0} P(\varepsilon) = 0$, there exists an $\varepsilon^* \in (0, 1]$ such that for any $\varepsilon \in (0, \varepsilon^*]$, $\|u_i(t)\|_\infty \leq \Delta$. Therefore, one has $\text{sat}_\Delta(u_i(t)) = u_i(t)$. Then, the closed-loop system is formulated as

$$\dot{\xi}_i = A \xi_i + B K \sum_{j=0}^N c_{ij} a_{ij}^{\sigma(t)} \tilde{\psi}_{ij} + B \sum_{j=0}^N c_{ij} a_{ij}^{\sigma(t)} \mu_{ij} \left(K \tilde{\psi}_{ij} \right) + B d_{pi}. \tag{6.14}$$

Firstly, we show that $\dot{V}(t) \leq 0$ at any non-switching instants. The derivative of V can

be calculated as

$$\begin{aligned}
\dot{V} = & \sum_{i=1}^N \xi_i^T P(\varepsilon) A \xi_i + \sum_{i=1}^N \sum_{j=0}^N c_{ij} a_{ij}^{\sigma(t)} \xi_i^T P(\varepsilon) B K \tilde{\psi}_{ij} \\
& + \sum_{i=1}^N \sum_{j=0}^N c_{ij} a_{ij}^{\sigma(t)} \xi_i^T P(\varepsilon) B \mu_{ij} \left(K \tilde{\psi}_{ij} \right) \\
& + \sum_{i=1}^N \frac{c_{i0} - \hat{c}}{2\kappa_{i0}} \dot{c}_{i0} + \sum_{i=1}^N \sum_{j=1}^N \frac{c_{ij} - \hat{c}}{4\kappa_{ij}} \dot{c}_{ij} \\
& + \sum_{i=1}^N \frac{\hat{c}}{2} \dot{\beta}_{i0} + \sum_{i=1}^N \sum_{j=1}^N \frac{\hat{c}}{2} \dot{\beta}_{ij} + \sum_{i=1}^N \xi_i^T P(\varepsilon) B d_{pi}.
\end{aligned} \tag{6.15}$$

It is noted that $\tilde{\psi}_{ij} = (\xi_i - \xi_j) + (e_{ij} - e_{ji})$ and $\tilde{\psi}_{i0} = \xi_i + e_{i0}$. By utilizing the Young's inequality, one has

$$\begin{aligned}
\sum_{i=1}^N \sum_{j=0}^N c_{ij} a_{ij}^{\sigma(t)} \xi_i^T P(\varepsilon) B K \tilde{\psi}_{ij} \leq & -\frac{1}{4} \sum_{i=1}^N \sum_{j=1}^N c_{ij} a_{ij}^{\sigma(t)} \tilde{\psi}_{ij}^T \Gamma \tilde{\psi}_{ij} + \sum_{i=1}^N \sum_{j=0}^N c_{ij} a_{ij}^{\sigma(t)} e_{ij}^T \Gamma e_{ij} \\
& - \frac{1}{2} \sum_{i=1}^N c_{i0} a_{i0}^{\sigma(t)} \tilde{\psi}_{i0}^T \Gamma \tilde{\psi}_{i0}.
\end{aligned} \tag{6.16}$$

Based on the definition of $\mu_{ij}(\cdot)$ in (6.7), one has

$$\begin{aligned}
\sum_{i=1}^N \sum_{j=0}^N c_{ij} a_{ij}^{\sigma(t)} \xi_i^T P(\varepsilon) B \mu_{ij} \left(K \tilde{\psi}_{ij} \right) \leq & -\frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N c_{ij} a_{ij}^{\sigma(t)} \left\| B^T P(\varepsilon) \tilde{\psi}_{ij} \right\| \\
& - \sum_{i=1}^N c_{i0} a_{i0}^{\sigma(t)} \left\| B^T P(\varepsilon) \tilde{\psi}_{i0} \right\| \\
& + \sum_{i=1}^N \sum_{j=0}^N c_{ij} a_{ij}^{\sigma(t)} \left\| B^T P(\varepsilon) e_{ij} \right\|.
\end{aligned} \tag{6.17}$$

From (6.6a) and (6.6b), we can have

$$\begin{aligned}
\sum_{i=1}^N \frac{c_{i0} - \hat{c}}{2\kappa_{i0}} \dot{c}_{i0} \leq & \frac{\hat{c}}{4} \sum_{i=1}^N a_{i0}^{\sigma(t)} e_{i0}^T \Gamma e_{i0} - \frac{\hat{c}}{8} \sum_{i=1}^N a_{i0}^{\sigma(t)} \xi_i^T \Gamma \xi_i \\
& - \frac{\hat{c}}{4} \sum_{i=1}^N a_{i0}^{\sigma(t)} \left(\tilde{\psi}_{i0}^T \Gamma \tilde{\psi}_{i0} + 4 \left\| K \tilde{\psi}_{i0} \right\| \right) \\
& + \frac{1}{2} \sum_{i=1}^N c_{i0} a_{i0}^{\sigma(t)} \left(\tilde{\psi}_{i0}^T \Gamma \tilde{\psi}_{i0} + 2 \left\| K \tilde{\psi}_{i0} \right\| \right),
\end{aligned} \tag{6.18}$$

and

$$\begin{aligned}
\sum_{i=1}^N \sum_{j=1}^N \frac{c_{ij} - \hat{c}}{4\kappa_{ij}} \dot{c}_{ij} &\leq \frac{\hat{c}}{2} \sum_{i=1}^N \sum_{j=1}^N a_{ij}^{\sigma(t)} e_{ij}^T \Gamma e_{ij} - \frac{\hat{c}}{8} \sum_{i=1}^N \sum_{j=1}^N a_{ij}^{\sigma(t)} \xi_i^T \Gamma (\xi_i - \xi_j) \\
&\quad - \frac{\hat{c}}{8} \sum_{i=1}^N \sum_{j=1}^N a_{ij}^{\sigma(t)} \left(\tilde{\psi}_{ij}^T \Gamma \tilde{\psi}_{ij} + 4 \left\| K \tilde{\psi}_{ij} \right\| \right) \\
&\quad + \frac{1}{4} \sum_{i=1}^N \sum_{j=1}^N c_{ij} a_{ij}^{\sigma(t)} \left(\tilde{\psi}_{ij}^T \Gamma \tilde{\psi}_{ij} + 2 \left\| K \tilde{\psi}_{ij} \right\| \right). \tag{6.19}
\end{aligned}$$

Assume that the subgraph $\mathcal{G}_p (p \in \mathcal{P})$ is activated at time t . For each $p \in \mathcal{P}$, we define $\delta_p = \rho(\mathcal{H}_p)$ as the smallest nonzero eigenvalue of $\mathcal{H}_p$. Because $\mathcal{P}$ is finite, one has $\{\delta_p : p \in \mathcal{P}\}$. Then, define $\delta_{\min} := \min \{\delta_p : p \in \mathcal{P}\}$. Since $\|d_{pi}\| \leq \gamma$ based on Assumption 11, we can have

$$\begin{aligned}
\sum_{i=1}^N \xi_i^T P(\varepsilon) B d_{pi} &\leq \frac{\gamma \sqrt{N}}{\delta_{\min}} \sum_{i=1}^N \sum_{j=0}^N a_{ij}^{\sigma(t)} \left\| B^T P(\varepsilon) \tilde{\psi}_{ij} \right\| \\
&\quad + \frac{2\gamma \sqrt{N}}{\delta_{\min}} \sum_{i=1}^N \sum_{j=1}^N a_{ij}^{\sigma(t)} \left\| B^T P(\varepsilon) e_{ij} \right\| \\
&\quad + \frac{\gamma \sqrt{N}}{\delta_{\min}} \sum_{i=1}^N a_{i0}^{\sigma(t)} \left\| B^T P(\varepsilon) e_{i0} \right\|. \tag{6.20}
\end{aligned}$$

Then, substituting (6.16)-(6.20) into (6.15), letting $\hat{c} = \max \left\{ \frac{2\gamma \sqrt{N}}{\delta_{\min}}, \frac{1}{\iota} \right\}$, and based on the triggering conditions (6.10) and the updating laws of dynamic variables in (6.11), one has

$$\dot{V} \leq \frac{1}{2} \xi^T \left[I_N \otimes (P(\varepsilon) A + A^T P(\varepsilon)) - \frac{\hat{c}}{4} (\mathcal{H}_p \otimes \Gamma) \right] \xi.$$

Based on [106], the N eigenvalues of matrix $\mathcal{H}_p$ are labeled as $\{\lambda_p^1, \lambda_p^2, \dots, \lambda_p^N\}$. Let $h(p) = \{m : \text{the eigenvalue corresponding to } \lambda_p^m \text{ is nonzero}, m = 1, \dots, N\}$. $\mathcal{H}_p$ is symmetric, and thus there is an orthogonal matrix T_p such that $T_p \mathcal{L}_p T_p^T = \text{diag}(\lambda_p^1, \lambda_p^2, \dots, \lambda_p^N)$.

Let $\bar{\xi} = (T_p \otimes I_6) \xi$, and we can have

$$\begin{aligned}
\dot{V} &\leq \frac{1}{2} \bar{\xi}^T (T_p \otimes I_6) \left[I_N \otimes (P(\varepsilon)A + A^T P(\varepsilon)) - \frac{\hat{c}}{4} (\mathcal{H}_p \otimes \Gamma) \right] (T_p^T \otimes I_6) \bar{\xi} \\
&\leq \frac{1}{2} \sum_{i \in h(p)} \bar{\xi}_i^T \left[(P(\varepsilon)A + A^T P(\varepsilon)) - \frac{\hat{c}}{4} (\lambda_p^i \Gamma) \right] \bar{\xi}_i \\
&\leq -\frac{1}{2} \sum_{i \in h(p)} \varepsilon \bar{\xi}_i^T \bar{\xi}_i.
\end{aligned} \tag{6.21}$$

where $\hat{c} = \max \left\{ \frac{2\gamma\sqrt{N}}{\delta_{\min}}, \frac{4}{\delta_{\min}}, \frac{1}{\iota} \right\}$. Since $V(t) \geq 0$ and $\dot{V}(t) \leq 0$, one has that $V(t)$ is bounded and $\lim_{t \rightarrow \infty} V(t)$ exists.

When the topology switches during the triggering period, the change of topology leads to the triggering of the event condition based on (6.10), and this switching time instant will be a new event time, and thus it turns to the aforementioned case. Then, we show that $\lim_{t \rightarrow \infty} \xi_i = 0$. According to Lemma 9, one has

$$\int_{t_s^0}^{t_s^1} [-\dot{V}(t)] dt + \cdots + \int_{t_s^{\mathcal{W}_s-1}}^{t_s^{\mathcal{W}_s}} [-\dot{V}(t)] dt < \Omega.$$

For each subinterval $[t_s^j, t_s^{j+1})$, $j = 0, \dots, \mathcal{W}_s - 1$, one has

$$\int_{t_s^j}^{t_s^{j+1}} [-\dot{V}(t)] dt \geq \frac{\varepsilon}{2} \int_{t_s^j}^{t_s^j + \bar{\tau}} \sum_{i \in h(\sigma(t_s^j))} \bar{\xi}_i^T \bar{\xi}_i dt. \tag{6.22}$$

Hence

$$\Omega > \frac{\varepsilon}{2} \left[\int_{t_s^0}^{t_s^0 + \bar{\tau}} \sum_{i \in h(\sigma(t_s^0))} \bar{\xi}_i^T \bar{\xi}_i dt + \cdots + \int_{t_s^{\mathcal{W}_s-1}}^{t_s^{\mathcal{W}_s-1} + \bar{\tau}} \sum_{i \in h(\sigma(t_s^{\mathcal{W}_s-1}))} \bar{\xi}_i^T \bar{\xi}_i dt \right]. \tag{6.23}$$

It is noted that there exist finite switches during the time interval $[t_s, t_{s+1})$, and $\mathcal{W}_s$ is also finite for $s = 0, 1, \dots$, so we can have

$$\lim_{t \rightarrow \infty} \int_t^{t+\bar{\tau}} \sum_{i \in h(\sigma(t_s^j))} \bar{\xi}_i^T(\varpi) \bar{\xi}_i(\varpi) d\varpi = 0, \quad j = 0, \dots, \mathcal{W}_s - 1,$$

which implies that

$$\lim_{t \rightarrow \infty} \int_t^{t+\bar{\tau}} \left[\sum_{i \in h(\sigma(t_s^0))} \bar{\xi}_i^T(\varpi) \bar{\xi}_i(\varpi) + \cdots \sum_{i \in h(\sigma(t_s^{W_s-1}))} \bar{\xi}_i^T(\varpi) \bar{\xi}_i(\varpi) \right] d\varpi = 0. \quad (6.24)$$

During the time interval $[t_s, t_{s+1})$, the graphs are jointly connected, and thus we can have that $\bigcup_{t \in [t_s, t_{s+1})} h(\sigma(t)) = \{1, \dots, N\}$. Then, one has

$$\lim_{t \rightarrow \infty} \int_t^{t+\bar{\tau}} \left[\sum_{i=1}^N \tilde{a}_i \bar{\xi}_i^T(\varpi) \bar{\xi}_i(\varpi) \right] d\varpi = 0, \quad (6.25)$$

where $\tilde{a}_i > 0, i = 1, \dots, N$. Based on the fact that $\dot{V}(t) \leq 0$, one has that $\bar{\xi}$ and c_{ij} are bounded. Noting Assumption 9, we can have that $\dot{\bar{\xi}}$ is bounded based on (6.14). Furthermore,

$$\frac{d^2}{dt^2} \int_t^{t+\bar{\tau}} \left\{ \sum_{i=1}^N \tilde{a}_i \bar{\xi}_i^T(\varpi) \bar{\xi}_i(\varpi) \right\} d\varpi = 2 \sum_{i=1}^N \tilde{a}_i \bar{\xi}_i^T(t) \dot{\bar{\xi}}_i(t),$$

which is also bounded. According to Barbalat's lemma [101], it can be obtained that $\lim_{t \rightarrow \infty} \left\{ \sum_{i=1}^N \tilde{a}_i \bar{\xi}_i^T(t) \bar{\xi}_i(t) \right\} = 0$, which further implies that $\lim_{t \rightarrow \infty} \bar{\xi}_i = 0$, and thus we have $\xi(t) \rightarrow 0$ as $t \rightarrow +\infty$. This completes the proof.

6.2.2 Zeno-Free Triggering

Then, Zeno behavior is proved to be avoided for the system.

Theorem 10. *For the closed-loop multi-quadrotor system described by Theorem 9, the minimum interevent interval of edge $(i, 0)$ and edge (i, j) is strictly positive in finite time and the Zeno behavior is eliminated.*

Proof: To exclude the Zeno behavior for the edge (i, j) , four triggering cases are considered:

- (i) In this case, both triggering instants t_k^{ij} and t_{k+1}^{ij} are determined by $\mathcal{H}_{ij}(t) \geq \beta_{ij}(t)$.

We first use contradiction argument to eliminate the Zeno phenomenon. Suppose there exists the Zeno phenomenon for the edge (i, j) , which implies that a finite time T exists, such that $\lim_{k \rightarrow +\infty} t_k^{ij} = T$. Note that $\tilde{\psi}_{i0}$ and $\tilde{\psi}_{ij}$ are bounded for any finite time, so as c_{i0} , c_{ij} , πh_i and d_{pi} . There exist positive constants $\bar{c}_1$ and $\bar{c}_2$ satisfying $|1 + 2\iota c_{ij}| \leq \bar{c}_1$ and $|2 + 2\iota c_{ij}| \leq \bar{c}_2$, respectively. The Dini derivative of e_{ij} is calculated as

$$D^+ e_{ij} = A e_{ij} - BK \sum_{j=0}^N c_{ij} a_{ij}^{\sigma(t)} \tilde{\psi}_{ij} - B \sum_{j=0}^N c_{ij} a_{ij}^{\sigma(t)} \mu_{ij} (K \tilde{\psi}_{ij}) - B \pi h_i - B d_{pi}.$$

It follows that $D^+ \|e_{ij}\| \leq A \|e_{ij}\| + \epsilon_{ij}$, where ϵ_{ij} is denoted as

$$\epsilon_{ij} = \sup \left\{ \left\| BK \sum_{j=0}^N c_{ij} a_{ij}^{\sigma(t)} \tilde{\psi}_{ij} + B \sum_{j=0}^N c_{ij} a_{ij}^{\sigma(t)} \mu_{ij} (K \tilde{\psi}_{ij}) + B \pi h_i + B d_{pi} \right\| \right\}$$

for $t \in [t_k^{ij}, t_{k+1}^{ij})$. Then, define a nonnegative function $\Xi(t) : [0, +\infty) \rightarrow \mathbb{R}_{\geq 0}$, satisfying $\dot{\Xi}(t) = \|A\| \Xi(t) + \epsilon_{ij}$, and $\Xi(0) = \|e_{ij}(t_k^{ij})\| = 0$, where $\Xi(t)$ is the analytical solution, given by $\Xi(t) = \frac{\epsilon_{ij}}{\|A\|} (e^{\|A\|t} - 1)$. Then, we have $\|e_{ij}(t)\| \leq \Xi(t - t_k^{ij})$. Furthermore, we can verify that the triggering condition (6.10b) satisfies $\mathcal{H}_{ij}(t) \leq \beta_{ij}(t)$ if

$$\|e_{ij}\| \leq \frac{-\bar{c}_2 + \sqrt{\bar{c}_2^2 + 4\bar{c}_1 \beta_{ij}}}{2\bar{c}_1 \|K\|} \leq \frac{\sqrt{\beta_{ij}(0)} e^{-\frac{1}{2}(\phi_{ij} + a_{ij}^{\sigma(t)} \frac{\epsilon_{ij}}{\omega_{ij}})T}}{\sqrt{\bar{c}_1} \|K\|}. \quad (6.26)$$

Let $\varphi = \frac{\sqrt{\beta_{ij}(0)} e^{-\frac{1}{2}(\phi_{ij} + a_{ij}^{\sigma(t)} \frac{\epsilon_{ij}}{\omega_{ij}})T}}{\sqrt{\bar{c}_1} \|K\|}$. On one hand, choose $\tilde{\varepsilon} < \ln \left(\frac{\|A\|\varphi}{\epsilon_{ij}} + 1 \right)$ such that $t_k^{ij} \in [T - \tilde{\varepsilon}, T]$. On the other hand, one has $\beta_{ij}(t) \geq \beta_{ij}(0) e^{-\left(\phi_{ij} + a_{ij}^{\sigma(t)} \frac{\epsilon_{ij}}{\omega_{ij}}\right)t}$ with $t \geq 0$, and the time interval $\bar{\Delta}$ for $\Xi(t)$ evolving from 0 to φ is given by $t_{k+1}^{ij} - t_k^{ij} \geq \bar{\Delta}$. Thus, it is obtained that $\bar{\Delta} = \ln \left(\frac{\|A\|\varphi}{\epsilon_{ij}} + 1 \right) > 0$. This contradicts the assumption of $t_{k+1}^{ij} - t_k^{ij} \leq \tilde{\varepsilon} < \ln \left(\frac{\|A\|\varphi}{\epsilon_{ij}} + 1 \right) = \bar{\Delta}$. Thus, there is no Zeno behavior.

- (ii) In this case, t_k^{ij} is determined by $\mathcal{H}_{ij}(t) \geq \beta_{ij}(t)$, while t_{k+1}^{ij} is determined by $a_{ij}^{\sigma(t)} \neq a_{ij}^{\sigma(t_k^{ij})}$. It is noted that only a finite number of switches exist in finite time. Thus, the minimum of the finite interval $\bar{\tau}_k^{ij} = t_{k+1}^{ij} - t_k^{ij}$ is nonzero, and a minimum interevent time exists.
- (iii) In this case, t_k^{ij} is determined by $a_{ij}^{\sigma(t)} \neq a_{ij}^{\sigma(t_k^{ij})}$, while t_{k+1}^{ij} is determined by $\mathcal{H}_{ij}(t) \geq \beta_{ij}(t)$. Because the measurement error e_{ij} is reset to zero at t_k^{ij} , the proof to exclude the Zeno behavior is similar to case (i) and thus is omitted here.
- (iv) In this case, t_k^{ij} and t_{k+1}^{ij} are determined by $a_{ij}^{\sigma(t)} \neq a_{ij}^{\sigma(t_k^{ij})}$. It can be found that the interval is not less than the dwell time $\bar{\tau}$.

Based on the analysis of these four cases, it can be concluded that there exists no Zeno behavior for the edge (i, j) and the minimum interevent interval is strictly positive in finite time. Similarly, the Zeno behavior for the edge $(i, 0)$ can also be avoided, and the details are omitted here.

6.3 Experimental Studies

Experiments are conducted to validate the effectiveness of the developed fully distributed ETC scheme. Fig. 6.2 shows the experimental setup. The unknown perturbations are provided by a Lasko box fan with three speed levels (L1, L2, and L3).

The quadrotor is modeled with $m_i = 1.121$ kg, $g = 9.8$ m/s² and $K_1 = K_2 = K_3 = 0.01$ N·s/m. In our experiments, proportional derivative (PD) controllers are developed as in [3] and [15] to stabilize the attitude of the multi-quadrotor system. Four quadrotors firstly took off and hovered at the initial position. At $t = 0$ s, they began to track the desired geometry. At $t = 20$ s, the quadrotors entered the landing stage. During $t \in [0, 20]$ s, possible communication topologies, as shown in Fig. 6.3, switched as $\mathcal{G}_1 \rightarrow \mathcal{G}_2 \rightarrow \mathcal{G}_3 \rightarrow \mathcal{G}_1$

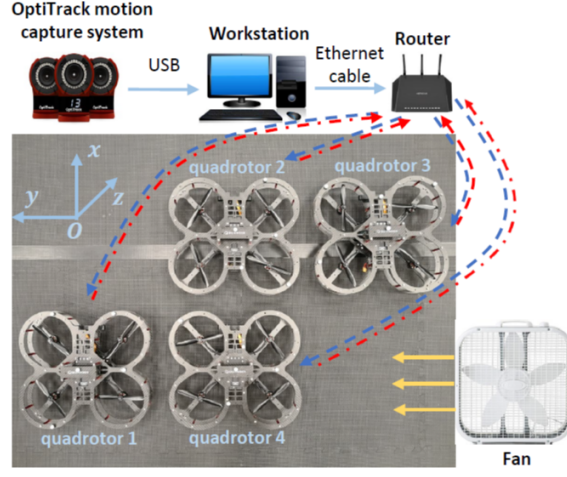


Figure 6.2: Experimental multi-quadrotor system.

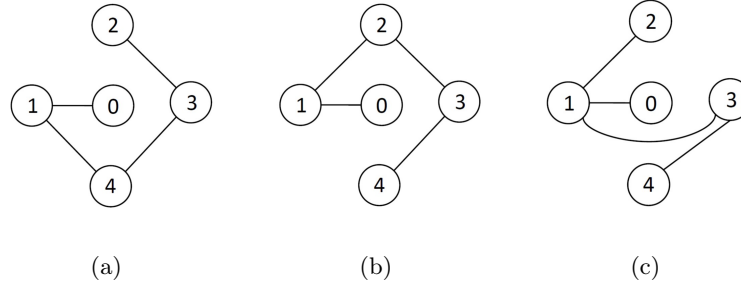


Figure 6.3: Possible interaction topologies. (a) $\mathcal{G}_1$. (b) $\mathcal{G}_2$. (c) $\mathcal{G}_3$.

with the dwell time $\bar{\tau} = 5$ s. Note that node 0 is the virtual leader, while nodes 1-4 represent four quadrotors. A circular formation was formed by four quadrotors with the following geometry

$$h_{ip}(t) = \begin{bmatrix} 1.2 \sin(0.5t + \pi(i-1)/2) \\ 1.2 \cos(0.5t + \pi(i-1)/2) \\ 0.9 \end{bmatrix}, i = 1, 2, 3, 4.$$

It can be found that the feasible condition in Theorem 1 is satisfied for all quadrotors by choosing

$$\pi = \begin{bmatrix} -\frac{1}{2} & -f_p & 0 & f_p & -\frac{1}{2} & 0 \\ f_p & -\frac{1}{2} & 0 & \frac{1}{2} & f_p & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The reference values provided by the virtual leader were $p_0 = [0, 0, 0.9]^T$ m and $v_0 = 0_{3 \times 1}$ m/s. The initial states of quadrotors were selected as $p_1(0) = [0, 1, 0.9]^T$ m, $p_2(0) = [1, 0, 0.9]^T$ m, $p_3(0) = [0, -1, 0.9]^T$ m, $p_4(0) = [-1, 0, 0.9]^T$ m, and $v_i(0) = 0_{3 \times 1}$ m/s, $i = 1, 2, 3, 4$. Note that the initial states of quadrotors are not in the desired formation. Since a virtual leader is considered in this chapter, and its information is stored in its connected agent, quadrotor i , the periodic time-triggered mechanism is implemented for the edge $(i, 0)$ by setting $\omega_{i0} = 0$ in (6.9a) and $\beta_{i0}(t) = 0$ in (6.10a) for the convenience of the experiments. The initial values of the time-varying gains c_{i0} and c_{ij} in (6.5-6.6) are chosen as $c_{i0}(0) = c_{ij}(0) = 0.01$. The parameters κ_{i0} , κ_{ij} and ε are chosen as $\kappa_{i0} = \kappa_{ij} = 0.03$ and $\varepsilon = 0.01$. In addition, we set $\phi_{ij} = 1$, $\iota = 0.1$, $\varrho_{ij} = 1.2$, and $\omega_{ij} = 0.06$ for the edge-based dynamic triggering strategy (6.9-6.11). The initial values of the dynamic variables β_{ij} are selected as $\beta_{ij}(0) = 2$.

To validate the effectiveness of the designed fully distributed event-based formation control algorithm for multiple quadrotors with external perturbations and input saturation on switching topologies, comparisons are conducted for different triggering mechanisms and control schemes.

Firstly, two synchronous edge-based triggering schemes and one node-based triggering strategy are compared:

- (i) *Edge-based dynamic event-triggered mechanism (EBDETM)*. The triggering detector is constructed in each communication channel. When edge (i, j) is triggered in response to a dynamic threshold $\beta_{ij}(t)$, quadrotor i will communicate with quadrotor j .
- (ii) *Edge-based static event-triggered mechanism (EBSETM)*. The triggering detector is developed in each edge with a permanent fixed triggering threshold. By setting $\beta_{ij}(t) = 0$ in (6.10b), this case can be achieved.

Table 6.1: Triggering Numbers of Various Triggering Schemes

Quadrotors $i \rightarrow j$	EBDETM	EBSETM	NBSETM
$1 \rightarrow 2$	50	87	103
$1 \rightarrow 3$	32	47	103
$1 \rightarrow 4$	68	71	103
$2 \rightarrow 1$	50	87	101
$2 \rightarrow 3$	96	103	101
$3 \rightarrow 1$	32	47	116
$3 \rightarrow 2$	96	103	116
$3 \rightarrow 4$	114	157	116
$4 \rightarrow 1$	68	71	125
$4 \rightarrow 3$	114	157	125

- (iii) *Node-based static event-triggered mechanism (NBSETM) in [2]*. The triggering detector is constructed at each node. When the triggering condition with the fixed triggering threshold is satisfied for the node i , the information will be transmitted to all its neighbours. We keep the parameters of triggering functions the same.

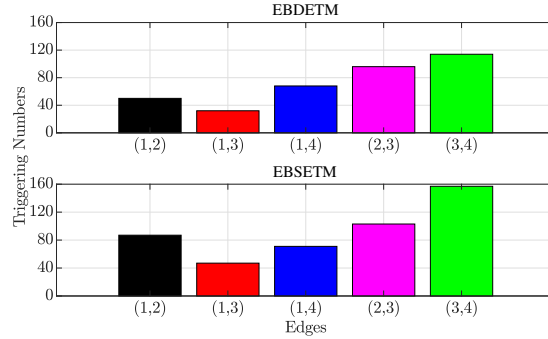


Figure 6.4: Triggering numbers for the edge (i, j) under two edge-based ETM.

During the experimental period $t \in [0, 20]$ s, there are 1000 points for the periodic time-triggered sampling. Table 6.1 provides a quantitative comparison with respect to the triggering numbers under different triggering mechanisms. For example, for quadrotor 1, the transmissions under the EBDETM to its neighbours 2, 3, 4 are 50, 32 and 68, respectively. Under the EBSETM, the transmissions are 87, 47, 71 for quadrotor 2, 3, and

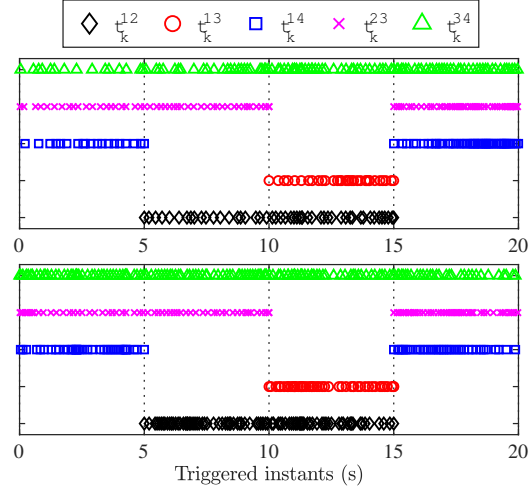


Figure 6.5: Triggering instants for the edge (i, j) under two edge-based ETM. Top: EBDETM. Bottom: EBSETM.

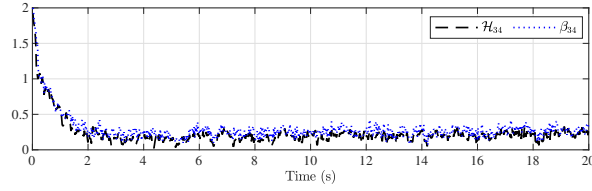


Figure 6.6: Evolution of interval error and triggering threshold for edge $(3, 4)$.

4, respectively, while under the NBSETM, 103 is for quadrotors 2, 3 and 4. Moreover, the triggering numbers and event release instants for each edge under the edge-based ETM are shown in Fig. 6.4 and Fig. 6.5, respectively. The relationship between the triggering function (6.9b) and the triggering threshold (6.11b) under the designed EBDETM is shown in Fig. 6.6, where edge $(3, 4)$ is used for illustration.

Secondly, the corresponding control schemes are compared:

- (i) *EBDETC*. The developed fully distributed control scheme (6.5) with the edge-based dynamic ETC strategy (5.10) is implemented to perform the time-varying formation for multiple quadrotors with input saturation and external perturbations on switching

topologies.

- (ii) *Edge-based static event-triggered control (EBSETC)*. The designed fully distributed control law (6.5) with the edge-based static ETC scheme is applied to perform the time-varying formation for multiple quadrotors with input saturation, external perturbations, and switching topologies.
- (iii) *Controller proposed in [2]*. The fully distributed node-based static ETC strategy developed in [2] is applied to achieve the time-varying formation for multiple quadrotors under input saturation and external perturbations with switching topologies, and the parameters of the controllers and triggering functions are kept the same.

Note that the unknown perturbations are provided by the fan with the highest speed (L3). The time-varying formation results in three dimensions under the EBDETC are demonstrated in Fig. 6.7, and it can be seen that the drones can track and maintain the desired trajectory. Fig. 6.8 shows the tracking errors for each quadrotor under three different event-based control strategies. Figs. 6.8(a) and (b) provide the tracking errors under EBDETC, where the drones can fly along the desired trajectory within an allowable error. Note that $e_{pi} = p_i(t) - h_{ip}(t) - p_0$ is the position tracking error and $e_{vi} = v_i(t) - h_{iv}(t) - v_0$ is the velocity tracking error. Figs. 6.8(c) and (d) show the tracking errors under EBSETC, whereas Figs. 6.8(e) and (f) show the tracking errors under the controller proposed in [2]. Furthermore, Table 6.2 compares the control performance in terms of Mean Squared Error (MSE), Mean Absolute Error (MAE), and Steady-State Error (SSE). Note that MSE can give more weight to the outliers, MAE can provide the real tracking errors, while SSE can show the tracking errors in steady states. All indicators are independent of the sampling numbers. The following conclusions can be obtained:

- (i) In terms of the transmission number, compared with the node-based ETC (NB-

SETM), the designed edge-based ETC (EBSETM) can save more communication resources. Especially, for quadrotor 1, the transmissions under the edge-based ETC to its neighbors are 205, while under the node-based ETC, 309 is in total for its neighbors. In this case, the communication resources are saved by 33.7% under the edge-based ETC. Moreover, the transmissions under the developed dynamic triggering strategy (EBDETM) to its neighbours are 150. In comparison with the static triggering scheme (EBSETM), the dynamic ETC (EBDETM) has larger triggering intervals and lower sampling frequency, and thus has potential to reduce more unnecessary resources consumption (saved by 26.8%).

- (ii) In terms of MSE, MAE and SSE, the designed EBDETC performs better than the control protocol proposed in [2]. For example, the average position tracking errors under EBDETC for each drone are 1.22 cm (MSE), 13.5 cm (MAE), and 2.6 cm (SSE), while they are 1.6 cm, 15.6 cm, and 4.7 cm, respectively, under the controller in [2]. The tracking errors have been improved by 23.8% (MSE), 13.5% (MAE), and 44.7% (SSE). Moreover, the EBDETC achieves similar tracking results as the EBSETC but with less transmissions. For the SSE, the EBDETC performs better than the EBSETC and the controller in [2] because the EBDETC has a higher triggering frequency in the steady-state, and the triggered data contains useful information for the desired formation.

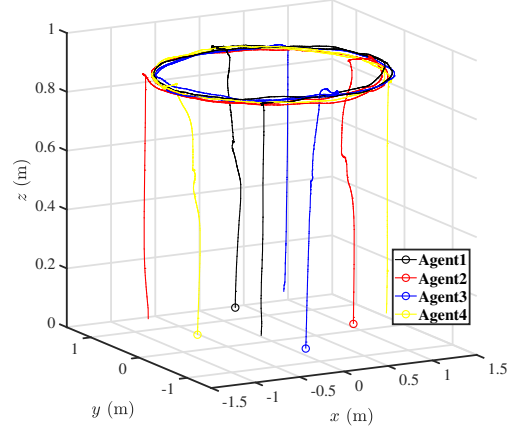
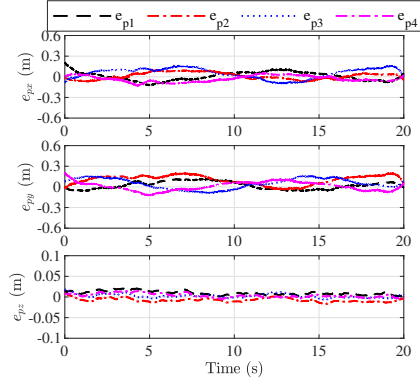


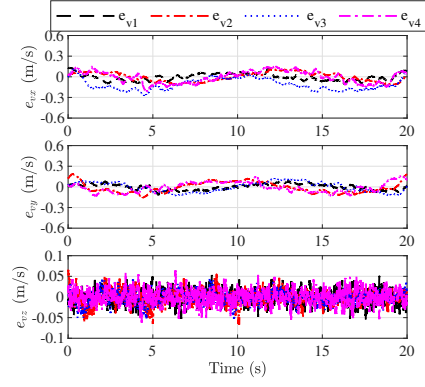
Figure 6.7: Time-varying formation trajectories in 3D of four quadrotors under the EBDETC. The takeoff positions of quadrotors are denoted by the circles.

Table 6.2: Tracking Comparison of Different Control Algorithms

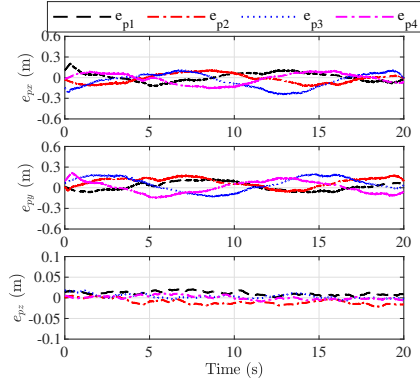
	EBDETC		EBSETC		Controller in [2]	
	Pos. (m)	Vel. (m/s)	Pos. (m)	Vel. (m/s)	Pos. (m)	Vel. (m/s)
MSE of Qd1	0.008	0.0049	0.017	0.0044	0.0084	0.0086
MSE of Qd2	0.0175	0.0108	0.0177	0.0149	0.016	0.0183
MSE of Qd3	0.0163	0.0242	0.0081	0.0199	0.0263	0.0011
MSE of Qd4	0.0069	0.0128	0.0151	0.0206	0.0139	0.0183
MAE of Qd1	0.1166	0.0949	0.1672	0.0907	0.1195	0.1235
MAE of Qd2	0.1591	0.1435	0.159	0.1637	0.1626	0.1773
MAE of Qd3	0.1594	0.1332	0.1134	0.1422	0.1943	0.0386
MAE of Qd4	0.1043	0.15	0.1404	0.1893	0.1473	0.1763
SSE of Qd1	0.0183	0.0336	0.0194	0.0719	0.0604	0.0386
SSE of Qd2	0.0308	0.0778	0.038	0.1214	0.0264	0.07
SSE of Qd3	0.0382	0.031	0.0133	0.0209	0.0346	0.0704
SSE of Qd4	0.0176	0.0493	0.0423	0.1101	0.0675	0.058



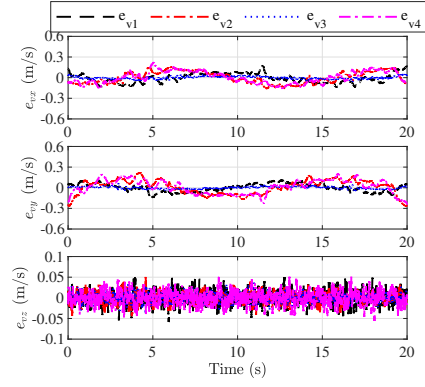
(a)



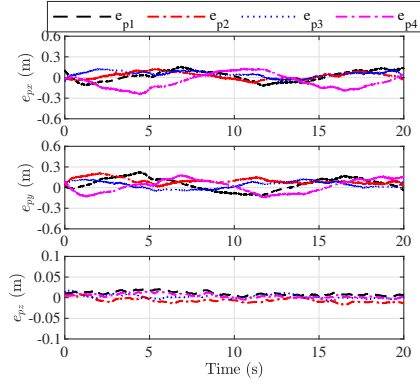
(b)



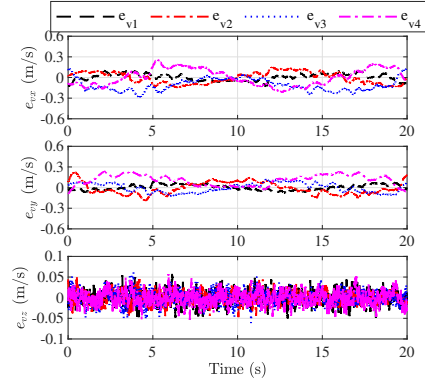
(c)



(d)



(e)



(f)

Figure 6.8: Position tracking errors (e_{pi}) and velocity tracking errors (e_{vi}) for quadrotor i under three controllers. (1) EBDTC (a, b); (2) EBSETC (c, d); (3) controller in [2] (e, f).

Thirdly, to investigate the influence of perturbations on the tracking performance of the developed EBDETC scheme, experiments with three fan speeds (L1, L2 and L3) are conducted. The triggering numbers for each edge are 48, 34, 71, 87, and 102 under L1; 53, 38, 59, 93, and 106 under L2, increased by 2% on average. The triggering numbers under L3 are 50, 32, 68, 96, 114, which only increase 3% compared with those under L2. Therefore, the triggering numbers under different perturbation levels are comparable. Moreover, it can be observed from the results in Table 6.3 that the tracking error differences are negligible. In terms of MSE, MAE, and SSE, the maximum position (or velocity) tracking errors under various perturbations are 2 cm (or 2.5 cm/s), 16 cm (or 15 cm/s), and 5 cm (or 8 cm/s), respectively. This verifies that the proposed EBDETC strategy can handle unknown perturbations effectively.

Table 6.3: Tracking Comparison of Different Perturbations

	L1		L2		L3	
	Pos. (m)	Vel. (m/s)	Pos. (m)	Vel. (m/s)	Pos. (m)	Vel. (m/s)
MSE of Qd1	0.0092	0.0067	0.0113	0.0085	0.008	0.0049
MSE of Qd2	0.0105	0.0117	0.0103	0.0091	0.0175	0.0108
MSE of Qd3	0.0138	0.0119	0.0134	0.0136	0.0163	0.0242
MSE of Qd4	0.0116	0.0157	0.0144	0.015	0.0069	0.0128
MAE of Qd1	0.1294	0.1001	0.1338	0.1174	0.1166	0.0949
MAE of Qd2	0.132	0.1419	0.1287	0.1315	0.1591	0.1435
MAE of Qd3	0.1431	0.1375	0.1377	0.1208	0.1594	0.1332
MAE of Qd4	0.1272	0.1467	0.1478	0.1434	0.1043	0.15
SSE of Qd1	0.0188	0.0102	0.0287	0.0481	0.0183	0.0336
SSE of Qd2	0.0265	0.0344	0.0436	0.0241	0.0308	0.0778
SSE of Qd3	0.0199	0.0247	0.0271	0.0396	0.0382	0.031
SSE of Qd4	0.0382	0.0413	0.0494	0.0641	0.0176	0.0493

Finally, to further validate the proposed asynchronous dynamic edge-event-triggered robust control strategy for multi-quadrotor systems, different formation tracking speeds are chosen and compared: 1) Speed 1: $h_{iv}(t)$; 2) Speed 2: $3 \times h_{iv}(t)$; and 3) Speed 3: $5 \times h_{iv}(t)$. The triggering numbers for the multi-quadrotor system with the asynchronous dynamic edge-event-triggered strategy are shown in Table 6.4, where it can be seen that

the triggering numbers for $i \rightarrow j$ and $j \rightarrow i$ are different. The triggering numbers for each edge are demonstrated in Fig. 6.9. It is observed from Table 6.4 and Fig. 6.9 that the triggering numbers increase when the tracking speed is faster, since the system needs more samples to maintain the time-varying formation, which is reasonable. By comparing Table 6.1 with Table 6.4, it can be seen that the triggering numbers of the designed asynchronous triggering strategy for each edge are less than that of the synchronous mechanism at the same tracking speed. For example, for the edge (1, 3), the triggering numbers are 32 using the synchronous strategy, while the events are triggered 24 times using the asynchronous method. On average, the asynchronous triggering scheme can lower the communication frequency for each edge by 13.3%, i.e., the developed asynchronous edge-event-triggered control scheme can save more communication resources. In addition, compared with the NBSETM, the asynchronous EBDSTM can reduce 71.9% communication resources consumption in terms of the transmission number.

Table 6.4: Triggering Numbers for Different Tracking Speeds

Quadrotors $i \rightarrow j$	$h_{iv}(t)$	$3 \times h_{iv}(t)$	$5 \times h_{iv}(t)$
$1 \rightarrow 2$	21	33	39
$1 \rightarrow 3$	13	20	26
$1 \rightarrow 4$	25	36	40
$2 \rightarrow 1$	28	31	43
$2 \rightarrow 3$	40	47	54
$3 \rightarrow 1$	11	20	24
$3 \rightarrow 2$	44	44	50
$3 \rightarrow 4$	48	51	76
$4 \rightarrow 1$	29	37	35
$4 \rightarrow 3$	53	58	71

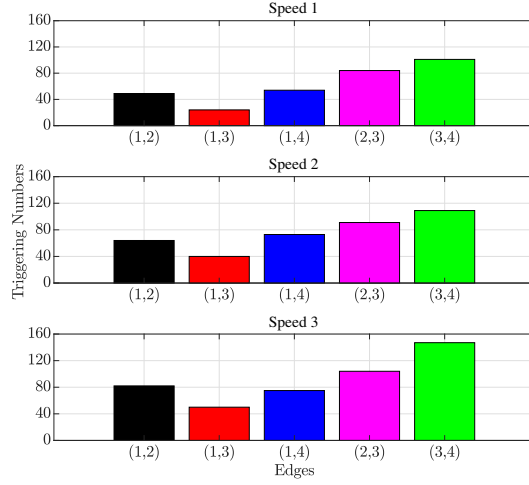


Figure 6.9: Triggering numbers for the edge (i, j) under three tracking speeds.

Table 6.5 shows the tracking results at different tracking speeds. It can be seen from Table 6.2 and Table 6.5 that in terms of MSE, MAE, and SSE, the tracking results of the synchronous and asynchronous methods with the same tracking speed are comparable. For example, the average position tracking errors under the asynchronous schemes are 1.32 cm, 13.4 cm, and 2.8 cm, which are close to those using the synchronous schemes. Compared with the NBSETC, the tracking errors are improved by 14.1% in terms of MAE. Therefore, it validates that the designed asynchronous triggering scheme can further reduce the communication frequency without degrading the system performance. It can also be observed from Table 6.5 that with faster formation speed the tracking errors are larger. However, the tracking results are still acceptable. For example, compared with Speed 1 and Speed 2, the average position tracking errors for each drone in terms of MAE under the fastest speed (Speed 3) only increase by 3.5 cm and 1.4 cm, respectively. Moreover, in terms of MSE, MAE, and SSE, the maximum position (or velocity) tracking errors are 2.5 cm (or 2.2 cm/s), 19.3 cm (or 19.8 cm/s), and 4.8 cm (or 6.3 cm/s), respectively.

Therefore, the designed asynchronous dynamic edge-event-triggered robust controller can handle faster tracking formation.

Table 6.5: Tracking Comparison for Different Tracking Speeds

	$h_{iv}(t)$		$3 \times h_{iv}(t)$		$5 \times h_{iv}(t)$	
	Pos. (m)	Vel. (m/s)	Pos. (m)	Vel. (m/s)	Pos. (m)	Vel. (m/s)
MSE of Qd1	0.0112	0.0058	0.0159	0.0127	0.0148	0.0101
MSE of Qd2	0.0143	0.0078	0.0166	0.0169	0.0249	0.0267
MSE of Qd3	0.015	0.0161	0.0238	0.0204	0.0172	0.0161
MSE of Qd4	0.0124	0.0145	0.0165	0.0142	0.0167	0.0175
MAE of Qd1	0.1281	0.1044	0.1541	0.143	0.1502	0.1437
MAE of Qd2	0.1315	0.1272	0.1607	0.1741	0.1923	0.1975
MAE of Qd3	0.146	0.1458	0.1906	0.1872	0.1673	0.1543
MAE of Qd4	0.1305	0.1364	0.157	0.1475	0.1664	0.1748
SSE of Qd1	0.0201	0.0232	0.0185	0.0174	0.035	0.0319
SSE of Qd2	0.0346	0.0527	0.0385	0.051	0.0478	0.0624
SSE of Qd3	0.0411	0.0497	0.0463	0.0617	0.0286	0.0337
SSE of Qd4	0.0165	0.0338	0.0372	0.0355	0.0219	0.0205

Based on the aforementioned discussion, it is concluded that the proposed fully distributed asynchronous edge-based dynamic ETC algorithm can guarantee the performance of time-varying formation of multiple quadrotors with input saturation and external perturbations on switching topologies, while save bandwidth and improve the resources utilization effectively. The complete experimental video can be accessed at https://youtu.be/dfmi_gVFSGQ.

6.4 Summary

In this chapter, a fully distributed time-varying formation control problem was studied for a leader-follower multi-quadrotor system under unknown perturbations, input saturation, limited bandwidth, and switching graphs. To reduce the communication resources consumption, an asynchronous edge-based adaptive dynamic triggering mechanism (ADTM) was proposed by introducing dynamic variables and adaptive triggering parameters associated with each edge. To attenuate the effects of external disturbances and input saturation,

an event-based adaptive robust control scheme was developed by using the low gain feedback technique, which does not require the upper bounds of perturbations. Moreover, under the developed edge-based ADTM and the robust control strategy, fully distributed time-varying formation control protocols were developed. Based on the designed controllers, the asymptotic convergence of the multi-quadrotor system was achieved, and Zeno behavior was avoided. It is demonstrated through experiments that the communication resources have been significantly saved using the developed fully distributed asynchronous edge-based dynamic event-triggered control strategy without degrading the system performance in comparison with the event-triggered controller in [2].

However, we need to point out that the communication among agents are commonly realized through public networks leading to high risks of being attacked by malicious adversaries. Therefore, we will extend our methods by integrating intelligent control to efficiently ensure network security in the future.

7 Conclusion and Future Work

7.1 Conclusion

This dissertation tackles pressing issues in the development of fully distributed event-based robust control strategies for multi-quadrotor system, approaching the challenges from the following distinct perspectives.

First, a fully distributed event-triggered formation control framework was developed for multi-quadrotor systems with limited communication and external disturbances. By introducing a dynamic sliding manifold, the fully distributed adaptive event-based sliding-mode control approach was proposed. According to the developed control framework, the chattering behavior of the controller was improved, Zeno phenomenon was eliminated, and the uniformly ultimately bounded (UUB) stability was achieved. The proposed methods were validated through simulations and experiments with four quadrotors, showing advantages in faster convergence and lower energy consumption. Compared to the time-triggered formation controller in [1], the developed fully distributed event-triggered control framework achieves a 50.5% reduction in resources consumption and a 48.6% improvement in average position tracking errors (steady-state error), as demonstrated by experimental results. These results contribute to advancing robust and efficient cooperative control in practical multi-agent systems.

Second, a fully distributed adaptive dynamic event-triggered robust control approach based on an integral sliding manifold was developed to handle the formation tracking problem for a leader-follower disturbed multi-quadrotor system, where the upper bounds of perturbations were not required. Asymptotic convergence of the sliding-mode dynamics states was guaranteed by using the proposed method, while actuator wear and unnecessary energy consumption were reduced through a dynamic triggering scheme. Moreover, the triggering intervals are adaptively adjusted, and Zeno behavior has been successfully avoided. The effectiveness of the proposed robust controller has been confirmed by both simulations and experiments using three quadrotors. Experimental results demonstrate that, compared with the fully distributed static ETC protocol and the time-triggered control scheme in [1], the developed fully distributed dynamic ETC scheme achieves reductions in resources consumption by 24.9% and 70.9%, respectively, and improvements in average position tracking errors by 14.5% (MAE) and 54.9% (MAE), respectively. Under various perturbations, the maximum position tracking error is 10.5 cm (MAE).

In addition, the fully distributed consensus tracking control problem was studied for multiple networked Euler–Lagrange systems under noisy state measurements, unknown perturbations, parameter uncertainties, and limited communication resources. A fully distributed dynamic ET scheme was proposed to reduce resources consumption without requiring relative velocity measurements, and Zeno behavior was effectively avoided. To handle imprecise system states, a nested adaptive sliding-mode estimator was developed. Based on this and the triggering mechanism, a fully distributed adaptive dynamic event-based TVSMC scheme was designed for EL systems, without relying on any global information or prior knowledge of perturbation bounds. Asymptotic convergence of the closed-loop system was also guaranteed. Experiments were conducted using four quadrotors, and the results validated the robustness and effectiveness of the developed control

strategy, demonstrating improved tracking accuracy and significant resource savings. As compared to the static ETC scheme and the event-based controller in [2], the developed adaptive dynamic ETC protocol reduces the resources consumption by 21.5% and 26.8%, respectively, and improves the average position tracking errors by 10.8% (MAE) and 47.4% (MAE), respectively.

Finally, the fully distributed time-varying formation control problem was investigated for leader-follower multi-quadrotor systems subject to unknown disturbances, input saturation, limited communication bandwidth, and switching network topologies. An asynchronous edge-based adaptive dynamic triggering mechanism was proposed to reduce communication resources consumption by introducing dynamic variables and adaptive triggering parameters associated with each edge. Moreover, an event-based adaptive robust control scheme using low-gain feedback was developed to handle disturbances and input saturation without requiring prior knowledge of perturbation bounds. Based on the designed control protocols, asymptotic convergence of the system was guaranteed, and Zeno behavior was avoided effectively. Experimental results confirm that the proposed approach significantly reduces resources usage and improves tracking accuracy while ensuring robust stabilization under practical uncertainties and network switching. As compared to the node-based ETC in [2], the proposed approach decreases the resources consumption by 35%, and improves the average position tracking errors by 13.5% (MAE). In addition, the designed control scheme can guarantee the robust stabilization of the system for fast time-varying formation tracking under unknown external disturbances, input saturation and switching graphs.

7.2 Future Work

In this dissertation, the fully distributed event-triggered robust cooperative control problem is discussed for multiple quadrotors. While the results are promising, we still identified several issues that require further exploration. The following are potential future research directions to enhance this work.

- Design of fully distributed control protocols from the perspective of attack immunity. In many multi-agent systems, distributed control protocols are often designed using only relative output information rather than full relative state information. Taking the quadrotors formation control studied in Chapter 5 as an example, relative distances between quadrotors can be obtained via onboard sensors, whereas relative velocity information is generally unavailable. Therefore, compared to protocols relying on relative state information, control protocols based on distributed observers/estimators that utilize only relative output information are more practical. However, existing observer-based protocols have inherent limitations, since some observer information needs to be transmitted over communication networks, making them vulnerable to potential cyber attacks. To overcome this limitation, a class of attack-immune control protocols can be developed. Such protocols would rely solely on relative output information and prohibit information exchange between neighboring agents via communication networks, thereby inherently enhancing resilience against network attacks.
- Design of fully distributed control protocols under heterogeneous time-varying communication delays. In practical distributed multi-quadrotor systems, communication channels often experience delays that vary over time, influenced by factors such as the distance between neighboring agents. Moreover, communication delays differ

across various agent pairs due to network heterogeneity. Therefore, studying the impact of heterogeneous time-varying communication delays on distributed cooperative control systems is more aligned with real-world scenarios. To address this challenge, a distributed compensator can be designed that is robust to communication delays, enabling each follower to obtain the leader's state information. Subsequently, tracking controllers can be developed based on the observed information for each follower to achieve coordination. The designed distributed protocol should satisfy two key requirements: first, the communication topology information is not required, thereby exhibiting robustness to changes in the network structure; second, it should accommodate heterogeneous communication delays that can be arbitrarily large and time-varying.

References

- [1] H. Du, W. Zhu, G. Wen, Z. Duan, and J. Lü, “Distributed formation control of multiple quadrotor aircraft based on nonsmooth consensus algorithms,” *IEEE Trans. Cybern.*, vol. 49, no. 1, pp. 342–353, 2017.
- [2] H. Wang and J. Shan, “Fully distributed event-triggered formation control for multiple quadrotors,” *IEEE Trans. Ind. Electron.*, vol. 70, no. 12, pp. 12 566–12 575, 2023.
- [3] X. Dong, B. Yu, Z. Shi, and Y. Zhong, “Time-varying formation control for unmanned aerial vehicles: Theories and applications,” *IEEE Trans. Control Syst. Technol.*, vol. 23, no. 1, pp. 340–348, 2014.
- [4] T. Chen and J. Shan, “Distributed tracking of a class of underactuated lagrangian systems with uncertain parameters and actuator faults,” *IEEE Trans. Ind. Electron.*, vol. 67, no. 5, pp. 4244–4253, 2019.
- [5] D.-W. Zhang, G.-P. Liu, and L. Cao, “Constrained cooperative control for high-order fully actuated multiagent systems with application to air-bearing spacecraft simulators,” *IEEE/ASME Trans. Mechatron.*, pp. 1–12, 2022.
- [6] X. Dong, Y. Zhou, Z. Ren, and Y. Zhong, “Time-varying formation tracking for second-order multi-agent systems subjected to switching topologies with application

- to quadrotor formation flying,” *IEEE Trans. Ind. Electron.*, vol. 64, no. 6, pp. 5014–5024, 2016.
- [7] B. Ning, Q.-L. Han, Q. Lu, and J. G. Sanjayan, “Cooperative control of multi-robot systems subject to control gain uncertainty,” *IEEE Trans. Ind. Informat.*, 2022.
- [8] W. Xu, G. Hu, D. W. C. Ho, and Z. Feng, “Distributed secure cooperative control under denial-of-service attacks from multiple adversaries,” *IEEE Trans. Cybern.*, vol. 50, no. 8, pp. 3458–3467, 2020.
- [9] W. Ren and R. W. Beard, *Distributed consensus in multi-vehicle cooperative control*. London, U.K.: Springer-Verlag, 2008.
- [10] D. V. Dimarogonas, E. Frazzoli, and K. H. Johansson, “Distributed event-triggered control for multi-agent systems,” *IEEE Trans. Autom. Control*, vol. 57, no. 5, pp. 1291–1297, 2011.
- [11] X. Meng and T. Chen, “Event based agreement protocols for multi-agent networks,” *Automatica*, vol. 49, no. 7, pp. 2125–2132, 2013.
- [12] Y. Fan, G. Feng, Y. Wang, and C. Song, “Distributed event-triggered control of multi-agent systems with combinational measurements,” *Automatica*, vol. 49, no. 2, pp. 671–675, 2013.
- [13] D. P. Borgers and W. M. H. Heemels, “Event-separation properties of event-triggered control systems,” *IEEE Trans. Autom. Control*, vol. 59, no. 10, pp. 2644–2656, 2014.
- [14] R. R. Nair, L. Behera, and S. Kumar, “Event-triggered finite-time integral sliding mode controller for consensus-based formation of multirobot systems with disturbances,” *IEEE Trans. Control Syst. Technol.*, vol. 27, no. 1, pp. 39–47, 2017.

- [15] H. Wang and J. Shan, “Distributed adaptive dynamic event-triggered control for multiple quadrotors,” *IEEE/ASME Trans. Mechatron.*, vol. 28, no. 4, pp. 1900–1910, 2023.
- [16] D. Qian, C. Li, S. Lee, and C. Ma, “Robust formation maneuvers through sliding mode for multi-agent systems with uncertainties,” *IEEE-CAA J. Automatica Sin.*, vol. 5, no. 1, pp. 342–351, 2017.
- [17] C. Yang, D. Huang, W. He, and L. Cheng, “Neural control of robot manipulators with trajectory tracking constraints and input saturation,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 32, no. 9, pp. 4231–4242, 2020.
- [18] H. Wang and J. Shan, “Adaptive dynamic event-based robust control for multiple networked euler–lagrange systems,” *IEEE Trans. Ind. Inform.*, 2025, in press.
- [19] Y. Wang, X.-M. Li, W. Yuan, D. Yao, and H. Li, “Dynamic event-triggered finite-time control for multiple euler-lagrange systems using integral terminal sliding mode,” *Sci. China Technol. Sci.*, vol. 66, no. 11, pp. 3164–3173, 2023.
- [20] S.-L. Du, T. Liu, and D. W. Ho, “Dynamic event-triggered control for leader-following consensus of multiagent systems,” *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 50, no. 9, pp. 3243–3251, 2018.
- [21] S. Zhao, Z. Li, and Z. Ding, “Bearing-only formation tracking control of multiagent systems,” *IEEE Trans. Autom. Control*, vol. 64, no. 11, pp. 4541–4554, 2019.
- [22] S. Zhao, “Affine formation maneuver control of multiagent systems,” *IEEE Trans. Autom. Control*, vol. 63, no. 12, pp. 4140–4155, 2018.

- [23] J. Mei, W. Ren, and G. Ma, “Distributed coordinated tracking with a dynamic leader for multiple euler-lagrange systems,” *IEEE Trans. Autom. Control*, vol. 56, no. 6, pp. 1415–1421, 2011.
- [24] H. Haghshenas, M. A. Badamchizadeh, and M. Baradarannia, “Containment control of heterogeneous linear multi-agent systems,” *Automatica*, vol. 54, pp. 210–216, 2015.
- [25] W. He, B. Xu, Q.-L. Han, and F. Qian, “Adaptive consensus control of linear multiagent systems with dynamic event-triggered strategies,” *IEEE Trans. Cybern.*, vol. 50, no. 7, pp. 2996–3008, 2019.
- [26] C. Xu, H. Xu, Z.-H. Guan, and Y. Ge, “Observer-based dynamic event-triggered semiglobal bipartite consensus of linear multi-agent systems with input saturation,” *IEEE Trans. Cybern.*, vol. 53, no. 5, pp. 3139–3152, 2023.
- [27] B. Chang, J. Fang, and X. Mu, “Fully distributed dynamic event-triggered control for multi-agent systems with input saturation and actuator failure,” *Eur J Control*, vol. 60, pp. 1–10, 2021.
- [28] T. Balch and R. C. Arkin, “Behavior-based formation control for multirobot teams,” *IEEE Trans. Robot. Autom.*, vol. 14, no. 6, pp. 926–939, 1998.
- [29] H. Zhao, J. Shan, L. Peng, and H. Yu, “Distributed event-triggered bipartite consensus for multiagent systems against injection attacks,” *IEEE Trans. Ind. Inform.*, vol. 19, no. 4, pp. 5377–5386, 2022.
- [30] H. Liu, T. Ma, F. L. Lewis, and Y. Wan, “Robust formation control for multiple quadrotors with nonlinearities and disturbances,” *IEEE Trans. Cybern.*, vol. 50, no. 4, pp. 1362–1371, 2018.

- [31] W. Jasim and D. Gu, “Robust team formation control for quadrotors,” *IEEE Trans. Control Syst. Technol.*, vol. 26, no. 4, pp. 1516–1523, 2017.
- [32] K. Harikumar, J. Senthilnath, and S. Sundaram, “Multi-uav oxyrrhis marina-inspired search and dynamic formation control for forest firefighting,” *IEEE Trans. Autom. Sci. Eng.*, vol. 16, no. 2, pp. 863–873, 2018.
- [33] T. Lee, “Geometric control of quadrotor uavs transporting a cable-suspended rigid body,” *IEEE Trans. Control Syst. Technol.*, vol. 26, no. 1, pp. 255–264, 2017.
- [34] N. Nigam, S. Bieniawski, I. Kroo, and J. Vian, “Control of multiple uavs for persistent surveillance: Algorithm and flight test results,” *IEEE Trans. Control Syst. Technol.*, vol. 20, no. 5, pp. 1236–1251, 2011.
- [35] K.-K. Oh, M.-C. Park, and H.-S. Ahn, “A survey of multi-agent formation control,” *Automatica*, vol. 53, pp. 424–440, 2015.
- [36] F. Mehdifar, C. P. Bechlioulis, F. Hashemzadeh, and M. Baradarannia, “Prescribed performance distance-based formation control of multi-agent systems,” *Automatica*, vol. 119, p. 109086, 2020.
- [37] Y. Huang and Z. Meng, “Bearing-based distributed formation control of multiple vertical take-off and landing uavs,” *IEEE Trans. Control Netw. Syst.*, vol. 8, no. 3, pp. 1281–1292, 2021.
- [38] Y.-B. Bae, Y.-H. Lim, and H.-S. Ahn, “Distributed robust adaptive gradient controller in distance-based formation control with exogenous disturbance,” *IEEE Trans. Autom. Control*, vol. 66, no. 6, pp. 2868–2874, 2021.

- [39] D. Van Vu, M. H. Trinh, P. D. Nguyen, and H.-S. Ahn, “Distance-based formation control with bounded disturbances,” *IEEE Control Syst. Lett.*, vol. 5, no. 2, pp. 451–456, 2021.
- [40] M. H. Trinh, S. Zhao, Z. Sun, D. Zelazo, B. D. O. Anderson, and H.-S. Ahn, “Bearing-based formation control of a group of agents with leader-first follower structure,” *IEEE Trans. Autom. Control*, vol. 64, no. 2, pp. 598–613, 2019.
- [41] T. Han, Z. Lin, R. Zheng, and M. Fu, “A barycentric coordinate-based approach to formation control under directed and switching sensing graphs,” *IEEE Trans. Cybern.*, vol. 48, no. 4, pp. 1202–1215, 2018.
- [42] M. Ranjbar, M. T. H. Beheshti, and S. Bolouki, “Event-based formation control of networked multi-agent systems using complex laplacian under directed topology,” *IEEE Control Syst. Lett.*, vol. 5, no. 3, pp. 1085–1090, 2021.
- [43] L. Chen, J. Mei, C. Li, and G. Ma, “Distributed leader–follower affine formation maneuver control for high-order multiagent systems,” *IEEE Trans. Autom. Control*, vol. 65, no. 11, pp. 4941–4948, 2020.
- [44] C. Yan, W. Zhang, H. Su, and X. Li, “Adaptive bipartite time-varying output formation control for multiagent systems on signed directed graphs,” *IEEE Trans. Cybern.*, vol. 52, no. 9, pp. 8987–9000, 2021.
- [45] X. Liu, F. Deng, W. Wei, and F. Wan, “Formation tracking control of networked systems with time-varying delays and sampling under fixed and markovian switching topology,” *IEEE Trans. Control Netw. Syst.*, vol. 9, no. 2, pp. 601–612, 2022.
- [46] L. Tian, Y. Hua, X. Dong, J. Lü, and Z. Ren, “Distributed time-varying group formation tracking for multiagent systems with switching interaction topologies via

- adaptive control protocols,” *IEEE Trans. Ind. Electron.*, vol. 18, no. 12, pp. 8422–8433, 2022.
- [47] W. Xu, D. W. C. Ho, L. Li, and J. Cao, “Event-triggered schemes on leader-following consensus of general linear multiagent systems under different topologies,” *IEEE Trans. Cybern.*, vol. 47, no. 1, pp. 212–223, 2017.
- [48] H. Zhao, X. Dai, Q. Zhang, and J. Ding, “Robust event-triggered model predictive control for multiple high-speed trains with switching topologies,” *IEEE Trans. Veh. Technol.*, vol. 69, no. 5, pp. 4700–4710, 2020.
- [49] T. Chen and J. Shan, “Fixed-time consensus control of multiagent systems using input shaping,” *IEEE Trans. Ind. Electron.*, vol. 66, no. 9, pp. 7433–7441, 2018.
- [50] Z. Li, W. Ren, X. Liu, and L. Xie, “Distributed consensus of linear multi-agent systems with adaptive dynamic protocols,” *Automatica*, vol. 49, no. 7, pp. 1986–1995, 2013.
- [51] W. Yu, W. Ren, W. X. Zheng, G. Chen, and J. Lü, “Distributed control gains design for consensus in multi-agent systems with second-order nonlinear dynamics,” *Automatica*, vol. 49, no. 7, pp. 2107–2115, 2013.
- [52] J. Mei, W. Ren, and J. Chen, “Distributed consensus of second-order multi-agent systems with heterogeneous unknown inertias and control gains under a directed graph,” *IEEE Trans. Autom. Control*, vol. 61, no. 8, pp. 2019–2034, 2015.
- [53] J. Mei, W. Ren, B. Li, and G. Ma, “Distributed containment control for multiple unknown second-order nonlinear systems with application to networked lagrangian systems,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 26, no. 9, pp. 1885–1899, 2014.

- [54] J. Liang, Y. Chen, N. Lai, B. He, Z. Miao, and Y. Wang, “Low-complexity prescribed performance control for unmanned aerial manipulator robot system under model uncertainty and unknown disturbances,” *IEEE Trans. Ind. Informat.*, vol. 18, no. 7, pp. 4632–4641, 2021.
- [55] H. Shen and Y.-J. Pan, “Tracking synchronization improvement of networked manipulators using novel adaptive nonsingular terminal sliding mode control,” *IEEE Trans. Ind. Electron.*, vol. 68, no. 5, pp. 4279–4287, 2021.
- [56] F. Plestan, Y. Shtessel, V. Bregeault, and A. Poznyak, “New methodologies for adaptive sliding mode control,” *Int. J. Control*, vol. 83, no. 9, pp. 1907–1919, 2010.
- [57] J. Baek, M. Jin, and S. Han, “A new adaptive sliding-mode control scheme for application to robot manipulators,” *IEEE Trans. Ind. Electron.*, vol. 63, no. 6, pp. 3628–3637, 2016.
- [58] D. Yang, W. Ren, and X. Liu, “Fully distributed adaptive sliding-mode controller design for containment control of multiple lagrangian systems,” *Systems Control Lett.*, vol. 72, pp. 44–52, 2014.
- [59] P. Li, F. Jabbari, and X.-M. Sun, “Containment control of multi-agent systems with input saturation and unknown leader inputs,” *Automatica*, vol. 130, p. 109677, 2021.
- [60] X. Wang, H. Su, X. Wang, and G. Chen, “Fully distributed event-triggered semiglobal consensus of multi-agent systems with input saturation,” *IEEE Trans. Ind. Electron.*, vol. 64, no. 6, pp. 5055–5064, 2016.
- [61] X. Li, X. Dong, Q. Li, and Z. Ren, “Event-triggered time-varying formation control for general linear multi-agent systems,” *J. Franklin Inst. Eng. Appl. Math.*, vol. 356, no. 17, pp. 10 179–10 195, 2019.

- [62] Z. Huang, R. Bauer, and Y.-J. Pan, “Event-triggered formation tracking control with application to multiple mobile robots,” *IEEE Trans. Ind. Electron.*, vol. 70, no. 1, pp. 846–854, 2023.
- [63] X. Jin, Y. Tang, Y. Shi, W. Zhang, and W. Du, “Event-triggered formation control for a class of uncertain euler–lagrange systems: Theory and experiment,” *IEEE Trans. Control Syst. Technol.*, vol. 30, no. 1, pp. 336–343, 2022.
- [64] D. Yao, H. Li, and Y. Shi, “Adaptive event-triggered sliding-mode control for consensus tracking of nonlinear multiagent systems with unknown perturbations,” *IEEE Trans. Cybern.*, vol. 53, no. 4, pp. 2672–2684, 2022.
- [65] A. Girard, “Dynamic triggering mechanisms for event-triggered control,” *IEEE Trans. Autom. Control*, vol. 60, no. 7, pp. 1992–1997, 2014.
- [66] L. Cao, Y. Pan, H. Liang, and C. K. Ahn, “Event-based adaptive neural network control for large-scale systems with nonconstant control gains and unknown measurement sensitivity,” *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 54, no. 11, pp. 7027–7038, 2024.
- [67] R. Cao and L. Cheng, “Distributed dynamic event-triggered control for euler–lagrange multiagent systems with parametric uncertainties,” *IEEE Trans. Cybern.*, vol. 53, no. 2, pp. 1272–1284, 2021.
- [68] S. Guo, Y. Pan, H. Li, and L. Cao, “Dynamic event-driven adp for n-player nonzero-sum games of constrained nonlinear systems,” *IEEE Trans. Autom. Sci. Eng.*, 2024.
- [69] B. Cheng and Z. Li, “Coordinated tracking control with asynchronous edge-based event-triggered communications,” *IEEE Trans. Autom. Control*, vol. 64, no. 10, pp. 4321–4328, 2019.

- [70] H. Shen, Y.-J. Pan, U. Ahmad, and B. He, “Pose synchronization of multiple networked manipulators using nonsingular terminal sliding mode control,” *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 12, pp. 7497–7509, 2021.
- [71] Z. Wang, Y. Zou, Y. Liu, and Z. Meng, “Distributed control algorithm for leader–follower formation tracking of multiple quadrotors: Theory and experiment,” *IEEE/ASME Trans. Mechatron.*, vol. 26, no. 2, pp. 1095–1105, 2020.
- [72] L. Brinón-Arranz, A. Seuret, and C. Canudas-de Wit, “Cooperative control design for time-varying formations of multi-agent systems,” *IEEE Trans. Autom. Control*, vol. 59, no. 8, pp. 2283–2288, 2014.
- [73] B. Cheng and Z. Li, “Fully distributed event-triggered protocols for linear multiagent networks,” *IEEE Trans. Autom. Control*, vol. 64, no. 4, pp. 1655–1662, 2018.
- [74] X. Li, Y. Tang, and H. R. Karimi, “Consensus of multi-agent systems via fully distributed event-triggered control,” *Automatica*, vol. 116, p. 108898, 2020.
- [75] W. Xu, D. W. C. Ho, J. Zhong, and B. Chen, “Event/self-triggered control for leader-following consensus over unreliable network with dos attacks,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 10, pp. 3137–3149, 2019.
- [76] H. Wang, J. Shan, and H. Alkomy, “Fully distributed edge-based dynamic event-triggered control for multiple quadrotors,” *IEEE/ASME Trans. Mechatron.*, vol. 29, no. 4, pp. 3203–3214, 2024.
- [77] X. Zhang, Q. Yang, F. Xiao, H. Fang, and J. Chen, “Linear formation control of multi-agent systems,” *Automatica*, vol. 171, p. 111935, 2025.

- [78] C. Binglong, L. Xiangdong, and C. Zhen, “Exponential time-varying sliding mode control for large angle attitude eigenaxis maneuver of rigid spacecraft,” *Chin. J. Aeronaut.*, vol. 23, no. 4, pp. 447–453, 2010.
- [79] D. Qin, Z. Jin, A. Liu, W.-a. Zhang, and L. Yu, “Event-triggered distributed predictive cooperation control for multi-agent systems subject to bounded disturbances,” *Automatica*, vol. 157, p. 111230, 2023.
- [80] X. Song, C. Wu, V. Stojanovic, and S. Song, “1 bit encoding–decoding-based event-triggered fixed-time adaptive control for unmanned surface vehicle with guaranteed tracking performance,” *Control Eng. Practice*, vol. 135, p. 105513, 2023.
- [81] J. Song, Y.-K. Wang, Y. Niu, H.-K. Lam, S. He, and H. Liu, “Periodic event-triggered terminal sliding mode speed control for networked pmsm system: A ga-optimized extended state observer approach,” *IEEE/ASME Trans. Mechatron.*, vol. 27, no. 5, pp. 4153–4164, 2022.
- [82] J. Wang, H. Pan, and W. Sun, “Event-triggered adaptive fault-tolerant control for unknown nonlinear systems with applications to linear motor,” *IEEE/ASME Trans. Mechatron.*, vol. 27, no. 2, pp. 940–949, 2022.
- [83] B. Cheng, Z. Wu, and Z. Li, “Distributed edge-based event-triggered formation control,” *IEEE Trans. Cybern.*, vol. 51, no. 3, pp. 1241–1252, 2021.
- [84] Y. Cai, H. Zhang, H. Su, J. Zhang, and Q. He, “The bipartite edge-based event-triggered output tracking of heterogeneous linear multiagent systems,” *IEEE Trans. Cybern.*, vol. 53, no. 2, pp. 967–978, 2021.

- [85] W. Xu, W. He, D. W. Ho, and J. Kurths, “Fully distributed observer-based consensus protocol: Adaptive dynamic event-triggered schemes,” *Automatica*, vol. 139, p. 110188, 2022.
- [86] M. Cao, F. Xiao, and L. Wang, “Second-order consensus in time-delayed networks based on periodic edge-event driven control,” *Syst. Control Lett.*, vol. 96, pp. 37–44, 2016.
- [87] X. Meng, L. Xie, and Y. C. Soh, “Event-triggered output regulation of heterogeneous multiagent networks,” *IEEE Trans. Autom. Control*, vol. 63, no. 12, pp. 4429–4434, 2018.
- [88] D. Yao, H. Li, and Y. Shi, “Event-based average consensus of disturbed mass via fully distributed sliding mode control,” *IEEE Trans. Autom. Control*, vol. 69, no. 3, pp. 2015–2022, 2024.
- [89] P. H. S. Coutinho and R. M. Palhares, “Codesign of dynamic event-triggered gain-scheduling control for a class of nonlinear systems,” *IEEE Trans. Autom. Control*, vol. 67, no. 8, pp. 4186–4193, 2022.
- [90] X. Song, Y. Song, V. Stojanovic, and S. Song, “Improved dynamic event-triggered security control for t–s fuzzy lpv-pde systems via pointwise measurements and point control,” *Int. J. Fuzzy Syst.*, vol. 25, no. 8, pp. 3177–3192, 2023.
- [91] W. Li, H. Zhang, Y. Zhou, and Y. Wang, “Bipartite formation tracking for multi-agent systems using fully distributed dynamic edge-event-triggered protocol,” *IEEE-CAA J. Automatica Sin.*, vol. 9, no. 5, pp. 847–853, 2022.
- [92] Y. Hong, J. Hu, and L. Gao, “Tracking control for multi-agent consensus with an active leader and variable topology,” *Automatica*, vol. 42, no. 7, pp. 1177–1182, 2006.

- [93] S. P. Bhat and D. S. Bernstein, “Finite-time stability of continuous autonomous systems,” *SIAM J. Control Optim.*, vol. 38, no. 3, pp. 751–766, 2000.
- [94] Z. Li, J. Yan, W. Yu, and J. Qiu, “Adaptive event-triggered control for unknown second-order nonlinear multiagent systems,” *IEEE Trans. Cybern.*, vol. 51, no. 12, pp. 6131–6140, 2020.
- [95] H. Li, X. Liao, T. Huang, and W. Zhu, “Event-triggering sampling based leader-following consensus in second-order multi-agent systems,” *IEEE Trans. Autom. Control*, vol. 60, no. 7, pp. 1998–2003, 2014.
- [96] H. K. Khalil, *Nonlinear Systems*. Prentice Hall, 2002.
- [97] Z.-G. Wu, Y. Xu, Y.-J. Pan, P. Shi, and Q. Wang, “Event-triggered pinning control for consensus of multiagent systems with quantized information,” *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 11, pp. 1929–1938, 2018.
- [98] Z. Huang, Y.-J. Pan, and R. Bauer, “Sampled-data event-triggered control with application to quadcopter formations,” *Int. J. Control*, vol. 96, no. 1, pp. 12–23, 2023.
- [99] V. I. Utkin, *Sliding modes in control and optimization*. Springer Science & Business Media, 2013.
- [100] Z.-H. Guan, F.-L. Sun, Y.-W. Wang, and T. Li, “Finite-time consensus for leader-following second-order multi-agent networks,” *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 59, no. 11, pp. 2646–2654, 2012.
- [101] P. A. Ioannou and J. Sun, *Robust adaptive control*. PTR Prentice-Hall Upper Saddle River, NJ, 1996, vol. 1.

- [102] R. R. Nair, L. Behera, and S. Kumar, “Event-triggered finite-time integral sliding mode controller for consensus-based formation of multirobot systems with disturbances,” *IEEE Trans. Control Syst. Technol.*, vol. 27, no. 1, pp. 39–47, 2019.
- [103] Q. Song, F. Liu, J. Cao, and W. Yu, “Pinning-controllability analysis of complex networks: an m-matrix approach,” *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 59, no. 11, pp. 2692–2701, 2012.
- [104] S. Sastry and M. Bodson, *Adaptive Control: Stability, Convergence and Robustness*. Prentice Hall, 1989.
- [105] S.-B. Choi, D.-W. Park, and S. Jayasuriya, “A time-varying sliding surface for fast and robust tracking control of second-order uncertain systems,” *Automatica*, vol. 30, no. 5, pp. 899–904, 1994.
- [106] W. Ni and D. Cheng, “Leader-following consensus of multi-agent systems under fixed and switching topologies,” *Syst. Control Lett.*, vol. 59, no. 3-4, pp. 209–217, 2010.
- [107] Z. Lin, *Low Gain Feedback*. London, U.K.: Springer, 1998.
- [108] Y. Su and J. Huang, “Cooperative output regulation with application to multi-agent consensus under switching network,” *IEEE Trans. Cybern.*, vol. 42, no. 3, pp. 864–875, 2012.

List of Publications

1. **H. Wang** and J. Shan, “Fully distributed event-triggered formation control for multiple quadrotors,” *IEEE Transactions on Industrial Electronics*, vol. 70, no. 12, pp. 12 566–12 575, 2023, DOI 10.1109/TIE.2023.3239870.
2. **H. Wang** and J. Shan, “Distributed adaptive dynamic event-triggered control for multiple quadrotors,” *IEEE/ASME Transactions on Mechatronics*, vol. 28, no. 4, pp. 1900–1910, 2023, DOI 10.1109/TMECH.2023.3273395.
3. **H. Wang**, J. Shan, and H. Alkomy, “Fully distributed edge-based dynamic event-triggered control for multiple quadrotors,” *IEEE/ASME Transactions on Mechatronics*, vol. 29, no. 4, pp. 3203–3214, 2024, DOI 10.1109/TMECH.2024.3427692.
4. **H. Wang** and J. Shan, “Adaptive dynamic event-based robust control for multiple networked euler–lagrange systems,” *IEEE Transactions on Industrial Informatics*, in press, 2025, DOI 10.1109/TII.2025.3567383
5. **H. Wang** and J. Shan, “Secure dynamic event-triggered formation tracking control for multi-agent systems,” *IEEE Transactions on Industrial Informatics*, in press, 2025.