Module 6 Practice Problems – Functions in Practice

1. Compute

   $g(f(x)) = x$, $f(g(x)) = x$, so f and g are inverse functoins of each other.

2. (a) $x^3 - 6x + x^3 = 2x^3 - 6x = 2x(x^2 - 3)$

   (b) $6x$

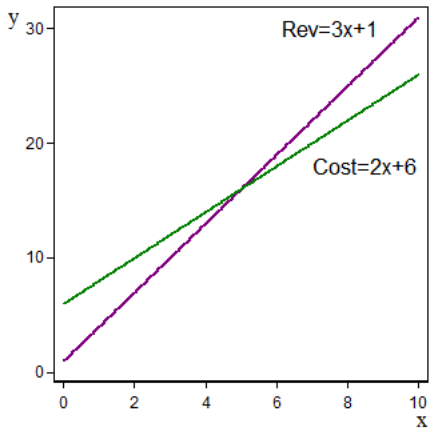
   (c) $x^3(6x - x^3) = 6x^4 - x^6$

   (d) $\frac{x^3}{6x - x^3} = \frac{x^2}{6 - x^2}$

   (e) $\frac{6x - x^3}{x^3} = \frac{6 - x^2}{x^2}$

   (f) $(6x - x^3)^3 = (6 - 1)^3 = 125$

   (g) $6x^3 - (x^3)^3 = 6x^3 - x^9 = 6 - 1 = 5$

3. 

4. When $x = 4096$, the two processes are equal. If $x > 4096$, then process 2 outperforms process 1.

5. 400 hours of security

6. (a) $p = \frac{100 - D}{2}$ where D is quantity demanded.

   (b) $p = 2(S - 20)$ where S is quantity supplied.

   (c) Remember that price is on y-axis and quantity is on the x-axis. The y-axis intercept is $p = 50, D = 0$ and the x-axis intercept is $p = 0, D = 100$

   (d) Remember that price is on y-axis and quantity is on the x-axis. The y-axis intercept is $p = -40, S = 0$ and the x-axis intercept is $p = 0, S = 20$

   (e) Set $p = p$, so we have $\frac{100 - D}{2} = 2(S - 20)$ and set $S = D = Q$ so we have $\frac{100 - Q}{2} = 2(Q - 20)$ and solve for Q. $Q^* = 36$ and $p = \frac{100 - 36}{2} = 36$

(f) See spreadsheet, "FunctionsExcelSolutions.xlsx", sheet "Q6f"

7. (a) $D^{-1}(p) = \left(\frac{100-p}{2}\right)^{1/2}$

   (b) $S^{-1}(p) = \left(\frac{p-2}{3}\right)^{3/2}$

   (c) See spreadsheet, "FunctionsExcelSolutions.xlsx", sheet "Q7c"

   (d) See spreadsheet, "FunctionsExcelSolutions.xlsx", sheet "Q7d"

8. (a) See spreadsheet, "FunctionsExcelSolutions.xlsx", sheet "Q8a"

   (b) See spreadsheet, "FunctionsExcelSolutions.xlsx", sheet "Q8b"

   (c) See spreadsheet, "FunctionsExcelSolutions.xlsx", sheet "Q8c"

   (d) See spreadsheet, "FunctionsExcelSolutions.xlsx", sheet "Q8d"

9. (a) one-to-one function

   (b) neither a function nor a one-to-one function

   (c) a function which is not one-to-one

   (d) a function which is not one-to-one

10. $150 = \frac{a}{5}, a = 750$, $r = \frac{750}{10}, r = 75$

11. $6 = a\frac{180}{10}$ Solve for $a = 1/3$. It takes 15 days for 16 workers to canvas 720 households.

12. , wage is \$18 per hour. If an individual had human capital of 2 and a wage of \$24, how much time did she spend unemployed?

    $w = a\frac{h^3}{u}$ solve for the constant parameter: $a = 4$ then solve $24 = 4\frac{2^3}{u}, u = 1/3$

13. (a)

$$\begin{aligned} C &= b\left(Y - tY\right) \\ C &= bY\left(1-t\right) \end{aligned}$$

$$\begin{aligned} Y &= bY\left(1-t\right) + \bar{G} + \bar{I} \\ Y - bY\left(1-t\right) &= \bar{G} + \bar{I} \\ Y\left(1 - b\left(1-t\right)\right) &= \bar{G} + \bar{I} \\ Y^* &= \frac{\bar{G} + \bar{I}}{1 - b\left(1-t\right)} \end{aligned}$$

$$\begin{aligned} C &= bY\left(1-t\right) \\ C &= b\left(\frac{\bar{G} + \bar{I}}{1 - b\left(1-t\right)}\right)\left(1-t\right) \\ C^* &= \left(\frac{b\left(1-t\right)\left(\bar{G} + \bar{I}\right)}{1 - b\left(1-t\right)}\right) \end{aligned}$$

(b) Consider Y. In short, a decrease in t increases Y. Notice that a smaller t makes $b(1-t)$ larger which in turn makes $1-b(1-t)$ smaller. A smaller right-hand-side (RHS) denominator always makes the left-hand-side (LHS) variable larger.

(c) An increase in $\bar{I}$ causes C to increase. Notice that $\bar{I}$ is in the RHS numerator. An increase in the RHS numerator will always increase the LHS variable.

14. (a) $2\left(23+w^2\right)^{1/2}$

(b) $L=144, Y=24$

(c) $Y(L(r))=2\left(32-2r\right)^{1/2}$

(d) $Y^{-1}(L(r))=16-\frac{r^2}{8}$

(e) $Y^{-1}(Y(L(r)))=16-\frac{\left(2(32-2r)^{1/2}\right)^2}{8}=r$