Module 6 Practice Problems – Functions in Practice

1. Compute $g(f(x))$ and $f(g(x))$ when $g(x) = 2x, f(x) = \frac{1}{2}x$ What do you notice about your answer?

2. When $v(x) = x^3, w(x) = 6x - x^3$ compute:

   (a) $v(x) - w(x)$

   (b) $v(x) + w(x)$

   (c) $v(x)w(x)$

   (d) $\frac{v(x)}{w(x)}$

   (e) $\frac{w(x)}{v(x)}$

   (f) $v(w(1))$

   (g) $w(v(1))$

3. A firm earns revenue of \$$3x - 1$ with costs $2x + 6$ where x is quantity produced. Illustrate on a graph the point at which revenue is equal to cost.

4. A firm uses x units of inputs to produce output, y. There are two functions that represent the manufacturing processes available:

$$\begin{aligned} (1)\ y &= 32x^{3/2} \\ (2)\ y &= \frac{1}{2}x^2 \end{aligned}$$

   For what levels of input, x, does process 1 outperform process 2?

5. It costs a private security company \$30 to provide one hour of security. There is also a fixed cost in security equipment of \$2,000. The company earns revenue of \$75 per hour. How many hours of security must the company provide to earn a profit of \$16,000?

6. Consider a demand function and a supply function that determine quantities. $S(p)$ is the quantity supplied and $D(p)$ is the quantity produced. The quantity depends on price, p.

$$\begin{aligned} D(p) &= 100 - 2p \\ S(p) &= 20 + \frac{p}{2} \end{aligned}$$

   (a) Re-arrange and isolate p to find the inverse demand function.

   (b) Re-arrange and isolate p to find the inverse supply function.

   (c) Find the x and y intercepts for the inverse demand function.

   (d) Find the x and y intercepts for the inverse supply function.

   (e) At what quantity is supply (S) equal to demand (D)? At what price is supply equal to demand?

   (f) Graph the inverse supply and demand functions using a spreadsheet program such as Excel. Identify equilibrium and the y-axis intercepts.

7. Consider a demand function and a supply function that determine quantities given price, $S(p), D(p)$.

$$
\begin{aligned}
S(p) &= 2 + 3p^{2/3} \\
D(p) &= 100 - 2p^2
\end{aligned}
$$

(a) Find the inverse function, $D^{-1}(p)$. Demonstrate that your answers are correct by showing $D(D^{-1}(p)) = p$.

(b) Find the inverse function, $S^{-1}(p)$. Demonstrate that your answers are correct by showing $S(S^{-1}(p)) = p$.

(c) Graph the equations $S(p)$ and $S^{-1}(p)$ using a spreadsheet program such as Excel. Comment on your result.

(d) Graph the equations $D(p)$ and $D^{-1}(p)$ using a spreadsheet program such as Excel. Comment on your result.

8. Sketch the following graphs using a spreadsheet program such as Excel.

(a) $x^2 - 10x$

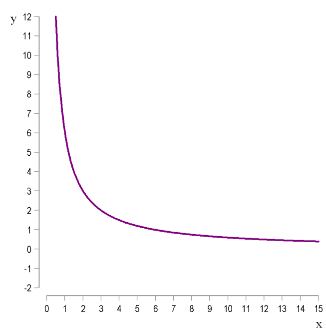
(b) $x - \sqrt{x}$

(c) $e^{-x} + x$

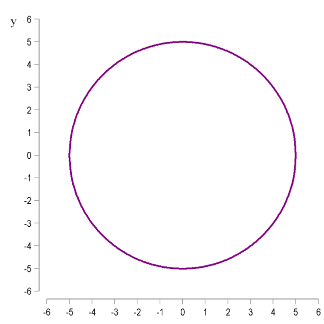
(d) $e^x - x$

9. Identify whether each of the following mathematical relations is a function, a one-to-one function, or neither.

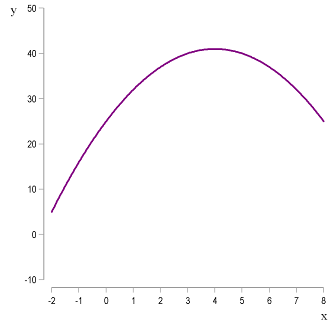
(a)



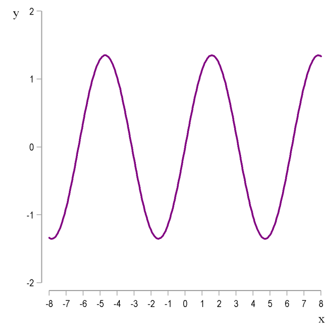
(b)



(c)



(d)



10. Businesses often find thre are decreasing benefits to hiring an additional workers. Suppose the additional benefit of a new employee at a retail clothing store, r, is inversely related to the number of current employees, n. The relation is $r = \frac{a}{n}$ where a is a constant parameter. There are currently 5 workers, and the additional benefit to hiring a new worker is 150. If there were currently 10 workers, what would be benefit of hiring an additional worker?

11. A campaign manager knows the number of days, t, it takes to canvas an area varies directly with the number of households, h and inversely with the number of volunteers, v, so the equation is $t = a\frac{h}{v}$ where a is a constant parameter. If 10 volunteers can canvas 180 households in 6 days, how many days long would it take for 16 workers to canvas 720 households?

12. Suppose wage, w, varies directly with the cube of an individual's human capital, h. Wage also varies inversely with time spent unemployed, u. Wage is also impacted by a constant multiplicative parameter, a. When an individual has $h = 3, u = 6$ wage is \$18 per hour. If an individual had human capital of 2 and a wage of \$24, how much time did she spend unemployed?

13. Consider the following system of linear equations for a simple macroeconomy. Y is GDP; C is consumption; T are taxes; G is government spending; t is the tax rate; b is the marginal propensity to consume.

$$\begin{aligned} Y &= C + \bar{G} + \bar{I} \\ C &= b\,(Y - T) \\ T &= tY \end{aligned}$$

Y, C, T are unknown variables. $\bar{G}, \bar{I}, t, b,$ are known variables. $0 \le t \le 1$ and $0 \le b \le 1$.

(a) Solve for Y and C as functions of the known variables.

(b) What happens to Y if t decreases?

(c) What happens to C if $\bar{I}$ decreases? Explain.

14. Consider the macroeconomy that will produce economic activity Y which depends on the quantity of hours worked, L. The function that represents this relationship is $Y(L) = (4)L^{1/2}$. Hours worked depends on how much you are paid. This is represented by the following function: $L(w) = 21 + w^2$.

(a) Write a composite function, $Y(L(w))$.

(b) If $w = 10$, what are Y and L?

Now suppose instead that hours worked, L, is inversely related to leisure time (i.e. time spent sleeping, having fun, cooking, etc.). Call this r for rest. We now have $L(r) = 4(24 - r)$.

(c) Write a composite function, $Y(L(r))$.

(d) Write down the inverse function for $Y(L(r))$?

(e) Prove that your inverse function is correct.