

**PARKING POLICIES IN DENSE URBAN AREAS: A COMPARISON OF
HOURLY, PROGRESSIVE, AND TIME-OF-DAY PRICING**

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Abstract

Parking supply and demand are often imbalanced in urban areas, causing adverse consequences such as excessive search times and long walking distances. Many parking authorities use parking pricing as a demand management strategy by charging either a fixed daily fee or an hourly price for parking. An emerging alternative is progressive pricing, where drivers pay an hourly price that increases if their parking duration is longer than a predetermined threshold.

An econometric model is developed to investigate the optimal design of hourly and progressive pricing under revenue and social welfare maximization. The equilibrium properties of the generalized econometric model are then examined. A parking capable micro-simulation model of a study area in downtown Toronto, Canada, is also developed to validate the econometric model. Results show that when the market is composed of two distinct driver groups, progressive pricing yields a greater revenue compared to hourly pricing under revenue maximization.

Dedication

I am grateful to all those who encouraged me and supported me to accomplish this incredible goal in my life. I want to dedicate my thesis work to my parents for their endless love, support, and encouragement. Also, to my siblings for always being present and for the moral support they gave me throughout this stage of my life. And lastly, to those I am honoured to call family, who support me at all times during my journey and are the most priceless gift that God has gifted me. To all of them, many thanks.

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Abbreviations

Blvd	Boulevard
CMS	Curbside Management Strategy
Crt	Court
CVs	Commercial Vehicles
EUR	Euros
GPS	Global Positioning System
MAPC	Metropolitan Area Planning Council
ORCA	One Region Card
St.	Street
TPA	Toronto Parking Authority
TTC	Toronto Transit Commission

Chapter 1

Introduction

In this chapter, we provide the motivation for the research followed by research goals, objectives, and scope. Following this, the thesis structure is briefly described to give an overview of topics covered.

1.1 Motivation

High demand for downtown cores and insufficient parking supply lead to adverse consequences, including long parking search times and walking distances to final destinations. Parking authorities often utilize parking management strategies to increase the performance of their parking system and achieve an optimal allocation of parking resources. Parking management strategies refer to policies or programs that will yield a more efficient parking system [1]. Two commonly applied parking pricing strategies are fixed daily fees (e.g. \$3 paid at the entrance) and hourly pricing (e.g. \$1/hr), chosen and adjusted depending on the time of day and location of parking.

Pricing is used to manage off-street facilities (parking garages and parking lots) and on-street spaces commonly aggregated into clusters (e.g., all spaces on the side of a block can be labelled as a parking cluster). Daily commuters demanding long stays often park off-street with a fixed fee, whereas leisure travellers and commercial delivery vehicles prefer on-street parking at an hourly price [2]. Hourly pricing is always superior to fixed fees because it segmentizes demand, such that each driver pays per number of hours parked unique to that

driver [3].

Progressive pricing (also known as escalating or graduated pricing) is an emerging alternative, where the price per hour increases if parking duration (dwell time) exceeds a certain threshold. Figure 1.1 presents a progressive pricing profile example, where drivers are charged the hourly *base price* if they park less than the cutoff time and the *augmented price* for any hours parked beyond the cutoff time.

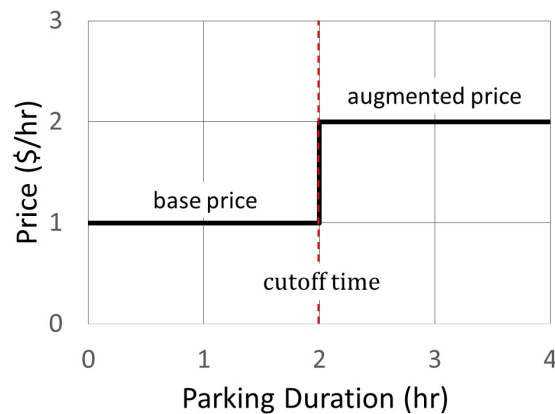


Figure 1.1: Step-wise pricing profile implemented in Whistler, British Columbia

Progressive pricing is less often implemented when compared to hourly pricing. Cities in the United States, including New York (New York), Jackson (Mississippi), Sacramento (California), and Aspen (Colorado), are currently implementing progressive pricing for some on-street parking locations in their jurisdiction. In Canada, only two instances of the implementation of progressive pricing were found. The central metropolitan area of Vancouver (British Columbia) and touristic city Whistler (British Columbia) are the only two known examples of progressive pricing as of the year 2021 in Canada. Other major metropolitan areas, such as Toronto and Montreal, are yet to implement progressive pricing.

Existing studies of progressive pricing for parking include [4] and [5]. Marcucci et al. [4] took a qualitative approach to argue that progressive pricing can increase parking turnover and improve availability, while generating higher revenues. Shoup [5] discussed the implementation

of progressive parking pricing in Auckland and its superiority to hourly pricing and parking time limits. Compared to parking time limits, drivers have more flexibility over their dwell time in progressive pricing. Progressive pricing is also used in other applications outside of parking pricing, including household utility billing plans such as hydro and power. For example, water management authorities in Sao Paulo (Brazil) control water demand by charging less per litre if consumption is below a certain threshold and more if consumption exceeds the threshold [6]. In South Korea and China, provincial governments have implemented a progressive pricing system for household electricity consumption [7, 8]. Although progressive pricing is found beneficial in many studies, including sectors of water and hydro management, the optimal design of the parking management policy and its impacts on parking search times, occupancy, and social welfare remains to be analyzed.

This research's methodology is based on a supply-demand econometric model. The econometric model is formulated to represent two pricing structures, hourly and progressive pricing. Since travellers react differently to pricing policies by choosing their individual preferred dwell times, we consider two representative groups of travellers. The optimal progressive and hourly price profile that maximizes revenue or social welfare is then derived from the model. The optimal design of pricing policies for parking would benefit the transport system from both the parking authorities' and policymakers' perspectives. From the standpoint of parking authorities, the optimal policy can maximize revenue. Similarly, from the perspective of public authorities, an optimal design can maximize social welfare. As a drawback, the optimal policy can directly increase drivers' parking costs due to higher price levels in areas where parking is under-priced. However, drivers will indirectly benefit from the optimal pricing policy as it can reduce network travel times and parking search cost. Parking search cost is defined as the monetary value of the time spent searching for parking. Parking search times and the benefit of its reduction are later considered in this research as a negative externality in the formulation of social welfare as a means to alleviate excessive

parking search.

1.2 Research Goal

The goal of this research is to study the intricacies of multiple pricing strategies (hourly pricing, time-of-day pricing and progressive pricing) and analyze its optimal design under revenue and social welfare (the sum of revenue and consumer surplus) maximization. Consumer surplus, a social welfare component, is defined as the excess social value gained over the price paid for a product (parking) [9]. Both objective functions are considered in the research as they are standard economic measures that, at times, can be mutually supportive or conflicting. The goal is achieved by first developing an econometric model to mathematically formulate the relationship between parking pricing and drivers' marginal utility. The Law of Diminishing Marginal Utility states that as consumption increases, the marginal utility (added satisfaction) from consumption of each additional unit will decrease [10]. Then, data analysis is completed for a case study in the downtown area of the City of Toronto. Lastly, the results from data analysis and the developed model are applied to a micro-simulation of the case study area.

1.3 Research Objectives

This research sets four main objectives to examine the properties of hourly, progressive and time-of-day pricing policies. The five objectives include:

1. Develop an econometric model to serve as a basis of comparison for multiple pricing policies.
2. Evaluate and compare the economic and social impacts of progressive pricing against the hourly and time-of-day pricing policies.

3. Characterize the properties of progressive pricing, specifically the impacts on drivers' parking behaviour.
4. Estimate the relative performance of an optimal progressive pricing policy for a case study in Toronto, Canada using parking measures including parking occupancy and congestion.

This research develops a framework that parking authorities can utilize, whether public or private, as a preliminary assessment to determine the pricing policies that can best fit under their given circumstances.

1.4 Scope & Thesis Organization

Although the developed econometric model can be applied to any parking facility type, this research's methodology is tailored towards on-street parking. We further validate the findings of the supply-demand econometric model and examine the practical implications of progressive pricing with a simulation of parking behaviour in the downtown core of Toronto. The simulation is calibrated with data from the parking payment machines in the study area, where each machine monitors payments for a cluster of on-street parking spaces. A flow chart of the research is seen in Figure 1.2. It includes the development of an econometric model and the optimization of the model. After this, the optimized model is utilized in conjunction with the results of analyzed data from the study area to serve as an input for the micro-simulation model.

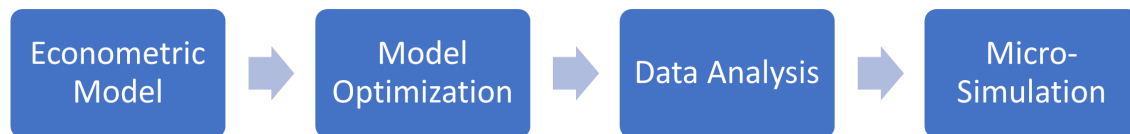


Figure 1.2: Flowchart representing key research milestones

The remainder of this thesis is organized as follows. A review of the related works is presented in Chapter 2. A description of the study area and used data is given in Chapter 3. An econometric model for the two pricing policies, hourly and progressive pricing, is presented in Chapter 4. Additionally, the optimal design and analysis of the pricing policies are also presented in Chapter 4. A case study of 54 parking clusters in the City of Toronto used for model validation is presented in Chapter 5. The simulation results from the case study are presented in Chapter 6. Lastly, the study's conclusions are presented in Chapter 7.

Chapter 2

Literature Review

In this chapter, we review literature completed on parking management strategies. After reviewing parking management strategies used by parking authorities, we further analyze parking pricing, including hourly, progressive, dynamic, fixed-rate, time-of-day, and permit pricing. Lastly, we review studies that have utilized micro-simulation models within the parking context to compare different network scenarios.

2.1 Parking Management Strategies

Parking management strategies refer to policies or programs that will yield a more efficient parking system [1]. Parking authorities often utilize parking management strategies to increase the performance of their parking system and achieve an optimal allocation of parking resources. Parking management strategies that parking authorities commonly use include shared parking, parking pricing, providing financial incentives, and utilizing parking information. Weinberger et al. [11] conducted a report where they investigated parking management strategies in cities across the US, including San Francisco (California), Portland (Oregon), Boulder (Colorado), Chicago (Illinois), New York (New York), and Cambridge (Massachusetts). The report concludes that using sustainable parking principles to create smarter parking management strategies can benefit drivers and businesses by increasing time and monetary savings. Furthermore, they also benefit society by creating more livable and attractive communities.

Litman [1] identified ten parking principles that can guide decision-makers in the planning phase of their parking projects. These ten principles are seen in Figure 2.1.

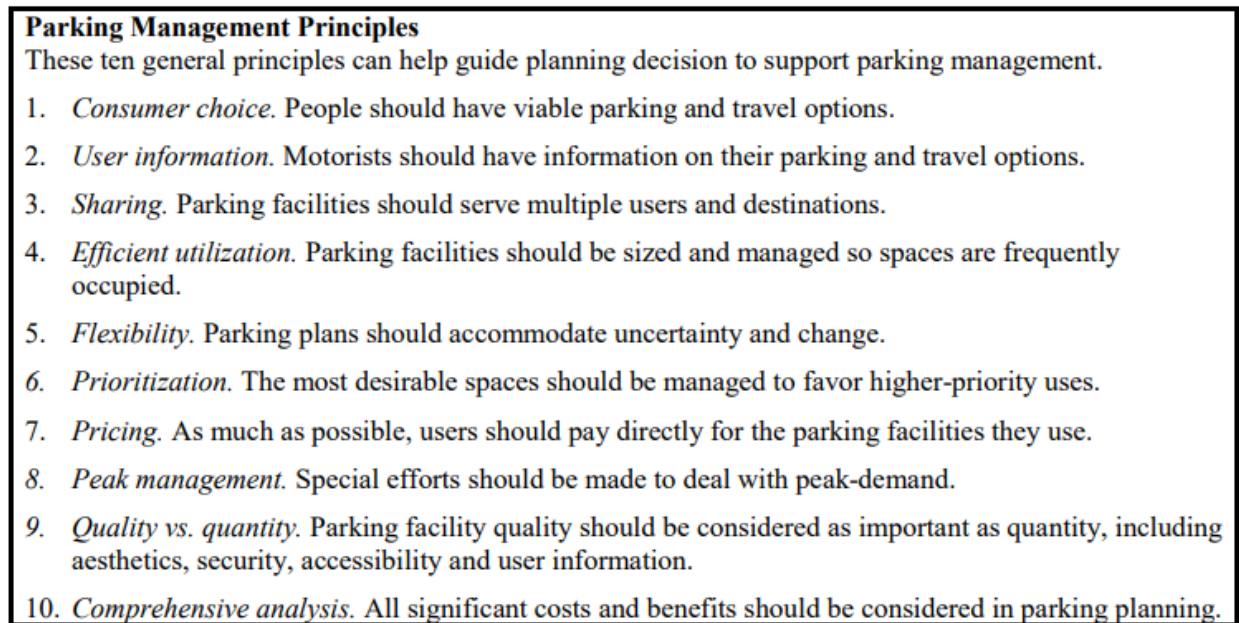


Figure 2.1: Ten parking management principles [1]

Considering these ten parking management principles during the planning phase is vital as it will eliminate many of the shared parking issues seen in the present day. As the population of urban areas continues to rise, governmental authorities of these areas have set a higher level of importance to the topic of parking. Banerjee [12] conducted a study in the United States and identified the common parking issues among communities. Some of the parking issues and a brief description are seen below.

- **Inadequate information for motorists:** In many places, motorists (especially those who are not familiar with the area) are frustrated with the available information of parking clusters. First, they do not have any information regarding the location of available parking spaces leading them to drive around their area looking for an on-street or an off-street facility. This cruising for parking will waste the driver's time and is

seen as an inconvenience. Secondly, once they do end up finding a parking facility, the pricing at the location might not be what they were expecting (i.e., too expensive).

- **Inefficient use of existing parking capacity:** Local development practices can result in an oversupply of parking spaces. For example, MAPC [13] conducted a study to measure the supply and demand of residential parking in the Inner Core sub-region. This region, located in Massachusetts, United States, can be seen in Figure 2.2 and contains Boston (Massachusetts). The study conducted overnight counts of parking spaces and vehicles parked at nearly 200 multifamily residential developments in the region. Findings of the study indicate that, on average, 30% of the available parking was not being used during the overnight period.
- **Excessive automobile use:** A significant problem seen across communities is the excessive use of automobiles. The dependence on cars comes with a high cost to society. Costs include increased vehicle costs, increased residential parking costs, increased congestion, and increased accident risk, among others [12].
- **Economic, environmental, and aesthetic impacts of parking facilities:** Parking facilities come at a tremendous economical cost [12]. First, parking facilities require a large amount of space that can be alternatively used for other higher economic generating purposes. Additionally, parking facilities with a large footprint can negatively impact the environment.
- **Parking spaces that are inconvenient:** In many of the communities finding parking spaces at their destination was seen as an inconvenience for users. Primarily due to the unavailability of parking at their trip destination [12].
- **Demand for handicapped parking spaces:** Handicapped parking spaces are generally located in off-street parking facilities; however, on-street parking is usually

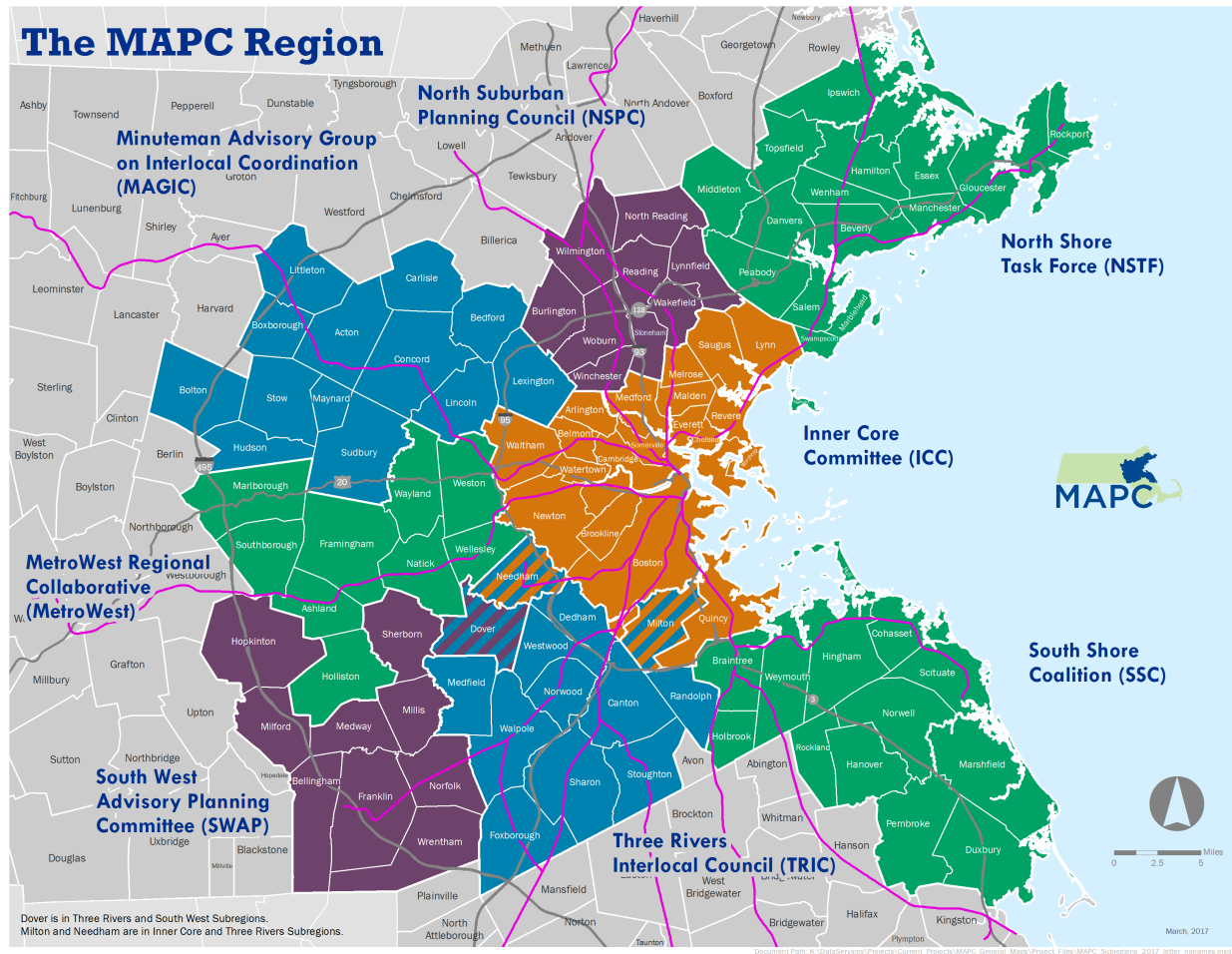


Figure 2.2: Map of the MAPC region in Massachusetts, United States [14]

not easily accessible to handicapped drivers due to the lack of ramps and curb cuts [15].

- **Impact of increasing parking supply:** Increasing parking supply is commonly seen as a simple solution to accommodate for the increase in travel demand. However, congestion and traffic issues may arise due to this increase in supply.
- **Parking spillover problem:** When the parking demand of one location is not being appropriately accommodated, the demand can cause issues at nearby parking spaces.
- **Out-of-town parking:** In many large urban areas where many people commute to work for, many of the drivers parked in a neighbourhood are from an outside area [12].

If the demand attracted to the location is not considered, parking supply will have issues accommodating all the travel demand.

- **Loading and unloading zones:** Loading and unloading zones has been a parking topic that has recently gained traction with metropolitan authorities. In the United States for the year 2020, online sales increased by 32.4% compared to 2019 [16]. The increase in online sales has also increased demand for loading and unloading zones as they are commonly used by postal delivery trucks in large urban areas to deliver packages. This increase in demand for loading and unloading zones can lead to severe consequences if not handled properly. The most notable consequence is double parking or illegal parking of delivery vehicles. As seen in Figure 2.3, since delivery vehicles must deliver packages at the location, the lack of available parking can lead to illegal parking or double parking. This action of illegal parking can have severe consequences to the road network, mainly obstructing the flow of vehicles and generating congestion.



Figure 2.3: Postal delivery vehicle double parking[17]

- **Inadequate pricing methods:** On many occasions, the pricing methods utilized are

inadequate and do not provide flexibility to the user. For example, if a driver wishes to park at the location, they must prepay based on their estimated parking duration. As a result, the driver will generally pay more than they need to pay to avoid any potential fines. Additionally, many on-street parking spaces are not properly priced or priced at all. The lack of parking pricing leads to indirect costs to society [15]. As Shoup [15] mentioned, when drivers do not pay for parking, everyone else will indirectly pay for it. Whether indirectly through higher prices for shoppers or travel time costs for drivers, society will pay the price of parking if parking is not priced effectively.

By comparing the before-mentioned parking management principles seen in Figure 2.1 and the shared parking issues discussed, we notice that the parking issues discussed above can be avoided by following some of the ten parking management principles. Table 2.1 a summary of what common issues each of the ten parking management principles can mitigate.

Table 2.1: Parking issues that each parking principle can mitigate [15]

Parking Principle	Issues that can be mitigated
Consumer choice	- Excessive automobile use - Parking spaces that are inconvenient - Out-of-town parking
User information	- Inadequate information for motorists - Inefficient use of existing parking capacity - Parking spaces that are inconvenient
Sharing	- Impact of increasing parking supply - Out-of-town parking - Loading and unloading zones
Efficient utilization	- Inefficient use of existing parking capacity - Loading and unloading zones
Flexibility	- Parking spaces that are inconvenient - Demand for handicapped parking spaces
Prioritization	- Parking spill over problem - Loading and unloading zones
Pricing	- Inadequate pricing methods
Peak Management	- Inadequate information for motorists - Inadequate pricing methods
Quality vs Quantity	- Excessive automobile use - Economic, environmental and aesthetic impacts of parking - Parking spaces that are inconvenient - Impact of increasing parking supply
Comprehensive analysis	- Inefficient use of existing parking capacity - Parking spaces that are inconvenient - Parking spill over problem - Loading and unloading zones - Out-of-town parking

The table above notes that all of the previously discussed common parking issues can be mitigated by implementing at least one of the ten parking management principles. This shows the importance of thoroughly planning the parking management strategies based on the ten parking principles to avoid future issues. In the following sections, we will review common parking strategies used throughout the world and discuss some of the advantages and disadvantages of each parking management strategy.

2.1.1 Pricing

Parking pricing refers to direct charges that a driver receives in exchange for using a parking space for a predetermined period. Calthrop et al. [18] argues that there are two primary sources of inefficiency in the transportation network; transport prices fail to reflect the external costs of travel and a large percentage of drivers parking for free. Efficient parking pricing can provide multiple benefits directly to the driver and indirectly to other transportation network users. Benefits of efficient parking pricing include but are not limited to parking facility cost savings, increased revenues, traffic congestion reduction, increased turnover and decrease in cruising for parking [19].

Deakin et al. [20] conducted a study of five pricing strategies in California, including parking pricing, congestion pricing, fuel taxes, and vehicle miles travelled fees. The study concludes that these pricing measures can effectively relieve congestion and increase revenue generation. D’Acerno et al. [21] argued that high congestion levels could be attributed to users’ preference for a private vehicle system. Moreover, to obtain a more balanced modal split between private and public transport, fees can be set on the use of private cars through road pricing or parking pricing. Ferrari [22] states that parking pricing strategies are easier to implement since road pricing requires more capital investment and the installation of advanced technologies.

In addition to congestion reduction, parking pricing can also significantly affect emissions generated from the transport industry. Proost and Van Dender [23] compared three policy types in the context of reducing emissions. The three investigated policies include transport management, emission technology, and fuel efficiency. Findings show that transport policies produce the best welfare results as they directly address two sources of inefficiencies, excessive congestion in peak hours and free parking. Čuljković [24] studied the influence that parking pricing can have on energy consumption and carbon dioxide emission. Results from the case

study completed for Belgrade (Serbia) showed that adequate on-street parking pricing could reduce cruising for parking, provide fuel savings, and reduce pollutant emissions [24].

Arnott and Rowse [25] argued that the inefficient management of parking leads to searching for parking. Shoup [26] conducted a study considering cruising for parking and concluded that on average, around 30% of traffic in the studied cities was cruising for parking. Cities examined in this study include Detroit, New York, London, San Francisco and Sydney. Parking pricing can reduce the number of vehicles that cruise for parking in large urban areas. Cats et al. [27] conducted a study that examined the effects of parking policies on parking occupancy levels for on-street parking clusters in the city of Stockholm, Sweden. Results from this study indicated that the pricing policies increased the ease of finding vacant parking spaces by achieving a reduction in parking occupancy [27].

There are multiple ways that parking management authorities can implement parking pricing. Pricing schemes include hourly, progressive, dynamic, fixed-rate, and time-of-day pricing. Literature on these pricing strategies will be further examined in Section 2.2 of this Chapter.

2.1.2 Shared Parking

Shared parking is a parking management concept under which private parking owners such as hotels, universities, hospitals and individuals opt to share their parking spaces with the public in exchange for monetary compensation. Shared parking is an emerging management strategy that aims to close the gap between parking demand and parking supply in dense urban areas. Although parking has become a hot topic over the last decade, many parking spaces remain unused, especially in dense urban areas. For example, nearly 800,000 private parking spaces in Beijing, China remained unused during work hours (9 am to 5 pm) [28].

This ample supply of parking remains unused as the private owners of these parking

spaces drive their cars out to work, leaving the space empty for the duration of their work shift. Shared parking leverages these temporarily vacant parking spaces to alleviate the high demand for public parking. Examples of parking platforms that offer shared parking services include spot-park.com, parkhound.com.au, parkcirca.com, roverparking.com, and justpark.com.

As described by Xiao et al. [28] the main difference between shared parking and public parking is the ownership of the rule-making right. The individuals or authorities in charge of setting the rules for the parking spaces differ for these two parking types. In public parking, the local government sets the rules, including pricing and parking limits. Contrary to this for shared parking, the parking rules can be set by the parking space owner or parking platform if a platform temporarily controls the parking space. Given this, parking platforms that rent out shared parking spaces from private owners can dynamically set the prices. Under these circumstances, shared parking will attract a portion of the parking demand that public parking cannot meet. Effectively, shared parking can be used to supplement the public parking supply by temporally using privately-owned parking spaces [28].

Guo et al. [29] developed a model to determine the price and time limit for which the private parking space can be temporarily rented. The model utilized was a Gaussian Mixture model, which uses multiple Gaussian probability distribution functions. This model was proposed to describe the time-varying behaviours of drivers under a profit maximization problem [29]. The model consists of 4 stages. In the first step, parking space owners designated leave the parking space. This will generally occur in the morning as the users leave for work. In the second stage, the model allows short-term drivers to park for a parking duration up to time t_{stop} . After this, on stage three, the model stops accepting short-term drivers and they begin vacating the parking spaces. Lastly, at the fourth stage, the last remaining T-users leave the parking spaces while parking space owners return and park at the location.

Under a shared parking system, there are two primary users, drivers who own parking

spaces and drivers who pay to park for a short period. At any given time, only one of the two user groups will park at a given location. Therefore, a significant element of the shared parking problem is to account for both users' punctuality and unpunctuality. This is essential since the parking platform (operator) will incur capital losses if the two users do not respect their assigned parking time. Jiang and Fan [30] develops an optimal allocation of shared parking that takes into account the unpunctuality of users. This is done by setting a parking probability function and then transforming it into an expectation model. From numerical experiments, the study concludes that when considering parking unpunctuality, the satisfaction of consumers decreases when the supply is limited, or the consumer demands are excessive [30].

As a parking management strategy, shared parking can alleviate some of the demand incurred by public parking by renting out privately-owned parking spaces that would otherwise be unused. Moreover, private owners will be incentivized to rent out their parking spaces and generate revenue from a limited resource that they already own and otherwise be unused.

2.1.3 Parking Information

Cruising for parking, which is the action of driving around your destination to find available parking, is a costly and time-consuming side effect of not having any available parking spaces at the drivers' destination. Hampshire and Shoup [31] estimated that 15% of traffic in central Stuttgart (Germany) was cruising for parking. This estimate was developed using 876 observations of newly vacated on-street spaces. Both Panja et al. [32] and Yoo et al. [33] argue that parking space availability should be collected and utilized to guide better drivers who seek parking since driving into an off-street parking lot only to find that there is no available parking is not only time consuming, but also maddening. Both studies consider using wireless sensors to collect and then distribute information to nearby drivers.

In San Francisco, SFPark collects information regarding parking availability using fixed sensors to use this data then to dynamically set the price of parking to better balance parking supply and demand [54]. More importantly, the information on parking availability is made available through their website and allows drivers to get a sense of parking available at the given on-street parking cluster. However, a major drawback to using fixed sensors to collect parking availability in the same manner that SFPark collects them can come with high setup and maintenance costs.

Bock et al. [34] proposes to monitor parking occupancy using a fleet of vehicles with onboard sensors. They propose to utilize high-mileage vehicles, such as taxis, to gather parking utilization data through crowd-sensing. Crowd-sensing refers to collecting data by recruiting a large number of people or, in this case, taxis. The study examines different taxi fleet sizes and their suitability for collecting data for 579 road segments in San Francisco, United States [34]. Deploying sensors embedded in vehicles can collect information from a set of spaces, whereas stationary sensors collect data from a single space. Nonetheless, a drawback to the solution is that the sensors will not survey highly congested areas. That is to say, the taxi driver will seek to avoid congested areas to reduce their travel time, and no parking availability will be gathered from these areas.

Xu et al. [35] developed Phone Park, a mobile application that utilizes travellers' GPS and accelerometer sensors to determine the location at which the traveller has parked. The application also detects when the traveller leaves the area making the parking space available. Data collection systems that leverage mobile devices can quickly and inexpensively be implemented. However, users must compromise to allow data collection with their phones. With increased privacy concerns around app data collection, users could opt out and decide not to provide data. A drawback of this application is that drivers do not have an incentive to provide their parking data.

Yan et al. [36] incentivized drivers to provide their departure times by developing a

crowd-sourcing platform called CrowdPark. The platform rewards drivers that provide their estimated departure time and “sell” the information to drivers who seek to park at this destination. Since this system employs a monetary reward for users, drivers will be encouraged to participate. A major drawback to this system is that from the moment that the platform receives the information to the time that the second driver arrives at the parking cluster, no information is provided regarding the availability of the parking space. Therefore, the parking space could potentially be taken when the second driver arrives at the parking cluster. These are a few of the multiple systems that seek to manage parking using parking information.

2.1.4 Financial Incentives

Recently, especially in large urban areas, organizations and governments have shifted to greener and sustainable transport modes such as cycling, public transport, and carpooling. This movement is no surprise since the ownership, and use of personal vehicles for work-based trips is unsustainable with current population growth in urban areas. To incentivize users to change to more sustainable modes of transit, governments and organizations worldwide began to offer financial incentives. In this section, we discuss financial incentives provided to commuters to reduce parking demand.

Shoup [37] argues that commuters are more likely to drive to work when employers offer free parking. For 1995 in the United States, 95% of automobile commuters parked free at work [38]. Shoup [37] states that this could be the very reason why 91% of commuters drove to work in that same year. If it were not for employer-provided parking, many of these commuters would choose an alternate mode of transport such as carpooling, taking public transit, walking or biking if it were to be available.

As seen in Figure 2.4, seven studies were examined by Shoup [37] to determine how free

employer-provided parking will influence driver mode choice. On average, we see that the percentage share of drivers who commute to work in single-occupancy vehicles increases by 25%. From the perspective of this analysis, free employer-provided parking can be considered a benefit that an employee can claim by driving to work. As Shoup [37] states, "Employer-paid parking is a tax-exempt fringe benefit you qualify for only by driving to work." Therefore, many commuters see free employer-provided parking as an invitation to drive to work.

Location and date of case study	Solo driver mode share			Cars driven to work per 100 employees				
	Driver pays for parking	Employer pays for parking	Percentage point increase	Driver pays for parking	Employer pays for parking	Increase	Percent increase	Price elasticity of demand
(1)	(2)	(3)	(4)=(3)-(2)	(5)	(6)	(7)=(6)-(5)	(8)=(7)/(5)	(9)
1. Civic Center, Los Angeles, 1969	40%	72%	+32%	50	78	+28	+56%	-0.22
2. Downtown Ottawa, Canada, 1978	28%	35%	+7%	32	39	+7	+22%	-0.10
3. Century City, Los Angeles, 1980	75%	92%	+17%	80	94	+14	+18%	-0.08
4. Mid-Wilshire, Los Angeles, 1984	8%	42%	+34%	30	48	+18	+60%	-0.23
5. Warner Center, Los Angeles, 1989	46%	90%	+44%	64	92	+28	+44%	-0.18
6. Washington, D.C., 1991	50%	72%	+22%	58	76	+18	+31%	-0.13
7. Downtown Los Angeles, 1991	48%	69%	+21%	56	75	+19	+34%	-0.15
Average of case studies	42%	67%	+25%	53	72	+19	+36%	-0.15

Figure 2.4: Solo commuter trips depending on available parking type [37]

In recent years, instead of inviting commuters to drive to work by providing free parking, employers have started to offer financial incentives to employees if they utilize a sustainable mode of transportation to get to work. An example of this is parking cash-outs. Parking cashouts are financial incentives given to commuters who are provided subsidized parking if they opt out and use an alternate mode of transport to commute. For example, users who are offered free parking at their workplace will be allowed to receive the cash equivalent of parking if they commute via public transit.

Seattle Children's Hospital used parking cash-outs to reduce the percent share of solo

driving commutes by 35%. A summary of the offered incentives can be seen in Figure 2.5. By providing these incentives, the Seattle Children’s Hospital reduced the number of solo driving commutes and consequently managed demand for parking at the location.

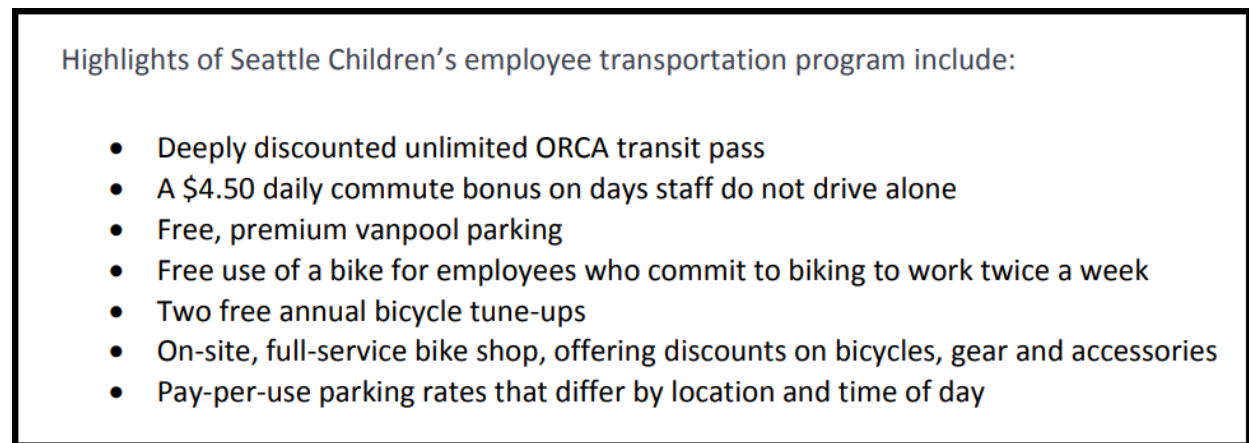


Figure 2.5: Highlights of Seattle Children’s employee transportation program [39]

2.2 Parking Pricing

In this section, we further analyze and review the literature on parking pricing as this will be the primary parking strategy examined in the thesis. The six parking pricing policies include hourly pricing, progressive pricing, dynamic pricing, fixed-rate pricing, time-of-day pricing and parking permits.

2.2.1 Fixed-Rate Pricing

In fixed-rate pricing, drivers pay a given fee for a set number of hours in the day. This pricing strategy is common for privately-owned parking garages that serve daily commuters. The main benefit for the drivers is in their parking duration; those that park for longer durations benefit from economies of scale of paying less per hour overall. In contrast, this system is inconvenient for short parking duration (e.g., leisure trips) as drivers are forced to pay a hefty

fee for hours extended beyond the required parking usage.

2.2.2 Hourly Pricing

Hourly pricing is a common pricing mechanism for parking management in many urban cities, where drivers are charged a fixed rate per hour of parking. Glazer and Niskanen [40] studied the intricacies of hourly pricing and showed that a higher hourly price shortens dwell time, leading to higher parking availability and parking demand. In contrast, Nourinejad and Roorda [3] showed that a decrease in hourly prices could also induce traffic because the lower cost of parking leads to a lower generalized cost of travel and ultimately higher demand. According to these studies, the effect of hourly parking pricing on demand depends on factors such as the dwell time elasticity with respect to the hourly parking price. Lehner and Peer [41] reviewed 50 parking studies and showed that price elasticity is highly specific to individual circumstances, and future studies should focus on the willingness to pay for parking. Focusing on the willingness to pay can yield a generalized formulation of the relationship between parking dwell time and parking pricing.

Hourly parking pricing affects drivers differently according to their desired dwell times. Simićević et al. [42] investigated setting the hourly parking price according to driver attitudes. The study showed that drivers who can decrease their dwell times are more likely to continue parking after an hourly price increase. This indicates that for on-street parking, where a predominant share of drivers are leisure travellers and commercial delivery vehicles, drivers reduce their parking dwell time instead of parking frequency (demand). For off-street parking lots where the predominant share of drivers are commuters that park for more extended periods, an increase in the hourly price can reduce parking demand instead of dwell time. Khordagui [43] analyzed morning commute patterns in California and showed that a 10% increase in parking prices for long-stay drivers reduces the probability of driving to work by

1% to 2%. In addition, Simićević et al. [42] studied parking patterns in the Central Business District of Belgrade (Serbia) and showed that an hourly price of 0.42 EUR resulted in a 10.4% parking reduction for leisure trips and a 14.9% parking reduction for commuter trips.

Hourly parking pricing can implicitly affect parking search times. Najmi et al. [44] studied the effects of parking policies, including time restrictions and expanding shared parking spaces using a behavioural pricing formulation influenced by local parameters. The study found that parking restrictions can increase parking search time while a higher hourly price can decrease it. Van Ommeren et al. [45] carried empirical research of parking patterns in the Netherlands and showed that parking search time increases with dwell time. Van Ommeren et al. [45] concluded that searching for parking is more common in large urban areas, particularly those with shopping and leisure attractions, and showed that an increase in hourly prices decreases parking search time.

Commonly, hourly parking rates are optimized by developing a model that determines the hourly price that can achieve a parking occupancy of around 85%, yielding optimal network conditions [46]. Several studies consider a dynamic parking management problem that optimizes parking prices over time [47]. Under an active parking management strategy, parking authorities will regularly alter hourly price levels to achieve an hourly price that will yield a predetermined targeted parking occupancy. Within the optimization context, Liu [48] considered minimizing total system costs, including both travel cost and the social cost (e.g. opportunity cost of utilizing the land as parking) of parking supply. In this study, aside from optimizing our model for maximum revenue generation, the model will also be optimized with the scope of maximizing social welfare to account for utility gained by the whole system as opposed to only the utility gained by parking authorities in the form of revenue.

2.2.3 Progressive Pricing

Progressive pricing is an innovative approach to balancing supply and demand in parking management and other applications subject to such an imbalance. As previously mentioned, progressive pricing is a robust tool for managing demand in utility services [6, 7]. Kang and Jia [8] studied the past and present electricity consumption policies in China. The study included both hourly pricing and progressive pricing policies. Results showed that progressive pricing did not affect the power usage of 75% of low-income households but decreased usage of high consumption households.

Examples of progressive pricing rates from public and private parking facilities are presented in Table 2.2 for United States, Netherlands, Canada and New Zealand. Parking rates in the table were current prices as of September 2021. The table shows that New York City, for instance, charges \$4.5 for the first hour and \$7.5 for any additional hour at parking clusters where progressive pricing is implemented. Progressive pricing is also implemented in private parking facilities; Atlanta's Hartsfield Jackson airport charges \$2 for the first two hours and \$3 for any additional hour. According to Table 2.2, progressive pricing is occasionally implemented in large urban areas with high parking demand or low parking supply, such as tourist attractions or highly urbanized cities. In New York City, progressive parking pricing is specific to the vehicle type, where commercial vehicles are charged an initial rate of \$6 for the first hour, \$7 for the second hour, and \$8/hr for any parking dwell time after that. In contrast, passenger vehicles are charged a rate of \$4.5 for the first hour and \$7.5/hr for any subsequent hours with a parking limit of 3 hours. In urban cities where parking is of small supply (New York and Auckland), the augmented prices are an order of magnitude larger than the base price. Progressive pricing can persuade travellers to truncate their parking dwell time to avoid the higher price incurred if they stay longer than the cutoff time.

Table 2.2: Progressive pricing applications for two parking types

Location	Country	Parking Type	Hourly Rate (\$/hr)						Time Limit
			30 minutes	1st hour	2nd hour	3rd hour	4th hour	5th hour	
Albany, New York [49]	United States	On-street	-	\$1.25	\$1.25	\$1.75	\$2.25	\$2.75	
Aspen, Colorado [50]		On-street	\$1.00	\$3.00	\$7.50	\$13.50	\$21.00	\$21.00	
Atlanta's Hartsfield-Jackson Airport [51]		Off-street	-	\$2.00	\$2.00	\$3.00	\$3.00	\$3.00	
Berkeley, California [52]		On-street	-	\$2.00	\$2.50	\$3.00	-	-	3 hrs.
Chicago's O'Hare Airport [53]		Off-street	-	\$2.00	\$3.00	\$0.00	\$5.00	\$19.00	
Queens, New York [54]		On-street	-	\$1.50	\$2.50	\$3.00	\$3.00	\$3.00	
Jackson, Mississippi [55]		On-street	-	\$1.00	\$1.00	\$3.00	\$4.00	\$4.00	
New York City core (commercial vehicles) [54]		On-street	-	\$6.00	\$7.00	\$8.00	\$8.00	\$8.00	
New York City core (passenger vehicles) [54]		On-street	-	\$4.50	\$7.50	\$7.50	-	-	3 hrs.
Sacramento, California [56]		On-street	-	\$1.75	\$3.00	\$3.75	\$3.75	\$3.75	
Victoria, British Columbia [57]	Canada	On-street	-	\$1.00	\$2.00	\$2.00	\$2.00	\$2.00	
Whistler, British Columbia [58]		On-street	-	\$1.00	\$1.00	\$2.00	\$2.00	\$2.00	
Antwerp, Flemish Region [59]	Netherlands	On-street	-	€ 1.50	€ 2.50	€ 3.50	-	-	3 hrs.
Auckland, North Island [60]	New Zealand	On-street	-	\$4.50	\$9.00	\$13.50	\$18.00	\$22.50	

Li et al. [61] developed a progressive pricing model for residential electricity consumption to maximize energy savings. Their study used a Genetic Algorithm for the model and conducted a case study using consumption data from Chengdu (China). The optimal progressive pricing policy included six price levels. The study found that total power consumption could be reduced by 27%. More importantly, electricity costs for more than 79% of the consumers were lowered or unaffected. Similarly, a progressive pricing policy will only increase parking costs for drivers who park longer than the defined threshold when applied to parking pricing. Wilson [62] pointed out that these positive effects of progressive pricing policy promotes are due to more efficient utilization of resources.

As an intelligent parking management mechanism, progressive pricing can prohibit long

dwell times as drivers must pay more per hour if they overstay. Lowering parking dwell times through progressive pricing can increase parking availability and reduce the number of vehicles searching for parking, consequently reducing traffic congestion [4]. Progressive parking pricing is currently implemented in many cities, including those presented in Table 2.2. Amongst them, urbanized cities like New York customize the policy for different blocks and vehicle types. For example, commercial vehicles are charged an initial rate of \$6/hr for the first hour, \$7/hr for the second hour, and \$8/hr for any parking dwell time after that. In contrast, passenger vehicles are charged a rate of \$4.50/hr for the first hour and \$7.50/hr for any subsequent hours with a parking limit of 3 hours.

This research presents the optimal design of a progressive pricing policy under revenue and social welfare maximization. Our model, which sets parking price based on drivers marginal utility, is distinguished from Simićević et al. [42] which sets parking price according to drivers attitudes. Despite existing implementations of progressive parking pricing, fewer studies focus on the reactive behaviour of customers in place of progressive pricing. Likewise, the trade-offs of progressive pricing on profit and social welfare remain to be investigated. This paper contributes to the literature on progressive pricing by formulating driver reactions to this policy and modelling its optimal profile design. Furthermore, it will allow parking authorities to follow a framework for setting parking pricing as opposed to just following common practices.

2.2.4 Dynamic Pricing

Dynamic pricing requires adjustments in the pricing structure via real-time monitoring of the parking occupancy status. When occupancy is high (or vacancy is low), a large dynamic price reduces occupancy (increase vacancy). This allows cities to keep a certain number of spaces available at all times, reduce the possibility of cruising for parking, and improve total network

travel time. Dynamic pricing is also implemented in other applications such as ridesharing. Service providers such as Uber and Lyft have a surge pricing mechanism in which ridership fares are increased if demand is high or supply is low, such as in adverse weather conditions like rain and snow [63].

SFPark, a pilot project in San Francisco, used parking occupancy data gathered from sensors to adjust hourly parking rates [64]. Rates were adjusted not more than one time per month by 0.25 to 0.50 dollars to achieve the desired parking occupancy of 85 percent. Such an approach requires parking occupancy sensors, which can be expensive to implement and a trial-and-error system to achieve optimal price convergence. Instead of varying the price and then observing driver's parking behaviour, this research will predict driver behaviour under varying parking price levels.

Although dynamic pricing can efficiently balance the supply and demand for parking by dynamically changing the price for parking, its implementation is associated with high fixed and variable costs. To dynamically increase or decrease the price levels at a location, information regarding parking occupancy at the location must be known. This parking occupancy is commonly collected using cameras and sensors, which come at a high cost. Not only that, but their implementation in locations with challenging climates (i.e. rain, snow, hail) makes the maintenance of sensors and cameras costly.

2.2.5 Time-of-Day Pricing

Increasing parking prices is often followed by public outcry. In parking pricing, a prominent strategy is to charge more only during peak hours when demand is high. The number of vehicles who want to park decreases when charging more during busy hours, allowing the parking authorities to decrease average parking occupancy. Time-of-day pricing is not only a pricing scheme that is already in use for on-street parking in Toronto but also a popular

pricing method in other industries such as the household electricity industry. In Ontario, the electricity service provider Hydro One implements a time-of-day (also known as time-of-use) pricing scheme where the pricing rate takes on three price levels based on time of consumption. This pricing scheme better reflects the cost of producing electricity at a different time of day based on demand. Since electricity production at a given time is capped at a limit, instead of attempting to produce more electricity, the pricing scheme shifts the demand from peak times to off-peak times by incentivizing consumers with a lower price. Similarly, when the number of parking spaces that can be provided is limited due to space constraints, implementing a time-of-day policy will incentivize some of the demand to park in the off-peak periods instead of during peak hours.

2.2.6 Permits

As the City of Toronto implements stricter parking enforcement in the city's downtown core, commercial vehicles (CVs) have become targets of increased ticketing and towing, often without alternate legal means of parking and loading. CV parking permits are a solution to provide legal and affordable parking options that maintain a source of revenue for the municipality.

Parking permits around the world are reviewed on the basis of their cost and scope. Studies of historical parking citations in Toronto indicate clear patterns of parking behaviour for which a permit would be beneficial. The trade-off between permit revenue and parking ticket revenue shows that optimal permit pricing of \$300 annually can improve municipal revenue and achieve widespread adoption [65]. An improvement in social welfare is also completed with permit adoption by reducing the cost of congestion, as permit holders are encouraged to park in legal zones away from congested arterial roads [65].

2.3 Parking Micro-Simulation

This study utilizes a parking micro-simulation to validate the developed econometric model. For this section, we will be analyzing previous literature that has previously developed micro-simulation models for parking. With the developments in computational power, the built-in capabilities of parking micro-simulation software have increased. The first set of used models could only simulate the behaviour of one single driver due to computational limitations [66]. However, more advanced models more recently developed like PARKAGENT and VISSIM can simulate the behaviour of each individual driver for thousand of drivers at a given time [67, 68].

Now, modellers not only have the option to utilize already developed modelling software but are also able to create new micro-simulation models to meet their intended use. As an example, Ramadan and Roorda [69] conducted a study to estimate the impact that illegally on-street will have on the network. The study utilizes Quadstone Paramics software to incorporate illegal parking Toronto Waterfront Network simulation model [69]. On the same topic of on-street parking, Madushanka et al. [70] studied the effects roadside parking manoeuvres have on traffic flow. In this case, the simulation was complete using VISSIM [68] micro-simulation software.

Considerations that need to be taken before either choosing or developing a micro-simulation software with parking capabilities will include the scope of the vehicle's trip. For example, Vo et al. [71] developed an agent-based simulation tool to study driver movements through off-street parking lots. In this model, the simulated process begins as the vehicle enters the parking lot and then finishes when the car leaves the parking lot after parking at a designated space. There also exists micro-simulation software that simulates a more extended portion of a driver's trip. For example, Benenson et al. [67] utilized the micro-simulation software PARKAGENT, which considers every aspect of the driver's trip from the moment

the vehicle enters the network to the moment the driver leaves the study area.

Additionally, the driver attributes used as input in the simulation will differ from one software to another. The PARKAGENT model considers driver attributes that include driver's knowledge of the area, parking habits, and willingness-to-pay for parking [67]. More advanced models can even implement technologies to provide drivers with parking availability information. PARKIT, a parking choice simulator, allows agents in the simulation to react to the simulation environment after being supplied with parking availability data via roadside messages [72]. For these models, the models assign parking choices using a probabilistic input. The models determine a percentage/number of drivers who desire to park at a location (calibrated from collected data). With regards to utility gained from each parking spot, drivers will choose to park at the closest parking spot to their destination, maximizing utility. Measures extracted from parking micro-simulations will differ based on the study's objectives. For simulations where the impact of on-street parking on traffic is studied, commonly used measures include vehicle travel times, link delay, average link speed, and link flow [69, 71, 70]. For parking specific measures studied, collect data on parking occupancy, search time, walking distance, and parking costs [67, 71]. For parking specific measures, data can even be divided for different driver groups in the network [67].

2.4 Econometric Modelling

Econometrics deals with the measurement of economic relationships, most of the time tied to a monetary value [73]. Econometrics integrates mathematics and statistics, intending to provide value to economic relationships. Economic models are sets of governing equations that seek to describe a specific phenomenon. In the context of economics, this commonly shows the quantities of goods purchased by consumers at different price levels [74]. The main data used in developing econometric models are historical data on prices and quantities

purchased. The information on purchases of goods is used to infer the underlying structure of consumer tastes and behaviour. The developed model can be used to predict the quantity of a particular good that will be purchased at a specific price point [74].

Within the transportation field, econometric modelling has been previously used to represent various phenomena, including a consumer deciding whether to purchase a new or used car and the extent of utilization, a traveller choosing a route and the speed of travel, and a shopper deciding where to shop and how much to spend [75]. In this research, econometrics will be utilized to formulate the relationship between drivers behaviour and pricing. Intuitively, as price for parking increases, drivers are expected to decrease their average parking duration since the cost of parking will be higher. Although we expect that the drivers will be parking by a lower amount, the exact relationship is to be further examined. For simplicity researchers such as He and Peeta [76] utilize a linear marginal utility function although other functions such as a step-wise or negative exponential function can be suitable for this phenomena.

2.5 Research Gap and Study Contribution

Despite existing implementations of progressive parking pricing, few studies focus on the reactive behaviour of customers in place of progressive pricing. There are parking authorities currently implementing progressive pricing policies, and studies have shown its efficacy in reducing parking occupancy due to a higher parking turnover. This occurs since progressive pricing lowers the drivers' average parking (dwell) time. To the best of our knowledge, there is no extensive research completed in the econometric modelling of progressive pricing. Econometric models have been developed in the transportation field; however, none have been created to compare progressive and hourly pricing. Additionally, the trade-offs of progressive pricing on profit and social welfare remain investigated. Furthermore, our econometric model,

which sets parking prices based on drivers' marginal utility, is distinguished from reviewed literature that sets parking prices according to drivers' attitudes.

This study contributes to the literature on progressive pricing by formulating drivers' reactions to this policy and modelling its optimal profile design. With the development of our model, parking authorities will have a framework that can be followed to more efficiently manage their parking supply. In this study, aside from optimizing our model for maximum revenue generation, the model will also be optimized with the scope of maximizing social welfare. This is completed to account for utility gained by the whole system instead of only the utility gained by parking authorities in the form of revenue. Previous studies have studied social welfare for hourly pricing but not for progressive pricing. This paper will build on the literature and provide an extensive mathematical comparison of both objective functions for the discussed pricing policies, which is lacking in the current literature available. As a summary, the major contribution of this research is the customization of the knowledge base available for hourly pricing to include a progressive pricing policy.

2.6 Chapter Summary

In this chapter, we review literature completed in parking management strategies. Parking management strategies refer to policies that will yield more efficient parking systems. We review commonly used parking management strategies, including shared parking, parking pricing, providing financial incentives, and utilizing parking information. After reviewing parking management strategies used by parking authorities, we further analyze parking pricing as this thesis focuses on this parking management strategy in proceeding sections.

The review conducted on parking management policies notes that using sustainable parking principles to create smarter parking management strategies can benefit drivers and businesses by increasing time and monetary savings. Additionally, we note that common parking issues

in some dense urban areas can be mitigated by considering the ten parking management principles discussed in this chapter. The parking management strategies discussed include shared parking, parking pricing, parking information, and financial incentives. From these parking management policies, parking pricing is the most commonly implemented strategy for on-street parking clusters.

Since this thesis focuses on the intricacies of parking pricing, mainly hourly, progressive, and time-of-day pricing, we further examined the literature of different pricing policies. Hourly pricing is a common pricing mechanism for parking management in many urban cities, where drivers pay an hourly rate according to their parking dwell time. Progressive pricing is an innovative approach to balancing supply and demand not just in parking management but other applications subjected to supply and demand disparities. In progressive pricing, drivers pay a base hourly rate up to a predetermined threshold. After this threshold is reached, an augmented hourly rate is paid for any additional hours parked. Dynamic pricing adjusts the parking price level via real-time monitoring of the parking occupancy status. A large dynamic price is applied to reduce occupancy when occupancy is high. Under time-of-day pricing, parking authorities charge more during peak hours when demand is high. The number of vehicles who want to park decreases by charging more during busy hours, allowing the city to decrease average parking occupancy during these busy times.

Lastly, this section also discussed previous parking-related studies that have used micro-simulation. Micro-simulation for parking studies has become quite popular with recent advancements in technology as it allows models to simulate each individual agent in the network and provide very granular data. Measures extracted from parking micro-simulations will differ based on the study's objectives. Measures utilized in review literature include vehicle travel times, link delay, average link speed, link flow, parking occupancy, search time, walking distance, and parking costs.

Chapter 3

Study Data

Parking transaction and vehicular volume data are required to build a site-specific model and micro-simulation. This chapter discusses the chosen case study area in the City of Toronto and the various data sources utilized in the thesis.

3.1 Study Area

The study area selected for this thesis is located in the downtown core of Toronto, Ontario, Canada. The study area, seen in Figure 3.1, is bounded by Bathurst Street (West), Lake Shore Boulevard (South), Jarvis Street (East), Queen Street (North), and Dundas Street (North). This study will examine 54 on-street parking clusters within the area. The area is currently important to the City of Toronto and is being used for the City of Toronto Curbside Management Strategy (CMS) [77]. The City of Toronto Curbside Management Strategy is a high-level policy approach and implementation plan that motivates the consideration of change for the use of the curbside. Within this strategy, two important goals considered when considering curbside policies are managing congestion and supporting economic activity. These two goals are of importance and relevance to this study since we consider the economic effects of pricing policies and their effect on congestion of the surrounding area.

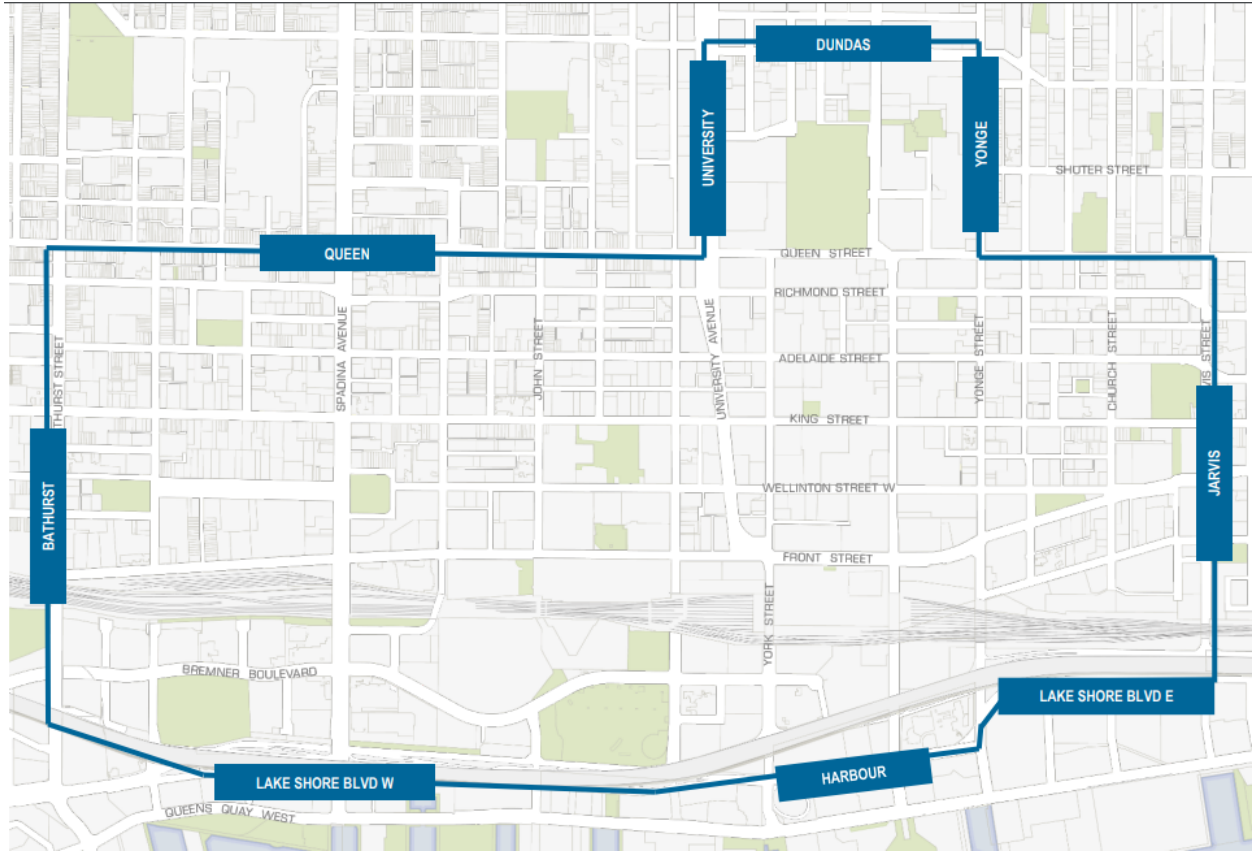


Figure 3.1: City of Toronto study area [77]

The City of Toronto Curbside Management Strategy outlines multiple proposed short-term and medium-term implementations that can be used to achieve the proposed goals. These implementations are seen in Figure 3.2. We see that the tactics target multiple users. Groups include businesses, delivery vehicles, passenger vehicles, taxis, and busses. We note in Figure 3.2 that one of the proposed tactics for the medium term is exploring variable pricing options. Currently, in the City of Toronto, there exist multiple parking strategies, including hourly pricing, time limits, rush hour parking restrictions, and monthly parking permits. The same study area as the City of Toronto Curbside Management Strategy is chosen to provide insights to the City of Toronto parking authorities regarding the three analyzed pricing strategies in this paper, including hourly pricing, progressive pricing, and time-of-day pricing.

IMPLEMENTATION - SHORT TERM (0-2 YEARS)

USER	TACTIC	THEME
	Support the expanded use of off-peak deliveries by building on the success of the Toronto 2015 Pan Am/Parapan Am Games experience.	Reduce Use
VARIOUS	Improve sign legibility and user friendliness, including undertaking a pilot project to present parking regulations as a graphic on a sign.	Effectiveness
	Provide supportive information to couriers and other service delivery vehicles to guide where and where not to park.	Effectiveness
	Implement messaging that communicates alternative off-street parking locations and rates	Reduce Use

IMPLEMENTATION - MEDIUM TERM (3-5 YEARS)

USER	TACTIC	THEME
	Explore a courier/delivery vehicle permit system	Value
	Explore variable pricing options.	Value
	Rationalize and /or introduce new designated motor coach pick-up/drop-off and parking zones locations in high demand areas.	Equitable User Priority
	Rationalize and introduce new taxicab stands, where necessary.	Appropriate Street Use
VARIOUS	Advance automated parking enforcement methods.	Effectiveness

Figure 3.2: Short-term and medium-term goals from the City of Toronto CMS [77]

One of the City of Toronto Curbside Management Strategy goals is the management of congestion in the area. As the population and economy of large urban areas such as Toronto continue to grow, the travel demand within their downtown cores increases. However, existing infrastructure in these areas has limited growth for various reasons, including space limitation and large infrastructure costs. The space that was once only shared between pedestrians and vehicles is now shared with alternate modes of transportation, including cycling and public

transit.

Spatial limitation in downtown cores increases the complexity of allocating enough transportation infrastructure to the ever-growing travel demand. When travel demand exceeds the transportation network infrastructure, congestion will be present. Although in this study we do not discuss how to efficiently allocate resources to ensure that the transportation infrastructure can meet travel demand, we explore how pricing policies can be used to mitigate adverse consequences caused by congestion. Congestion management is crucial for a large urban area as its mismanagement can come at a high cost. Dachis [78] estimates the net cost of congestion for the Greater Toronto and Hamilton Area to be somewhere between \$1.5 billion and \$5 billion a year.

3.1.1 Land Use

Land use, which is the primary purpose or activity for which the population uses the land area, can significantly affect the transportation network since the difference in economic activity will generate differences in transportation infrastructure use [79]. For example, if a location is labelled as an institutional area, we would expect that majority of the trips to the destination will be work trips. In contrast, if a site is of commercial land use, the trips to the location can consist of passenger vehicles going shopping or freight trucks delivering commercial goods. When planning transportation infrastructure, considering land use is essential.

The governmental authority (City of Toronto) divides land use into 11 categories in Toronto [80]. The categories include:

- **Neighbourhoods:** Area composed of predominately low-density residential housing including single-detached houses, semi-detached houses, townhouses and low-rise apartments.

- **Apartment Neighbourhoods:** Area composed of predominately high-density residential housing including mid-rise and high-rise apartments.
- **Natural Areas:** Areas developed by natural growth with minimal urbanization.
- **Parks:** Open green space developed and maintained by the governing authority. It commonly contains trees, benches, statues, and playgrounds.
- **Other Open Space Areas:** Areas including golf courses, cemeteries, and public utilities.
- **Institutional Areas:** Areas which serve a communities' social, educational, cultural, health and recreational needs. This includes schools, museums, recreational facilities, and hospitals.
- **Regeneration Areas:** Areas under redevelopment and revitalization. Generally with developments aiming to improve working and living environments.
- **General Employment Areas:** Areas where the primary purpose includes places of business and economic activities such as business offices.
- **Core Employment Areas:** Areas containing a predominant share of specialized industries including manufacturing, processing, warehousing, wholesaling, distribution, storage, transportation facilities, vehicle repair and services, etc.
- **Mixed Use Areas:** Areas composed of multiple land uses, including apartment neighbourhoods, institutional areas and employment areas.
- **Utility Corridors:** Corridors are mainly used for the movement or transfer of energy, information, people, and goods.

A map of the designated land uses for the study area is seen in Figure 3.3. The map notes that land uses in the study area include institutional, mixed-use, regeneration areas, utility corridors, and parks. The distribution of these areas is not equal; we note that one land use area is geographically close to areas with the same land use. This leads to a clustering of land uses. On the east side of the map majority of the area is designed as a regeneration area, while the west side of the map is assigned as a mixed-use area and the north side of the map as an institutional area.

Additionally, there is also a utility corridor running in the east-west direction on the south side of the study area, primarily composed of a rail network and hydro lines. From this description of the land uses in the study area, we can obtain land uses of interest that can be further examined later under the data analysis portion of this thesis. Later on, we will further analyze parking characteristics in mixed-use, institutional and regeneration land use types as these are the primary land uses of the study area.

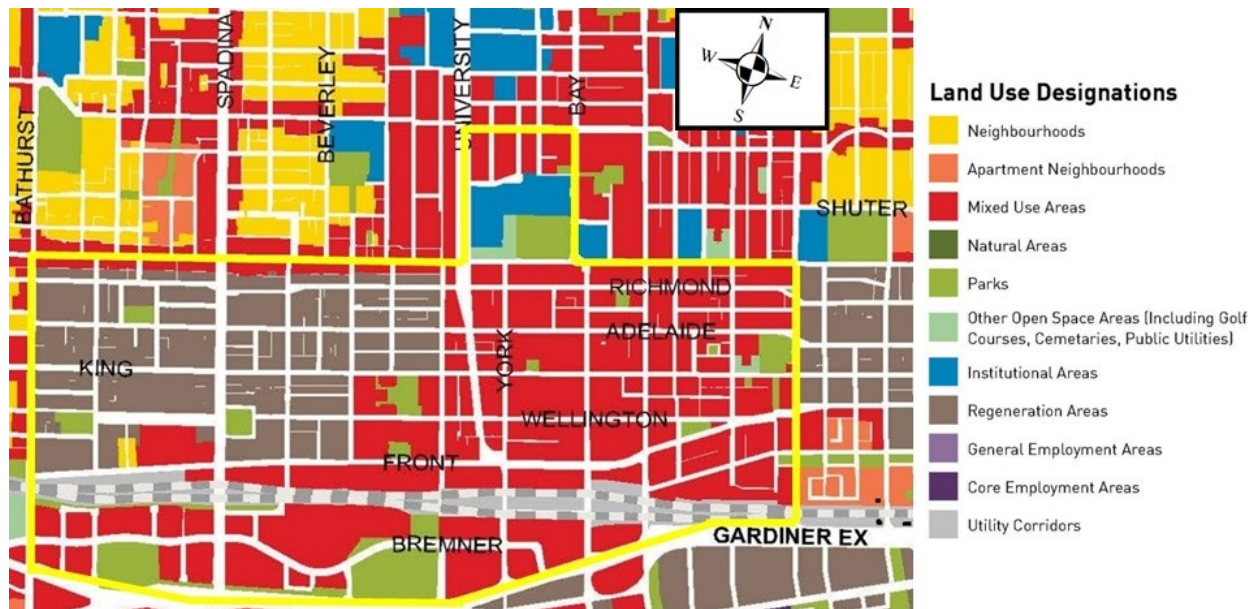


Figure 3.3: Land uses of the study area [81]

3.1.2 Road Type

Transportation networks are commonly composed of multiple road types. These road types are categorized based on their capacity to transport travel demand. The capacity of each road type is calculated based on numerous parameters, including the number of lanes, size of lanes, speed limit, and built material. Low-capacity road types include streets, boulevards, avenues, and collector roads. Examples of larger roads include highways and expressways.

For the study area located in downtown Toronto, the City of Toronto implements a road classification system with six road types. The road types include provincial expressway, city expressway, major arterial, minor arterial, collector and local roads. Figure 3.4 shows the classified roads in the study area. The study area, outlined by a yellow line in Figure 3.4, is mainly composed of a combination of major arterial roads, minor arterial roads and collectors.

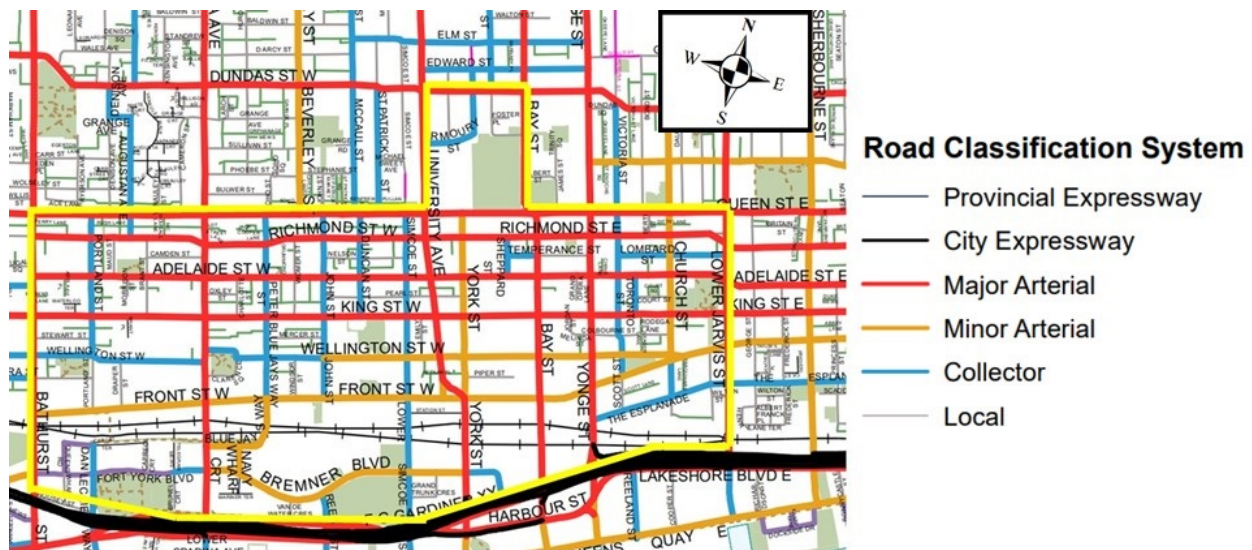


Figure 3.4: Road classifications of the study area [81]

Roads on the north side of the study area mainly consist of major arterial roads with collector roads located in between the major roads. For the south side of the study area, there are only a few major arterial roads, and the main roads in this area include minor arterial and collector roads. Additionally, at the south boundary of the study area, the Gardiner

Expressway is present and labelled in black colour on Figure 3.4. This expressway, classified as a city expressway, will not be included in the study area since there is no on-street parking on this road. However, its presence is noted and will be considered when roads in proximity to this expressway are analyzed.

Streets in cities can be arranged into systems, each with different shapes, some of which can be seen in Figure 3.5[82]. The road system for the study area consists of a combination between the gridiron and loose grid. On the north side of the study area, where there are mainly major arterial roads, the roads can be classified as a gridiron system. On the south side of the study area, where the roads mainly include minor arterial and collector roads, the road system can be classified as a loose grid system.

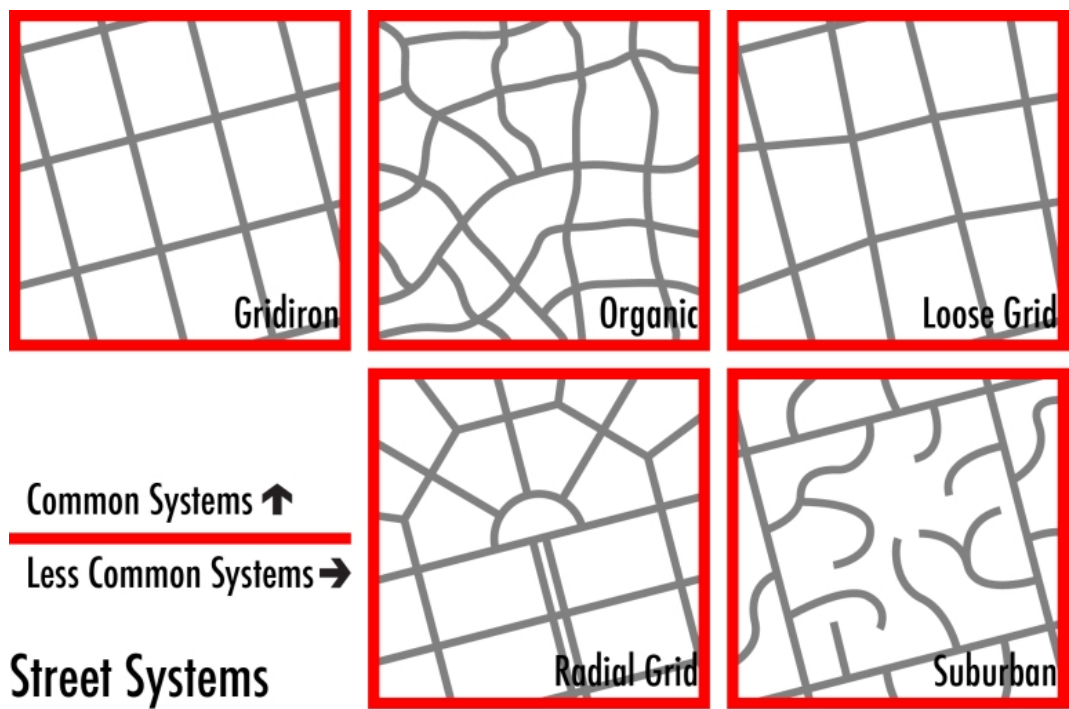


Figure 3.5: Examples of street systems [82]

Table 3.1 shows the major arterial, minor arterial and collector roads in the study area. There are fourteen major arterial roads, seven minor arterial roads, and eighteen collector roads. With this list of street within the study area, the transportation network will be built

for the micro-simulation.

Table 3.1: Road class of streets

Name	Road Type	Name	Road Type
Richmond St. W.	major arterial	Navy Wharf Crt	minor arterial
Richmond St. E.	major arterial	Armoury St.	collector
University Ave.	major arterial	St Patrick St.	collector
Bay St.	major arterial	McCaul St.	collector
Queen St. E.	major arterial	Augusta Ave	collector
Queen St. W.	major arterial	Portland St.	collector
Adelaide St. W.	major arterial	Wellington St. W.	collector
Adelaide St. E.	major arterial	Fort York Blvd	collector
King St. W.	major arterial	Dan Leckie Way	collector
King St. E.	major arterial	Peter St.	collector
Yonge St.	major arterial	Blue Jays Way	collector
Spadina Ave.	major arterial	John St.	collector
Bathurst St.	major arterial	Simcoe St.	collector
Lower Jarvis St.	major arterial	Sheppard St.	collector
York St.	minor arterial	Victoria St.	collector
Church St.	minor arterial	Lombard St.	collector
Wellington St. W.	minor arterial	Toronto St.	collector
Front St. W.	minor arterial	Scott St.	collector
Bremner Blvd	minor arterial	The Esplanade	collector
Blue Jays Way	minor arterial		

3.1.3 Modes of Transportation

As seen in the table below, the study area contains alternate modes of transportation, most notably cycling lanes and a streetcar system. Cycling lanes will not be considered as they are out of scope for this study. Their implementation in the micro-simulation model will not yield any beneficial results as we only focus on-street parking characteristics. Additionally, there is also the presence of a light rail transit system in the study area in addition to bus lines. The public transit agency in charge of these two modes of transport is the Toronto Transit Commission, also known as the TTC. The TTC public transport network in the study area can be seen in Figure 3.6.

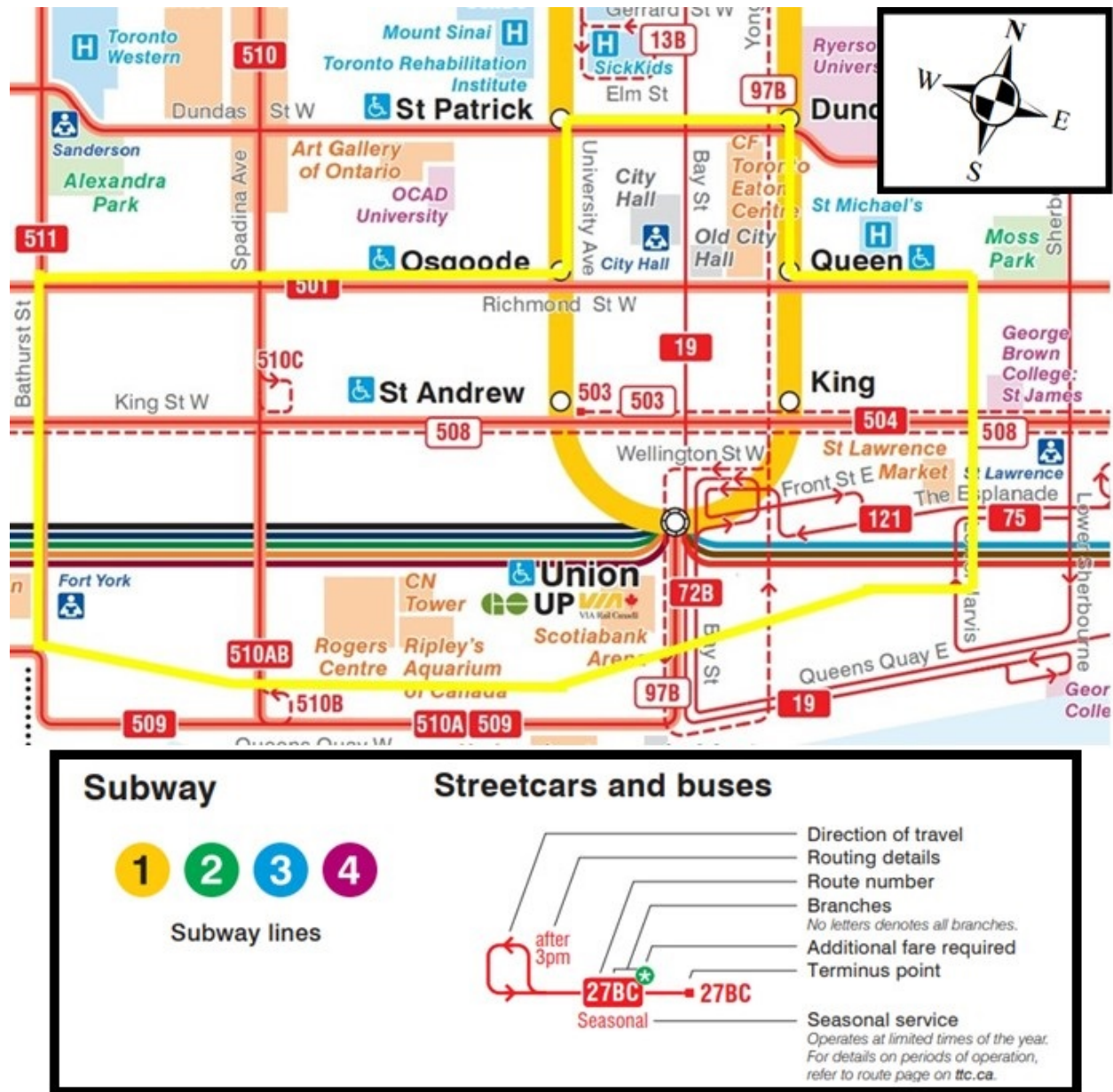


Figure 3.6: TTC public transit network [83]

Table 3.2 shows the list of TTC transit lines within the study area. There exist three types of transport modes, subway, busses and streetcars. For this study, we will only consider the bus and streetcar lines as the subway does not directly impact the congestion of the roads.

Table 3.2: TTC transit lines [83]

Line Number	Transit Type
72	bus
97	bus
121	bus
501	streetcar
503	streetcar
504	streetcar
510	bus
511	streetcar
514	streetcar

In addition to the transportation type, we must also consider the frequency at which these lines provide service to determine whether or not it will be beneficial to include the line within the micro-simulation model. If the transit line provides service often, the public transit line will significantly affect the congestion of the area. In contrast, if the service is not frequent, we assume that congestion due to public transport will not be significant. From the weekday schedule for the TTC line 97 as of January 1, 2022, it can be seen that line 97 has a service spacing of 30 minutes [83]. The same spacing can be seen for all other bus lines listed in Table 3.2 [83]. These bus lines have a high service spacing since, at this location, the predominant share of public transport travel demand is handled by the TTC Subway line and streetcar network.

From the weekday schedule for TTC line 501, the service spacing for this streetcar line is of 8-minute intervals. After examining the rest of the lines in Table 3.2 we conclude that the lines with the transport mode of streetcars offer frequent service while bus lines usually have a service frequency of 30 minutes. For the rest of this thesis, we assume that only the TTC streetcars will significantly affect the results provided by the micro-simulation and therefore only consider TTC lines that consist of streetcars. The transit lines included in this study are lines 501, 504, 503, 514, and 511.

3.2 Data Sources

This thesis utilized two primary sources of data. The first source of data used to gather information regarding the geometric design and current conditions of roads was Google Maps [84]. The second source of information was data provided by the Toronto Parking Authority (TPA) [85].

3.2.1 Google Maps

The first set of data used in this thesis was road characteristic data collected from google maps. This data was manually collected using google maps browser for all streets in the study area. The first characteristic which was collected was the intersection type. In the study area, there exist 74 signalized intersections, as seen in Figure 3.7. All other intersections on the map are minor intersections where the traffic control measure is a stop sign. Intersections composed of a minor/major arterial road and collector road have a two-way stop sign for traffic control. A four-way stop sign controls intersections containing only collector or local roads. Additionally, the allowed turning movements were collected at an intersection level.

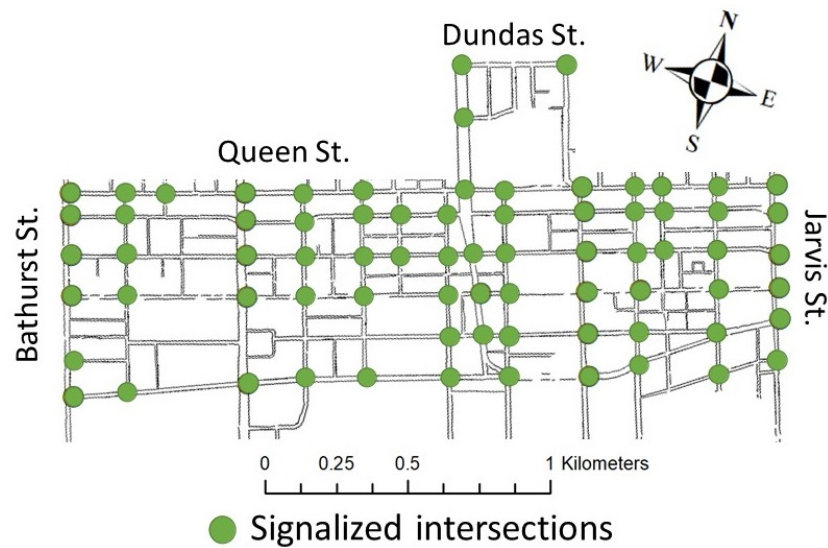


Figure 3.7: Signalized intersections in the study area

Lastly, road characteristics were collected at the street level. This collected data can be seen in Table 3.3. Features include the number of driving lanes, the direction of flow, bike lanes presence and streetcar presence. Overall, the streets in the study area contain either 1 or 2 driving lanes in each direction of flow except for University Avenue, which contains three driving lanes in each direction of flow.

Table 3.3: Collected data for 39 streets in the study area

Name	Road type	Number of Driving Lanes	Direction of Flow	Bike Lanes	Streetcar
Richmond St. W.	major arterial	2	one-way	yes	
Richmond St. E.	major arterial	2	one-way	yes	
University Ave.	major arterial	6	two-way	yes	
Bay St.	major arterial	4	two-way	yes	yes
Queen St. E.	major arterial	2	two-way		yes
Queen St. W.	major arterial	2	two-way		yes
Adelaide St. W.	major arterial	2	one-way	yes	
Adelaide St. E.	major arterial	2	one-way	yes	
King St. W.	major arterial	1	one-way		yes
King St. E.	major arterial	1	one-way		yes
Yonge St.	major arterial	2	two-way	yes	
Spadina Ave.	major arterial	4	two-way	yes	yes
Bathurst St.	major arterial	4	two-way		yes
Lower Jarvis St.	major arterial	4	two-way		
York St.	minor arterial	2	one-way		
Church St.	minor arterial	4	two-way		
Wellington St. W.	minor arterial	2	two-way	yes	
Front St. W.	minor arterial	4	two-way		
Bremner Blvd	minor arterial	2	two-way		
Blue Jays Way	minor arterial	1	one-way	yes	
Navy Wharf Crt.	minor arterial	2	two-way		
Armoury St.	collector	2	two-way		
St Patrick St.	collector	2	two-way		
McCaul St.	collector	2	two-way	yes	
Augusta Ave.	collector	1	one-way		
Portland St.	collector	2	two-way		
Wellington St. W.	collector	2	two-way	yes	
Fort York Blvd	collector	2	one-way	yes	
Dan Leckie Way	collector	2	two-way		
Peter St.	collector	2	two-way	yes	
Blue Jays Way	collector	2	two-way		
John St.	collector	2	two-way		
Simcoe St.	collector	2	one-way	yes	
Sheppard St.	collector	2	two-way		
Victoria St.	collector	2	two-way	yes	
Lombard St.	collector	2	two-way		
Toronto St.	collector	2	two-way		
Scott St.	collector	2	two-way		
The Esplanade	collector	2	two-way		

3.2.2 Toronto Parking Authority

The Toronto Parking Authority (TPA) provided three dataset [85]. The data included turning movement counts of vehicles, parking occupancy and parking payment transactions.

Turning Movement Counts of Vehicles

The first data set contained turning movement counts of vehicles for 23 intersections of the study area. The locations of these intersections are shown in Figure 5.12. This data will be utilized to develop the VISSIM micro-simulation in Chapter 5 of this study.

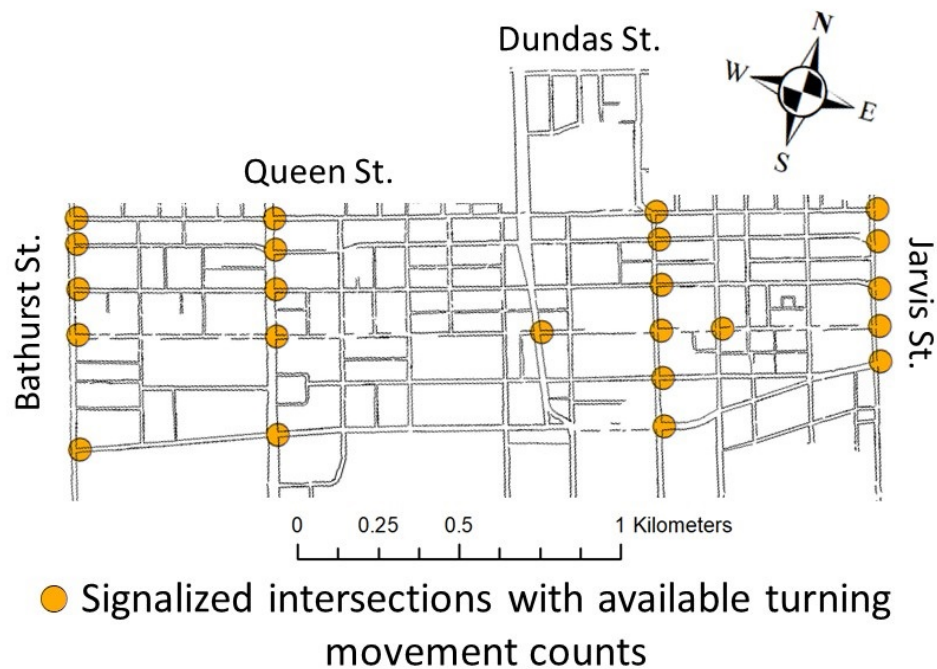


Figure 3.8: Signalized intersections with available turning movement counts

The data set contained two files; the first file was ".csv," and the second file was ".shp." The data on the first file was organized in 8 columns and contained 69,458 rows. The data consisted of average hourly volume counts for two vehicle types (cars and trucks) and two-day types (weekend and weekday). A preview of the first 7 rows of the raw data can be seen in

Table 3.4 followed by a description of each column.

Table 3.4: Turning movement vehicle count

Intersection ID	Day	Class	Month	Hour	Leg	Movement	Average
1	Weekday	Cars	Nov-19	0	N	Left	42.38095238
1	Weekday	Cars	Nov-19	0	N	Through	188.047619
1	Weekday	Cars	Nov-19	0	N	U-Turn	0
1	Weekday	Cars	Nov-19	0	S	Right	51.14285714
1	Weekday	Cars	Nov-19	0	S	Through	170.2857143
1	Weekday	Cars	Nov-19	0	S	U-Turn	0
1	Weekday	Cars	Nov-19	0	W	Left	18.76190476

- **Intersection ID:** Intersection number ID which is used as a link between both files in the data set. Values are integers and range from 1 to 23.
- **Day:** Categorical field for day type with two possible values, which include weekday and weekend.
- **Class:** Categorical field for vehicle type with two possible values; cars and trucks.
- **Month:** Month and year of data. Data spans the three months September, October and November for the year 2019.
- **Hour:** Hour of the day for which the average volume is calculated. Values are integers ranging from 0 to 23.
- **Leg:** Leg of intersection for which the average volume is calculated. Categorical values with letters N, S, E, and W refer to the intersection's legs located North, South, East, and West, respectively.
- **Movement:** Movement type for which the average volume is calculated. Each leg will have four movement types. Types include Left, Right, Through, and U-Turn.
- **Average:** Average vehicle volume count.

Note from Table 3.4 that some entries under the column "Average" may have a value of 0. After further examining the data and cross-referencing with google maps data, we concluded

that the value of 0 means that the movement at that intersection is not permitted. Therefore when using the data to build the micro-simulation model, we ensured that movements with average volumes of 0 were implemented as not permitted within the simulation software.

Payment Machines in Study Area

There was a second data set provided by Toronto Parking Authority [85] was data on the on-street parking payment machines in the study area. The data was divided into two ".xlsx" files named "Parking Machines in the Study Area" and "On Street Parking Review". A preview of the first file named "Parking Machines in the Study Area" can be seen in Table 3.5. This file contained information on the location of parking machines in the study area and characteristics, including parking spaces, hourly rate, and parking cluster usage. This data is utilized to calibrate econometric model developed in this thesis.

Table 3.5: Parking payment machines in the study area

Street	Side of Street	From	To	Location ID	Spaces	Rate/hr	Usage 2019
Adelaide St. W.	North	Bathurst St.	Spadina Ave.	4310	33	\$3.00	High
Adelaide St. W.	North	Simcoe St.	Spadina Ave.	3106	30	\$5.00	Medium
Adelaide St. E.	North	Church St.	Jarvis St.	4109	10	\$4.00	Medium
Adelaide St. E.	North	Victoria St.	Church St.	4108	14	\$5.00	High
Armoury St.	North	Centre St.	University Ave.	3012	4	\$5.00	Medium

The data contained 8 columns and 60 rows. The first 4 columns described the street location where the parking machine is located and the number of on-street parking spaces served by the machine. For example, the first parking machine is located on the north side of Adelaide St. W. from Bathurst St. to Spadina Ave. Column 5 contains a unique payment machine identifier that can cross-reference the data with other data sets. Column 6 and

7 contains the number of parking spaces that are served by the payment machine and the hourly rate for parking, respectively. Lastly, column 8 contains a measure of usage for the payment machine in 2019. Usage is given based on the average parking occupancy of the location. Low, medium and high usage pertain to ranges 0% to 50%, 51% to 79%, and 80% to 100% respectively.

The second file in the data set named "On-Street Parking Review" contained information regarding the revenue generation of each parking cluster for the year 2019. A preview of the data can be seen in Table 3.6. The data is composed of 8 columns and contains 986 entries. We note that the data had significantly more entries than the data described in Table 3.5. Table 3.5 only included parking clusters (described by Location ID) in the study area. In contrast, Table 3.6 is a master file with data of all parking clusters in Toronto. Recall, a parking cluster is a collection of parking spots at a given on-street parking location.

Table 3.6: Revenue generated by parking machines in 2019

Location ID	Spaces	Rate/hr	Revenue 2019	Revenue per Space 2019	Percent of Available Hours Purchased 2019	USAGE 2019
3001	32	5	\$335,683	\$10,490	46.91%	High
3002	33	5	\$203,299	\$6,161	33.37%	Medium
3003	9	5	\$82,561	\$9,173	41.03%	Medium
3004	20	5	\$145,362	\$7,268	32.50%	Medium
3005	30	5	\$262,490	\$8,750	39.13%	Medium
3006	8	5	\$76,921	\$9,615	43.00%	Medium
3007	6	5	\$36,169	\$6,028	26.96%	Medium
3008	24	5	\$229,309	\$9,555	42.73%	Medium
3009	8	5	\$47,750	\$5,969	26.69%	Medium
3010	28	5	\$193,825	\$6,922	30.96%	Medium

Similarly to Table 3.5, the first three columns in Table 3.6 are Location ID, number of parking spaces, and hourly rate. In addition to this, column 4 and 5 contain values for the revenue generated by the parking machine in the year 2019. Column 4 includes the total revenue generated, while column 5 normalizes the revenue based on the number of parking

spaces the payment machine serves. Column 6 gives values for the percentage of available hours purchased in 2019. We note that values are generally below 50% (44 out of 60). This observation is as expected because we do not expect many parking spaces to be occupied during off-peak hours, especially during nighttime.

Parking Transaction Data

The last data set that was provided by the Toronto Parking Authority [85] and was used in this thesis pertained to parking transaction data. The data set named "TPA Data 2019 10 TD" was of file type ".zip". This folder contained 989 ".xlsx" type files. The naming format for these files began with the prefix "sm201810" followed by a number suffix corresponding to the Location ID number similar to the ones seen in Tables Table 3.5 and Table 3.6. For example, the first .xlsx file was named "sm2019103001" representing the parking transaction data for Location ID 3001 for the tenth month (October) of 2019. This data is utilized to calibrate econometric model developed in this thesis.

Each of the 989 .xlsx files contained transaction data organized into a table containing 18 columns and 31 rows. Each of the 31 rows pertained to the averaged values of each day of October. The title and description of each column are seen below.

- **Location ID:** Identifying ID for each parking cluster. ID can be cross-referenced with the payment machines' data set.
- **Year:** Year for which the data was collected. All entries for this column will be 2019.
- **Month:** Month of the year for which the data is collected. All entries in this column will have a value of 10, meaning the information is all from October.
- **Day:** Day of the month. Integer values will range from 1 to 31.
- **Trans:** Total number of transactions at the specified Location ID for the given day.
- **Peak:** Peak occupancy for the given day. The greatest number of vehicles parked

during the peak hour in a day.

- **Peakmin:** Peak occupancy in minutes. It can be divided by 60 to peak occupancy in hours.
- **Peakshort:** Number of vehicles parking under 3 hours.
- **Shortfees:** Revenue generated from those vehicles that have parked under 3 hours.
- **Longpeak:** Revenue generated from those parking a minimum of 4 hours and between 6 am to 7 pm.
- **Nondiscpeak:** Number of vehicles that park at least 4 hours during the non-discretionary peak period. This period is between 10 am and 7 pm.
- **Commuter:** Number of commuter vehicles. Vehicles are considered commuters if they park between 6 am – 10 pm for at least 6 hours.
- **Late:** Number of vehicles parking between 10 pm and 6 am.
- **Morning:** Number of vehicles parking between 6 am and 10 am.
- **Midday:** Number of vehicles parking between 10 am and 6 pm.
- **Afternoon:** Number of vehicles parking between 2 pm and 6 pm.
- **Evening:** Number of vehicles parking between 6 pm and 10 pm.
- **Revenue:** Total revenue generated at the Location ID for the given day.

3.3 Data Limitations

Building a micro-simulation model and calibrating the developed econometric model requires a large amount of data. In Sections 3.2.1 and 3.2.2 we have described the collected/acquired data. However, the collected/acquired data is not perfect for the study and has limitations. To begin, the data that was collected manually with the use of Google maps pertained to data collected in October of 2020. However, for this thesis, we mainly utilize transaction data

from October 2019. Between these two periods, there could have been changes to the road infrastructure, especially since October 2019 occurred before the City of Toronto officials implemented changes to the infrastructure to accommodate transportation users during the COVID-19 pandemic.

Secondly, the parking transaction data received from the Toronto Parking Authority [85] was aggregated daily for October 2019. Ideally, we would have liked to receive raw data that can then be extensively analyzed; only a limited number of insights can be derived from aggregated data. Nonetheless, the provided data was essential for the project. Lastly, the provided parking transaction data was not divided based on vehicle types. The data received was aggregated and only contained one vehicle type. For our analysis, we assume that this data pertains to passenger vehicles only, although there might be a proportion consisting of parcel trucks.

3.4 Chapter Summary

In this chapter, we discuss the characteristics of the chosen study area and the data used in this thesis. First, we give the background information on the study area. The study area selected for this thesis is located in the downtown core of Toronto, Ontario, Canada. This study will examine 54 on-street parking clusters within the area. This study area was selected for this thesis. The site is currently important to the City of Toronto and is being used for the City of Toronto Curbside Management Strategy (CMS) [77]. The same study area as the City of Toronto Curbside Management Strategy is chosen to provide insights to the City of Toronto parking authorities regarding the three analyzed pricing strategies in this paper, including hourly pricing, progressive pricing, and time-of-day pricing. Future studies can further explore these insights for other dense urban areas.

For this study area, we discuss important characteristics that can be important when

performing data analysis in a future section of this thesis. These characteristics include land use and road types and the presence of alternate modes of transportation. Land use is taken into consideration as it can greatly affect the transportation network since the difference in economic activity will generate differences in transportation infrastructure use. The governmental authority (City of Toronto) divides land use into 11 categories in Toronto [80]. Furthermore, we examine the road types found in the area as their presence will give an idea of the travel demand that will flow through those areas. There are 14 major arterial roads, 7 minor arterial roads, and 18 collector roads. A road system type can describe the arrangement of these roads. The study area consists of a combination between the gridiron and loose grid systems. On the north side of the study area, where there are mainly major arterial and collector roads, the roads can be classified as a gridiron system. On the south side of the study area, where the roads mainly include minor arterial and collector roads, the road system can be classified as a loose grid system.

Additionally, the alternate modes of transportation in the study area were considered. Cycling was not thoroughly examined since the micro-simulation model will not contain bike lanes. However, the possible interactions that public transport (busses and streetcars) can have with the vehicles in the area were considered. After examining the bus and streetcar lines in the study area, we conclude that the lines with the transport mode of streetcars offer frequent service while bus lines usually have a service frequency of 30 minutes. For the rest of this thesis, we TTC streetcars since the potential impact that busses can have on the congestion of the model are assumed to be minor. The TTC transit lines included in this study are lines 501, 504, 503, 514, and 511.

Regarding data utilized, two primary data sources were described in this section. These two sources include Google Maps [84] and the Toronto Parking Authority [85]. Data collected from Google Maps [84] pertained to road characteristics. This data was manually collected. The first characteristic which was collected was the intersection type. Data collection showed

that there existed 74 signalized intersections in the study area. Data including the number of lanes and allowed turning movements for each intersection was also collected. Lastly, road characteristics were also collected at the street level. Characteristics include the number of driving lanes, the direction of flow, bike lanes presence and streetcar presence.

With regards to the data received from the Toronto Parking Authority [85], the three data sets provided included turning movement counts of vehicles, parking occupancy and parking payment transactions for the 54 on-street parking clusters of the study area. A description of each data type can be found in Section 3.2.2. This collected and acquired data will be analyzed and utilized in Chapter to develop the micro-simulation model. Although in Chapter 4 the econometric model derives general equations and this data is not directly utilized, the data is used for data validation and micro-simulation development.

Chapter 4

Modelling Methodology

Consider a parking facility with a parking demand of T vehicles per hour. The facility may either be an off-street parking lot or a cluster of on-street parking spaces in close vicinity and managed by the same authority. We consider a time period of interest during which the parking demand is known such as the morning peak hours. A proportion α of the demand are low value drivers, and the remaining $1 - \alpha$ are high value drivers. Let $T_l = T\alpha$ and $T_h = T(1 - \alpha)$ be the demand of high and low value drivers, respectively, where “ h ” and “ l ” are mnemonics for the two groups. The terms “low” and “high” are indicative of the relative notion of the driver group’s initial marginal utility. That is to say, the driver group with a higher initial marginal utility will be denoted high value while the other driver group will be denoted low value. We normalize the total demand to one (i.e., $T = 1$) for exposition purposes, therefore $T_l = \alpha$ and $T_h = 1 - \alpha$. Drivers benefit from time spent parking as this time can be used to engage in utility-generating activities. Let $u_i(d)$ be a group i driver’s surplus from the d^{th} hour of parking. $u_i(d)$ is regarded as the *marginal utility* of parking, which satisfies $\partial u_i(d)/\partial d < 0$ representing the law of diminishing returns, indicating for example that the first hour of parking for shopping purposes provides a greater benefit the second hour. We further assume the marginal utility function is convex and satisfies $\partial^2 u_i(d)/\partial d^2 > 0$ for both groups. This type of marginal utility function can capture both work and leisure-related trips as long as the function parameters are well-calibrated. Nevertheless, they are more suitable for non-work-related trips where the dwell times are sensitive to the parking pricing policy.

4.1 Pricing Policies

We define two pricing policies: hourly and progressive pricing. We describe the impacts of these policies on behavior of the drivers in choosing their preferred dwell times. We then derive the optimal revenue and social welfare maximizing structure of the policies.

Hourly pricing:

The price of parking is p dollars per hour. Let $U_i(d)$ be the *utility* of group i drivers with parking dwell time d . The utility is the benefit from engaging in activities (the integral of marginal utility from zero to d) minus the paid price of parking, pd , given as

$$U_i(d) = \int_0^d u_i(w)dw - pd. \quad 4.1$$

We note that the parking search cost (i.e., the monetary value of the time spent searching for parking) is not included in the total utility because Equation 4.1 accounts for the utility gained from the moment a driver finds parking. We later consider the search cost as a negative externality in social welfare to alleviate excessive searching and its adverse impacts on traffic, emissions, and welfare.

On average, drivers maximize their utility by choosing an optimal parking dwell time denoted by d_i^* for group i . From the first order condition, setting the derivative of Equation 4.1 to zero gives

$$d_i^* = u_i^{-1}(p), \quad 4.2$$

where $u_i^{-1}()$ is the inverse of the marginal utility function. Given the concavity of $u_i(d)$ w.r.t. d , there exists a single and unique d_i^* for each group of drivers. Note that drivers reduce their dwell time if p increases.

Progressive pricing:

In progressive parking pricing, drivers pay p_1 per hour if their dwell time is less than q . For those with a dwell time larger than q , the price is p_1 for the first q hours and p_2 for the remaining hours (from q to d). In line with other applications of progressive parking, we assume $p_2 \geq p_1$ as this policy is stipulated to reduce excessive dwell times [4]. The utility of group i drivers is

$$U_i(d) = \int_0^d u_i(w)dw - C(d), \quad 4.3$$

where

$$C(d) = \begin{cases} p_1 d & d \leq q \\ p_1 q + p_2(d - q), & d > q \end{cases} \quad 4.4$$

is the cost of parking for drivers.

Drivers maximize their utility by choosing an optimal dwell time denoted by d_i^* for group i . From the first order condition, setting the derivative of Equation 4.3 to zero gives

$$d_i^* = \begin{cases} u_i^{-1}(p_1) & u_i(q) \leq p_1 \\ q & p_1 < u_i(q) \leq p_2 \\ u_i^{-1}(p_2) & p_2 < u_i(q). \end{cases} \quad 4.5$$

We note that hourly pricing is a special case of progressive pricing if we set $q = 0$ and $p_1 = p_2 = p$. Thus, the latter outperforms the former regardless of if the policy's objective is to maximize revenue or social welfare. Hereafter, we focus only on progressive pricing and consider the special case of hourly pricing wherever needed.

4.2 Revenue Maximization

Parking policies are designed to achieve a specific objective depending on decision-maker preferences. Private parking operators (e.g., garage owners) seek to maximize their revenue, whereas public agencies (e.g., parking authorities) maximize the social welfare of the community which accounts for the surplus gained by drivers and revenue generated from parking minus the cost of negative externalities. We consider both objectives in the optimal design of parking policies [86].

We maximize revenue by choosing the optimal prices, p_1 and p_2 , and the threshold q . The revenue, denoted by π , is the sum of payments from each group, denoted by π_i , such that $\pi = \pi_l + \pi_h$. The revenue of each group is

$$\pi_i = \begin{cases} T_i p_1 d_i^* & u_i(q) \leq p_2 \\ T_i (p_1 q + p_2 (d_i^* - q)) & u_i(q) > p_2. \end{cases} \quad 4.6$$

According to Equation 4.6, one of three cases may occur as shown in Figure 4.1. In case 1 (Figure 4.1a), both marginal utility functions only cross the horizontal p_2 line and both groups pay the augmented price p_2 . In case 2 (Figure 4.1b), the low value group pays the base price, p_1 , and the high value group pays the augmented price, p_2 . In case 3 (Figure 4.1c), both groups pay the base price, making progressive pricing ineffective as both user groups only pay p_1 and no driver's dwell time is longer than the threshold q . We do not consider case 3, because this case is essentially the same as hourly pricing.

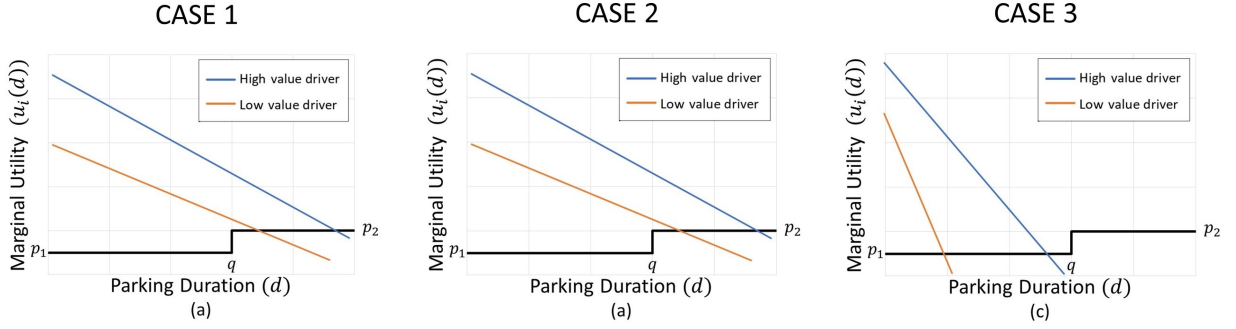


Figure 4.1: Relationships between dwell time and the policy design

4.3 Social Welfare Maximization

We now seek to design the progressive pricing policy to maximize social welfare. We first define the average parking search cost as follows. Let s denote the supply of parking in the study area, i.e., the number of parking spaces. According to Little's Law, parking occupancy is the product of arrival rate and parking dwell times [87]. Parking occupancy is denoted by o , which under steady state conditions is

$$o = T_h d_h^* + T_l d_l^*. \quad 4.7$$

Empirical and theoretical evidence shows that search time increases with the ratio of parking occupancy to supply [87]. In line with [88] the search time is strictly increasing with the ratio o/s . We define search time as $(o/s)^2$. The marginal cost of search is γ and the average search cost per driver is $\gamma o/s$.

Social welfare is the sum of the revenue generated from parking payments, and the utility of drivers (consumer surplus), minus the negative externality of searching. Consumer surplus, a component of social welfare, is defined as the excess social value gained over the price paid

for a product (parking). We present social welfare, denoted by W , as

$$W = T_l U_l(d_l^*) + T_h U_h(d_h^*) + \pi - \gamma T \left(\frac{(T_h d_h^* + T_l d_l^*)}{s} \right)^2, \quad 4.8$$

where the first two terms are the total utility of the low and high value drivers, the third term is the revenue, and the last term is the negative externality of searching for parking.

4.4 Optimal Policies

We now analyze the optimal step-wise structure of the progressive pricing policy to maximize either the revenue or social welfare. To this end, we consider a special case of the marginal utility function which allows the derivation of closed-form relationships that yield managerial insights. The marginal utility function can be of any form; however, some common functions are negative exponential, power, and linear [89]. For exposition purposes, we consider a linear marginal utility function because it has properties that enable closed-form derivations of the optimal policies.

Assume the marginal utility function of group i drivers is linear and defined as

$$u_i(d) = a_i - b_i d, \quad 4.9$$

where $a_i, b_i > 0$ are parameters and d is the dwell time. The linear marginal utility function is convex and exhibits diminishing returns. Without loss of generality, we let $u_l(d) \leq u_h(d)$ for all d , indicating that the high value drivers always obtain a larger marginal utility than low value drivers for the same dwell time. Note that the maximum dwell time according to Equation 4.9 is b_i/a_i for group i drivers, which is the dwell time at which the marginal utility function is zero. We use the equilibrium conditions of the previous section to derive

the dwell time of group i drivers as

$$d_i^* = \begin{cases} (a_i - p_1)/b_i & a_i - b_i q \leq p_1 \\ q & p_1 < a_i - b_i q \leq p_2 \\ (a_i - p_2)/b_i & p_2 < a_i - b_i q. \end{cases} \quad 4.10$$

For a given policy defined by p_1, p_2 , and q , one of three cases may occur as explained previously and depicted in Figure 4.1 and Equation 4.10 . In case 3 both groups pay the base price. We do not consider case 3 as explained above.

4.4.1 Revenue Under Linear Marginal Utilities

We separately consider the above two cases in revenue maximization, compare the revenue of the two, and pick the case with the largest revenue. Following Equation 4.6 the optimal step-wise pricing structure in case 1 is the solution of the following mathematical model

$$\begin{aligned} \max_{q, p_1, p_2} \quad & \pi = T_l(qp_1 + (d_l^* - q)p_2) + T_h(qp_1 + (d_h^* - q)p_2) \\ \text{s.t.} \quad & 0 \leq q \leq (a_l - p_2)/b_l, \end{aligned} \quad 4.11$$

where d_i^* is obtained from Equation 4.10. In the objective function given in Equation 4.11 the two terms are the revenue of the low and high value groups, respectively. The single constraint ensures case 1 occurs. We show in the appendix that the optimal solution of Equation 4.11 is

$$\begin{aligned} q^* &= 0, \\ p_1^* &= p_2^* = \frac{a_h b_l (1 - \alpha) + \alpha a_l b_h}{2b_l(1 - \alpha) + 2\alpha b_h}, \end{aligned} \quad 4.12$$

which shows we have a single-step hourly pricing structure, where $p_2^* = p_1^*$ and $q^* = 0$. Note that q^* can have any value here but by definition, we set $q = 0$ as explained above. In other words, the first step of the price profile does not exist, and we have an hourly pricing policy instead. It is evident from the structure of this optimal policy that progressive pricing is the same as hourly pricing if one seeks to maximize revenue while ensuring both user groups pay the augmented price. According to Equation 4.12, price decreases with α (proportion of low value drivers) because of their lower willingness to pay (in terms of intercept and slope of the marginal utility of low value drivers). Because of the structure of Equation 4.12, we hereafter refer to case 1 as hourly pricing.

Similarly, following Equation 4.6 the optimal policy in case 2 is the solution of the following mathematical model

$$\begin{aligned} \max_{q, p_1, p_2} \quad & \pi = T_l p_1 d_l^* + T_h (q p_1 + (d_h^* - q) p_2) \\ \text{s.t.} \quad & (a_l - p_1)/(b_l) \leq q \leq (a_h - p_2)/(b_h), \end{aligned} \tag{4.13}$$

where d_i^* is obtained from Equation 4.10. The two terms in the objective function given in Equation 4.13 are the revenue of the low and high value groups, respectively. The single constraint ensures case 2 occurs. We show in the appendix that the optimal solution of Equation 4.13 is

$$\begin{aligned} q^* &= \frac{\alpha a_l}{b_l(1 + \alpha)}, \\ p_1^* &= \frac{a_l}{1 + \alpha}, \\ p_2^* &= \frac{a_h}{2} - \frac{\alpha a_l b_h}{2b_l(1 + \alpha)}, \end{aligned} \tag{4.14}$$

which, contrary to case 1, has a step-wise structure with distinct base and augmented prices.

We compare the revenue of the two cases in the following proposition. An increase in the proportion of low value drivers α decreases price levels p_1 and p_2 due to the lower willingness to pay (in terms of intercept and slope of the marginal utility) of the low value drivers.

Proposition 1. In the optimal revenue maximizing progressive parking pricing, the low value group pays the base price and the high value group pays the augmented price.

According to Proposition 1 and its proof in the appendix, case 2 yields a larger or equal maximum revenue than case 1. Thus, progressive parking pricing for revenue maximization is a tool of market segmentation in the sense that it targets one group to pay the base price and the other to both prices (base price for the initial q hours and the augmented price thereafter), as opposed to having both groups paying both price levels.

We now discuss some properties of the optimal policy. Using Equation 4.12 and Equation 4.14 and comparing the optimal revenue from the two cases, progressive pricing (case 2) yields a larger revenue than hourly pricing (case 1) if the following condition is satisfied:

$$\frac{a_l}{a_h} < \frac{b_l(1 + \alpha)}{2b_l + \alpha b_h}. \quad 4.15$$

The above equation stipulates that the initial marginal utility ratio, $\frac{a_l}{a_h}$, of both driver groups must be below a threshold for the progressive pricing policy to yield a larger revenue. This implies that when the initial marginal utilities significantly differ, progressive pricing is the superior pricing structure under revenue maximization. This argument and the impacts of market share proportion, α , are further analyzed in Section 4.4.4.

4.4.2 Social Welfare Under Linear Marginal Utilities

We analyze the social welfare with linear marginal utilities. The following lemma expresses the impact of the cutoff time on social welfare.

Lemma 1. *The cutoff time, q , does not affect social welfare.*

According to Lemma 1 and its proof in the appendix any changes in the cutoff time affects revenue and consumer surplus, but the sum of the two remains constant. This occurs because the trade-off between revenue and consumer surplus is of zero sum. We note the searching externality in social welfare is related to the equilibrium dwell times that are sensitive only to the augmented price in both case 1 and 2. Therefore, the cutoff time also does not affect the negative externality of searching.

Lemma 2. *The base price, p_1 , does not affect social welfare in case 1 where both groups pay the augmented price, p_2 .*

Interpretation of Lemma 2 is similar to Lemma 1. Any changes in the base price of case 1 affect the revenue and consumer surplus, but the sum of the two remains constant. The base price also does not affect the negative externality of searching. We note that Lemma 1 holds in general whereas Lemma 2 only applies to case 1, because in case 2 the low value group pays the base price and any changes in this price affect social welfare.

We now separately consider the two cases for social welfare maximization. The optimal policy in case 1 is obtained from the following mathematical model:

$$\begin{aligned} \max_{q, p_1, p_2} \quad & W = T_l U_l(d_l^*) + T_h U_h(d_h^*) + \pi - (\gamma T(T_h d_h^* + T_l d_l^*)/s)^2 \\ \text{s.t.} \quad & 0 \leq q \leq (a_l - p_2)/b_l, \end{aligned} \tag{4.16}$$

where d_i^* is from Equation 4.10. The first two terms of the objective function, $T_l U_l(d_l^*)$ and $T_h U_h(d_h^*)$, indicate the consumer surplus of the groups, the third term is the revenue, and the last term is the negative externality of searching. The single constraint ensures case 1 occurs.

We show in the appendix that the optimal solution of Equation 4.16 is

$$p_1^* = p_2^* = \frac{2(a_h b_l (1 - \alpha) + a_l \alpha b_h)}{2\alpha b_h + b_l (2 - 2\alpha + b_h s^2)}. \tag{4.17}$$

indicating that social welfare is only a function of a single price (Lemma 2), and is insensitive to the cutoff time (Lemma 1).

The social welfare maximizing policy of case 2 is the solution of the following mathematical model:

$$\begin{aligned} \max_{q, p_1, p_2} \quad & W = T_l U_l(d_l^*) + T_h U_h(d_h^*) + \pi - (\gamma T(T_h d_h^* + T_l d_l^*)/s)^2 \\ \text{s.t.} \quad & (a_l - p_1)/b_l \leq q \leq (a_h - p_2)/b_h, \end{aligned} \quad 4.18$$

where d_i^* is obtained from Equation 4.10. The first two terms of the objective function, $T_l U_l(d_l^*)$ and $T_h U_h(d_h^*)$, indicate the consumer surplus of the groups, the third term is the revenue, and the last term is the negative externality of searching. The single constraint ensures case 2 occurs.

We show in the appendix that the optimal solution of Equation 4.18 is

$$p_1^* = p_2^* = \frac{2(a_h b_l (1 - \alpha) + a_l \alpha b_h)}{2\alpha b_h + b_l (2 - 2\alpha + b_h s^2)}, \quad 4.19$$

which has a single price and is insensitive to the cut-off time as explained above.

We compare the social welfare of the two cases in the following proposition.

Proposition 2. Progressive parking pricing does not improve social welfare over hourly pricing.

4.4.3 Second-Best Social Welfare Price & Pareto-Optimality

We further analyze the trade-offs between the two conflicting objectives of revenue and social welfare maximization. We consider the two objectives simultaneously through a bi-objective maximization problem [90]. According to Proposition 2, given that social welfare is maximized when the base and augmented price are equal, we explore the impacts of a single hourly price

(Case 3) on the properties of Pareto-efficient solutions. Pareto efficiency is defined as an economic state where resources cannot be better allocated without making an individual worse off. Let p^{sw} and p^* be the optimal revenue and social welfare maximizing prices, respectively, and $\pi(p, s)$ be the revenue when hourly price is p and supply is s . We show in the appendix that there exists some supply level $\hat{s}$ defined as

$$\hat{s} = \sqrt{\frac{2(b_h(1 - \alpha) + b_l)}{b_l b_h}}, \quad 4.20$$

for which $p^* = p^{\text{sw}}$ yielding the same revenue in both objectives. When $s \leq \hat{s}$, then $p^{\text{sw}} > p^*$ because insufficient supply pushes the social welfare maximizer to increase the hourly price to reduce dwell times and consequently the external negativity of searching for parking. In contrast, when $s > \hat{s}$, then $p^{\text{sw}} < p^*$ because ample parking supply allows the social welfare maximizer to reduce the hourly price (compared to p^*) and allows drivers to benefit from longer dwell times. In both cases, as long as $s \neq \hat{s}$, then $\pi(p^*, s) > \pi(p^{\text{sw}}, s)$.

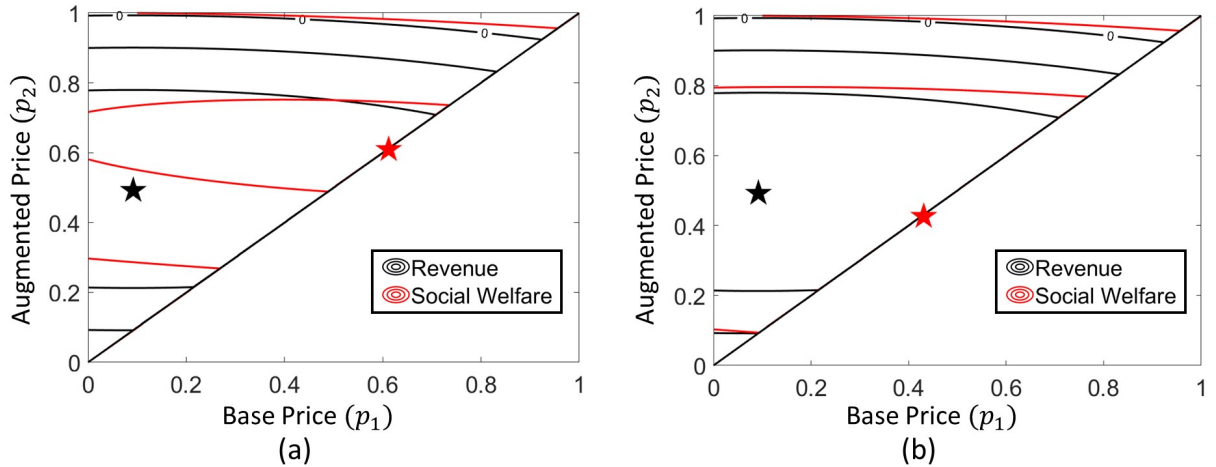
We now present the second-best social welfare price at which we seek to set the prices and cutoff time to maximize social welfare while ensuring a non-negative revenue. Such regulatory policies enhance sustainability by making the system financially self-sufficient.

In our analysis, we keep the cutoff time constant and equal to its revenue-maximizing expression presented in Equation 4.14. We fix the revenue-maximizing cutoff time because the cutoff time does not affect social welfare according to Lemma 1. Figure 4.2 shows the revenue and social welfare from Equations 4.13 and 4.16 for varying base and augmented prices, and the star signs represent the optimal objectives. Note that the figure only contains values for the upper diagonal half where $p_2 \geq p_1$. According to Equation 4.19, social welfare is maximized at a single hourly price where $p_1 = p_2 = p^{\text{sw}}$ as shown by the blue star in Figure

4.2. The revenue obtained at the social welfare-maximizing profit is

$$\pi(p^{\text{sw}}, s) = \frac{2s^2(a_h(\alpha - 1)b_l - a_l\alpha b_h)^2}{(b_l(-2\alpha + b_h s^2 + 2) + 2\alpha b_h)^2} \quad 4.21$$

which is strictly positive indicating that the social welfare-maximizing policy yields a non-negative revenue and can be self-sustaining. This result, however, holds when parking demand is fixed and driver utility is inelastic.



(Fixed parameters: $\alpha = 0.1$, $a_l = 0.1$, $b_l = 1.2$, $a_h = 1$, and $b_h = 1$)

Figure 4.2: Revenue and Social Welfare plots for values $s = 1$ (a) and $s = 1.5$ (b)

In the case of elastic demand, the hourly price affects the negative externality of searching in two conflicting ways: an increase in the hourly price may lower dwell times but at the same time increase parking demand. Thus, depending on demand characteristics, the searching externality may increase or decrease. Another influential factor is the cost of parking provision and maintenance, which would affect the overall profit of the policy but is not considered in our analysis because we assume the supply, s , is fixed and so the cost of parking maintenance would also be fixed and not affect the optimal hourly prices.

4.4.4 Optimal Policy Analysis and Comparison

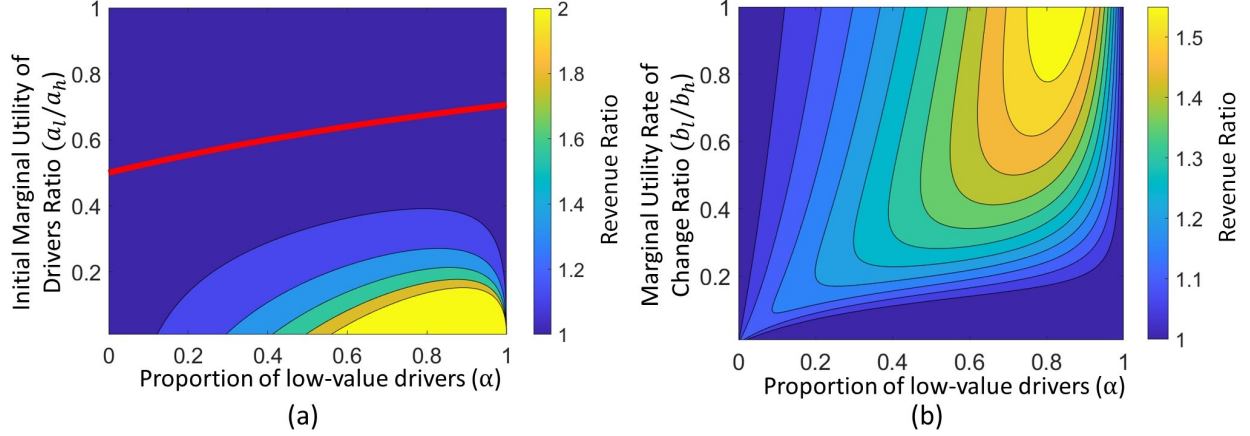
We define the *revenue (or social welfare) ratio* as the ratio of progressive pricing revenue (social welfare) to the hourly pricing revenue (social welfare). This ratio is indicative of the performance of progressive pricing compared to hourly pricing. Under revenue maximization this ratio is always larger than 1.

In the proceeding analysis, we normalize the high-value group's marginal utility intercept to one such that $a_h = 1$, therefore the ratio a_l/a_h is bounded by one and zero. The revenue ratio is presented in Figure 4.3a for varying initial marginal utility ratios (a_l/a_h) and proportion of low-value drivers (α). According to Figure 4.3a when the ratio a_l/a_h is large, the two policies are close in revenue, and when a_l/a_h is small, progressive pricing yields a larger revenue than hourly pricing. Furthermore, the revenue ratio is higher when the parameter α is close to a value of 1. This observation indicates that revenue from progressive pricing is larger than hourly pricing when a high proportion of the demand is composed of low-value drivers.

Equation 4.15 shows the threshold that determines whether the progressive pricing policy is equal or superior to the hourly pricing policy with regards to revenue generation. When $\frac{a_l}{a_h} < \frac{b_l(1+\alpha)}{2b_l+\alpha b_h}$ the progressive pricing policy yields a larger revenue and when $\frac{a_l}{a_h} \geq \frac{b_l(1+\alpha)}{2b_l+\alpha b_h}$ both policies yield the same revenue. This threshold is represented by the red line in Figure 4.3a. Conceptually, the ratio a_l/a_h represents the initial marginal utility of drivers. When a_l/a_h is high (close to 1), both user groups have a similar initial marginal utility for parking, while the a_l/a_h ratio is low (close to 0), their initial marginal utilities differ drastically.

Figure 4.3b shows that when the ratio b_l/b_h initially increases from 0, the revenue ratio increases as well. However, after a threshold is reached, the revenue ratio decreases with a b_l/b_h ratio increase. This phenomenon can be more clearly explained by taking into consideration what happens to the price levels of the progressive pricing structure under these conditions. From Figure 4.4b we can notice that at low values of b_l/b_h , the price levels p_1

and p_2 are equal. Since the price levels are equal, the hourly pricing and progressive pricing structures both yield the same revenue as noted in Figure 4.3b.

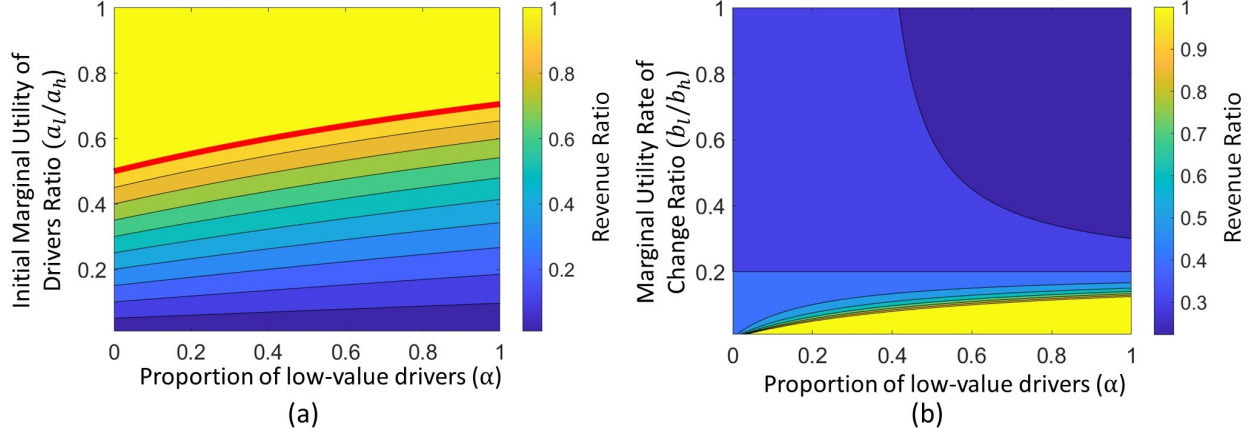


(Fixed parameters: $b_l = 1.2$ and $b_h = 1$)

Figure 4.3: Revenue ratio policy plots for varying α , $\frac{b_l}{b_h}$ and $\frac{a_l}{a_h}$

According to Figure 4.3a, progressive pricing outperforms hourly pricing when both user groups have a large difference in their initial marginal utility value. This can be explained by the relationship between the two price levels and the progressive pricing structure as seen in Figure 4.4a. As a_l increases, the optimal base price p_1^* increases while the optimal augmented p_2^* decreases. Given this, there exists a reference ratio a'_l/a'_h at which the two price levels p_1^* and p_2^* are equal. The ratio a'_l/a'_h is equal to $\frac{b_l(1+\alpha)}{2b_l+\alpha b_h}$ and it is represented in Figure 4.4a as the lower bound of the contour line for $p_1/p_2 = 1$.

According to Figure 4.5a and 4.5b, the price levels p_1 and p_2 of the progressive pricing policy are equal when the condition outlined in Equation 4.15 is not met. When p_1 and p_2 are equal, the progressive pricing structure will be equal to the hourly pricing structure since the driver will pay the same hourly rate no matter the parking duration. From both contour maps in Figure 4.5 it can be noted that an increase in the value of α will lead to a decrease in both price levels when the condition outlined in Equation 4.15 is not met. On the contrary, an increase in the ratio $\frac{a_l}{a_h}$ will increase the price levels.

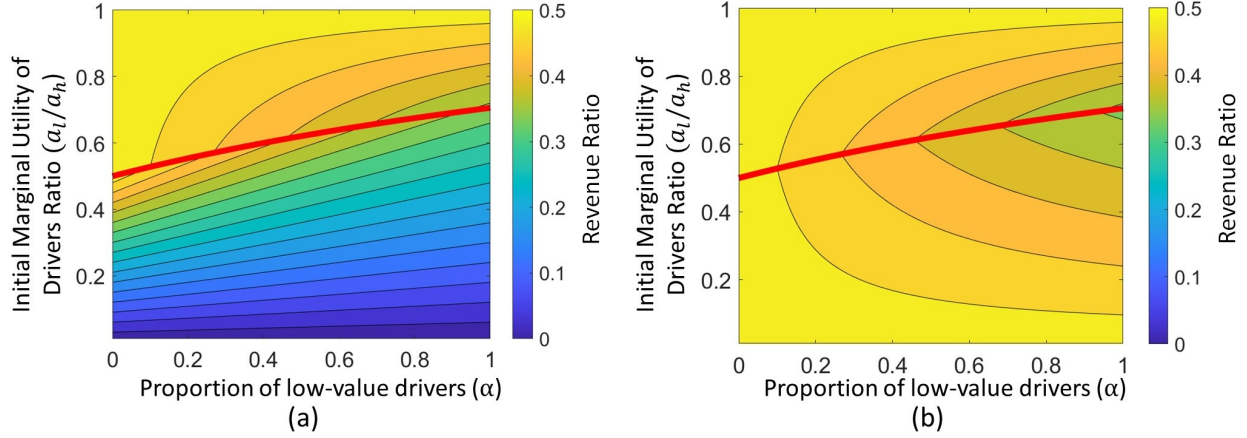


(Fixed parameters: $a_l = 0.2$, $b_l = 1.2$, $a_h = 1$, and $b_h = 1$)

Figure 4.4: Price ratio p_1/p_2 plots of optimal revenue policies

When the condition outlined in Equation 4.15 is met (area under the red lines of Figure 4.5a and 4.5b), price levels p_1 and p_2 will differ. Price level p_1 , seen in Figure 4.5a, decreases with an increase in the proportion of low-value users (α). Furthermore, an increase in the ratio $\frac{a_l}{a_h}$ leads to an increase in price level p_1 . Price level p_2 , seen in Figure 4.5b, decreases with an increase in both α and $\frac{a_l}{a_h}$.

In general, as discussed in this section the optimal progressive pricing structure outperforms the hourly pricing structure when the initial marginal utility of both driver groups differs to a large extent and the market is composed of predominantly low-value users. In addition to this, when the initial marginal utility of both driver groups differs, price levels p_1 and p_2 will also differ as the progressive pricing policy will segmentize the market. However, when both the initial marginal utilities are similar, the progressive pricing policy will yield price levels that are close to the hourly pricing policy. Policymakers who seek to determine whether or not implementing a progressive pricing policy can improve the system, can compare the initial marginal utilities of both driver groups as a preliminary assessment.



(Fixed parameters: $b_l = 1.2$, $a_h = 1$, and $b_h = 1$)

Figure 4.5: Policy plots of optimal price levels p_1 (a) and p_2 (b)

4.5 Chapter Summary

In this chapter, we show formulate a model with the goal of maximizing two commonly used economic measures, revenue and social welfare. In this model we seek to determine the price levels of two pricing structures, hourly pricing and progressive pricing, that will maximize the two economic measures under question. To develop the model, we first derive the general mathematical equations representing the drivers' marginal utility under both pricing structures. An assumption made during the development of this model is that the drivers' marginal utility function can be represented by a linear function. This assumption was made to simplify the derivation of closed-form solutions but future work can be completed on this topic by performing sensitivity analysis utilizing other marginal utility functions. For example, alternative marginal utility functions that can be used include negative exponential and piece-wise functions.

The model considers a market with two user classes. A proportion α of the demand are low-value drivers, and the remaining $1 - \alpha$ are high-value drivers. The terms "low" and "high" are indicative of the relative notion of the driver group's initial marginal utility. That is to say, the driver group with a higher initial marginal utility will be denoted high value

while the other driver group will be denoted low value.

A major result of this section includes the optimal price levels that will maximize revenue or social welfare. For revenue maximization, these price levels are expressed in Equations 4.12 and 4.14 for the hourly and progressive pricing respectively. For social welfare maximization, findings indicate that both the hourly and progressive pricing policies will have the same price levels under optimal conditions. These price levels are seen in Equations 4.17 and 4.19.

In this section, we also analyze the trade-offs between the two conflicting objectives, revenue and social welfare. We consider the two objectives simultaneously through a bi-objective maximization problem. According to Proposition 2, given that social welfare is maximized when the base and augmented price are equal, we explore the impacts of a single hourly price on the properties of Pareto-efficient solutions. From this analysis, we show that there exists a supply level (labelled as $\hat{s}$) for which the optimal price levels under revenue and social welfare maximization (p^* and p^{sw}) yield the same revenue. Furthermore, when the existing supply is above or below this supply level ($\hat{s}$) the revenue generated by the revenue-maximizing price level (p^*) will always be larger than the revenue generated by the social welfare maximizing price level (p^{sw}).

Lastly, in Section 4.4.4 we develop and analyze policy plots from the general equations of revenue for the hourly and progressive pricing policies. Findings from this section indicate that the revenue generated by the optimal progressive pricing policy will always perform equally or better than the hourly pricing policy. In this section, we discuss the scenarios for which the implementation of progressive pricing will be beneficial. From the policy maps, we note that the progressive pricing policy will generate more revenue when compared to its counterpart when the two user classes in the market exhibit a large difference in their initial marginal utility. From these policy maps, policymakers will be able to get a sense of whether or not implementing a progressive pricing policy can increase revenue generated. The optimal solutions to the equations derived in this section will be used in Section 5 to

calculate the optimal price levels used in the developed micro-simulation.

Chapter 5

Model Validation

To further validate the described econometric model, a case study for the City of Toronto is conducted. In addition to validating previous results, the case study analyzes additional parking statistics commonly used as performance measures by parking authorities. Parking management authorities use parking pricing as a demand regulation strategy to reduce socially adverse outcomes. An adhoc approach is to price parking such that 85% of the supply is occupied, leaving the remaining 15% available for parking seekers. Parking utilization levels below 85% are often seen as a misuse of available resources by parking authorities [46]. Public parking authorities also use metrics such as cruising for parking to get a sense of system performance.

Cities often group parking spaces according to their spatial proximity such that every space in the same spatial group is subject to the same pricing regime and other regulatory policies (e.g., allowable dwell time). Spatial grouping also allows cities to ease the payment process by deploying payment machines near their designated spaces. For example, Toronto spatially groups parking spaces giving each cluster a parking ID. Drivers can pay their parking fees by either using the parking meter for the respective parking cluster or paying via a mobile app. Each group of parking spaces (parking cluster) contains an average of 18 parking spaces with a standard deviation of 15. The minimum and maximum number of spaces in these parking clusters are 3 and 82 respectively. Parking spaces are often grouped based on the side of the street at which the spaces are located. This prevents forcing drivers to cross the street to pay at the meter.

The case study completed for the City of Toronto aims to quantify the impacts of implementing progressive and time-of-day pricing policies for on-street parking. These impacts are quantified using parking statistics, including parking occupancy and cruising for parking. The following sections are organized as follows. In Sections 5.1 we analyze the utilized data which was provided by the Toronto Parking Authority (TPA) [85]. Following this in Sections 5.2 and 5.3 we describe the development and calibration of the study area’s road network using VSSIM [68] micro-simulation software. Lastly, in Section 5.4 we described the scenarios developed to test out the pricing policies under question.

5.1 Data Analysis

In this section, we analyze the provided data described in Section 3.2.2. The resulting outcome of this analysis will then be used to calibrate the econometric model and micro-simulation of the study area. In Section 5.1.1 we analyze parking data and in Section 5.1.2 we analyze vehicular flow data.

5.1.1 On-Street Parking

The study area located in Toronto’s downtown core is depicted in Figure 5.1. A selection of on-street parking spaces are operated by one payment machine and denoted as a parking cluster. The data acquired includes daily transactions at each payment machine, including the generated revenue, the total number of transactions, the number of transactions with a parking dwell time longer than three hours, etc., as previously described in Section 3.2.2.

Parking clusters (each operated by one payment machine) will display distinct parking behaviours based on several factors, including land use of the location. Consider three parking clusters within the specified study area. These three locations are selected based on their difference/similarities in location and land use. The first, depicted by the green line and

number 1 in Figure 5.1, consists of institutions, parks, retail, condos, and other open space areas. Location number 2, depicted with a red line, is labelled as regeneration and mixed use area. Lastly, location 3, shown with a blue line, is a mixed-use area. Land use areas are determined based on the map found in Section 3.1.1 created by the local governing agency. Recall that mixed land use areas are composed of multiple land uses, including apartment neighbourhoods, institutional areas and employment areas. These three parking clusters have an hourly rate of 5 \$/hr, 5 \$/hr, and 4 \$/hr respectively.

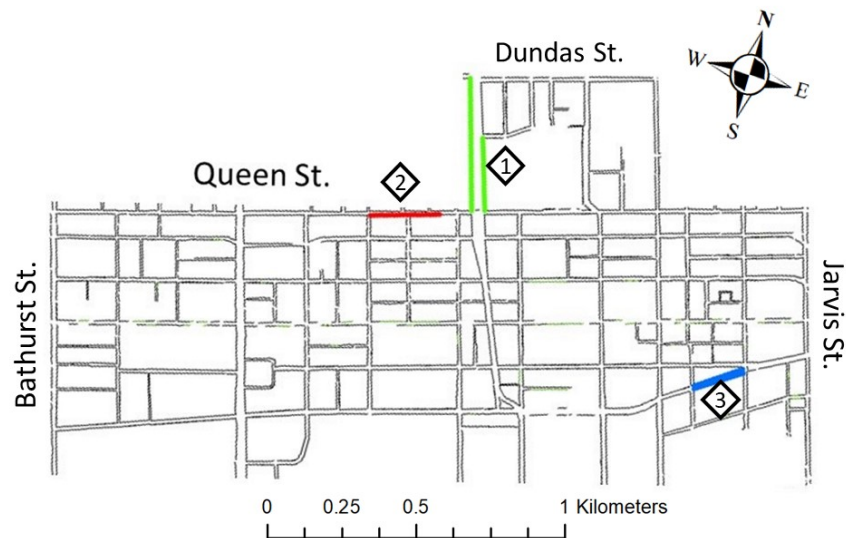


Figure 5.1: Three on-street parking clusters within the study area

The daily parking transaction for October 2019 of all 54 parking clusters was analyzed using an R-language script that can be found in Appendix C. Analysis results for the three areas mapped in Figure 5.1 (areas 1, 2, and 3) can be seen in Figures 5.2, 5.3, and 5.4.

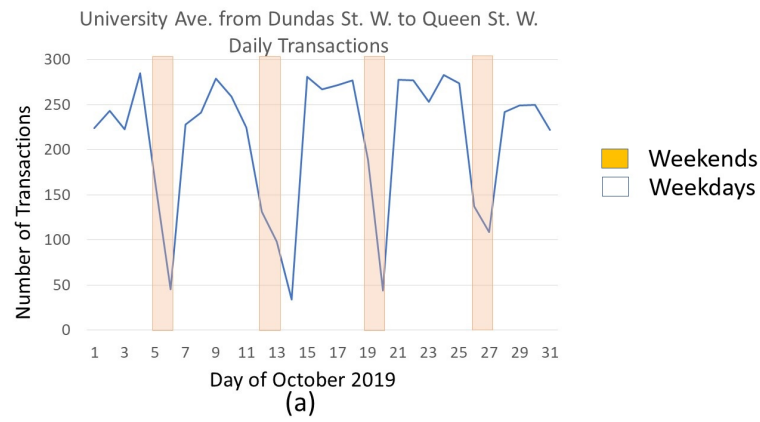


Figure 5.2: Daily transactions for parking cluster 1

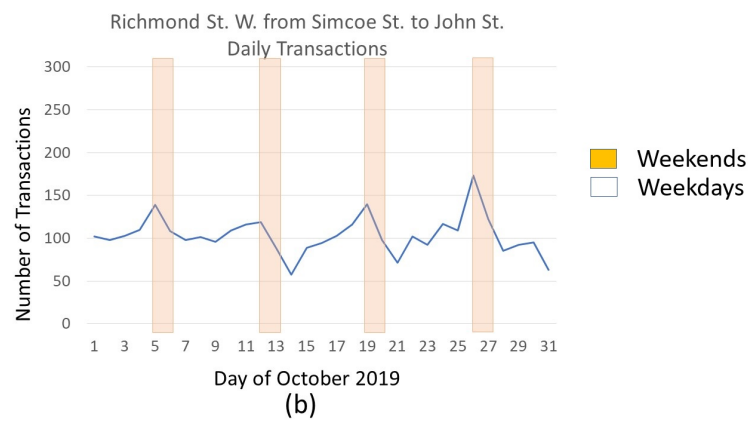


Figure 5.3: Daily transactions for parking cluster 2

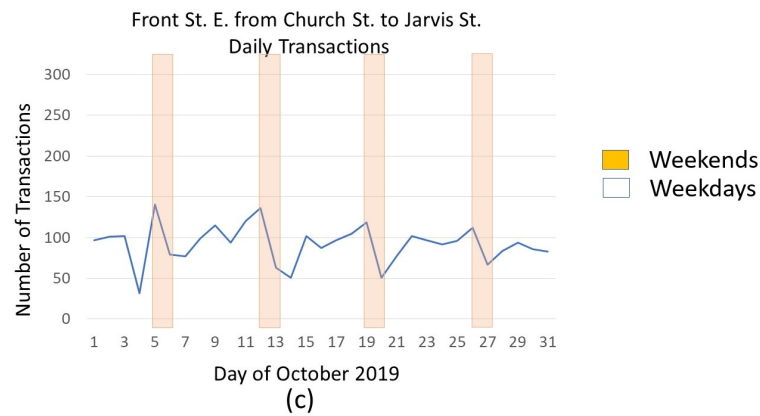


Figure 5.4: Daily transactions for parking cluster 3

As shown in Figure 5.2, the number of daily transactions for parking cluster 1 is approximately 250 during weekdays (Monday through Friday). It drops to around 50 transactions per day on Saturday and Sunday (shown in orange in Figure 5.2). In comparison, the two mixed land-use locations, parking clusters 2 and 3, shown in Figures 5.3 and 5.4 do not display this significant end-of-the-week drop. This can be explained by the fact that mixed land use contains commercial establishments that operate during non-business days and therefore still generate travel demand during these times of the week.

The daily transaction data can infer the dwell times for each parking cluster. The expected dwell time for each day is calculated by dividing daily revenue by the number of daily transactions and parking price. The expected dwell times throughout October 2019 for the three locations under question can be seen in Figures 5.5, 5.6, and 5.7. From the figures, it can be noted that the average dwell time throughout October remains relatively constant. Therefore, it can be inferred that the average dwell times accurately depict the length of stay of most drivers for each of the locations.

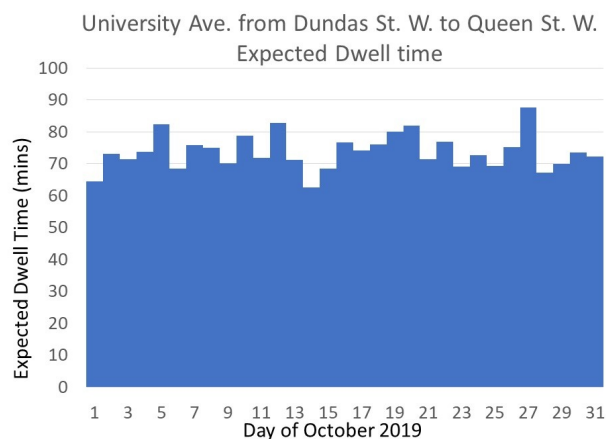


Figure 5.5: Expected dwell times for parking cluster 1

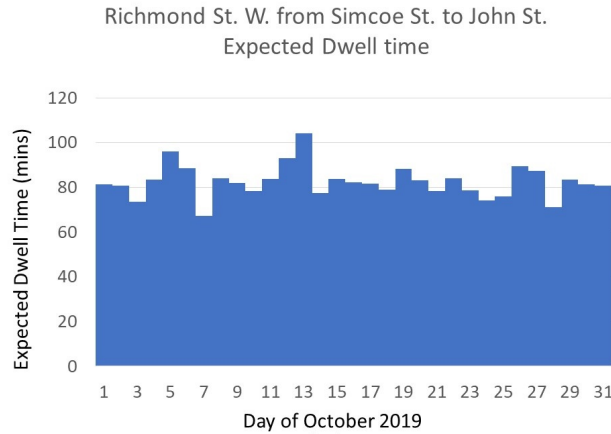


Figure 5.6: Expected dwell times for parking cluster 2

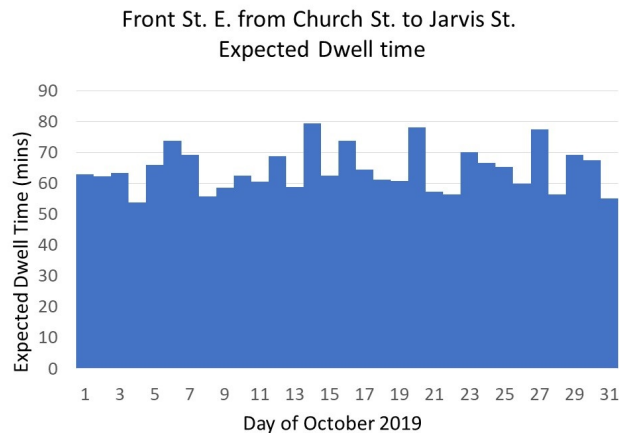


Figure 5.7: Expected dwell times for parking cluster 3

Using the R-language script found in Appendix C, we can generate an output file containing the mean and standard deviation of the dwell time for all 54 parking clusters for October 2019. A preview of this output can be seen in Figure 5.1. The information regarding the location of the each parking ID can be seen in Table .

Table 5.1: Mean and standard deviations of expected dwell time

Location ID	Mean Dwell Time	Standard Deviation
3008	59.11	16.53
3009	64.75	11.99
3010	67.23	7.39
3012	55.71	19.10
3013	75.17	5.14
3101	69.71	5.39
3102	78.61	12.15
3104	83.92	7.28
3106	77.41	6.38
3108	63.46	10.94
3111	86.87	38.85
3112	88.66	19.71
3116	43.45	4.36

Lastly, we graph the expected dwell time distribution for the three parking clusters. The expected dwell time distributions can be seen in Figures 5.8, 5.9, and 5.10. The dwell time distributions for parking clusters 1 and 2 resemble a normal distribution. The distribution of the third location can be described as a multimodal distribution as it contains two peaks. The first major peak occurs at an expected dwell time of 60 to 63 minutes, while the second occurs at 76 to 79 minutes. We use these distributions from the data to approximate the linear marginal utilities of two types of customers.

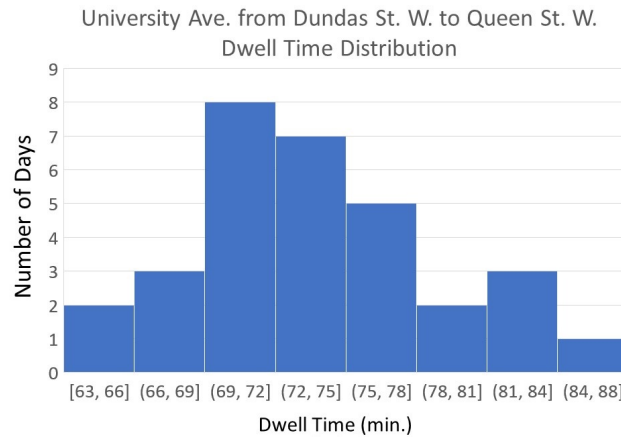


Figure 5.8: Average dwell time distribution for parking cluster 1

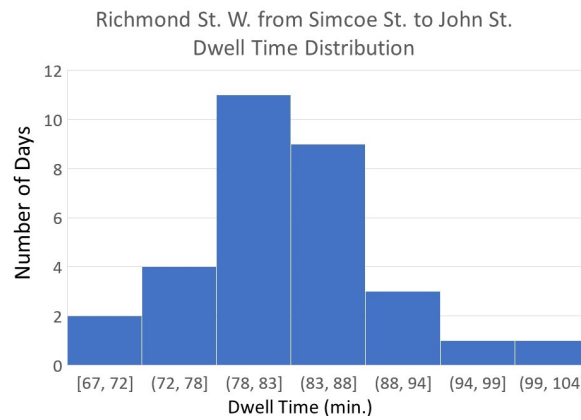


Figure 5.9: Average dwell time distribution for parking cluster 2

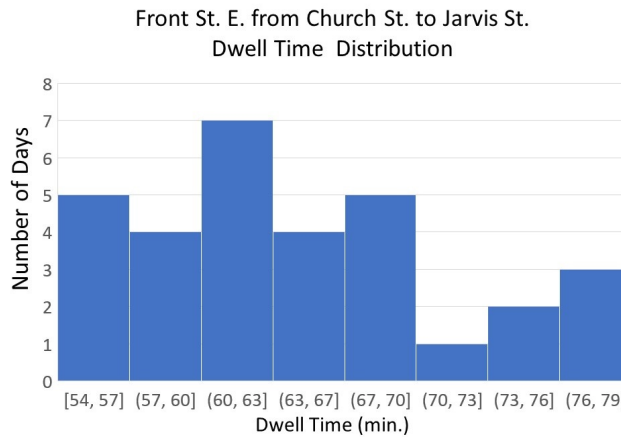


Figure 5.10: Average dwell time distribution for parking cluster 3

5.1.2 Vehicle Volumes

Vehicle turning movement counts were provided by the Toronto Parking Authority [85] and used in the micro-simulation model. The data provided is described in Section 3.2.2. As a summary, the data includes vehicle volumes for 23 intersections. The location of these intersections is shown in Figure 5.12. The nature of the data consists of averaged volume counts for October 2019 at an hourly level. The data set contained attributes defined by day type (weekend or weekday), vehicle type, the hour of the day, intersection leg, movement type (left, right, through), and hourly average volumes.

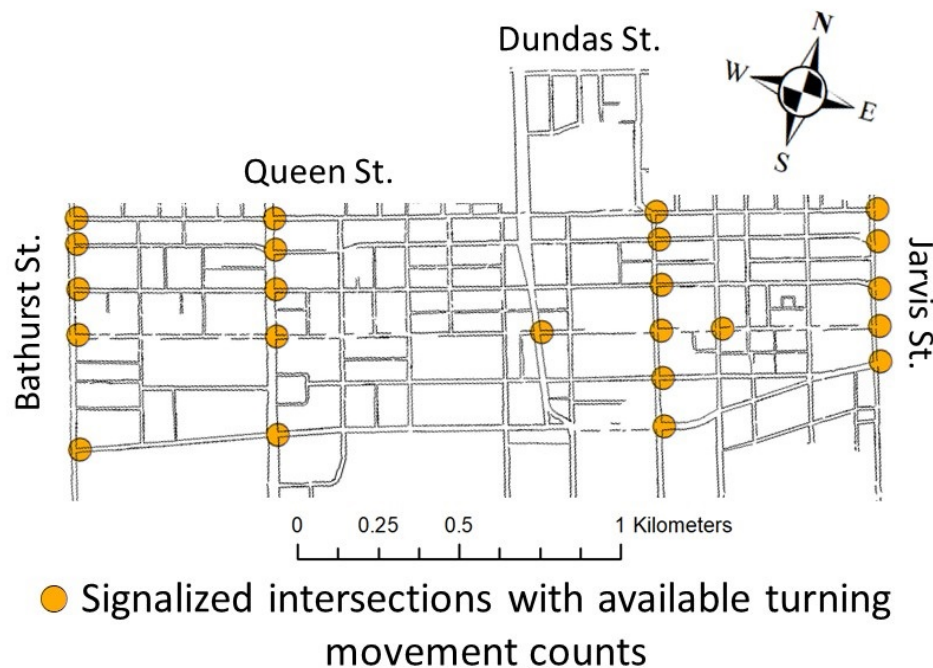


Figure 5.11: Signalized intersections with available turning movement count data

The data were analyzed to obtain variables that can be used for micro-simulation input. Since the data was provided in 1-hour intervals for all the 23 locations, the first step in the analysis was to determine the A.M. and P.M. peak hour intervals. This was done by summing up all the vehicle volumes of the 23 intersections for each hour of the day. This analysis

was completed using the developed R-language script that can be seen in Appendix C. After this was completed, the highest summation that occurred between 6:00 am to 10:00 am was chosen as the morning peak hour. Similarly, the highest summation between 3:00 pm to 7:00 pm was selected as the afternoon peak hour. Findings indicate that the peak hour conditions occur from 8:00 am to 9:00 am in the morning and from 5:00 pm to 6:00 pm in the afternoon.

The morning time frame is used as the designated peak hour time for the study. For this study, two vehicle groups are considered as previously described in Chapter 4. The two considered vehicle groups include passenger vehicles and trucks. The truck vehicle volume counts for each peak hour time frames under question were calculated using the same methodology as the passenger vehicle volumes. The percent share of truck-type vehicles in the network was estimated by summing up all turning movements for the peak hour scenario (car and truck type vehicles) and calculating the percent share of truck vehicles from this total. The resulting truck percent share in the network was 3.32%. This value for the percent share of trucks is used for all subsequent calculations and micro-simulation.

Lastly, regarding vehicle volumes, the data provided only contained volumes for 23 signalized intersections even though the study area comprises 74 signalized intersections. This is problematic as when we build the micro-simulation model, the vehicle movements at the intersection will need to be approximated. Although this will limit the model, these vehicle movements are approximated by assigning generic turning movement share at the unknown intersection. The assumed percent share of vehicle movements is composed of 50% through movement, 25% left-turn movement, and 25% left-turn movement. Although this was the general assumption, while calibration of the model was completed, percentage share were altered to ensure that the vehicle flow at links under questions matched with the expected volumes.

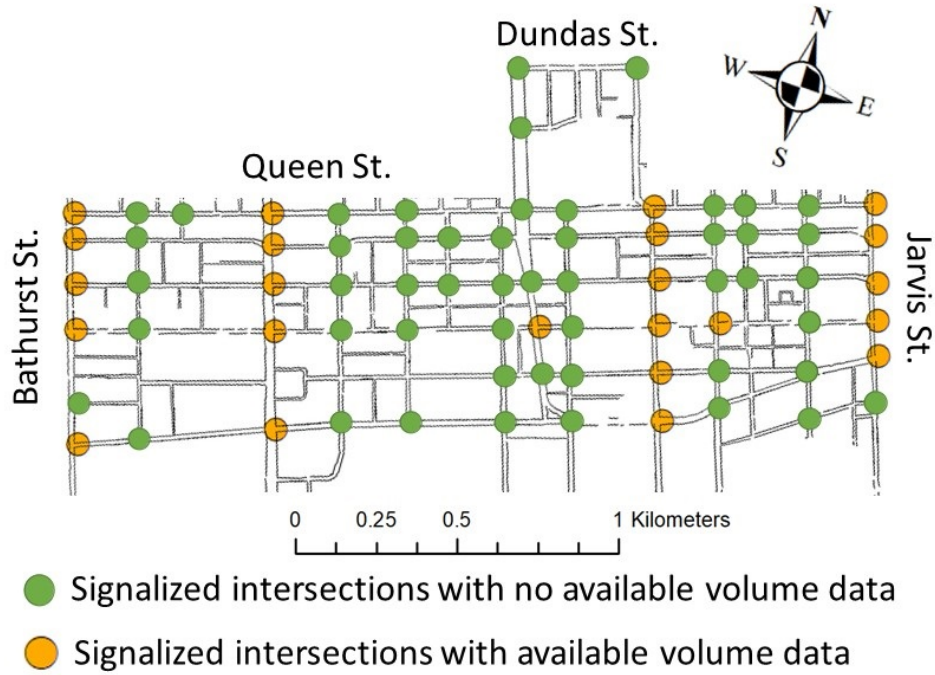


Figure 5.12: Signalized intersections in the study area

5.2 VISSIM Network

The road network was built in the VISSIM micro-simulation software using data gathered from google maps and provided by the Toronto Parking Authority [85]. This data has been described in Section 3.2.2. Attributes including the number of lanes, speed limit and permitted turning movements were collected for each road of the study area. Additionally, data pertaining to the number of parking spaces available at each road and their location was collected. Moreover, the TTC streetcar network was modelled in the micro-simulation to get a closer representation of the current network conditions of the study area as the TTC streetcars can have a significant traffic flow in the area of consideration. Figure 5.13 shows the developed network.

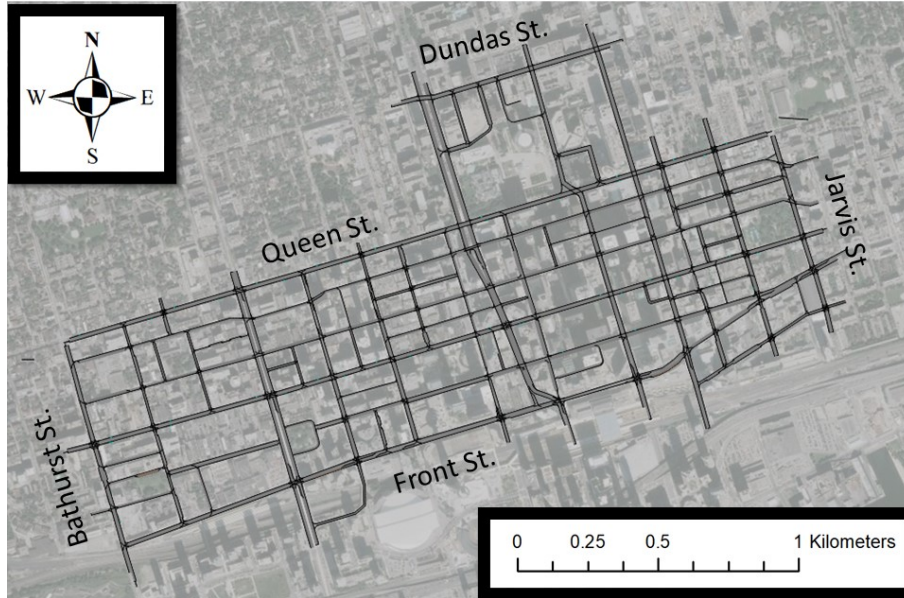


Figure 5.13: Aerial view of road the network built in VISSIM

Vehicles in the network were assigned travel routes based on the percent share of vehicles that performed a turning movement. This data and the lack of data for some intersections in the study area have been previously described in Sections 3.2.2 and 5.1.2. The percent share of vehicles that performed a left turn, right turn, and through movement were calculated and used as input in the simulation. Intersections with missing data were assumed to have a 50% through movement, 25% left-turn movement, and 25% left-turn movement split. Another input that was used in the simulation is traffic composition. As previously calculated in Section 5.1.2 the vehicular traffic inputs that were used were a share of 96.68% passenger vehicles and 3.32% truck vehicles.

Furthermore, signal timings were implemented into the network by creating a signal controller for each intersection. A ring barrier controller was used as the signal type and detectors were used for each movement. Since signal timing data was not available and out of scope for this project, a standard timing with a cycle length of 78 seconds was used.

This signal timing was used since the National Association of City Transportation Officials suggests a 60 to 90 second cycle time for urban areas nacto. Figure 5.14 shows the components included at each intersection of the network. A total of 74 signalized intersections were implemented in the study area.

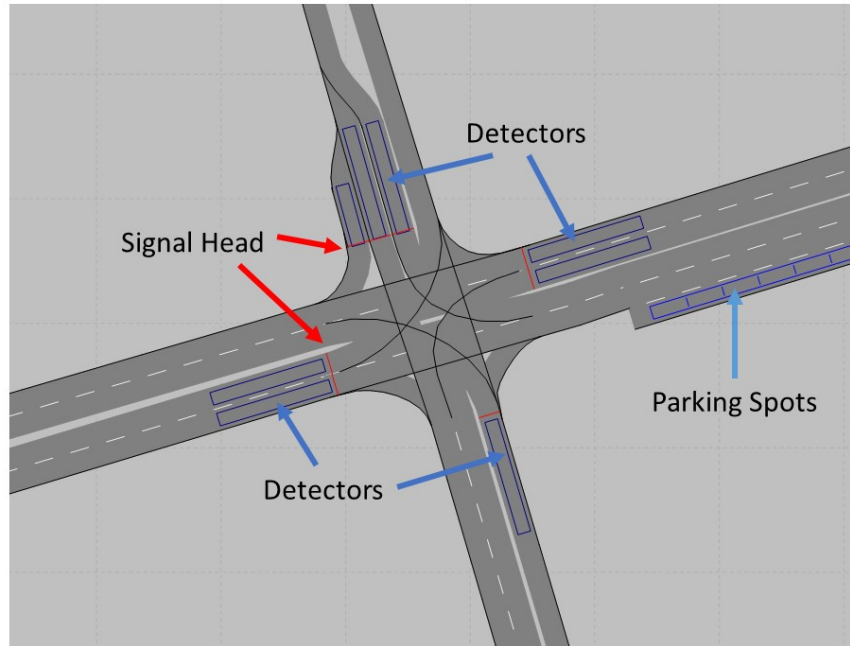


Figure 5.14: Example design of signalized intersections in VISSIM

5.3 Calibration

To ensure that the developed network accurately represents the current network conditions, the VISSIM simulation was calibrated. Calibration was completed with the use of available parking occupancy data. As mentioned in Section 3.2.2, parking occupancy was given as a percentage. The three ranges were 0% to 50%, 51% to 79%, and 80% to 100% for labels "low", "medium" and "high" respectively. Parking rates that serve as inputs for the micro-simulation model were calibrated to ensure that parking occupancy from the simulation matched the occupancy provided in the data set.

The simulation was run without any parking spots implemented to calculate the initial parking rates. As an output, this provided the vehicle volume on each of the streets in the network. After this was completed, the parking rates were calculated by multiplying the number of parking spots of the parking cluster times the parking occupancy and dividing by the total vehicle volume. This procedure allowed us to calculate the initial parking rate; however, this rate does not account for the fact that the number of vehicles seen downstream will decrease when implementing parking upstream. Therefore, the simulation was rerun with the initial parking rates to calibrate the parking rates properly. These parking rates are then calibrated in an iterative process to ensure the parking occupancy of the parking cluster matches the expected occupancy from the provided data. The final calibrated rates can be seen in Figure 5.2.

Table 5.2: Calibrated parking rates

Location ID	Parking Rate	Location ID	Parking Rate
3008	0.7094	4118	0.3560
3009	0.2574	4119	0.6038
3010	0.3050	4120	0.0212
3012	0.0684	4121	0.1727
3101	0.0963	4122	0.0252
3102	0.0466	4123	0.0256
3104	0.1716	4124	0.0091
3106	0.0729	4125	0.0202
3108	0.0044	4126	0.4501
3111	0.0112	4128	0.0811
3112	0.6601	4129	0.0895
3116	0.0063	4130	0.6151
3117	0.0060	4131	0.0802
3118	0.0546	4134	0.0074
3120	0.1500	4143	0.2138
3215	0.1486	4144	0.1177
4101	0.0510	4302	0.0419
4102	0.0028	4307	0.1250
4103	0.0034	4315	0.0249
4105	0.6038	4316	0.0634
4106	0.0267	4317	0.3471
4107	0.0570	4318	0.5629
4108	0.0232	4319	0.0041
4109	0.0162	4320	0.0742
4111	0.7129	4321	0.0339
4112	0.1026	4322	0.0104
4115	0.1144	4358	0.0175

The calibration, which was an iterative process, took a total of 7 iterations. The parking occupancy results from the simulation can be seen in Tables 5.3 and 5.4.

Table 5.3: Results from micro-simulation calibration part 1

Location ID	Occupancy from Simulation	Known Occupancy Bottom Range	Known Occupancy Top Range
3008	72%	51%	79%
3009	89%	51%	79%
3010	90%	51%	79%
3012	85%	51%	79%
3101	88%	80%	100%
3102	41%	51%	79%
3104	88%	80%	100%
3106	66%	51%	79%
3108	47%	51%	79%
3111	9%	0%	50%
3112	74%	51%	79%
3116	25%	0%	50%
3117	11%	0%	50%
3118	81%	80%	100%
3120	84%	51%	79%
3215	77%	80%	100%
4101	86%	51%	79%
4102	27%	0%	50%
4103	37%	0%	50%
4105	97%	80%	100%
4106	64%	51%	79%
4107	68%	51%	79%
4108	82%	80%	100%
4109	68%	51%	79%
4111	93%	80%	100%
4112	96%	80%	100%
4115	33%	0%	50%
4118	69%	80%	100%
4119	100%	80%	100%
4120	48%	0%	50%
4121	99%	80%	100%
4122	89%	80%	100%
4123	89%	80%	100%
4124	32%	0%	50%

Table 5.4: Results from micro-simulation calibration part 2

Location ID	Occupancy from Simulation	Known Occupancy Bottom Range	Known Occupancy Top Range
4125	95%	80%	100%
4126	100%	80%	100%
4128	69%	51%	79%
4129	88%	80%	100%
4130	66%	51%	79%
4131	85%	80%	100%
4134	15%	0%	50%
4143	78%	51%	79%
4144	32%	51%	79%
4302	65%	51%	79%
4307	67%	51%	79%
4315	90%	80%	100%
4316	68%	51%	79%
4317	84%	80%	100%
4318	81%	80%	100%
4319	32%	0%	50%
4320	73%	51%	79%
4321	7%	0%	50%
4322	7%	0%	50%
4358	91%	51%	79%

After calibration, 43 out of 54 parking clusters were calibrated to match their known parking occupancy, while the remaining 11 parking clusters showed a parking occupancy within 20% of the known parking occupancy. The parking clusters that did not yield the same parking occupancy in the simulation when compared to the known occupancy are highlighted in red on Tables 5.3 and 5.4.

To calculate the optimal price levels from the econometric model, the driver utility equations parameters a_h , a_l , b_h , and b_l were calibrated. This was completed by utilizing the calculated expected dwell time from each parking cluster and price for parking. For delivery vehicles (low-value drivers) the parameters were calculated by assuming a 15 minute average dwell time and the respective price for parking at the parking cluster.

5.4 Scenario Formulation

The first time frame of importance for the study was the a.m. peak scenario. As described in Section 5.1.2, the morning peak vehicular volume occurred from 8:00 am to 9:00 am. For this time frame, two scenarios were created within the simulation. The first scenario was labelled as hourly pricing, and it implemented an hourly pricing rate at each of the parking clusters. This hourly rate is the same as the current pricing rate of the parking clusters. This hourly price can be seen in column 5 of Table 5.5. Secondly, a progressive pricing scenario was also developed for the morning peak conditions. The progressive pricing rate was developed by inputting the analyzed data into the optimal answers of Equation 4.13 which are given in Equation 4.14. The optimal price level p_2 , which is the second level of the optimal progressive pricing policy, can be seen in Table 5.5. These price levels, are optimized under revenue maximization since under social welfare, the price levels of progressive pricing will equal hourly pricing price levels. Additionally, q was assumed to have a value of 15 minutes for all parking clusters. This assumption was made since there was no parking data available for delivery vehicle but the City of Toronto informed us that on average delivery vehicles parked for 15 minutes 3.2.2.

Table 5.5: Optimal price levels of the progressive pricing policy

Location ID	Street	From	To	Price #1	Price #2
3008	Elizabeth St.	Dundas St. W.	Hagerman St.	\$5.00	\$6.82
3009	Chestnut St.	Dundas St. W.	Armoury St.	\$5.00	\$9.05
3010	Centre Ave.	Dundas St. W.	Armoury St.	\$5.00	\$13.55
3012	Armoury St.	Centre St.	University Ave.	\$5.00	\$6.02
3101	Queen St. W.	Simcoe St.	Spadina Ave.	\$4.00	\$6.41
3102	Simcoe St.	Queen St. W.	Richmond St. W.	\$5.00	\$10.39
3104	Richmond St. W.	Simcoe St.	John St.	\$5.00	\$16.57
3106	Adelaide St. W.	Simcoe St.	Spadina Ave.	\$5.00	\$17.29
3108	Spadina Ave.	Adelaide St. W.	King St. W.	\$4.00	\$7.63
3111	Simcoe St.	Adelaide St. W.	King St. W.	\$4.00	\$5.23
3112	Pearl St.	Duncan St.	Simcoe St.	\$5.00	\$8.00
3116	Front St. W.	Spadina Ave.	Blue Jays Way	\$5.00	\$14.41
3117	Front St. W.	Simcoe St.	University Ave.	\$5.00	\$4.96
3118	York St.	King St. W.	Richmond St. W.	\$5.00	\$12.35
3120	Clarence Sq.	Spadina Avenue	Wellington St. W.	\$5.00	\$8.53
3215	Queen St. W.	Soho St.	Spadina Ave.	\$4.00	\$13.87
4101	Queen St. E.	Church St.	Jarvis St.	\$4.00	\$9.18
4102	Richmond St. E.	Victoria St.	Church St.	\$5.00	\$9.17
4103	Richmond St. E.	Church St.	Jarvis St.	\$4.00	\$7.90
4105	Victoria St.	Richmond St. E.	Adelaide St. E.	\$4.00	\$4.80
4106	Lombard St.	Victoria St.	Church St.	\$5.00	\$16.27
4107	Lombard St.	Church St.	Jarvis St.	\$4.00	\$17.06
4108	Adelaide St. E.	Victoria St.	Church St.	\$5.00	\$13.05
4109	Adelaide St. E.	Church St.	Jarvis St.	\$4.00	\$8.54
4111	Toronto St.	Adelaide St. E.	King St. W.	\$5.00	\$14.27
4112	Church St.	Adelaide St. E.	King St. W.	\$5.00	\$14.30
4115	Colborne St.	Yonge St.	Victoria St.	\$5.00	\$5.70
4118	Church St.	Colborne St.	Wellington St. E.	\$4.00	\$8.87
4119	Church St.	King St. W.	Wellington St. E.	\$4.00	\$11.57
4120	Wellington St. E.	Yonge St.	Scott St.	\$5.00	\$9.90
4121	Wellington St. E.	Scott St.	Church St.	\$4.00	\$8.65
4122	Front St. E.	Church St.	Jarvis St.	\$4.00	\$11.48
4123	Front St. E.	Church St.	Market St.	\$4.00	\$9.53
4124	Front St. W.	Bay St.	Yonge St.	\$5.00	\$10.76
4125	Front St. E.	Yonge St.	Scott St.	\$5.00	\$12.68
4126	Front St. E.	Scott St.	Church St.	\$4.00	\$9.53
4128	Scott St.	Front St. E.	The Esplanade	\$4.00	\$13.18
4129	Church St.	Front St. E.	The Esplanade	\$4.00	\$10.25
4130	Market St.	Front St. E.	The Esplanade	\$4.00	\$7.92
4131	The Esplanade	Scott St.	Church St.	\$4.00	\$6.82
4134	Jarvis St.	The Esplanade	South Limit Prkng.	\$4.00	\$6.89
4143	Victoria St.	Adelaide St. E.	Old Post Office Ln.	\$5.00	\$7.99
4144	Victoria St.	King St. E.	Colborne St.	\$5.00	\$7.66
4302	Queen St. W.	Bathurst St.	Spadina Ave.	\$4.00	\$11.00
4307	Richmond St. W.	Bathurst St.	Portland St.	\$4.00	\$6.29
4315	Bathurst St.	King St. W.	Wellington St. W.	\$3.00	\$8.18
4316	Stewart St.	Bathurst St.	Portland St.	\$3.00	\$9.62
4317	Portland St.	King St. W.	Wellington St. W.	\$3.00	\$7.27
4318	Wellington St. W.	Bathurst St.	Spadina Ave.	\$3.00	\$6.82
4319	Niagara St.	Bathurst St.	Portland St.	\$3.00	\$7.27
4320	Portland St.	Wellington St. W.	Front St. W.	\$3.00	\$9.79
4321	Front St. W.	Bathurst St.	Spadina Ave.	\$3.00	\$7.69
4322	Spadina Ave.	King St. W.	Front St. W.	\$3.00	\$3.39
4358	Augusta Ave.	Richmond St. W.	Queen St. W.	\$3.00	\$6.82

All parameters in the simulation remained the same for both scenarios except for the price levels of parking. Results from these scenarios will be discussed in Chapter 6 Section 6.1

The off-peak time frame is also of importance to the study. For this scenario, vehicular volumes were reduced by 50% to compensate for the reduced traffic flow during this time of the day. All other parameters remaining the same, three scenarios were created for the off-peak time frame. Similar to the A.M. peak time frame, the first two scenarios consisted of hourly and progressive pricing policies. These two policies remained the same, and no parameters were changed.

The third scenario for the off-peak time frame was an hourly pricing policy but with a 50% reduction in price. For parking clusters that charged \$5 per hour in the hourly policy will now have a rate of \$ 2.5 per hour in the second hourly pricing scenario. This scenario is labelled as "time-of-day pricing" for all subsequent sections of the report. The assumption of reducing the price by 50% has been made in accordance with various time-of-day pricing schemes implemented around the world. For example, in Sydney, Australia, the hourly rate decreases from \$7.2 to \$3.9 during off-peak times. Results from this scenario will be presented in Chapter 6 Section 6.2. For each scenario, 30 simulation runs were performed and averaged.

5.5 Chapter Summary

This chapter discusses the approach taken to validate the model developed in Section 4. This chapter is divided into four sections. In Section 5.1, the available data is analyzed. First, we analyze data pertaining to on-street parking transactions for the 54 parking clusters in the study area. We considered the number of daily transactions for the whole month of October 2019. From this analysis, we determine that land use of the parking cluster will affect the pattern of parking transactions seen throughout the month. If the parking cluster is located in an area where the travel demand attracted decreases in the weekends, i.e. institutional

land uses, the number of parking transactions will also drop. While other land uses that have uniform demand throughout the week, i.e. mixed land use, will not experience this drop in transactions on the weekends. This is an important finding because, for parking clusters that do not experience many transactions on the weekend, we could potentially implement a day-of-week pricing scheme with two different price levels for weekends and weekdays. An example of this technique is currently being used by Western University in Canada (institutional land use). In Western University for the time period spanning from 8:00 pm Friday to 7:00 am Monday parking is complimentary (free) while at any other times parking is priced [91].

Secondly, from the on-street parking transaction analysis completed we conclude that the average parking dwell time of drivers remains relatively constant for the month of October 2019 and the distribution of average dwell times follows a normal distribution. The parking dwell time is then used in Section 5.4 to calculate the optimal price levels used for our progressive pricing policy. Knowing that the dwell times remain relatively constant is an important finding because given this fact we expect that the calculated optimal price level (seen in Table 5.5) will be close to optimal for all days in the month of October 2019 as opposed to having two dwell times (i.e. one for weekdays and one for weekends) which will result in having two optimal price levels, one for weekends and one for weekdays.

Following this, we describe the steps taken to build and calibrate the VISSIM micro-simulation. When building the micro-simulation model the lack of vehicle volume data availability for all intersections led to us making the assumption an assumption for vehicle volumes in some of the signalized intersections. We assumed the percent share of vehicle movements at the intersection to have a split of 50% through movement, 25% left-turning movement, and 25% left-turning movement. Although this assumption introduces some error into the model, the assumption is acceptable to make since the measures extracted from the scenarios in the model will be compared to each other. Meaning that both scenarios that are

being compared will be exposed to the same errors and therefore the relative performance of scenarios will be a suitable comparison. Aside from these, this section also contained a discussion on micro-simulation calibration. The model was calibrated using the known parking occupancy of the 54 parking clusters.

Lastly, we describe the scenarios created in the micro-simulation. As described in Section 5.1.2, the morning peak vehicular volume occurred from 8:00 am to 9:00 am. This time frame will be assigned as our a.m. peak scenario and will include two pricing policies, hourly pricing and progressive pricing. For our off-peak scenario, all simulation parameters remain the same with the exception of the vehicle volumes. The vehicle volumes in the micro-simulation are reduced by 50% during the off-peak scenario. For this scenario, the simulated policies include hourly pricing, progressive pricing and time-of-day pricing.

Chapter 6

Simulation Results & Analysis

In this chapter, we discuss the results and relative performance of the before-mentioned pricing policies under the a.m. peak (Section 6.1) and off-peak (Section 6.2) Scenarios. Before results are presented, we recall the definitions of measures and terminology utilized in this section. To begin, average parking occupancy is defined as the percentage of spaces occupied during a specified time interval. Secondly, parking requests declined is measured as the percentage of vehicles that are rejected from parking at their desired parking cluster due to no parking spaces being available. Total network travel time is defined as the summation of travel times (in hours) experienced by all users in the network in a 1 hour time interval. Lastly, parking revenue is calculated as the summation of the product of the parking duration of each vehicle and the price paid for parking.

6.1 A.M. Peak Scenario

For each of the two policies examined in this section, hourly pricing and progressive pricing, 30 simulation runs were taken and averaged. Results from these simulations will be presented in two sections. First, the network-wide measures followed by parking specific measures.

6.1.1 Network-Wide Measures

The first measure given as an output from the simulation was the total network travel time. The results from the simulation results show that the progressive pricing policy consistently

achieved a lower value of total network travel time when compared to the hourly pricing policy in the 30 simulation runs. As noted from Figure 6.1, the general trend showcases a decrease in total travel time when implementing the progressive pricing policy. The results show a mean travel time of 2249 hours for the hourly pricing policy and 2224 hours for the progressive pricing policy.

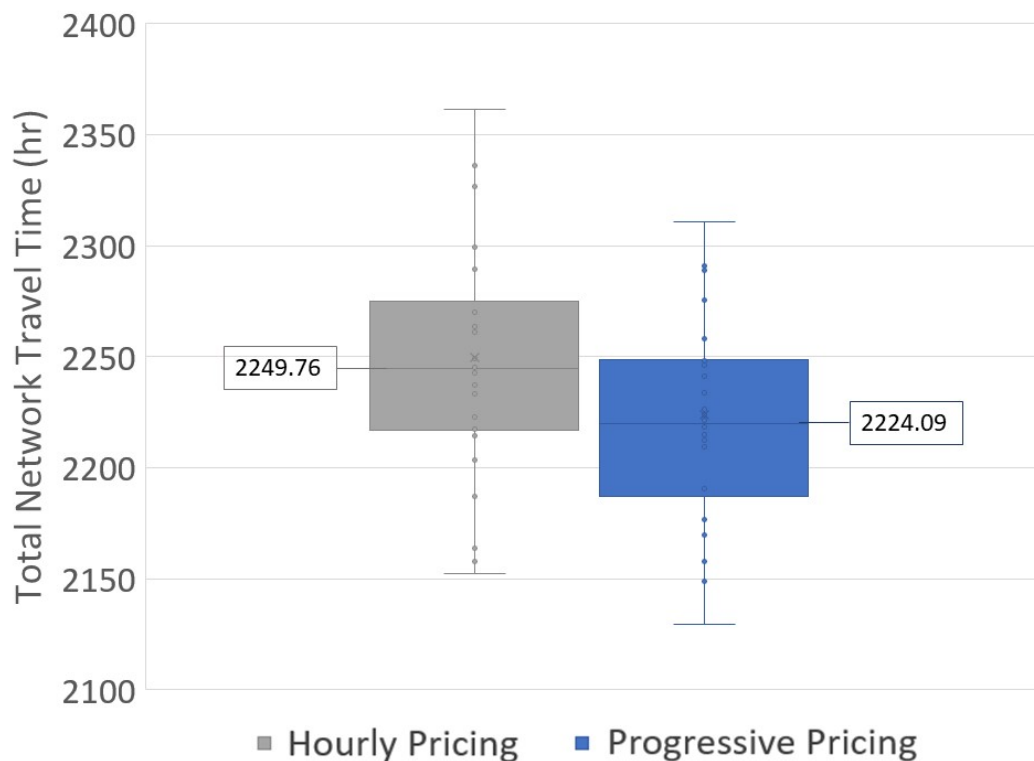


Figure 6.1: Box plots of total network travel time under a.m. peak conditions

Additionally, from the micro-simulation, average speed heat maps of the study area were extracted and can be seen in Figure 6.2. The heat maps showcase an increase in average speeds at various locations throughout the study area when implementing the progressive pricing policy. Examples of locations where the average speed differs from one policy to another are highlighted by black circles. Average speed increases can be explained by the

higher turnover of the progressive pricing policy which reduces the number of vehicles in the network, consequently reducing congestion and increasing mean average speeds.

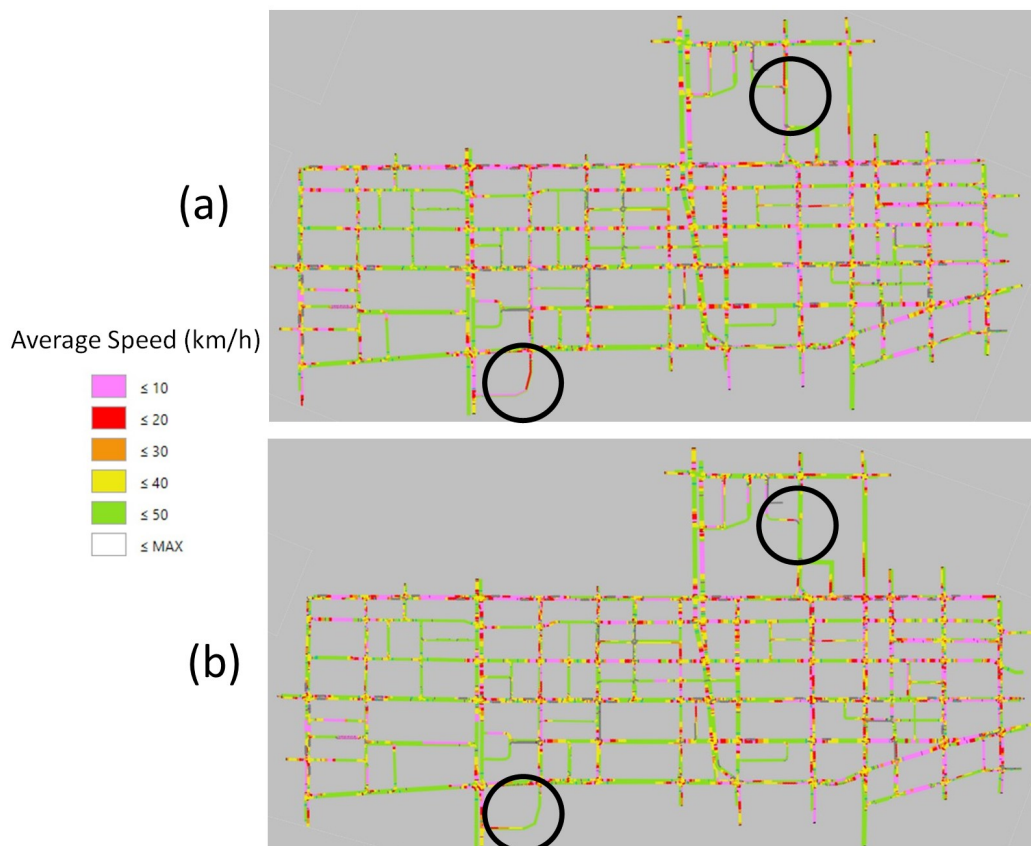


Figure 6.2: Average speed heat maps for hourly pricing (a) and progressive pricing (b)

Lastly, the last network-wide measure that was calculated was revenue generated by each parking clusters. Recall that prices used are with respect to the revenue maximizing policy. Table 6.1 shows the revenue generated by each parking cluster for the 1 hour time period that was run in the simulation. We note that the progressive pricing policy more than doubles the revenue generated by the hourly pricing policy.

Table 6.1: Revenue generated by each parking cluster during a.m. peak scenario

Location ID	Hourly Pricing Policy Revenue	Progressive Pricing Policy Revenue
3008	\$15.80	\$64.01
3009	\$36.03	\$59.56
3010	\$100.24	\$233.86
3012	\$17.64	\$20.42
3101	\$130.82	\$146.05
3102	\$10.09	\$17.73
3104	\$121.90	\$381.25
3106	\$80.80	\$318.33
3108	\$13.73	\$22.32
3111	\$17.46	\$26.18
3112	\$14.45	\$22.75
3116	\$13.73	\$28.33
3117	\$11.07	\$29.99
3118	\$12.76	\$24.48
3120	\$37.69	\$58.97
3215	\$53.45	\$173.13
4101	\$62.72	\$133.94
4102	\$8.06	\$12.23
4103	\$8.79	\$14.60
4105	\$23.95	\$37.10
4106	\$121.33	\$361.73
4107	\$103.10	\$396.17
4108	\$52.75	\$126.99
4109	\$24.69	\$43.13
4111	\$20.65	\$63.35
4112	\$58.35	\$145.31
4115	\$8.06	\$25.76
4118	\$7.74	\$17.41
4119	\$19.25	\$49.63
4120	\$94.58	\$171.21
4121	\$30.32	\$58.52
4122	\$42.17	\$105.38
4123	\$31.92	\$68.94
4124	\$38.87	\$76.25
4125	\$32.72	\$77.98
4126	\$58.83	\$132.20
4128	\$37.50	\$116.55
4129	\$46.09	\$115.61
4130	\$45.45	\$81.11
4131	\$110.38	\$407.58
4134	\$11.03	\$16.70
4143	\$29.21	\$45.95
4144	\$14.76	\$21.82
4302	\$226.90	\$519.69
4307	\$41.26	\$116.71
4315	\$24.61	\$58.71
4316	\$38.98	\$114.57
4317	\$18.42	\$42.32
4318	\$162.13	\$139.16
4319	\$8.74	\$8.37
4320	\$20.20	\$19.82
4321	\$24.29	\$21.99
4322	\$9.06	\$8.94
4358	\$5.21	\$5.30
Total	\$2,522.63	\$5,790.28

6.1.2 Parking Measures

Moving on to the parking specific measure, parking occupancies of the 54 parking clusters in the study area are shown in Figure 6.3. For ease of interpretation, the parking clusters are plotted by increasing occupancy. We note that the parking occupancy is lower under the progressive pricing policy when compared to hourly pricing. The parking occupancy decreases due to drivers' lower average dwell time under the progressive pricing scheme. In addition to this, all parking clusters experiencing a high occupancy under an hourly pricing policy show a decrease in occupancy with the implementation of progressive pricing. Furthermore, for low occupancy parking clusters, the implementation of a progressive pricing policy does not seem to decrease nor increase parking occupancy consistently. The resulting data from the 30 simulations lead to the proposition that progressive pricing reduces the average parking occupancy for parking clusters that experience a high occupancy under an hourly pricing policy.

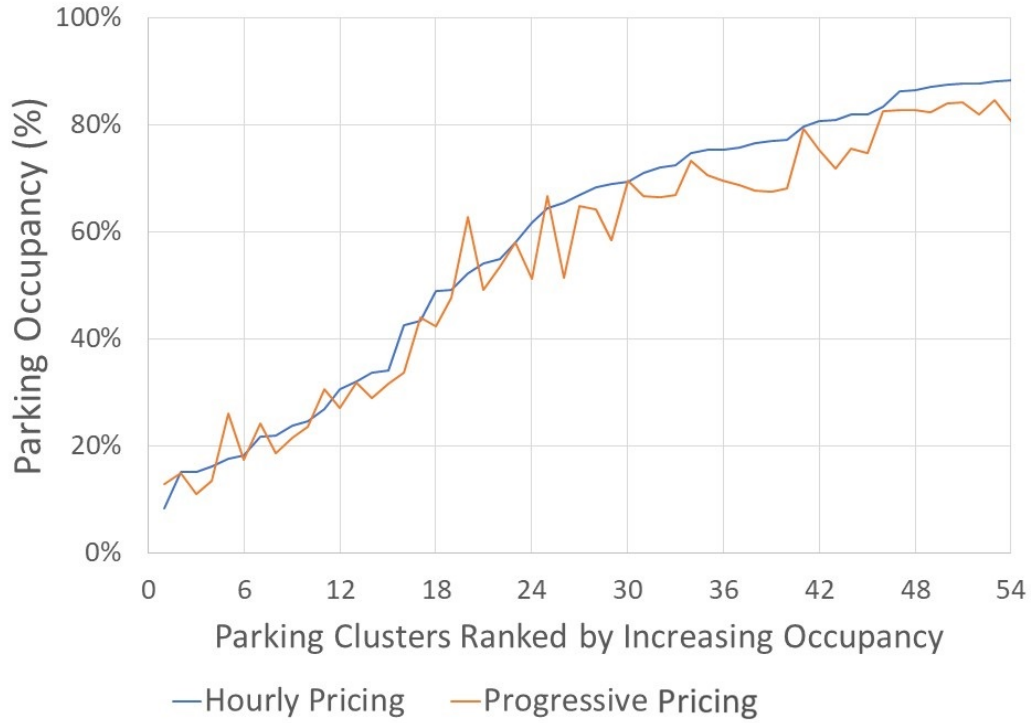


Figure 6.3: Parking occupancy under a.m. peak conditions

It is essential to consider the percentage of vehicles that cannot find parking as these vehicles remain in the network, cruise for parking, and increase congestion in the area. The measure parking requests declined is used to represent the percentage of vehicles rejected from parking due to no parking spaces being available. Although we will not be considering cruising for parking directly, the percentage of vehicles declined from parking will give us an idea of cruising for parking. As seen in Figure 6.4, the number of parking requests declined is shown as a percentage of all requests received. When comparing the hourly pricing policy to progressive pricing, the percentage of parking requests declined consistently decreases under progressive pricing. Since the progressive pricing policy decreases the driver's average dwell time and consequently reduces occupancy, the number of vehicles that can park in the same period of time increases. This increase in the number of vehicles that can park in a given period leads to fewer vehicles not finding parking.

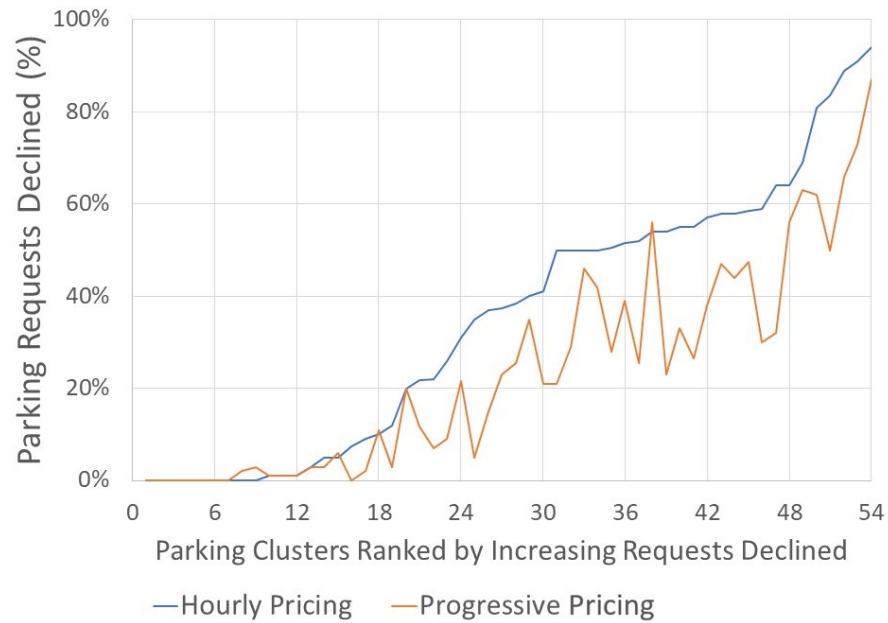


Figure 6.4: Percentage of parking requests declined under a.m. peak conditions

Furthermore, there exists a relationship between the two measures mentioned above, parking occupancy and parking requests declined. The relationship between both measures is displayed in Figure 6.5. As the average occupancy increases, the percent of parking requests declined increases at a higher rate. Although the same trend is seen for both policies, the rate at which parking requests declined increases is lower for the progressive pricing policy when compared to hourly pricing. This leads to the proposition that for a parking cluster that experiences a given parking occupancy, the number of vehicles that cannot find parking is larger under an hourly pricing policy than the progressive pricing policy.

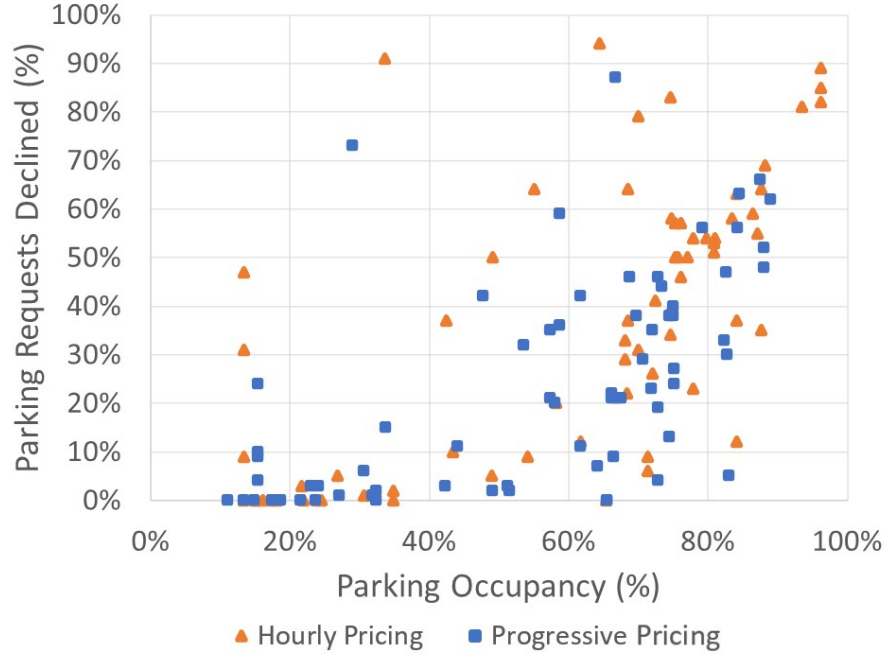


Figure 6.5: Relationship between parking occupancy and parking requests declined

6.1.3 A.M. Peak Scenario Summary

Table 6.4 shows a summary of results from the micro-simulation. The progressive pricing policy achieves a consistent reduction for all measures, including parking occupancy, searching for parking, and travel time. From Table 6.4 we note that revenue generated by the progressive pricing policy is more than double the revenue generated by the hourly pricing policy. Additionally, the total network travel time, which is defined as the sum of travel times for all vehicles in the network, decreases by 1.1%. Results for parking occupancy can also be seen in Table 6.4. We note that parking occupancy has been divided into two sections, high and low demand parking clusters. The threshold used for the separation was 50% parking occupancy. We numerically support the before-mentioned statement which states that the progressive pricing policy will achieve a higher reduction in parking occupancy when implemented in parking clusters that experience a high parking occupancy under hourly pricing. Lastly, the

percentage of vehicles who are declined parking due to no parking spaces being available at the cluster decreased from 32.3% to 19.8%.

Table 6.2: Results from VISSIM micro-simulation

Measure	Hourly Pricing	Progressive Pricing
Parking Occupancy (High Demand Clusters, occupancy >50%)	76.6%	71.0%
Parking Occupancy (Low Demand Clusters, occupancy <50%)	24.5%	24.0%
Parking Revenue	\$2,522.6	\$5,790.2
Percentage of Parking Requests Declined	32.3%	19.8%
Total Network Travel Time	2249.7	2224.1

6.2 Off-Peak Scenario

For off-peak conditions, the simulation consisted of 3 scenarios. The first scenario was the current hourly pricing policy for the study area labelled as "Hourly Pricing". Secondly, the scenario labelled as "Time-of-Day Pricing" consists of a reduction of 50% of the price levels of the "Hourly Pricing" scenario. Lastly, the scenario belled "Progressive Pricing" consists of the optimal progressive pricing levels calculated from the econometric model, these price levels can be seen in Table 5.5. This section will be organized in a similar manner as Section 6.1. Results from the 30 simulations will be presented in two sections. First the network-wide measures followed by parking-specific measures.

6.2.1 Network-Wide Measures

The progressive pricing policy achieved a decrease in network travel time when compared to the two hourly pricing policies for the 30 simulation runs as seen in Figure ???. The average total network travel times for hourly pricing, time-of-day pricing, and progressive hourly pricing are 1075 hrs, 1089 hrs, and 1060 hrs respectively as seen in Figure 6.6.

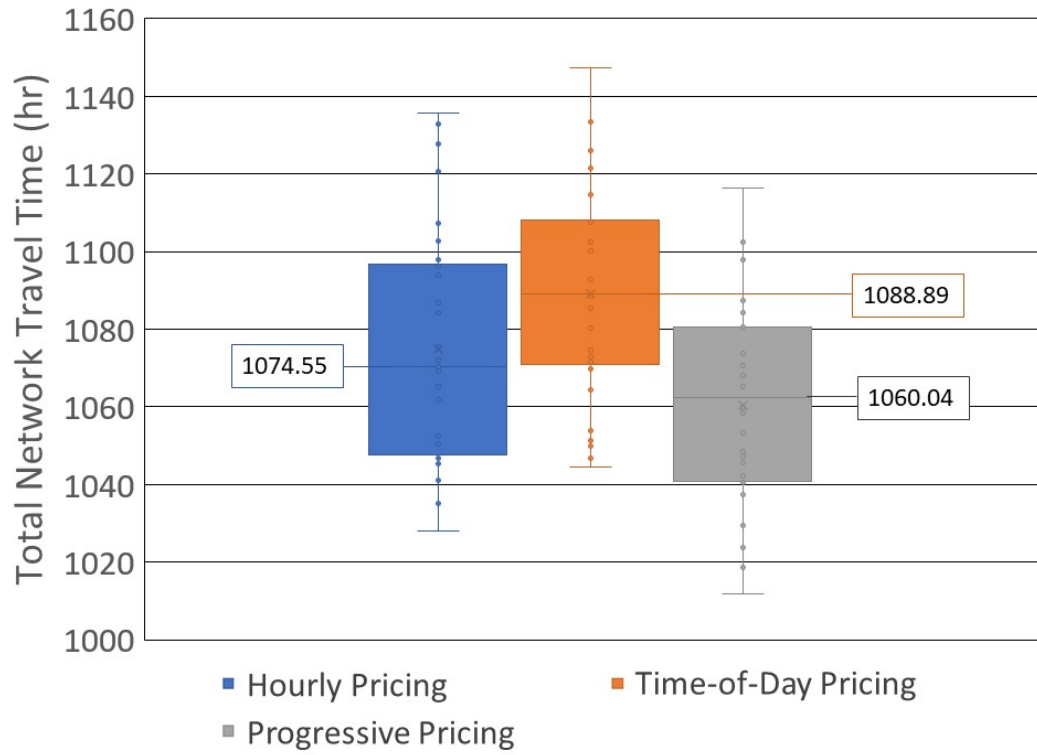


Figure 6.6: Box plots of total network travel time under off-peak conditions

Additionally, average speed heat maps of the study are extracted from the micro-simulation and can be seen in Figure ???. Although there is a minor decrease in total network travel times with the implementation of a progressive pricing policy, there is no noticeable reduction in the average speeds of the network. This is attributed to the fact that during off-peak conditions average speeds in the network are close to the posted speed limit and therefore cannot be further increased as congestion does not limit travel speed during off-peak conditions.

Lastly, revenue generated by each parking cluster was calculated for the three policies. Table 6.3 shows the revenue generated by each parking cluster for the 1 hour time period that was run in the simulation. The total revenue is equal to \$1,792.94, \$830.34, and \$4,468.04 for the hourly pricing, time-of-day pricing, and progressive pricing policies respectively.

Table 6.3: Revenue generated by each parking cluster during off-peak scenario

Location ID	Hourly Pricing Revenue	Time-of-day Pricing Revenue	Progressive Pricing Revenue
3008	\$9.96	\$22.34	\$44.58
3009	\$27.69	\$12.04	\$50.50
3010	\$58.32	\$18.42	\$128.17
3012	\$14.56	\$6.88	\$17.88
3101	\$107.89	\$47.06	\$126.42
3102	\$4.53	\$2.76	\$11.31
3104	\$97.99	\$46.14	\$313.87
3106	\$60.88	\$35.50	\$256.50
3108	\$7.72	\$2.86	\$11.25
3111	\$12.59	\$6.55	\$19.66
3112	\$4.02	\$1.40	\$4.89
3116	\$7.49	\$2.63	\$19.53
3117	\$3.55	\$1.97	\$11.79
3118	\$10.97	\$4.16	\$22.42
3120	\$29.67	\$12.86	\$43.68
3215	\$43.89	\$19.19	\$144.61
4101	\$59.89	\$25.57	\$129.94
4102	\$6.20	\$2.50	\$11.61
4103	\$5.27	\$2.34	\$10.10
4105	\$15.85	\$9.14	\$29.13
4106	\$92.90	\$41.79	\$293.08
4107	\$82.49	\$38.00	\$340.16
4108	\$42.40	\$19.86	\$106.00
4109	\$19.02	\$7.10	\$39.00
4111	\$8.12	\$5.38	\$34.64
4112	\$39.05	\$18.24	\$110.45
4115	\$3.32	\$2.93	\$15.23
4118	\$5.71	\$3.94	\$14.61
4119	\$16.98	\$7.38	\$47.13
4120	\$80.80	\$38.38	\$157.95
4121	\$17.41	\$8.06	\$36.89
4122	\$26.60	\$11.58	\$71.12
4123	\$19.36	\$8.83	\$47.27
4124	\$27.44	\$12.75	\$59.85
4125	\$28.31	\$13.77	\$70.64
4126	\$48.45	\$22.93	\$113.82
4128	\$23.78	\$11.94	\$78.29
4129	\$12.19	\$8.26	\$41.18
4130	\$27.55	\$11.19	\$47.93
4131	\$96.81	\$44.65	\$386.24
4134	\$5.63	\$2.30	\$9.79
4143	\$26.25	\$12.01	\$39.93
4144	\$12.19	\$5.85	\$18.98
4302	\$166.83	\$66.92	\$437.95
4307	\$25.02	\$9.82	\$87.21
4315	\$17.17	\$7.72	\$47.39
4316	\$22.32	\$9.80	\$69.58
4317	\$16.49	\$8.32	\$40.77
4318	\$146.06	\$65.34	\$148.29
4319	\$5.59	\$2.66	\$5.55
4320	\$17.11	\$8.61	\$18.01
4321	\$14.31	\$6.58	\$14.75
4322	\$7.36	\$3.68	\$7.33
4358	\$2.99	\$1.50	\$3.18
Total	\$1,792.94	\$830.34	\$4,468.04

6.2.2 Parking Measures

With regards to parking occupancy, as expected the off-peak conditions resulted in a decrease in parking occupancy when compared to the A.M. conditions due to a decrease in the vehicles present in the network. The three policies under question hourly pricing, time-of-day pricing, and progressive pricing resulted in similar parking occupancy for most of the parking clusters. Parking occupancy for the three policies can be seen in Figure 6.7. For two parking clusters, parking clusters ranked number 2 and 7 in Figure 6.7, the parking occupancy significantly increased with the implementation of time-of-day pricing and progressive hourly pricing. These results are an anomaly in the data as the general trend for all other 52 parking clusters does not showcase the same trend. The results from these two clusters are labelled as outliers, a possible reason for this inconsistency could be improper calibration of parking rates during the calibration phase.

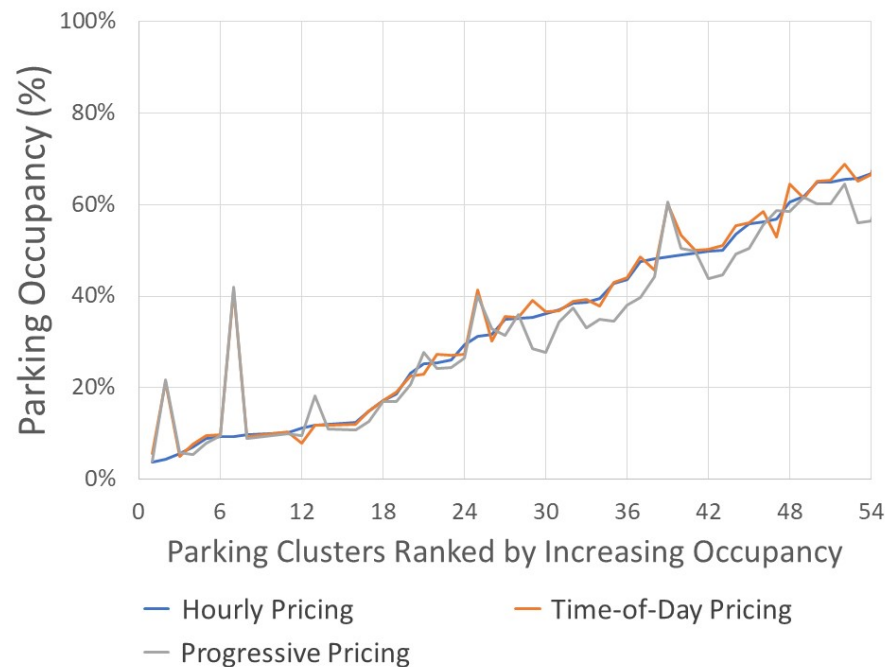


Figure 6.7: Parking occupancy under off-peak conditions

Additionally, the two parking clusters denoted as outliers only contain 11 and 3 parking spaces each while the average number of parking spaces is 17 for all parking clusters in the study area. The small number of parking spaces can drastically influence the parking occupancy since, for a parking cluster with 3 spaces, the arrival of one additional vehicle represents a 33% increase in parking occupancy. Nonetheless, the general trend observed from the three policies is that the three pricing policies perform similarly when parking occupancy is low. Notably, the progressive pricing policy achieved a marginal reduction of parking occupancy when parking occupancy was high.

The percent of vehicles who are declined parking due to no spaces being available for the three policies can be seen in Figure 6.8. The time-of-day and hourly pricing policies yield similar results in terms of the percentage of vehicles that are declined parking while the progressive pricing policy was able to consistently decrease the percentage of vehicles declined parking for each parking cluster. From all the before-mentioned figures it can be seen that during off-peak conditions a parking price reduction (in this case 50%) does not have a significant effect on the parking occupancy, parking requests declined, and total network travel time. However, revenue generation does significantly decrease. With respect to the progressive pricing policy, implementation of this policy will consistently perform equally or better than the hourly pricing and time-of-day pricing policies.

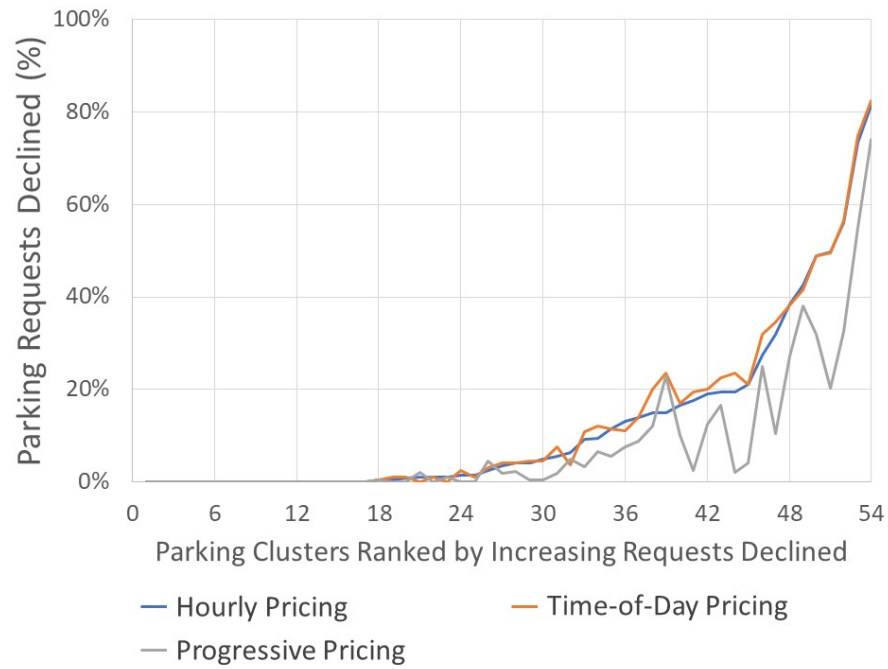


Figure 6.8: Percentage of parking requests declined under off-peak conditions

As seen in Figure 6.9, as the average parking occupancy increases, the percentage of parking requests declined will exponentially increase. This exponential relationship outlines the importance of maintaining an acceptable parking occupancy, commonly determined to be around 85% [46]. Additionally, it can be noted from Figure 6.9 that the exponential rate at which the percentage of parking requests increases is lower for the progressive hourly pricing when compared to the two hourly pricing policies.

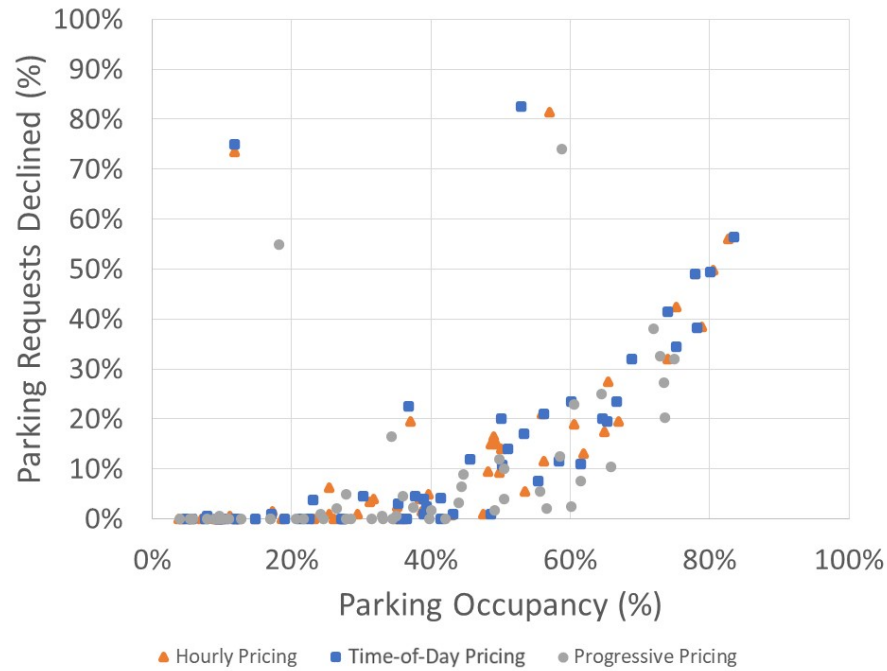


Figure 6.9: Relationship between parking occupancy and parking requests declined

6.2.3 Off-Peak Scenario Summary

Table 6.4 contains a summary of all measures taken into consideration. With regards to parking revenue, the progressive pricing policy once again generates more revenue as expected. Additionally, the time-of-day pricing policy which lowers the hourly rate by 50% generates the lowest revenue of all policies. Table 6.4 also contains numerical data pertaining to the before-mentioned parking occupancy results. We note that parking occupancy has been divided into two sections, high and low demand parking clusters. The threshold used for the separation was 50% parking occupancy. The progressive pricing policy achieves a higher reduction in parking occupancy when implemented in clusters that experience a high parking occupancy under hourly pricing when compared to lower demand clusters. For all measures, time-of-day pricing is marginally outperformed by hourly pricing. Additionally, progressive pricing achieves better values for all measures when compared to hourly pricing

and time-of-day pricing.

Table 6.4: Results from VISSIM micro-simulation during off-peak conditions

Measure	Hourly Pricing	Time-of-Day Pricing	Progressive Pricing
Parking Occupancy (High Demand Clusters, occupancy >50%)	67.1%	67.3%	61.6%
Parking Occupancy (Low Demand Clusters, occupancy <50%)	28.5%	30.5%	27.7%
Parking Revenue	\$1,792.9	\$873.3	\$4,468.0
Percentage of Parking Requests Declined	11.1%	11.5%	6.1%
Total Network Travel Time	1075.6	1088.9	1060.0

Chapter 7

Conclusion

7.1 Summary

This research aims to examine the properties of multiple pricing strategies (hourly pricing, time-of-day pricing and progressive pricing) and analyze their optimal design under revenue and social welfare maximization. This goal is achieved by first developing an econometric model to mathematically formulate the relationship between parking price and drivers' marginal utility. Then, data analysis is completed for a case study in the downtown area of Toronto. Lastly, the results from data analysis and the developed model are applied to a micro-simulation of the case study to validate the developed econometric model and calculate parking specific measures.

This research develops a framework that can be utilized by parking authorities, whether public or private, as a preliminary assessment to determine the pricing policies that can best fit under their given circumstances. The optimal design of pricing policies for parking would benefit the transport system from both the parking authorities' and policymakers' perspectives. From the perspective of parking authorities, the optimal policy can maximize revenue. Conversely, from the perspective of public authorities, an optimal design can maximize social welfare.

Literature is completed in the sector of parking management strategies. From the literature review conducted on parking management policies, we note that using sustainable parking principles to create smarter parking management strategies can benefit drivers and businesses

by increasing time and monetary savings. We review commonly used parking management strategies, including shared parking, parking pricing, providing financial incentives, and utilizing parking information. After reviewing parking management strategies used by parking authorities, we further analyze parking pricing as this thesis focuses on this parking management strategy. Parking pricing policies reviewed include hourly pricing, progressive pricing, dynamic pricing, fixed rate pricing, parking limits, and time-of-day pricing. From our review, we note that previous literature completed for hourly pricing is far superior in volume when compared to progressive pricing. The lack of literature that examines the intricacies of progressive pricing motivates this study's methodology.

This thesis's methodology is based on a supply-demand econometric model. The econometric model is formulated to represent two pricing structures, hourly and progressive pricing. The econometric model is derived from a step-wise pricing profile comprised of the base price, an augmented price, and the cutoff time. In the model, two driver groups benefit from each hour of parking as this time is used to engage in utility-generating activities. We characterize the market equilibrium point and optimize the step-wise structure of the progressive pricing policy composed of a base price, an augmented price and a cutoff time as well as the hourly pricing policy composed of a single price level.

The model yields the following insights. First, hourly pricing is a special case of progressive pricing where the augmented price equals the base price and the cutoff time can be any arbitrary threshold. Thus, progressive pricing outperforms hourly pricing in terms of achieving a larger objective function due to an additional lever of decision-making. Second, the optimal progressive price profile in the developed model for revenue maximization is chosen such that the low-value traveller pays the base price and the high-value travellers pay the augmented price. This structure represents a segmentizing strategy compared to other alternatives of having both groups pay the augmented price. Thirdly, in social welfare maximization, the optimal base price and augmented price are equal, representing an hourly pricing structure.

Thus, progressive pricing is not required for social welfare maximization as the common hourly pricing mechanism is sufficient.

Further analysis of the optimal solutions found in this thesis shows that when the initial marginal utility of each user group differs to a large extent, implementing a progressive pricing policy can significantly increase revenue generation compared to hourly pricing (Chapter 4.4.4). Regarding social welfare, the optimal price levels under both revenue and social welfare maximization are equal. Furthermore, this study also formulates inequalities and shows policy plots that policymakers and parking operators can utilize to set optimal parking management strategies.

Results from the micro-simulation model, which are limited to the study area, show that the progressive pricing under least optimal conditions (conditions where the market demand cannot be effectively segmentized) yields similar results as hourly pricing. Results from the a.m. peak scenario show that parking occupancy and search time decreased by 5.6% and 12.5% respectively under optimal conditions. Additionally, under off-peak conditions parking occupancy and search time decreased by 5.5% and 5% respectively. Furthermore, under off-peak conditions, the implementation of a time-of-day pricing structure results in similar parking occupancy, cruising for parking and network travel time as the hourly pricing policy. The major notable difference resulting from both hourly and time-of-day pricing policies will be the revenue generated.

As mentioned, the results show that implementing a progressive pricing policy can reduce parking occupancy and cruising for parking. Implementing a progressive price level at all locations of the study area could be impractical as the price difference from one location to another could introduce confusion to the parking system, especially for drivers who are less familiar with the area. A recommendation is made to parking authorities based on the results of the research completed. We encourage parking authorities to consider implementing a progressive pricing scheme at a location in the city where parking occupancy is high, and

a high turnover rate is desired. A suitable location for the City of Toronto will be the St. Lawrence Market neighbourhood since parking occupancy for these on-street parking spots is high. Based on results from this implementation, a strategy can be developed for the rest of the city based on lessons learned.

7.2 Literature Contribution

Our model, which sets parking prices based on drivers' marginal utility, is distinguished from reviewed literature that sets parking prices according to drivers' attitudes. Despite existing implementations of progressive parking pricing, fewer studies focus on the reactive behaviour of customers in place of progressive pricing. Additionally, the trade-offs of progressive pricing on profit and social welfare remain to be investigated. This paper contributes to the literature on progressive pricing by formulating drivers' reactions to this policy and modelling its optimal profile design. With the development of our model, parking authorities will have a framework that can be followed to more efficiently manage their parking supply.

In this study, aside from optimizing our model for maximum revenue generation, the model will also be optimized with the scope of maximizing social welfare. This is completed to account for utility gained by the whole system as opposed to only the utility gained by parking authorities in the form of revenue. In contrast to other reviewed models, the developed econometric model provides flexibility to parking authorities by considering two objectives that can be conflicting at times.

7.3 Research Limitations

This thesis made use of all available and provided data. The data utilized had some limitations that will limit the study. Building a micro-simulation model and calibrating the developed

econometric model requires a large amount of data. In Sections 3.2.1 and 3.2.2 we have described the collected/acquired data and its limitations. First, the data collected manually with the use of Google maps mostly pertained to October 2020. However, for this thesis, we mainly utilize transaction data from October 2019. Between these two periods, there could have been changes to the road infrastructure, mainly since October 2019 occurred before the City of Toronto officials implemented changes to the infrastructure to accommodate transportation users during the COVID-19 pandemic.

Secondly, the parking transaction data received from the Toronto Parking Authority [85] was aggregated daily for October 2019. Ideally, we would have liked to receive raw data that can then be extensively analyzed; only a limited number of insights can be derived from aggregated data. Nonetheless, the provided data was essential for the project. Additionally, the provided parking transaction data was not divided based on vehicle types. The data received was aggregated and only contained one vehicle type. We assume that this data pertains to passenger vehicles only for our analysis, although there might be a small proportion of trucks. Lastly, vehicular volume data was only provided for 23 out of 74 intersections in the study area. In lieu of this limited data, results obtained from the micro-simulation should only be taken as early results, and further analysis should be completed to validate the results.

7.4 Future Work

For future work on this topic, multiple paths of future research can be taken. The first future work that can be completed is to collect the missing data and update the micro-simulation model. Alternatively, a more accurate model for a smaller area can be built since the study area utilized in this thesis is very large and requires a lot of data to calibrate the simulation properly.

In addition, we recommend that the econometric model be utilized with different data sets to validate its efficacy. This study validated the model using a micro-simulation and available data for a case study in Toronto, Canada. Therefore, results from the simulation are limited to the examined study area, and further investigation is recommended for other sites.

To build on the theory and analysis completed in this study, the authors recommend that future work be conducted to validate further and scrutinize the developed model. We recommend that future work be completed to validate the marginal utility function used in this report. Although this work utilizes a linear marginal utility function, which is commonly used in econometric modelling, it is recommended that sensitivity analysis is completed on the model by considering alternative functions, including a negative exponential function or a step-wise function. For this study, drivers were assumed to follow a linear marginal utility function to simplify the model and derive optimal closed-form solutions that can be mathematically analyzed. For future studies, this assumption can be further examined.

In this study the developed econometric model is utilized to determine the optimal price levels based on historical parking transaction data. However, by calculating the price levels using this data (transaction data for October 2019) we assumed that demand remains the same in future years. To further build on the analysis completed in this thesis, future work can be completed in the prediction of future parking demand at the locations. This will then allow us to calculate the optimal price levels for future expected demand.

In addition for future studies, the developed model can be further extended to account for the demand induced by the price changes. For this study, the demand component of the simulation is assumed to stay constant. However, in a real life scenario this demand will vary and can increase or decrease based on the change in price. To account for this induction of demand, a demand model can be developed to predict the new demand under the new price levels given by the econometric model.

Lastly, I would like to recommend developing a dynamic pricing system based on parking

transaction data for future studies. As discussed in the literature review of this paper, dynamic pricing is called by many professionals in the industry the most efficient pricing strategy as it allows parking authorities to dynamically change price levels to better match the supply and demand for parking. As a drawback, such a policy will require parking sensors or cameras to determine real-time parking occupancy, which can be very costly. I would recommend that future work be completed in developing a dynamic pricing system that utilizes parking transaction data as a real-time demand measure. Parking authorities, including the Toronto Parking Authority (TPA), have access to parking transaction data. The model developed in this paper can be extended to accept real-time transaction data to obtain the current optimal price level that can be set dynamically.

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Appendices

Appendix A

Derivation of Equations in Econometric Model

First and second-order conditions

We consider the three conditions of Equation 4.5 individually for a given policy defined by the triplet (p_1, p_2, q) consisting of the base price p_1 , augmented price p_2 , and cutoff time q . For $u_i(q) < p_1$, the marginal utility function of group i intersects only the base price curve once indicating that $d_i^* < q$ and that the cost of parking according to Equation 4.4 is $p_1 d_i^*$. According to the first order condition of Equation 4.5, setting the derivative of Equation 4.3 to zero gives $u_i(d) = p_1$, and its inverse gives $d_i^* = u_i^{-1}(p_1)$.

Following the same procedure as above, for $u_i(q) > p_1$ the marginal utility function only intersects the augmented price curve once indicating that $d_i^* > q$ and that the cost of parking according to Equation 4.4 is $p_2 d_i^*$. According to the first order condition of Equation 4.5, $u_i(d) = p_2$ and its inverse gives $d_i^* = u_i^{-1}(p_2)$.

Therefore, drivers maximize their utility by choosing the dwell time given by 4.5. According to the second-order condition, since $u_i(d)$ is convex the optimal dwell time of drivers is also convex and unique.

From the second-order condition in hourly pricing, we have

$$\frac{\partial^2 \pi}{\partial p^2} = -\frac{2\alpha}{b_l} - \frac{2(1-\alpha)}{b_h}, \tag{A.1}$$

and for progressive pricing we have

$$\frac{\partial^2 \pi}{\partial p_1^2} = -\frac{2\alpha}{b_l}, \quad \text{A.2}$$

$$\frac{\partial^2 \pi}{\partial p_2^2} = -\frac{2(1-\alpha)}{b_h}. \quad \text{A.3}$$

Since Equations A.1 and A.2 are negative, the second order condition is met and revenue is convex.

According to the second-order condition of social welfare in hourly pricing we have

$$\frac{\partial^2 \pi}{\partial p^2} = -\frac{2\left(-\frac{\alpha}{b_l} - \frac{1-\alpha}{b_h}\right)^2}{s^2} - \frac{\alpha}{b_l} - \frac{1-\alpha}{b_h}, \quad \text{A.4}$$

and in progressive pricing, we get

$$\frac{\partial^2 \pi}{\partial p_1^2} = -\frac{2\alpha^2}{b_l^2 s^2} - \frac{\alpha}{b_l}, \quad \text{A.5}$$

$$\frac{\partial^2 \pi}{\partial p_2^2} = -\frac{2(1-\alpha)^2}{b_h^2 s^2} - \frac{1-\alpha}{b_h}. \quad \text{A.6}$$

Since Equations A.4 and A.5 are negative, the second order condition is met and social welfare is convex.

Derivation of 4.12

Consider 4.11 as the revenue of case 1 where $d_l^* = \frac{a_l - p_2}{b_l}$ and $d_h^* = \frac{a_h - p_2}{b_h}$. The partial derivative of 4.11 with respect to p_1 , p_2 , and q gives

$$\frac{\partial \pi}{\partial p_1} = q(1-\alpha) + q\alpha \quad \text{A.7}$$

$$\frac{\partial \pi}{\partial p_2} = \frac{a_h b_l (1-\alpha) + a_l b_h \alpha - 2b_l p_2 + 2\alpha p_2 (b_l - b_h) - b_l b_h q}{b_l b_h} \quad \text{A.8}$$

$$\frac{\partial \pi}{\partial q} = p_1 - p_2. \quad \text{A.9}$$

Setting A.8 to zero and solving for p_2 gives the optimal solution for the augmented price which is

$$p_2 = \frac{a_h b_l (\alpha - 1) - \alpha a_l b_h + b_l b_h q}{2b_l (\alpha - 1) - 2\alpha b_h}. \quad \text{A.10}$$

Knowing that $q = 0$ in case 1, A.10 can be further simplified to

$$p_2 = \frac{a_h b_l (\alpha - 1) - \alpha a_l b_h}{2b_l (\alpha - 1) - 2\alpha b_h}. \quad \text{A.11}$$

Setting A.9 to zero gives $p_1 = p_2$. Therefore, A.11 is the optimal solution of case 1 for both price levels.

Derivation of 4.14

Consider 4.13 as the revenue of case 2 where $d_l^* = \frac{a_l - p_1}{b_l}$ and $d_h^* = \frac{a_h - p_2}{b_h}$. Taking the derivative of 4.13 with respect to p_1 , p_2 , and q gives

$$\frac{\partial \pi}{\partial p_1} = \frac{\alpha(a_l - p_1)}{b_l} - \frac{\alpha p_1}{b_l} + q(1 - \alpha) \quad \text{A.12}$$

$$\frac{\partial \pi}{\partial p_2} = (1 - \alpha) \left(\frac{a_h - p_2}{b_h} - q \right) - \left(\frac{p_2(1 - \alpha)}{b_h} \right) \quad \text{A.13}$$

$$\frac{\partial \pi}{\partial q} = (1 - \alpha)(p_1 - p_2). \quad \text{A.14}$$

Knowing that maximum revenue occurs at the lower bound condition of q , then by equating q to its lower bound condition d_i^* , substituting into A.12 and A.13, and setting it to zero gives $p_1 = \frac{a_l}{1+\alpha}$ and $p_2 = \frac{a_h b_l - a_l b_h + b_h p_1}{2b_l}$. The latter equation can then be further simplified by substituting p_1 and getting $p_2 = \frac{a_h}{2} - \frac{\alpha a_l b_h}{2b_l(1+\alpha)}$

Substituting the the optimal p_1 into $q = \frac{a_l - p_1}{b_l}$ gives $q = \frac{\alpha a_l}{b_l(1+\alpha)}$. Therefore, the

before-mentioned equations are the optimal solutions for revenue under case 2 conditions.

Proof of Lemma 1

Recall that the equations for consumer surplus and revenue of case 2 are

$$\pi = \alpha p_1 d_l^* + \alpha(qp_1 + (d_h^* - q)p_2),$$

and

$$CS = \alpha(U_l(d_l^*) - d_l^* p_1) + \alpha(U_h(d_h^*) - qp_1 - (d_h^* - q)p_2) + \pi - \gamma, T((1 - \alpha)d_h^* + \alpha d_l^*)/s.$$

In addition, social welfare is the summation of revenue and consumer surplus, which gives

$$W = \alpha U_l(d_l^*) + (1 - \alpha)U_h(d_h^*) + \pi - \gamma T((1 - \alpha)d_h^* + \alpha d_l^*)/s.$$

Note that both revenue and consumer surplus are a function of the cutoff time q . However, when deriving social welfare from the summation of revenue and consumer surplus we see that the term q is dropped. This occurs because q enables the increase/decrease of revenue at the cost of consumer surplus and vice versa. The trade-off between revenue and consumer surplus is of zero sum. By this logic, social welfare only depends on marginal utility functions of drivers, price levels and the negative externalities of searching.

Proof of Proposition 1

Substituting 4.12 into 4.11 gives

$$\pi_{case1} = \frac{(a_l b_h \alpha + a_h (b_l - b_l \alpha))^2}{4b_h b_l (b_l + b_h \alpha - b_l \alpha)}. \quad A.15$$

Similarly, substituting 4.14 into 4.13 gives

$$\pi_{case2} = \frac{4a_l^2 \alpha b_l b_h + (1 - \alpha)(a_h (1 + \alpha) b_l - a_l \alpha b_h)^2}{4(\alpha + 1)^2 b_l^2 b_h}. \quad A.16$$

Setting $\pi_{case2} \geq \pi_{case1}$ and simplifying gives

$$\frac{(\alpha - 1)\alpha(a_h(\alpha + 1)b_l - a_l(\alpha b_h + 2b_l))}{(\alpha + 1)b_l((\alpha - 1)b_l - \alpha b_h)} \geq 0. \quad A.17$$

Simplifying Inequality A.17 by normalizing the high value group's marginal utility intercept and rate of change to one such that $a_h = 1$ and $b_h = 1$ gives,

$$\frac{(\alpha - 1)\alpha(-a_l(\alpha + 2b_l) + \alpha b_l + b_l)}{(\alpha + 1)b_l(\alpha(b_l - 1) - b_l)} \geq 0. \quad A.18$$

Given the conditions stated in Equations 4.5, 4.6, and 4.10 we note that α , $1 + \alpha$, and b_l are always non-negative, and $\alpha - 1 \leq 0$. The term $(\alpha(b_l - 1) - b_l)$ is non-positive since $\alpha b_l \leq \alpha + b_l$. Lastly, the term $(-a_l(\alpha + 2b_l) + \alpha b_l + b_l)$ is non-negative since $b_l(1 + \alpha) \leq a_l(\alpha + 2b_l)$. In lieu of this, Inequality A.18 holds, showing that the revenue generated under case 1 (hourly pricing) is be less than or equal to revenue generated in case 2 (progressive pricing).

Derivation of 4.16

Consider 4.16 as the social welfare of case 1 where $d_l^* = \frac{a_l - p_2}{b_l}$ and $d_h^* = \frac{a_h - p_2}{b_h}$. Taking the derivatives of 4.16 with respect to p_1 , p_2 , and q gives

$$\frac{\partial W}{\partial p_1} = 0, \quad A.19$$

$$\frac{\partial W}{\partial p_2} = \frac{((\alpha - 1)b_l - \alpha b_h)(2a_h(-1 + \alpha)b_l - 2a_l\alpha b_h + 2\alpha b_h p_2 + b_l p_2(2 - 2\alpha + b_h s^2))}{b_l^2 b_h^2 s^2}, \quad A.20$$

$$\frac{\partial W}{\partial q} = 0. \quad A.21$$

Setting equation A.20 to zero and solving for p_2 we get the optimal solution

$$p_2 = \frac{2(a_h b_l(1 - \alpha) + a_l \alpha b_h)}{2\alpha b_h + b_l(2 - 2\alpha + b_h s^2)}. \quad \text{A.22}$$

Derivation of 4.18

Consider 4.18 as the social welfare of case 2, where $d_l^* = \frac{a_l - p_1}{b_l}$ and $d_h^* = \frac{a_h - p_2}{b_h}$. The partial derivatives of 4.18 with respect to p_1 , p_2 , and q are

$$\frac{\partial \pi}{\partial p_1} = -\frac{\alpha(2a_h(-1 + \alpha)b_l + 2b_l p_2 - 2\alpha(a_l b_h - b_h p_1 + b_l p_2) + b_l b_h p_2 s^2)}{b_l^2 b_h s^2} \quad \text{A.23}$$

$$\frac{\partial \pi}{\partial p_2} = \frac{(-1 + \alpha)(2a_h(-1 + \alpha)b_l + 2b_l p_2 - 2\alpha(a_l b_h - b_h p_1 + b_l p_2) + b_l b_h p_2 s^2)}{b_l b_h^2 s^2} \quad \text{A.24}$$

$$\frac{\partial \pi}{\partial q} = 0. \quad \text{A.25}$$

Setting A.23 and A.24 to zero and solving for p_1 and p_2 gives

$$p_1 = -\frac{2(-a_h b_l + a_h \alpha b_l - a_l \alpha b_h + b_l p_2 - \alpha b_l p_2)}{2(2\alpha + b_l s^2)}, \quad \text{A.26}$$

$$p_2 = \frac{2(-a_h b_l + a_h \alpha b_l - a_l \alpha b_h + \alpha b_h p_1)}{b_l(-2 + 2\alpha - b_h s^2)}. \quad \text{A.27}$$

Substituting A.27 into A.26 and solving for p_1 gives

$$p_1^* = \frac{2(a_h b_l(1 - \alpha) + a_l \alpha b_h)}{2\alpha b_h + b_l(2 - 2\alpha + b_h s^2)}. \quad \text{A.28}$$

Similarly, substituting A.28 into A.27 gives

$$p_2^* = \frac{2(a_h b_l(1 - \alpha) + a_l \alpha b_h)}{2\alpha b_h + b_l(2 - 2\alpha + b_h s^2)}. \quad \text{A.29}$$

Derivation of 4.20

To determine the value of s for which the hourly pricing policy under revenue and social welfare maximization are the same, the revenue of both pricing policies are equated. By substituting the optimal price from 4.12 into the revenue expression 4.11 we obtain the following simplified equation for the maximum revenue under revenue maximization:

$$\pi(p^*) = -\frac{(a_h(\alpha - 1)b_l - a_l\alpha b_h)^2}{4b_l b_h((\alpha - 1)b_l - \alpha b_h)}. \quad \text{A.30}$$

Similarly, substituting 4.19 into the revenue expression 4.11 we obtain the following revenue under social welfare maximization:

$$\pi(p^{\text{sw}}) = \frac{2s^2(a_h(\alpha - 1)b_l - a_l\alpha b_h)^2}{(b_l(-2\alpha + b_h s^2 + 2) + 2\alpha b_h)^2}. \quad \text{A.31}$$

By equating A.30 and A.31 and solving for s we get

$$\hat{s} = \frac{\sqrt{2}\sqrt{-\alpha b_l + \alpha b_h + b_l}}{\sqrt{b_l}\sqrt{b_h}}. \quad \text{A.32}$$

It is then easy to show that when $s \leq \hat{s}$, then $p^{\text{sw}} > p^*$, and when $s > \hat{s}$, then $p^{\text{sw}} < p^*$.

Appendix B

R-language Scripts for Parking Transaction Data Analysis

```
rm(list = ls())

library(readxl)

#OBTAIN MACHINE ID'S OF STUDY AREA

data_machine_ID <- read_excel("C:/Users/daveo/Desktop/THESIS/Data
                               Manipulation R/Parking lot ID.xlsx")

machine_ID <- data_machine_ID[,5]

#CREATE DF FOR SAVING RESULTS

results <- data.frame()

#ITERATE TROUGH EACH MACHINE ID

for (i in 1:60){

  results[i,1] <- machine_ID[i,1]

  temp <- read.csv(paste("C:/Users/daveo/Desktop/THESIS/Green P/
TPA Data 2019 10 TD/sm201910",machine_ID[i,1],".csv", sep = ""))

  #SUBSET BASED UPON WEEKDAY OR WEEKENDS

  temp <- subset(temp,day == 5|day == 6|day == 12
|day == 13|day == 19|day == 20|day == 26|day == 27)
```

```

for (n in 1:nrow(temp)){
  if (temp[n,5] != 0){
    temp[n,19] <- (temp[n,18]*60)/(data_machine_ID[i,7]*
                                temp[n,5])

    colnames(temp)[19] <- paste("dwell")
  } else {
    temp[n,19] <- 0
  }
}

if (sum(temp[,19]) > 0){
  #CREATE HISTOGRAM
  hist(temp$'dwell',main = machine_ID[i,1])
}

#CALCULATE MEAN AND SD
results[i,2] <- mean(temp[,19])
results[i,3] <- sd(temp[,19])
}

#NAME COLUMNS
names(results) <- c("Machine ID","Mean Dwell Time","Standard Deviation")

#SAVE TO CSV
write.table(results, "C:/Users/daveo/Desktop/THESIS/Data
                Manipulation R/dwell time weekday.csv",sep=",")

## weekend USE CODE temp <- subset(temp,day == 5|day == 6|day == 12
                |day == 13|day == 19|day == 20|day == 26|day == 27)
## weekday USE CODE temp <- subset(temp,day != 5&day != 6&day != 12&

```


day != 13&day != 19&day != 20&day != 26&day != 27)

Appendix C

R-language Scripts for Vehicle Volume Data Analysis

C.1 Passenger Vehicle Volumes

```
#Import Data

perm_vehicles <- read.csv("C:/Users/daveo/Desktop/THESIS/Data
Manipulation R/perm_tmcs_vehicles no headings.csv", header=FALSE)

perm_vehicles <- subset(perm_vehicles,V7 != "U-Turn")

#Subset Data

carsampeak1 <- subset(perm_vehicles, V2 == "Weekday" & V4 == "Oct-19"
& V3 == "Cars" & V5 == 7)

carsampeak2 <- subset(perm_vehicles, V2 == "Weekday" & V4 == "Oct-19"
& V3 == "Cars" & V5 == 8)

carsampeak3 <- subset(perm_vehicles, V2 == "Weekday" & V4 == "Oct-19"
& V3 == "Cars" & V5 == 9)

#Determine AM Hour interval with highest Volume

sum1 <- sum(carsampeak1$V8)

sum2 <- sum(carsampeak2$V8)

sum3 <- sum(carsampeak3$V8)

#Save Data (in this case 8 AM was had the highest vehicle volume)
```

```

write.table(carsampeak2, "C:/Users/daveo/Desktop/THESIS/Data
Manipulation R/vehicle peak am.csv",sep=",")

carspmpeak1 <- subset(perm_vehicles, V2 == "Weekday" & V4 == "Oct-19"
& V3 == "Cars" & V5 == 16)

carspmpeak2 <- subset(perm_vehicles, V2 == "Weekday" & V4 == "Oct-19"
& V3 == "Cars" & V5 == 17)

carspmpeak3 <- subset(perm_vehicles, V2 == "Weekday" & V4 == "Oct-19"
& V3 == "Cars" & V5 == 18)

#Determine Hour interval with highest Volume

psum1 <- sum(carspmpeak1$V8)
psum2 <- sum(carspmpeak2$V8)
psum3 <- sum(carspmpeak3$V8)

#Save Peak hour (in this case 6PM was the peak)

write.table(carspmpeak3, "C:/Users/daveo/Desktop/THESIS/
Data Manipulation R/vehicle peak pm.csv",sep=",")

```

C.2 Truck Vehicle Volumes

```

#Import Data

perm_vehicles <- read.csv("C:/Users/daveo/Desktop/THESIS/Data
Manipulation R/perm_tmcs_vehicles no headings.csv", header=FALSE)

perm_vehicles <- subset(perm_vehicles,V7 != "U-Turn")

#Subset Data

truckampeak1 <- subset(perm_vehicles, V2 == "Weekday" & V4 == "Oct-19"
& V3 == "Trucks" & V5 == 7)

truckampeak2 <- subset(perm_vehicles, V2 == "Weekday" & V4 == "Oct-19"

```

```

& V3 == "Trucks" & V5 == 8)

truckampeak3 <- subset(perm_vehicles, V2 == "Weekday" & V4 == "Oct-19"
& V3 == "Trucks" & V5 == 9)

#Determine AM Hour interval with highest Volume

sum1 <- sum(truckampeak1$V8)
sum2 <- sum(truckampeak2$V8)
sum3 <- sum(truckampeak3$V8)

write.table(truckampeak1, "C:/Users/daveo/Desktop/THESIS/Data
Manipulation R/vehicle Trucks peak am.csv",sep=",")

#Subset Data

truckampeak1 <- subset(perm_vehicles, V2 == "Weekday" & V4 == "Oct-19"
& V3 == "Trucks" & V5 == 16)

truckampeak2 <- subset(perm_vehicles, V2 == "Weekday" & V4 == "Oct-19"
& V3 == "Trucks" & V5 == 17)

truckampeak3 <- subset(perm_vehicles, V2 == "Weekday" & V4 == "Oct-19"
& V3 == "Trucks" & V5 == 18)

#Determine PM hour interval with highest Volume

psum1 <- sum(truckampeak1$V8)
psum2 <- sum(truckampeak2$V8)
psum3 <- sum(truckampeak3$V8)

write.table(truckampeak3, "C:/Users/daveo/Desktop/THESIS/Data
Manipulation R/vehicle Trucks peak pm.csv",sep=",")

```

Appendix D

MATLAB Script for Policy Maps

D.1 Revenue Policy Plots

```
clear;
b1 = 1.2;
a2 = 1;
b2 = 1;
s= 1;
%alpha = 0.2;
%a1_upper = (a2*(1 + alpha)*b1)/(2*b1 + alpha*b2);
%b1_lower = (a1*alpha*b2)/(-2*a1 + a2 + a2*alpha);

[alpha,a1] = meshgrid(linspace(0,1,100),linspace(0.01,1,100));

%Progressive pricing
p1 = p1case1(alpha,a1);
p2 = p2case1(alpha,a1,a2,b1,b2);
q = qcase1(alpha,a1,b1);
p1star = p1case2(alpha,a1,a2,b1,b2);

temp1 = p1 <= p2;
```

```

temp2 = p1 > p2;
z5 = profitcase1(q,alpha,a1,a2,b1,b2,p2,p1);
z6 = profitcase22(alpha,a1,a2,b1,b2,p1star);
z1 = (temp1.*z5) + (temp2.*z6);

p1pro = (temp1.*p1) + (temp2.*p1star);
p2pro = (temp1.*p2) + (temp2.*p1star);

%Hourly pricing
p1 = p1case2(alpha,a1,a2,b1,b2);
priceonehourly = p1;
z2 = profitcase2(alpha,a1,a2,b1,b2,p1);

matrix = z1./z2;

%Figures
fig1 = figure();
contourf(alpha,a1,z1./z2,[linspace(0,2,10)]) %,[linspace(0,2,10)]
colorbar
title("Revenue Ratio of Hourly Pricing to Progressive Pricing")
xlabel("alpha")
ylabel("a1")
set(gca,'FontSize',18)
xlim([0 1])
ylim([0.01 1])

```

```

yi= linspace(0,1,100);
x = linspace(-0.2,1.2,1000);
y= (b1.*(1+x))./(2.*b1+x.*b2);
xi = interp1(y,x,yi);
hold on
plot(xi,yi,'r','LineWidth',5)

fig2 = figure();
contourf(alpha,a1,priceonehourly)
colorbar
title("P1")
xlabel("alpha")
ylabel("a1")

levels = 0:0.03:1;
fig3 = figure();
contourf(alpha,a1,p1pro,levels)
colorbar
    set(gca,'FontSize',18)
    xlim([0 1])
ylim([0.01 1])
caxis([0 0.5])
title("P1")
xlabel("alpha")
ylabel("a1")
    hold on

```

```

plot(xi,yi,'r','LineWidth',5)

fig2 = figure();
contourf(alpha,a1,p2pro,levels)
colorbar
    set(gca,'FontSize',18)
    xlim([0 1])
ylim([0.01 1])
caxis([0 0.5])
title("P2")
xlabel("alpha")
ylabel("a1")
    hold on
plot(xi,yi,'r','LineWidth',5)

fig3 = figure();
contourf(alpha,a1,p1pro./p2pro)
colorbar
title("Price Ratio P1/P2")
xlabel("alpha")
ylabel("a1")
set(gca,'FontSize',18)
    xlim([0 1])
ylim([0.01 1])
hold on
plot(xi,yi,'r','LineWidth',5)

```


D.2 Pareto-Optimality Plots

```
clear;
a1 = 0.1;
b1 = 1.2;
a2 = 1;
b2 = 1;
s = 1.5;
alpha = 0.1;
q = (a1.*alpha)./(b1+alpha.*b1);

[p1,p2] = meshgrid(linspace(0,1,1000),linspace(0.01,1,1000));

temp1 = p1 <= p2;
temp2 = p1 > p2;
z5 = profitcase1(q,alpha,a1,a2,b1,b2,p2,p1);
z6 = 0;
revenue = (temp1.*z5) + (temp2.*z6);

z55 = socialwelfarecase1(alpha,a1,a2,b1,b2,p2,p1,s);
z66 = 0;
social_welfare = (temp1.*z55) + (temp2.*z66);

fig2 = figure();
[C,h] = contour(p1,p2,revenue,'k','LevelList',0:0.075:1,
                'LineWidth',1.5);
```

```

set(gca,'FontSize',18)
hold on
v = [0];
clabel(C,h,v);
[M,j] = contour(p1,p2,social_welfare,'r','LevelList',0:0.15:1,
               'LineWidth',1.5);

set(gca,'FontSize',18)
colorbar off
hold on
yi= linspace(0,1,100);
x = linspace(0,1,100);
y= x;
xi = interp1(y,x,yi);
plot(xi,yi,'k','LineWidth',1.5)
%legend('FontSize',25)
hold on
[row, col] = find(ismember(revenue, max(revenue(:)))));
h = scatter(col/1000, row/1000,'p','filled','k');
h.SizeData = 500;
hold on
[row1, col1] = find(ismember(social_welfare, max(social_welfare(:)))));
h = scatter(col1/1000, row1/1000,'p','filled','r');
h.SizeData = 500;

```