

Fractional Exponents

$$\sqrt{4} = 4^{1/2}$$

$$\sqrt{4} \sqrt{4} = 4$$

$$(2)(2) = 4$$

$$= 4^{1/2} 4^{1/2}$$

$$= (4^{1/2})^2$$

$$= 4^{(1/2)(2)}$$

$$= 4^{2/2}$$

$$= 4$$

$$\sqrt{ab} = \sqrt{a} \sqrt{b}$$

$$a^{1/2} b^{1/2} = (ab)^{1/2}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} = \left(\frac{a}{b}\right)^{1/2}$$

$$(a/b)^{-1/2} = (a/b)^{(-1)(1/2)}$$

$$= \left(\frac{b}{a}\right)^{1/2}$$

$$= \sqrt{b/a}$$

Remember:

$$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$$

What real number multiplied by itself is equal to -4 ? None

$\sqrt{-4}$ is undefined

Remember: you can't find the square root of a negative

$$x^2 = 16$$

$$\sqrt{x^2} = \sqrt{16}$$

$$x = \pm 4$$

Fractional Exponents

no sq-root
in the denominator

Rationalize the denominators and simplify:

$$\frac{\sqrt{3}}{\sqrt{5}} = \left(\frac{\sqrt{3}}{\sqrt{5}} \right) \left(\frac{\sqrt{5}}{\sqrt{5}} \right) = \frac{\sqrt{3} \sqrt{5}}{5} = \frac{\sqrt{(3)(5)}}{5} = \frac{\sqrt{15}}{5}$$

$$\begin{aligned} \frac{1}{\sqrt{50}} &= \frac{\sqrt{50}}{50} = \frac{\sqrt{(25)2}}{50} = \frac{\sqrt{25} \cdot \sqrt{2}}{50} \\ &= \frac{5\sqrt{2}}{50} = \frac{\sqrt{2}}{10} \end{aligned}$$

The n-th Root

(What is the solution to $8^{1/3}$?)

$$\begin{aligned} 8^{1/3} &= \sqrt[3]{8} = \sqrt[3]{(2)(2)(2)} \\ &= \sqrt[3]{2^3} = (2^3)^{1/3} = 2^{3(1/3)} \\ &= 2 \end{aligned}$$

What is the solution to $(1/32)^{1/5}$?

$$\begin{aligned} \left(\frac{1}{32}\right)^{1/5} &= \sqrt[5]{1/32} \Rightarrow \text{What number multiplied} \\ &\quad \text{by itself 5 times} \\ &\quad \text{is equal to } 1/32? \\ &= \sqrt[5]{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)} \\ &= \sqrt[5]{\left(\frac{1}{2}\right)^5} = \left(\left(\frac{1}{2}\right)^5\right)^{1/5} = \left(\frac{1}{2}\right)^{5(1/5)} \\ &= 1/2 \end{aligned}$$

The n-th Root

Let a be a positive number and n be a positive integer: ✓

$$a^{1/n} = \sqrt[n]{a}$$

↑ ↑

The n-th Root

$$(a+b)^2 \neq a^2 + b^2$$

Not

Correct!

$$\sqrt{\frac{1}{16} + \frac{1}{25}}$$

$$\neq \sqrt{\frac{1}{16}} + \sqrt{\frac{1}{25}}$$

$$= \frac{1}{\sqrt{16}} + \frac{1}{\sqrt{25}}$$

$$= \frac{1}{4} + \frac{1}{5}$$

$$= \frac{5}{20} + \frac{4}{20} = \frac{9}{20} = 0.45$$

calculator result:

The real answer is 0.32 !?

BIDMAS

$$\begin{aligned}\sqrt{\frac{1}{16} + \frac{1}{25}} &= \left(\frac{1}{16} + \frac{1}{25}\right)^{1/2} \\&= \left[\left(\frac{1}{16}\right)\left(\frac{25}{25}\right) + \left(\frac{1}{25}\right)\left(\frac{16}{16}\right)\right]^{1/2} \\&= \left(\frac{25 + 16}{400}\right)^{1/2} = \sqrt{\frac{41}{400}} \\&= 0.32\end{aligned}$$

Political Science
model of voting
economic conservation on use
economic models
geography:
Useful modeling:
Financial models like
modeling - represent real world
with math
- Backbone of

Functions

What is a function?

A Less Mathematical Definition: A function is an expression, rule, or law that defines a relationship between one variable and another variable.

What is a function?

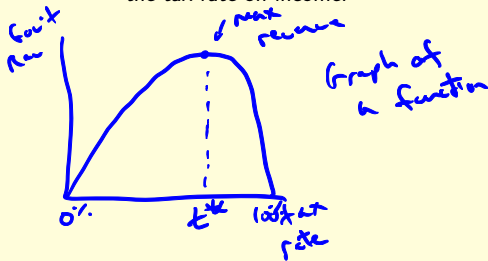
A Less Mathematical Definition: A function is an expression, rule, or law that defines a relationship between one variable and another variable.

input → Add \$2.00 to a vending machine to receive 1 soft drink.
↓ output function

What is a function?

A Less Mathematical Definition: A function is an expression, rule, or law that defines a relationship between one variable and another variable.

- ▶ Add \$2.00 to a vending machine to receive 1 soft drink.
- ▶ The distance a car can travel is a function of:
 - ▶ How much gasoline is in the fuel tank.
- ▶ Government revenue is a function of:
 - ▶ the tax rate on income.



Example of a Math Function

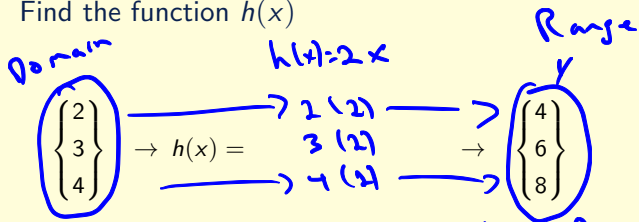
In math, there is a function that converts air temperature from the celcius scale (C) to fahrenheit (F):

$$F = 1.8C + 32$$

$$\text{Celsius} = \left\{ \begin{array}{c} -10^\circ \\ 20^\circ \\ 30^\circ \end{array} \right\} \xrightarrow{\substack{1.8(-10)+32 \\ 1.8(20)+32 \\ 1.8(30)+32}} \rightarrow \text{Fahrenheit} = \left\{ \begin{array}{c} 14^\circ \\ 68^\circ \\ 86^\circ \end{array} \right\}$$

C F

Find the function $h(x)$



x can take on any value in the Domain

$$x \in \{2, 3, 4\}$$

Each x "maps" to a value in the range.
Give the function an x -value from the Domain
and the function will find all related
 y -values in the Range.

Notation and Language

A general way to write a function is:

$$y = f(x)$$

- ▶ x is called the **independant variable** because it can take on any value in the domain without conditions.
- ▶ x is also called the **exogenous variable** because it is determined “outside” the model.

Notation and Language

A general way to write a function is:

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⇒ Freedom!

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- ▶ x is also called the **exogenous variable** because it is determined “outside” the model.
- ▶ y is the dependent variable because its value is determined by the value of x .

↘ ↘ not free
conditional on x , the
independent variable

Notation and Language

A general way to write a function is:

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- ▶ x is also called the **exogenous variable** because it is determined “outside” the model.
- ▶ y is the dependent variable because its value is determined by the value of x .
- ▶ $f(x)$ is the function pronounced “f of x” or “f-x”.
- ▶ $f(x)$ is also the value of y given the value x .

if $y = 4$ then $f(x) = 4$

Example: A model estimates the quality of wine (q) based on average air temperature at the vineyard (at):

$$q = v(at)$$

v = function

at = Ind. var.

q = dep. var.

$v(at) = q$!

Notation and Language

$$y = f(x)$$

Summary of Notation

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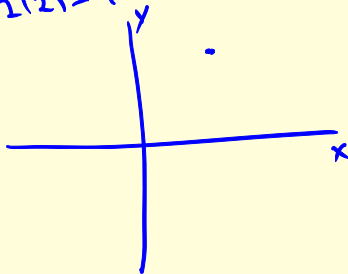
$$y = f(x)$$

$$y = 2x$$

$$\text{let } x = 2$$

$$y = 2(2) = 4$$

- ▶ x can take on any value in the **Domain**.
- ▶ $f(x)$ is the range of f .
- ▶ $(x, f(x))$ is an ordered pair in x, y .



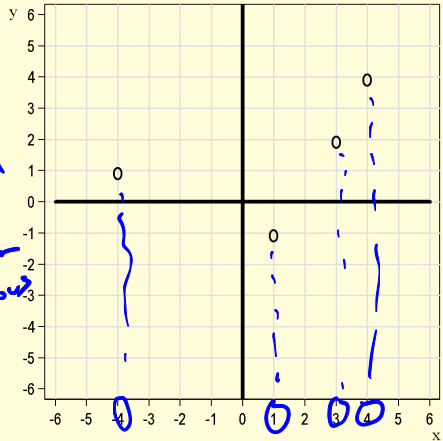
Domain of a Discrete Function

Based on the graph, what is the domain of this discrete function, k ?

$$x \in D\{-4, 1, 3, 4\}$$

4 ordered pairs
rather than
a continuous
line

A graph of the discrete function, k

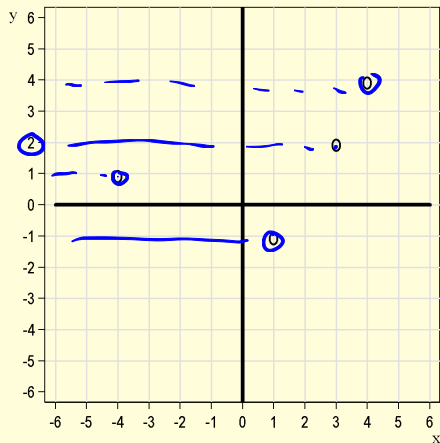


Range of a Discrete Function

Based on the graph, what is the range of this discrete function, k ?

$$y \in \mathbb{R} \{1, -1, 2, 4\}$$

A graph of the discrete function, k

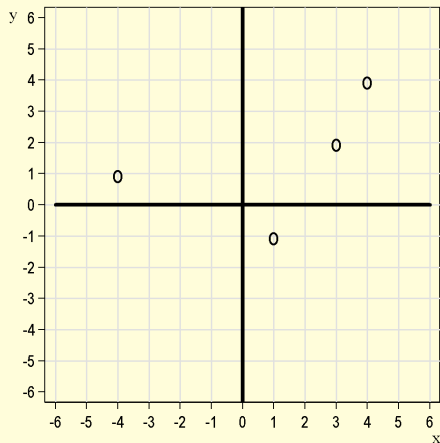


Application with a Discrete Function

Calculate: $4k(1) + k(-4)$

if $x=1$ in k

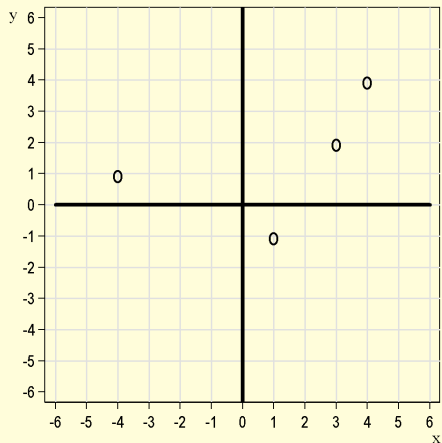
A graph of the discrete function, k



Domain of a Continuous Function

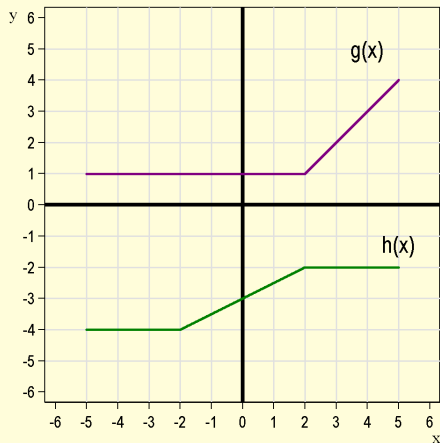
Based on the graph, what are the domains of $g(x)$ and $h(x)$, respectively?

A graph of the discrete function, k



Range of a Continuous Function

Based on the graph, what are the ranges of $g(x)$ and $h(x)$, respectively?



Application of a Continuous Function

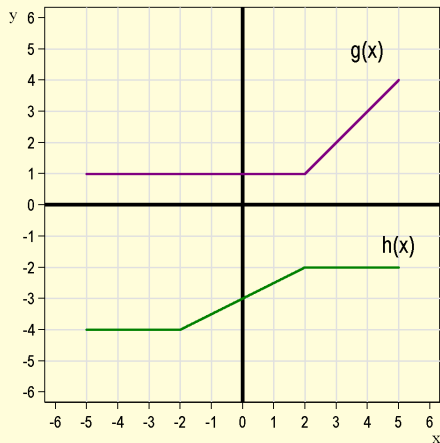
Calculate: $2h(0) - g(4)$



Application of a Continuous Function

Let $f(x) = g(x) + h(x)$

Draw $f(x)$ on the diagram



A More Mathematical Definition of a Function

- ▶ A **function** is a **relation or mapping** for which each value from the **domain** is associated with **exactly one value** from the range.

A More Mathematical Definition of a Function

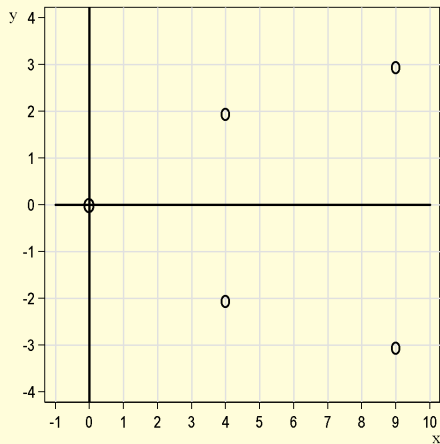
- ▶ A **function** is a **relation or mapping** for which each value from the **domain** is associated with **exactly one value** from the range.
- ▶ According to the definition, is g a function?

$$\begin{Bmatrix} 0 \\ 4 \\ 9 \end{Bmatrix} \rightarrow g \rightarrow \begin{Bmatrix} 0 \\ -2 \text{ or } 2 \\ -3 \text{ or } 3 \end{Bmatrix}$$

The Vertical Line Test

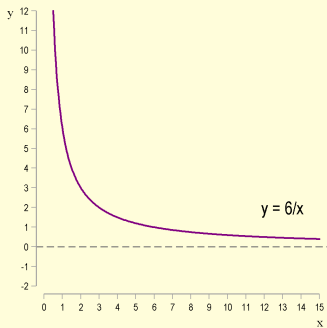
$$\begin{Bmatrix} 0 \\ 4 \\ 9 \end{Bmatrix} \rightarrow g \rightarrow \begin{Bmatrix} 0 \\ -2 \text{ or } 2 \\ -3 \text{ or } 3 \end{Bmatrix}$$

A graph of the Discrete Function g



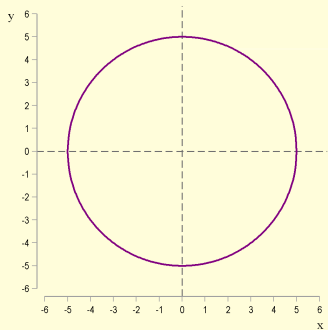
The Vertical Line Test

Is this expression a function?



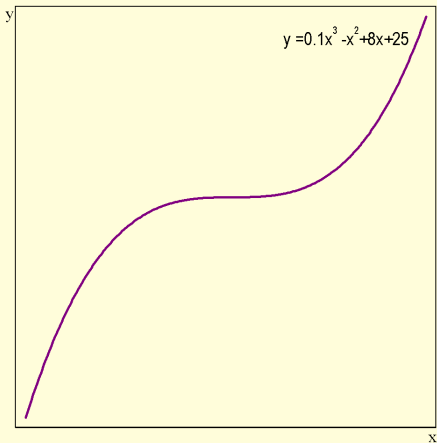
The Vertical Line Test

Is this a function?



The Vertical Line Test

Is this expression a function?



Find the domain of:

$$f(x) = \frac{1}{x+5}$$

$$f(x) = \sqrt{x-3}$$

Compositions of Functions

- Functions can be combined to produce interesting insights.

Compositions of Functions

- ▶ Functions can be combined to produce interesting insights.
- ▶ Let's take a closer look at the Laffer Curve.
- ▶ The Laffer Curve demonstrates that if income taxes are too high then government revenue falls.

Compositions of Functions: a simple Laffer Curve

Government income, g depends on hours worked h ^{- given}
and the income tax rate, t .

Compositions of Functions: a simple Laffer Curve

Government income, g depends on hours worked, h ,
and the income tax rate, t .

$$\underline{g(t)} = ht = h \cdot t$$

Compositions of Functions: a simple Laffer Curve

Government income, g depends on hours worked, h ,
and the income tax rate, t .

$$g(t) = \underline{ht}$$

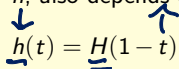
The insight from the Laffer Curve is that hours worked,
 h , also depends on the tax rate, t .

Compositions of Functions: a simple Laffer Curve

Government income, g depends on hours worked, h ,
and the income tax rate, t .

$$g(t) = ht$$

The insight from the Laffer Curve is that hours worked,
 h , also depends on the tax rate, t .



The diagram shows the equation $h(t) = H(1 - t)$ with blue annotations. A downward arrow points from the word 'depends' in the text above to the variable h in the equation. An upward arrow points from the variable t in the equation back to the word 'depends' in the text above. The variable h is underlined with a blue line, and the variable H is underlined with a blue double line.

$$\underline{h}(t) = \underline{H}(1 - t)$$

where H is some positive real number.

Compositions of Functions: a simple Laffer Curve

Next, let's Combine these two functions into one:

$$\begin{cases} g(t) = ht & \text{gov't rev.} \\ h(t) = H(1-t) & \text{hours worked} \end{cases}$$
$$h = h(t)$$

$$g(t) = h(t)t$$

$$g(t) = H(1-t)t$$

Compositions of Functions: a simple Laffer Curve

$$g(t) = (1 - t)(t)H$$

Assume that $H = 16$ and use Excel to find the tax rate, t^* , that maximizes government revenue.

Compositions of Functions: Example

$$\checkmark f(x) = 2x - 4$$

$$\checkmark g(x) = x^2$$

1. Find $f(5) + g(6)$

$$= f(5) + f(6)$$

$$= 6 + 36$$

$$= 42$$

$$f(5) = 2(5) - 4$$

$$= 10 - 4$$

$$= 6$$

$$g(6) = 6^2$$

$$g(6) = 36$$

Compositions of Functions: Example

$$f(5) = 2(5) - 4 \\ = 6$$

2. Find $g(f(5))$

$$g(f(5)) = g(6)$$

$$g(6) = 6^2 = 36$$

$$g(f(5)) = \underline{36}$$

plug $f(x)$
into $g(x)$

$$f(x) = 2x - 4$$

$$g(x) = x^2$$

$$g(f(5)) = ?$$

first find

$$g(f(x)) = (2x - 4)^2$$

$$= (2(5) - 4)^2$$

$$= 6^2 = \underline{36}$$

Compositions of Functions: Example

$$f(x) = 2x - 4$$

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1. Find $f(g(5))$

Compositions of Functions: Example

$$f(x) = 2x - 4$$

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1. Find $f(2g(x))$

Profit as a Composite Function

TS is a firm that sells Q units of superconductors. The price received from the sale of a superconductor is $P(Q) = 100 - \frac{1}{3}Q$. The price to produce one superconductor is $C(Q) = 20 + \frac{1}{5}Q$. Transporting the superconductors across the Pacific Ocean costs \$2 per superconductor.

- A) Combine the above into a profit function, $\pi(Q)$, where,
$$\pi(Q) = PQ - C(Q)$$

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C) Use Excel to find the profit maximizing quantity, Q , from parts A and B.

Systems of Equations

Two Methods for solving a system of linear equations:

1. Substitution
2. Elimination (Linear Combination or Addition)

Substitution Method

Solve for one variable in one equation. Then use that expression to solve for a variable in another equation.

$$5x + 2y = 3$$

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Elimination Method

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Systems of Equations Problem 1

$$\begin{aligned}0.01r + 0.21s &= 0.042 \\ -0.25r + 0.55s &= -0.47\end{aligned}$$

Systems of Equations Problem 2

$$\frac{2}{x} + \frac{3}{y} = 4$$

$$\frac{3}{x} - \frac{2}{y} = 19$$

Systems of Equations Problem 3

$$3\sqrt{x} + 2\sqrt{y} = 2$$

$$2\sqrt{x} - 3\sqrt{y} = \frac{1}{4}$$

Write the following as a linear system of two equations and solve the system:

Five meals and 20 drinks cost \$900. However, two meals and three drinks cost \$210. What is the price of each?

Write the following as a linear system of two equations and solve the system:

Rogers offers two subscription service plans: a and b . Each subscription a has a profit of \$600 while each subscription b has a profit of \$400. There are always 50% more b subscriptions than a subscriptions. If Rogers made a profit of \$520,000, how many of each subscription service were sold?

Compositions of Functions: Example

$$f(g(x)) \neq g(f(x))$$

in general

$$f(x) = 2x - 4$$

$$g(x) = x^2$$

3. Find $f(g(5))$

$$g(5) = 5^2$$
$$= 25$$

$$f(25) = 2(25) - 4$$
$$= 50 - 4$$
$$= 46$$

$$f(g(x)) = 2(x^2) - 4$$
$$= 2x^2 - 4$$
$$= 2(5^2) - 4$$
$$= 50 - 4$$
$$= 46$$

Compositions of Functions: Example

unknown
variable
↓

$$\begin{aligned} f(x) &= 2x - 4 \\ g(x) &= x^2 \end{aligned}$$

4. Find $f(2g(x))$

$$2g(x) = 2x^2$$

$$\begin{aligned} f(2g(x)) &= 2(2x^2) - 4 \\ &= 4x^2 - 4 \end{aligned}$$

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A) Combine the above into a profit function, $\pi(Q)$, where,

$$\pi(Q) = PQ - C(Q)$$

(profit = revenue - cost
revenue = (price)(quantity)

$$\pi(Q) = P(Q)Q - C(Q)$$

$$\pi(Q) = (100 - (\frac{1}{3})Q)Q - (\frac{1}{5})Q$$

$$\pi(Q) = 100Q - (\frac{1}{3})Q^2 - (\frac{1}{5})Q$$

$$\frac{1}{5} = 0.2 \quad = 99.8Q - \frac{Q^2}{3}$$

Profit as a Composite Function

TS is a firm that sells Q units of superconductors. The price received from the sale of a superconductor is $P(Q) = 100 - \frac{1}{3}Q$. The cost to produce superconductors is $C(Q) = \frac{1}{5}Q$.

- B) How would the profit function change if transporting each superconductor, V , cost an additional \$0.20?

$$\begin{aligned} \underline{V(Q)} &= 0.2Q \\ &= \left(\frac{1}{5}\right)Q \end{aligned}$$

$$\begin{aligned} \pi(Q) &= P(Q)Q - C(Q) - V(Q) \\ &= \left(100 - \frac{1}{3}Q\right)Q - \left(\frac{1}{5}\right)Q - \left(\frac{1}{5}\right)Q \\ &= 100Q - \frac{Q^2}{3} - \left(\frac{2}{5}\right)Q \\ &= 99.6Q - \frac{Q^2}{3} \end{aligned}$$

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$$5x + 2y = 3 \quad \Rightarrow \quad 2y = 3 - 5x$$

$$2x + 3y = -1$$

$$y = \frac{3 - 5x}{2}$$

$$2x + 3\left(\frac{3 - 5x}{2}\right) = -1$$

$$4x + 9 - 15x = -2$$

$$-11x = -11$$

$$x = 1$$

$$5x + 2y = 3$$

$$5(1) + 2y = 3$$

$$2y = 3 - 5$$

$$y = -1$$

Elimination Method

Add or subtract a multiple of one equation with another equation.

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Add or subtract a multiple of one equation with another equation.

$$2x + 3y = 3$$

$$\begin{pmatrix} 6x + 6y = -1 \end{pmatrix} \Rightarrow$$

$$4x + 6y = 6$$

$$- (6x + 6y = -1)$$

$$-2x + 0 = 7$$

$$x = -3.5$$

$$\text{if } x = -3.5$$

$$2(-3.5) + 3y = 3$$

$$-7 + 3y = 3$$

$$3y = 10$$

$$y = \frac{10}{3}$$

Systems of Equations Problem 1

Try yourself

$$0.01r + 0.21s = 0.042$$

$$-0.25r + 0.55s = -0.47$$

$$.25r + 5.25s = 1.05$$

$$+ -0.25r + 0.55s = -0.47$$

$$0 + 5.80s = 0.58$$

$$s = 0.1$$

$$0.01r + 0.21(0.1) = 0.042$$

$$r + 21(.1) = 4.2$$

$$r + 2.1 = 4.2$$

$$r = 2.1$$

Systems of Equations Problem 2

$$\frac{2}{x} + \frac{3}{y} = 4$$

$$\frac{3}{x} - \frac{2}{y} = 19$$

Trick:

$$\text{Let } \frac{1}{x} = v = \frac{2v + 3w = 4}{\frac{1}{y} = w = \frac{3v - 2w = 19}$$

$$3\left(\frac{4 - 3w}{2}\right) - 2w = 19 \quad \leftarrow \quad \begin{matrix} 2v = 4 - 3w \\ v = \frac{4 - 3w}{2} \end{matrix}$$

$$12 - 9w - 4w = 38$$

$$-13w = 26$$

$$w = -2$$

$$\rightarrow 2v + 3(-2) = 4$$

$$2v + (-6) = 4$$

$$v = 5$$

$$w = \frac{1}{y}$$

$$y = \frac{1}{w} = -$$

$$y = -2$$

$$v = \frac{1}{x}$$

$$x = \frac{1}{v}$$

$$x = \frac{1}{5}$$

Systems of Equations Problem 3

$$3\sqrt{x} + 2\sqrt{y} = 2$$

$$2\sqrt{x} - 3\sqrt{y} = \frac{1}{4}$$

$$6\sqrt{x} + 4\sqrt{y} = 4$$

$$6\sqrt{x} - 9\sqrt{y} = \frac{3}{4}$$

$$6\sqrt{x} + 4\sqrt{y} = 4$$

$$-(6\sqrt{x} - 9\sqrt{y} = \frac{3}{4})$$

$$\frac{16}{4} - \frac{3}{4}$$

$$0 + 13\sqrt{y} = \frac{13}{4}$$

$$\sqrt{y} = \frac{1}{4}$$

$$y = \frac{1}{16}$$

$$3\sqrt{x} + 2\sqrt{\frac{1}{16}} = 2$$

$$3\sqrt{x} + 2(\frac{1}{4}) = 2$$

$$3\sqrt{x} = 1.5$$

$$\sqrt{x} = \frac{1}{2}$$

$$x = 1/4$$

Write the following as a linear system of two equations and solve the system:

Five meals and 20 drinks cost \$900. However, two meals and three drinks cost \$210. What is the price of each?

$$\textcircled{1} 5m + 20d = 900$$

$$\textcircled{2} 2m + 3d = 210 \rightarrow 3d = 210 - 2m$$

$$d = \frac{210 - 2m}{3}$$

$$5m + 20\left(\frac{210 - 2m}{3}\right) = 900$$

$$15m + 4200 - 40m = 2700$$

$$4200 - 2700 = 40m - 15m$$

$$1500 = 25m$$

$$60 = m$$

$$2(60) + 3d = 210$$

$$120 + 3d = 210$$

$$3d = 90$$

$$d = 30$$

proof

$$5m + 20d = 900$$

$$5(60) + 20(30) =$$

$$300 + 600 = 900 \checkmark$$

Write the following as a linear system of two equations and solve the system:

Rogers offers two subscription service plans: a and b . Each subscription a has a profit of \$600 while each subscription b has a profit of \$400. There are always 50% more b subscriptions than a subscriptions. If Rogers has a profit of \$520,000, how many of each subscription service are sold?

$$540 \quad 600a + 400b = 540,000$$

$$1.5a = b$$

$$600a + 400(1.5a) = 540,000$$

$$600a + 600a = 540,000$$

$$1200a = 540,000$$

$$a = 450$$

$$b = 1.5a$$

$$b = 450(1.5)$$

$$b = 675$$

Proof

$$600(450) + 400(675) = \cancel{5}40,000$$