

**SMARTPHONE PRECISE POSITIONING IN URBAN
ENVIRONMENTS USING INTERNAL GNSS AND IMU
SENSORS**

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Abstract

The last decade has seen the modernization of Global Navigation Satellite Systems (GNSS) and the proliferation of multi-constellation, multi-frequency GNSS chipsets. This progress has boosted the surge of GNSS-enabled handsets and revolutionized receiver industries and Location-Based Services (LBS) for mass-market applications. These applications, including autonomous driving, social networking, health tracking, and personal/property security, increasingly demand higher levels of user positioning accuracy and reliability. In this context, a fundamental push for smartphone positioning came in 2016 after Google released the Android 7.0 platform, allowing users and developers the capability to access GNSS raw measurements, including carrier-phase observations, catalyzing the progress of smartphone precise positioning using Real-Time Kinematic (RTK) and Precise Point Positioning (PPP) technologies. However, challenges such as high received signal noise, limited multipath suppression capabilities, and frequent signal losses are prevalent in smartphone-grade GNSS receivers and antennas. These factors notably degrade the performance of smartphone positioning for precise applications.

In response to these challenges, this dissertation is dedicated to addressing these smartphone positioning limitations and aims to provide accurate, continuous, and resilient Positioning, Navigation, and Timing (PNT) solutions with a developed smartphone processing software. Given that ionospheric refraction at GNSS signals is a significant error source impacting the accuracy of, e.g., PPP solutions, this research commences with an investigation of smartphone PPP performance with external ionospheric constraints. To utilize smartphone observations effectively, a novel PPP algorithm tailored for smartphone navi-

gation is introduced. Further, this algorithm utilizes GPS, Galileo, GLONASS, and BeiDou single- and dual-frequency observations, while also integrating pseudorange-only measurements in scenarios where carrier-phase measurements are missing. Moreover, to overcome limitations in both the PPP and RTK techniques, a novel PPP/RTK hybridization algorithm with smartphone inertial measurement unit (IMU) integration is proposed. Validated through a series of vehicle experiments conducted in realistic driving environments, the enhanced GNSS/IMU smartphone positioning algorithm was evaluated across various urban multipath scenarios, including open-sky highways, vegetation roads, suburban roads, and underpasses. The results demonstrate a substantial improvement in reducing horizontal root mean square error (rmse), reducing it from approximately 10 m to less than 1.5 m compared to traditional Single Point Positioning (SPP) solutions, showing a noteworthy advancement in smartphone positioning capabilities.

Additionally, this research explores the utilization of the available Galileo HAS corrections for smartphone navigation. To fully leverage all available smartphone measurements and orbit and clock information directly from satellites, this research combines the corrections with broadcast ephemerides for smartphone PPP processing (HASandBRDC PPP). Investigated through vehicle experiments, the results show that the proposed HASandBRDC PPP solutions are highly comparable to PPP solutions utilizing Centre National d'Etudes Spatiales (CNES) ultra-rapid products with four GNSS constellations, achieving approximately 1.5 m of positioning performance. This finding underscores the great potential of real-time smartphone applications in pelagic and mountain areas with limited Internet connectivity.

Keywords: GNSS; Smartphone; PPP; PPP/IMU; Ionospheric constraints; Pseudorange enhancement; PPP/RTK Hybridization; Galileo High Accuracy Service; Urban environments

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List of Acronyms

ANTEX	ANTenna EXchange
APC	Antenna Phase Centre
AR	Ambiguity Resolution
ARKF	Adaptive Robust Kalman Filter
BDS	BeiDou Navigation Satellite System
BIE	Best Integer Equivariant
CAS	Chinese Academy of Sciences
CDF	Cumulative Distribution Function
CDMA	Code Division Multiple Access
CNES	Centre National d'Etudes Spatiales
CODE	Center for Orbit Determination in Europe
DCB	Differential Code Bias
DCM	Decoupled Clock Model
DPB	Differential Phase Bias
ECEF	Earth-Centred Earth-Fixed
EKF	Extended Kalman Filter
EUSPA	EU Agency for the Space Programme
FCB	Fractional Cycle Biases
FDMA	Frequency Division Multiple Access
GFZ	GeoForschungsZentrum
GIM	Global Ionospheric Maps
GNSS	Global Navigation Satellite Systems

GPS	Global Positioning System
GREC	GPS, GLONASS, Galileo, and BeiDou
HAS	High Accuracy Service
IC	Ionospheric Constraints
IDD	Internet Data Distribution
IGS	International GNSS Service
IMU	inertial measurement unit
INS	inertial navigation system
IRC	Integer Recovery Clock
LAMBDA	Least-squares AMBiguity Decorrelation Adjustment
LBS	Location-Based Services
MEMS	micro-electromechanical system
NavIC	Navigation with Indian Constellation
NRCan	Natural Resources Canada
OSB	Observable-specific Signal Biases
OSR	Observation State Representation
PCO	Phase Centre Offset
PCV	Phase Centre Variation
PDF	Probability Density Function
PDOP	Position Dilution Of Precision
PNT	Positioning, Navigation, and Timing
PPK	Post-Processed Kinematic
PPP	Precise Point Positioning
PPS	Precise Positioning Service
PSD	Power Spectral Density
QZSS	Quasi-Zenith Satellite System
RIM	Regional Ionospheric Maps
RTK	Real-Time Kinematic

rms	root mean square error
SHAKF	Sage-Husa Adaptive Extended Kalman Filter
SISRE	Signal-In-Space Ranging Error
SLS	Sequential Least Squares
SPP	Single Point Positioning
SPS	Standard Positioning Service
SSR	State Space Representation
TGD	Total Group Delay
UDUC	Undifferenced and Uncombined
ZHD	zenith hydrostatic delay
ZWD	zenith wet delay

Chapter 1

Introduction

Thanks to the maturity of Global Navigation Satellite Systems (GNSS) and the evolution of microelectronic technologies, GNSS-enabled smartphones and their navigation capability have gained intensive development over the last decade. These pocket-sized and affordable smart devices have become an inseparable part of our daily lives, allowing people to remain connected indefinitely and access information worldwide. As satellite navigation technology progresses, furnishing smartphone users with accurate, continuous, and reliable Positioning, Navigation, and Timing (PNT) solutions is the central focus of this dissertation.

1.1 The evolution of GNSS infrastructure and smartphones

The most renowned GNSS is the Global Positioning System (GPS), which stands as the first global satellite navigation system. GPS originated from the collaborative efforts of the United States Department of Defense. The genesis of GPS traces back to the early 1960s when the U.S. Navy initiated experiments with satellite navigation systems. The impetus behind these experiments stemmed from the military's imperative for a dependable and precise positioning system to enhance navigation capabilities across ships, aircraft, and terrestrial vehicles for military applications. The first experimental GPS satellite, named Navstar 1, was launched in 1978. Over the next decade, the U.S. Air Force orchestrated the launch of numerous satellites into orbit to bolster the GPS constellation. On April 27, 1995, the U.S. Air Force Space Command declared the attainment of full operational capability for the GPS constellation (NASA, 2024; GPS.gov, 2024). This milestone underscored the

system’s fulfillment of all requisite criteria, with 24 operational satellites occupying their designated orbital positions. This achievement marked the system’s ability to provide both the military Precise Positioning Service (PPS) and the civil Standard Positioning Service (SPS) performance standard (Annex, 1995).

GLONASS was conceived by the Soviet Union during the early 1970s and is now managed by Russia. The first GLONASS satellite was launched in 1982, marking the commencement of the system’s deployment. However, due to financial constraints and technical obstacles, progress was sluggish during the initial phase. Despite these challenges, GLONASS gradually attained operational status with a limited number of satellites in the 1990s (Urlichich et al., 2010). In 2010, GLONASS’s Initial Operational Capability (IOC) was achieved with 18 satellites, followed by the completion of a full 24-satellite constellation in 2011, catering to both civilian and military users. Unlike other satellite systems, initial GLONASS satellites use Frequency Division Multiple Access (FDMA) rather than Code Division Multiple Access (CDMA) to transmit signals. With the modernization of GLONASS, several GLONASS-M and GLONASS-K satellites began transmitting CDMA signals on the L3 band in 2011 (Urlichich et al., 2011).

Galileo is the European Union’s global satellite navigation system, designed to deliver precise PNT services across the globe. Its development commenced in the late 1990s in response to the increasing reliance on the GPS. The project received funding from the European Commission and the European Space Agency, and the first two Galileo satellites, named GIOVE-A and GIOVE-B, were launched in 2005 and 2008, respectively. Subsequent successful testing led to Galileo attaining its Initial Services status in December 2016 (ESA, 2016). Furthermore, Galileo High Accuracy Service (HAS) officially announced its initial service in January 2023, intending to provide Precise Point Positioning (PPP)-based services to users worldwide (EUSPA, 2023).

BeiDou, also known as the BeiDou Navigation Satellite System (BDS), is China’s indigenous global satellite navigation system. Its development commenced in the 1980s as part of China’s endeavour to establish an autonomous satellite navigation system, reducing

reliance on GPS. Initially, BeiDou served as a regional navigation system, offering PNT services primarily for users within China and neighbouring regions. The first generation of BeiDou satellites, BeiDou-1, was launched in 2000, followed by the deployment of the second generation, BeiDou-2, starting in 2007. By 2012, BeiDou-2 achieved operational capability, delivering navigation services to users across the Asia-Pacific region. Subsequently, China outlined plans to expand BeiDou into a global navigation system, referred to as BeiDou-3, aimed at providing global coverage through satellites in geostationary orbits, inclined geosynchronous orbits, and medium Earth orbits. The BeiDou-3 constellation was fully operationalized in June 2020 (China Satellite Navigation Office, 2019).

Regional satellite navigation systems are crucial for extending the reach and functionality of GNSS within specific geographic areas. These systems, typically developed and operated by individual countries or consortia, are designed to meet the navigation requirements of regions where coverage or accuracy provided by GNSS may be inadequate. Examples of regional satellite navigation systems include Japan's Quasi-Zenith Satellite System (QZSS) and India's Navigation with Indian Constellation (NavIC). Figure 1.1 shows the GNSS satellite frequency spectrum for each constellation.

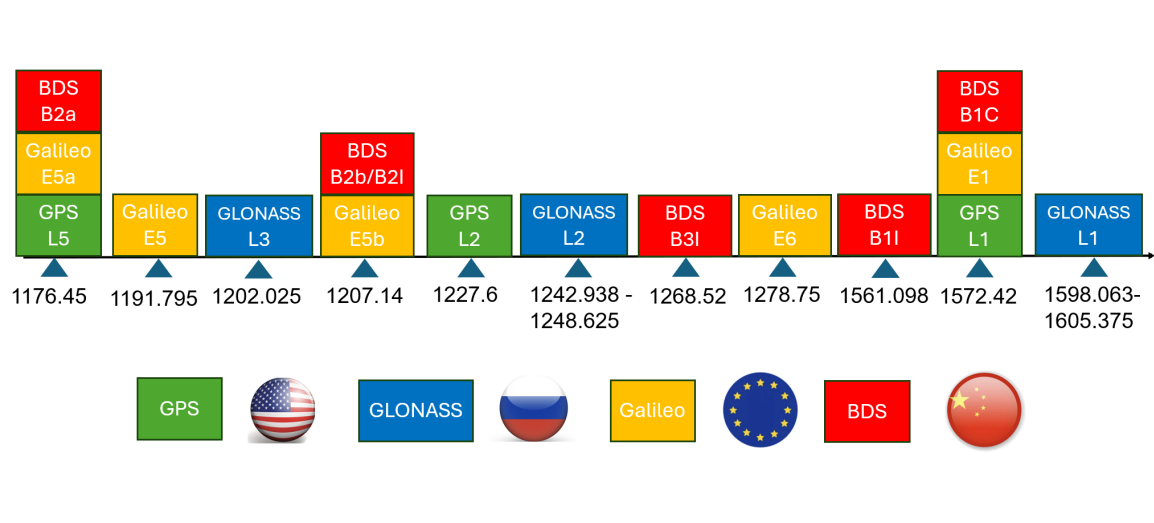


Figure 1.1: GNSS satellite signal frequency spectrum (unit: MHz)

The evolution of GNSS and the accessibility of multi-constellation, multi-frequency signals have propelled the growth of GNSS-enabled handsets within the GNSS downstream space application market. Progress in semiconductor technology has also facilitated the creation of smaller, more energy-efficient GNSS receivers. This advancement has allowed GNSS chipset manufacturers to prioritize miniaturization and cost reduction, thereby enabling the integration of GNSS capabilities into smartphones without substantial increases in size or cost. According to the EU Agency for the Space Programme (EUSPA) GNSS market report, smartphones continued to dominate the market, comprising approximately 80% of the roughly 1.5 billion unit shipments in 2022, which was significantly higher than other GNSS consumer devices. In 2023, the Consumer Solutions, Tourism, and Health segment accounted for 90% of the installed base of GNSS devices, primarily due to the substantial sales of smartphones and wearables. The global GNSS downstream market revenues from both device and service sales amounted to 260 billion euros, and revenues from smartphone apps represented a significant component, comprising over 60% of added-value services and over 40% of the total revenues (EUSPA, 2024). Recently, Bisnath and Aggrey (2024) explored the current state and limitations associated with GNSS smartphone positioning, highlighting that precise positioning is essential for smartphone-based applications. These figures underscore the extensive integration of GNSS technology into smartphones and highlight the significant economic influence of the smartphone market.

Figure 1.2 illustrates the evolutionary path of smartphones over the past few decades. The first commercially available phone with GPS support was the Benefon Esc! released in 1999. Manufactured by the Finnish company Benefon, this mobile phone was among the first to incorporate GPS technology for PNT services. Despite GPS technology being relatively new in consumer devices at that time, the Benefon Esc! played a pivotal role in paving the way for the integration of GPS capabilities into mainstream mobile phones in the subsequent years. In 2005, Samsung introduced the SCH-S310, the first phone equipped with a three-axis accelerometer, enabling users to dial phone numbers by gestures in the air. Subsequently, the first commercially available phone to operate on the Android operating

system was the HTC Dream, also known as the T-Mobile G1, which debuted in October 2008. In this context, a fundamental push for smartphone positioning came in 2016 after Google released the Android 7.0 platform (Malkos, 2016), allowing users and developers the capability to access GNSS raw measurements, including carrier-phase observations, catalyzing the progress of smartphone precise positioning using Real-Time Kinematic (RTK) and PPP technologies (Gill et al., 2017; Odolinski and Teunissen, 2019). In response to mass-market demands, Broadcom launched the first dual-frequency, multi-constellation GNSS chipset BCM47755 in 2017, ushering in a new era in smartphone dual-frequency GNSS processing to manage the ionospheric delay. In 2018, the first dual-frequency smartphone MI 8 was released with a BCM47755 chip (Elmezayen and El-Rabbany, 2019). Presently, smartphones integrated with multi-frequency, multi-constellation GNSS chipsets are increasingly prevalent in the market, reflecting the evolution toward enhanced positioning capabilities.

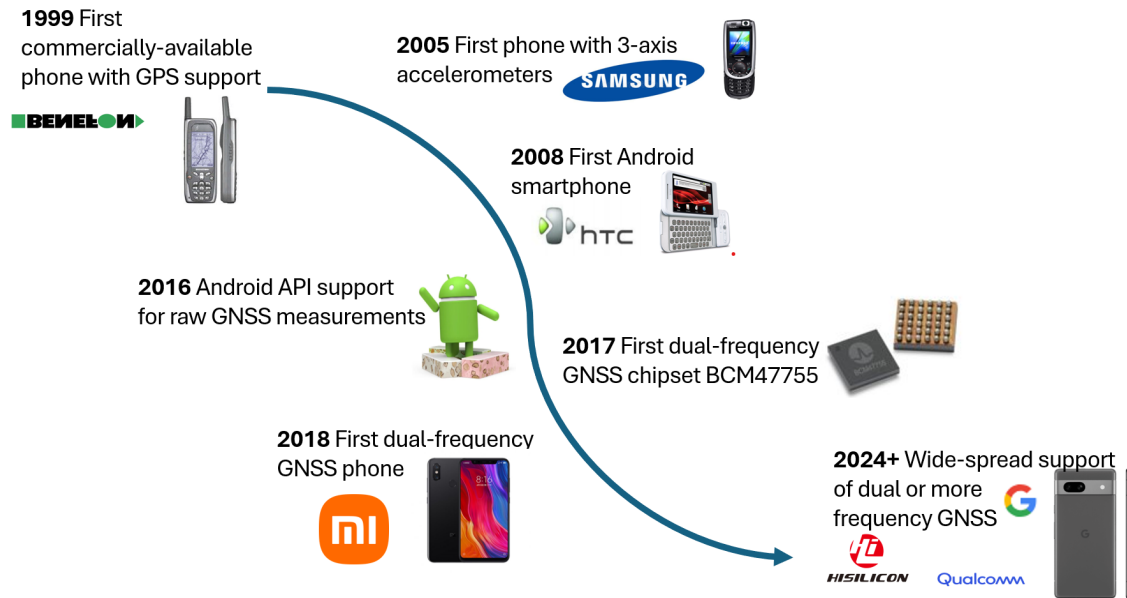


Figure 1.2: Evolutionary path of smartphones

GNSS-enabled smartphones have revolutionized our daily lives by providing access to a wide range of location-based applications and services. The potential applications that enrich our daily experiences are illustrated in Figure 1.3:

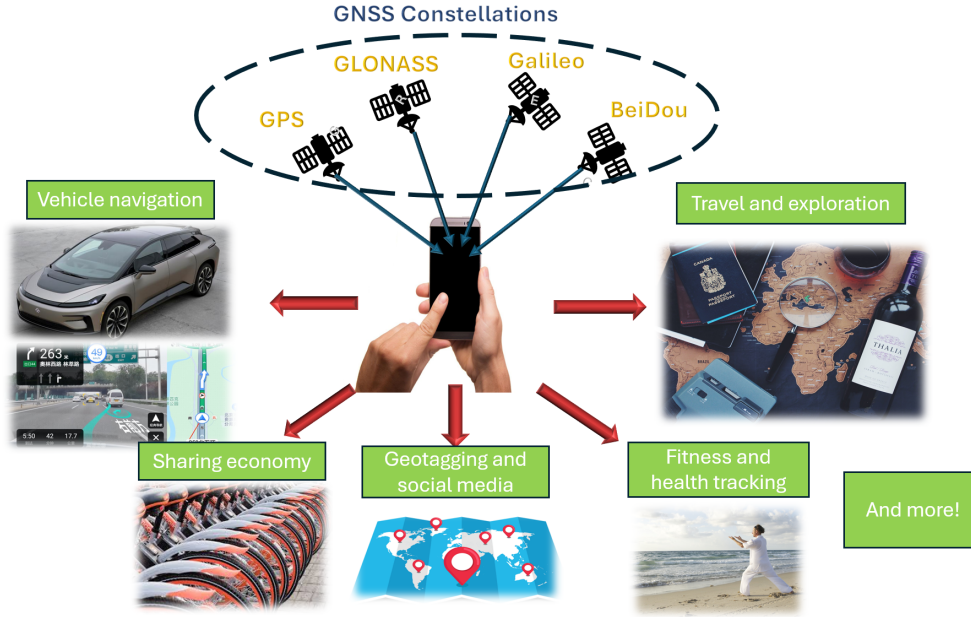


Figure 1.3: Smartphone applications for daily lives

1. **Vehicle navigation:** GNSS-enabled smartphones serve as portable navigation devices, offering turn-by-turn directions, real-time traffic updates, and route optimization to help users navigate unfamiliar environments.

Common smartphone applications: Google Maps, Apple Maps, Waze, Baidu map, Amap, Uber, Lyft, etc.

2. **Sharing economy:** The sharing economy is a peer-to-peer (P2P) economic system, smartphones serve as powerful tools for connecting users, facilitating transactions, and enabling access to shared resources such as bike-sharing, car-sharing, accommodation-sharing, etc.

Common smartphone applications: Hello bike, Zipcar, Getaround, etc.

3. **Geotagging and social media:** GNSS-enabled smartphones enable geotagging, allowing users to attach location information to photos, videos, and social media posts.

Common smartphone applications: Instagram, Facebook, Twitter, Wechat, etc.

4. **Fitness and health tracking:** Users can use their phones to monitor their outdoor activities such as walking, running, cycling, and hiking.

Common smartphone applications: Nike Run Club, MapMyFitness, Strava, etc.

5. **Travel and exploration:** GNSS-enabled smartphones facilitate travel and exploration by providing access to travel planning tools, offline maps, location-based recommendations, and hotel rentals.

Common smartphone applications: Airbnb, Booking, TripAdvisor, Meituan, etc.

1.2 Current limitations and challenges

Smartphone positioning techniques and their navigation capability based on GNSS have seen intensive development in the last decade. The proliferation of GNSS-enabled smartphones and wearables has boosted the GNSS downstream industries, around which a growing number of applications require greater positioning accuracy, resilience, and availability. However, the commercialization of the smartphone industry has produced the low-cost nature of GNSS chips and antennas on mobile devices, compromising the quality of observation data compared to that of geodetic receivers, which is challenging for high-precision positioning services.

Compared to a geodetic antenna, the omnidirectional polarized antenna used in smartphones has the advantage to receive reflected signals from all directions, but the tradeoff is that smartphone GNSS measurements are susceptible to significant multipath effects (Darugna et al., 2021; Pathak et al., 2003). It is well recognized that the smartphone GNSS antenna is the major restriction that limits smartphone positioning accuracy, and great efforts have been made to characterize smartphone GNSS observations and mitigate the impact of multipath and unexpected measurement noise. Liu et al. (2019b) characterized the smartphone observation errors with respect to the geodetic receiver through a short baseline and single-difference approach, and the authors found that unexpected measurement noise can reach 10-30 m in smartphone pseudorange measurements. Similar conclusions can be also drawn from recent studies (Zhang et al., 2018b; Li and Geng, 2019; Paziewski,

2020a). Continuing this research, it has been suggested that a carrier-to-noise density ratio (C/N_0)-based weighting scheme is superior to the traditional elevation-based stochastic weighting scheme for smartphone processing (Shinghal and Bisnath, 2021; Banville et al., 2019; Zhang et al., 2018b), as the noisy smartphone observations are prone to lower C/N_0 (Zhang et al., 2018a; Paziewski et al., 2019), the detailed explanations and determination of C/N_0 weighting scheme are discussed in Chapter 3. Recently, smartphone carrier-phase ambiguities have been successfully resolved for both PPP and RTK by using an external geodetic antenna (Geng et al., 2018; Wen et al., 2020; Geng and Li, 2019). This development suggests that achieving cm-level positioning accuracy with smartphones is indeed possible.

However, connecting an external antenna to smartphones, especially in real-world scenarios like in-car navigation where users typically mount their phones on the vehicle dashboard, is not practical. Figure 1.4 illustrates the probability distribution of smartphone range errors at different locations. The results indicate that the standard deviation values of range errors are 5.1 m on the car roof and 5.3 m on the dashboard for pseudorange measurement on the first frequency (P1). For pseudorange measurement on the second frequency (P5), the standard deviation values are 2.2 m on the car roof and 2.4 m on the dashboard (Hu et al., 2023). These findings underscore the challenges associated with achieving precise smartphone positioning due to signal noise and multipath effects.

Additionally, GNSS chipsets in smartphones have limited signal-tracking capabilities. For instance, the Xiaomi Mi 8 smartphone, equipped with the BCM47755 chipset, can track pseudorange and carrier-phase measurements for GPS, GLONASS, Galileo, and BeiDou (GREC) constellations, with dual-frequency support for GPS L5 and Galileo E5a signals. Although this capability exceeds that of many other smartphone chipsets, which typically only support single-frequency and pseudorange measurements, it still falls behind the capabilities of geodetic and low-cost receivers. This discrepancy is illustrated in Table 1.1, which compares the signal-tracking performance of the Xiaomi Mi 8 with that of the low-cost Septentrio Mosaic X5 receiver.

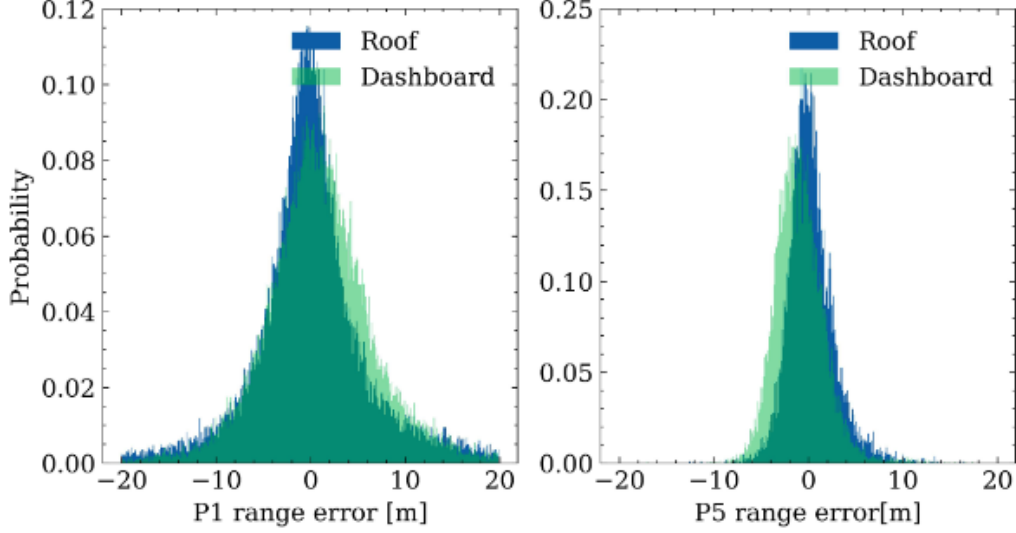


Figure 1.4: Smartphone range error distribution comparison (Hu et al., 2023)

Table 1.1: Signal tracking discrepancy between Xiaomi MI 8 and Septentrio Mosaic X5

Constellations	GPS					GLONASS				Galileo					BeiDou					
Signals	L1 C/A	L1P	L2 C/A	L2P	L5	L1 C/A	L2 C/A	L2P	L3	E1	E5a	E5b	E5	E6	B1I	B1C	B2a	B2I	B2b	B3I
Septentrio Mosaic X5	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Xiaomi MI 8	✓	×	✓	×	×	✓	×	×	×	✓	✓	×	×	×	✓	×	×	×	×	×

Figure 1.5 illustrates the number of observations, including pseudorange and carrier-phase measurements, tracked by the MI 8 smartphone and the Mosaic X5 receiver for GREC single- and dual-frequency processing during a kinematic test conducted on October 10, 2022. It is evident that the Mosaic X5 receiver can track a greater number of observations compared to the MI 8 smartphone. On average, the Mosaic X5 receiver tracks 83 observations, significantly more than the MI 8, which tracks only 56 observations. Additionally, both the Mosaic X5 and MI 8 experience a notable reduction in observations at times 20.44 hours and 20.65 hours, due to signal blockages caused by underpasses. These findings underscore the significant challenges faced by smartphone GNSS positioning, including limited observations, high signal noise, limited multipath suppression capabilities, and signal loss, which impact the accuracy of precise positioning solutions.

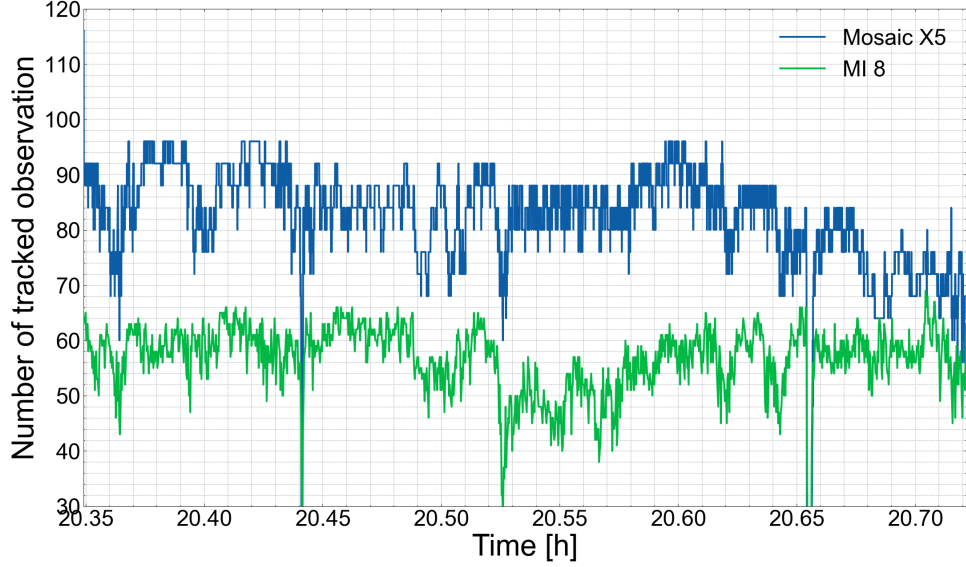


Figure 1.5: The number of observations (code and phase) comparison for Xiaomi MI 8 and Septentrio Mosaic X5 for GREC single- and dual-frequency processing

1.3 Research questions and contributions

To tackle the challenges outlined in Section 1.2, this research makes significant contributions across four key topics, each aiming at addressing specific research questions to enhance smartphone precise positioning.

Research topic 1: It is widely acknowledged that smartphone measurements are susceptible to low carrier-to-noise-density ratio (C/N_0) and significant multipath effects. These factors can have a notable impact on the accurate estimation of positioning errors, particularly in correctly estimating ionospheric delay, which is a major contributor to errors in GNSS positioning. Previous research has demonstrated that combining Ionospheric Constraints (IC) with Differential Code Bias (DCB) estimation is an effective approach to improve PPP solution initialization. However, none of these studies investigated the utility of PPP-IC with Global Ionospheric Maps (GIM) for the spectrum of next-generation GNSS hardware. Therefore, this research delves into this topic to address the following questions:

1. Whether the GIM ionospheric constraints help in terms of initialization for the dual-frequency smartphone, low-cost, and geodetic hardware PPP?

2. Whether the ionospheric constraints help under challenging environments for these three different grades of GNSS receivers?
3. How significant is the improvement of PPP-IC for the smartphone in comparison to low-cost and geodetic-grade receivers?
4. Do ionospheric constraints influence other PPP states, such as estimated receiver clock offset and ambiguities?

Research topic 2: Smartphone positioning is challenged by significant signal losses and insufficient satellite coverage, leading to suboptimal satellite geometry for GNSS positioning. Notably, previous studies have not fully utilized all available pseudorange and phase measurements for smartphone PPP processing. This is particularly evident when smartphone chipsets occasionally record only pseudorange measurements from satellites, while omitting corresponding phase measurements. Moreover, traditional quality control methods, such as post-fit residual rejection with fixed thresholds, are inadequate for meeting the dynamic positioning requirements of smartphones. Therefore, this research proposes an adaptive post-fit residual rejection method to mitigate the effects of noisy measurements. This method is combined with an enhanced PPP algorithm designed to utilize all available pseudorange and phase measurements for smartphone processing. The main novelties and contributions of topic 2 are to answer the following questions:

1. With regard to ultra-low-cost (smartphone-grade) hardware with an approximate cost of \$10, does pseudorange-enhanced PPP improve traditional PPP solutions and to what extent in harsh environments?
2. Assessed through the range error correlation analysis, which indicators among the satellite elevation, C/N_0 value, pre-fit residuals, and post-fit residuals can infer smartphone measurement quality?
3. How to form the adaptive post-fit residual rejection function according to the learning statistic? And what is the ‘best’ GNSS PPP solution with smartphone-grade GNSS chipset and antenna that can be attained in real-world driving environments?

Research topic 3: Despite notable advancements, the primary limitations hindering the use of smartphones for precise navigation applications are their low-quality, noisy measurements, and the degradation of positioning accuracy during GNSS outages. While the concept of integrating PPP or other GNSS techniques with inertial measurement unit (IMU) data is not new, it is worth noting that this research represents the first implementation of native inertial sensors from smartphones (such as Xiaomi MI 8 and Samsung Galaxy S21+) for tightly-coupled PPP/IMU integration, addressing not only benign environments but also realistic urban areas. While RTK may be considered a traditional positioning technique that requires a base station, it still offers a more reliable and accurate positioning solution during the initialization stage compared to PPP. This research introduces a novel PPP and RTK hybridization algorithm designed to leverage the strengths of both advanced positioning techniques in smartphone positioning. This is crucial for the automotive industry as it enables drivers to reach their destinations efficiently and safely while enhancing location-based services. Hence, the key novelties and contributions of topic 3 aim to address the following questions:

1. In realistic driving scenarios, how comparable is the ionospheric-constrained smartphone PPP solution to RTK performance?
2. How does tightly-coupled IMU integration behave on RTK and PPP modes under different multipath profiles? Does the inclusion of smartphone native IMU harm the PPP or RTK solutions?
3. How does the proposed hybridization algorithm perform in comparison to RTK and PPP? And what is the optimal solution that smartphone precise positioning can achieve now with GNSS and IMU measurements in real-world driving environments?

Research topic 4: While the pursuit of real-time precise positioning has been ongoing for decades, both RTK and traditional real-time PPP are constrained by the need for nearby infrastructures, such as a nearby base station for RTK or nearby Internet infrastructure for PPP, to receive real-time streams. This requirement is highly impractical and costly, partic-

ularly in rural and pelagic areas. However, the emergence of Galileo High Accuracy Service (HAS) offers a promising solution. Officially announced in January 2023, HAS provides precise orbit, clock, and bias information through the E6B signal globally and free of charge. This development eliminates the need for an Internet connection, making real-time PPP feasible without such limitations. In light of this, the present research explores smartphone HAS PPP processing and proposes an algorithm that leverages HAS corrections and broadcast ephemeris information for smartphone positioning. Therefore, the major novelties and contributions of topic 4 aim to answer the following questions:

1. In comparison to the initial phase of Galileo HAS in 2022, how do the available HAS corrections and associated broadcast ephemeris perform after the official HAS announcement in 2023, in terms of orbit and clock availability, Signal-In-Space Ranging Error (SISRE), and SISRE distribution?
2. For smartphone-based navigation, how does the HAS PPP solution perform compared to traditional Single Point Positioning (SPP) and PPP with broadcast ephemeris (BRDC PPP) with GPS and Galileo constellations?
3. In realistic driving conditions, how does the proposed smartphone PPP with the combination of HAS corrections and broadcast ephemeris (HASandBRDC) behave compared to the smartphone PPP solution using traditional Internet-based ultra-rapid products with a full four constellations? Additionally, how does the inclusion of the smartphone IMU measurements impact the overall performance?

Furthermore, this research has developed a high-precision mobile data positioning processing software named "JaGen", which integrates all the innovative algorithms and academic contributions of this research, as will be discussed in the next section. Additionally, many of the achievements and contributions of this research have been disseminated through publications in journals and conference proceedings. The following publications were produced during the preparation of this dissertation, and the first-authored and corresponding authored publications (major contributions) are listed below:

1. **Yi, D.**, Hu, J., and Bisnath, S. (2024). Improving PPP smartphone processing with adaptive quality control method in obstructed environments when carrier-phase measurements are missing. *GPS Solutions*, 28(1), 56.
2. **Yi, D.**, Bisnath, S., Naciri, N., de Blas, F. J., and Capua, R. (2023, September). Performance Assessment of Galileo High Accuracy Service (HAS) with Low-Cost GNSS/IMU Sensors in Urban Driving Environments. In *Proceedings of the 36th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2023)* (pp. 1403-1416).
3. **Yi, D.**, and Yang, S. (2023, September). Hybridization of Smartphone GNSS PPP/RTK with Native IMU in Realistic Driving Scenarios. In *Proceedings of the 36th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2023)* (pp. 2227-2241) (student paper award).
4. Hu, J., **Yi, D.**, and Bisnath, S. (2023). A Comprehensive Analysis of Smartphone GNSS Range Errors in Realistic Environments. *Sensors*, 23(3), 1631.
5. Yang, S., **Yi, D.**, Vana, S., and Bisnath, S. (2023). Resilient Smartphone Positioning using Native Sensors and PPP Augmentation. *NAVIGATION: Journal of the Institute of Navigation*, 70(2).
6. **Yi, D.**, Yang, S., and Bisnath, S. (2022). Native smartphone single-and dual-frequency GNSS-PPP/IMU solution in real-world driving scenarios. *Remote Sensing*, 14(14), 3286.
7. **Yi, D.**, Bisnath, S., Naciri, N., and Vana, S. (2021). Effects of ionospheric constraints in Precise Point Positioning processing of geodetic, low-cost and smartphone GNSS measurements. *Measurement*, 183, 109887.

with relevant co-authored publications:

1. Naciri, N., **Yi, D.**, Bisnath, S., de Blas, F. J., and Capua, R. (2023). Assessment of Galileo High Accuracy Service (HAS) test signals and preliminary positioning performance. *GPS solutions*, 27(2), 73.
2. Naciri, N., **Yi, D.**, Bisnath, S., de Blas, F. J., and Capua, R. (2022, September). Validation of a European High accuracy GNSS service for cadastral surveying applications. In *Proceedings of the 35th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2022)* (pp. 381-396).
3. Seepersad, G., Hu, J., Yang, S., **Yi, D.**, and Bisnath, S. (2022, September). Performance Assessment of Tightly Coupled Smartphone Sensors with Legacy and State Space Corrections. In *Proceedings of the 35th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2022)* (pp. 2235-2255).
4. Seepersad, G., Hu, J., Yang, S., **Yi, D.**, and Bisnath, S. (2021, September). Changing lanes with smartphones technology. In *Proceedings of the 34th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2021)* (pp. 3021-3036).

1.4 Dissertation outline

Chapter 2 provides an overview of advanced GNSS-based positioning techniques. It begins by discussing sources of GNSS errors that can be modelled or predicted using mathematical simulations and precise products. This chapter then compares SPP, RTK, and PPP for their observation equations, as well as their respective advantages and disadvantages. Additionally, Chapter 2 covers GNSS/IMU integration algorithms, including IMU initialization and alignment, IMU mechanization, and the tightly-coupled GNSS/IMU EKF model.

Chapter 3 is dedicated to addressing the research questions outlined in research topic 1. It focuses on utilizing the GIM as an external ionospheric constraint for PPP processing. This chapter introduces the PPP-IC model, which incorporates receiver DCB estimation, along with stochastic weighting schemes and processing settings tailored to various GNSS

hardware. Static and kinematic experiments are conducted using geodetic, low-cost, and smartphone GNSS receivers. The horizontal performance results with and without ionospheric constraints are compared across different scenarios, and the impacts of PPP-IC on PPP states are analyzed.

Chapter 4 delves into the analysis of the research questions raised by research topic 2. It introduces an enhanced smartphone PPP model designed to utilize all smartphone observations in both single- and dual-frequency processing. This algorithm includes incorporating pseudorange-only measurements, which are often disregarded in traditional PPP processing methods. Additionally, this chapter discusses the implementation of an adaptive post-fit residual threshold method aimed at effectively filtering out noisy measurements. Chapter 4 further explores these algorithms through ten kinematic vehicle experiments, presenting detailed insights into their impacts on positioning accuracy and performance.

Chapter 5 focuses on solving the research questions raised by research topic 3. It centres on evaluating the discrepancy in performance between RTK and PPP for smartphone positioning. This chapter introduces a novel PPP/RTK hybridization algorithm aimed at enhancing smartphone positioning accuracy. Additionally, it discusses the integration of native IMU raw measurements with PPP to provide robust and accurate solutions, particularly in challenging environments. This chapter concludes with the validation of these algorithms through realistic driving tests.

Chapter 6 aims to investigate the research questions posed by research topic 4. It begins with an exploration of the recent advancements in Galileo HAS and its implications for real-time PPP. Subsequently, this chapter conducts a thorough evaluation of HAS and assesses the errors associated with broadcast products. To leverage the satellite-disseminated information and smartphone observations effectively, a new algorithm is proposed. This algorithm is designed to utilize HAS and broadcast products for processing the GNSS signals from four constellations (GREC) for smartphone PPP and PPP/IMU integration. The algorithm's performance is evaluated across various kinematic environments featuring multiple multipath profiles.

Chapter 7 offers a summary of the research findings and the importance of the research's outcomes. Additionally, it provides recommendations for future work, highlighting potential areas for further exploration and improvement in the field of smartphone positioning and navigation.

Chapter 2

Advanced GNSS-based positioning techniques

Navigation solutions from smartphones and other smart devices, e.g., smart watches, equipped with GNSS and IMU sensors have become an integral part of our daily lives. This progress has, in turn, significantly boosted smartphone applications, such as augmented reality, social networking, gaming, mobile mapping, autonomous driving, etc. To address the research questions posed in Chapter 1, this chapter provides an overview of GNSS/IMU positioning techniques, processing software, as well as measurement campaigns for smartphone applications.

2.1 GNSS error sources

Multi-GNSS satellite positioning techniques require considering numerous error sources from satellites, receivers, and signal propagation. These errors can be categorized as follows: 1) errors that can be accurately predicted or modelled, and 2) errors that require estimation.

2.1.1 GNSS modelled errors

This section provides a brief overview of the GNSS errors that can be predicted or modelled using mathematical models or precise products such as SP3 and CLK files (Gurtner and Estey, 2007; Hilla, 2016). These errors are referred to as GNSS modelled errors.

Satellite orbit and clock errors

Satellite orbit errors arise from differences between the calculated orbit, obtained through the Lagrange or Chebyshev interpolation using broadcast or precise ephemeris, and the actual positions of a satellite. These discrepancies are influenced by factors like the distribution of ground tracking stations, orbit determination models, data processing methods, etc. Presently, the International GNSS Service (IGS) offers real-time satellite broadcast and ultra-rapid (predicted half), along with post-processed ultra-rapid (observed half), rapid, and final precise products (CDDIS, 2024). Typically, broadcast products offer an orbit accuracy of 100 cm, while ultra-rapid (predicted half) products can achieve accuracies of 5 cm in real-time. Currently, the best final precise orbit products can achieve accuracies of 2.5 cm, substantially reducing the impact of orbit errors on precise positioning (IGS, 2024).

Despite the presence of highly stable atomic clocks onboard satellites, the continuous clock drift unavoidably accounts for clock desynchronization between the time generated by the satellite atomic clock and the GPS time. Typically, a quadratic function is used to describe the variation of satellite clock error with corresponding satellite clock bias, drift, and aging parameters. The current final clock products can achieve clock root mean square error (rms) of 75 ps, with a standard deviation of 20 ps (IGS, 2024).

Tropospheric hydrostatic delay

The troposphere is the lowest layer of the Earth’s atmosphere, extending from the Earth’s surface up to 9 km over the poles and about 16 km over the equator. In GNSS processing, the troposphere effect will include the layers of the Earth’s atmosphere up to about 50 km, and it introduces delays of equal magnitude to both pseudorange and carrier-phase signals (Teunissen and Montenbruck, 2017). Approximately 90% of this delay is attributed to the dry or hydrostatic component. Given the consistent and homogeneous distribution of dry atmosphere constituents, tropospheric zenith hydrostatic delay (ZHD) can be corrected using empirical models such as the Hopfield model and Saastamoinen model, (Hopfield, 1969; Saastamoinen, 1972). The other crucial component of tropospheric delay is the tropospheric

wet delay, which is estimated in the GNSS processing and accounts for approximately 10% of the total tropospheric delay. This will be discussed in the subsequent section.

Satellite hardware delays

Satellite hardware delays stem from numerous processes occurring between signal generation and transmission to the satellite antenna. These delays impact both code and phase measurements and are correlated with signal frequency. The satellite hardware delays can be eliminated or mitigated by utilizing Total Group Delay (TGD) information, or products from Differential Code Bias (DCB), Differential Phase Bias (DPB), and Observable-specific Signal Biases (OSB) corrections.

Satellite antenna correction

The geometric range is measured between the phase centres of the satellite and receiver antennas. The GNSS receiver antenna serves as the intermediary component, filtering, amplifying, and converting the incoming signal from satellites into an electrical signal that can be processed by receivers. The point at which the satellite signal is received is referred to as the Antenna Phase Centre (APC). Typically, the APC does not align precisely with the geometric centre of the antenna and is influenced by various factors, as noted by Zhou et al. (2022). Therefore, accurately determining the position of the phase centre of the receiving GNSS antenna is crucial to meet the requirement of precise positioning. The antenna Phase Centre Offset (PCO) is defined as the average of phase centre locations relative to a physical reference point, which is usually provided by the manufacturer; if not, the determination of these coordinates is carried out by a calibration procedure. In contrast, Phase Centre Variation (PCV) refers to the deviation of the antenna phase centre beyond the antenna offset. In processing, PCO and PCV from satellite antenna and IGS station receivers can be corrected from the IGS ANTenna EXchange ANTenna EXchange (ANTEX) file (Schmid et al., 2016). Typically, PCO and PCV can lead to decimetre-to-centimetre level corrections for geodetic processing (Teunissen and Montenbruck, 2017). Currently, smartphone PCO and PCV corrections remain unavailable and are not applied in the PPP processing.

Phase wind-up

GNSS satellites transmit circularly polarized radio waves, meaning that the carrier-phase measurements depend on the relative orientation of the satellite and receiver antennas. If either the receiver or transmitter antenna rotates a full rotation around its boresight axis, it will result in a change in the carrier-phase measurement by one cycle. This phenomenon is known as phase wind-up (Teunissen and Montenbruck, 2017). Typically, the fractional part of the carrier-phase wind-up presents a minor correction, on the order of a few centimetres in terms of phase path, which is essential to consider for high-precision applications (Wu et al., 1992).

Relativistic effects

Einstein’s theory of relativity predicts that relativistic effects arise due to differences in gravitational potential and relative speeds between satellites and receiver clocks. Haustein (2009) noted that neglecting these effects could lead to significant errors, such as 12 km per day for position determination or 39 microseconds for time determination. The three primary effects of relativity on positioning are:

1. Fixed Frequency Offset Effect: a fixed frequency offset exists in the satellite’s clock rate when observed from Earth. This effect is intentionally compensated in the satellite clock products, eliminating the need for users to consider it.
2. Sagnac effect: the Sagnac effect occurs due to the Earth’s rotation during the propagation of GNSS signals. It represents a correction to accommodate the time dilation caused by a clock carried by a rotating object in non-inertial frames (Ashby and Spilker, 1996). This effect arises from the tendency of a moving clock to run slower than one at rest or moving at a slower speed. Consequently, it leads to a frequency offset that can be interpreted as a distance.
3. Periodic Clock Error Effect: the satellite orbits are not perfectly circular, and the slight eccentricity of each orbit introduces an additional periodic clock error. To ac-

count for this effect, a correction must be applied to the broadcast time of the signal transmission (Ashby and Spilker, 1996). This clock effect can be eliminated by double-differencing. However, it can introduce a maximum error of 6.9 m in distance for undifferenced positioning.

Solid Earth tides

Solid Earth tides result from gravitational forces exerted by celestial bodies such as the moon and sun, leading to displacements in the Earth's surface. These mostly vertical displacements vary depending on the gravitational potential at the site location, with typical ranges of 20 cm and occasional peaks exceeding 30 cm at certain stations (Zheng, 2007). The magnitude of solid Earth tides on GNSS horizontal positioning is typically on centimetre level or less. Therefore, the solid Earth tides are still significant for cm level accuracy.

Ocean loading

Similar to solid earth tides, ocean loading refers to the deformation of the Earth's crust due to the movement of large bodies of water, primarily the oceans. Ocean loading primarily affects areas close to the oceans, typically at the millimetre level (Agnew, 1997), and is negligible for smartphone applications.

2.1.2 GNSS estimated errors

Equation (2.1) presents the pseudorange and carrier-phase measurement equations.

$$\begin{aligned} P_{r,i}^s &= \rho_r^s + c(dt_r - dt^s) + \gamma_i I_{r,1}^s + M_r^s T_r + b_{r,P_i} - b_{P_i}^s + \varepsilon_{P_i} \\ L_{r,i}^s &= \rho_r^s + c(dt_r - dt^s) - \gamma_i I_{r,1}^s + M_r^s T_r + \lambda_i \left(N_i^s + b_{r,L_i} - b_{L_i}^s \right) + \varepsilon_{L_i} \end{aligned} \quad (2.1)$$

where P_i and L_i represent the pseudorange and carrier-phase measurements on frequency f_i , respectively; i refers to signal frequency index, and ρ_r^s refers the geometric distance between satellite s and receiver r ; dt_r and dt^s represent the receiver and satellite clock offset, respectively; c refers to the speed of light, and γ_i equals to f_1^2/f_i^2 ; M_r^s is the mapping function that converts the zenith troposphere delay T_r to slant troposphere delay, $I_{r,1}^s$ is the slant

ionospheric delay on the first frequency; $b_{r,P}$ and $b_{r,L}$ denote the receiver code and phase biases, respectively; b_P^s and b_L^s denote the satellite code and phase biases, respectively; λ is the signal wavelength, and N is the unknown ambiguity in cycles; ε_P and ε_L are unmodelled pseudorange and phase noises, respectively. It is important to note that the satellite clock errors and code biases are retained in the equations to provide a clear illustration and understanding of the measurement constituents. These states and errors are discussed briefly below:

Receiver clock errors

In general, GNSS receivers are equipped with low-cost crystal clocks, which have lower accuracy and stability compared to satellite clocks. Consequently, receiver clock errors tend to be larger than those of GNSS satellite clocks. RTK and Post-Processed Kinematic (PPK), or relative positioning techniques utilize the double-differencing method to eliminate receiver clock errors, while Single Point Positioning (SPP) and PPP estimate the receiver clock error as an additional estimate in the processing, posing a requirement of a minimum of four visible satellites to obtain a solution from a standalone receiver. In traditional PPP mode, the estimated receiver clock absorbs the receiver biases and they are highly correlated.

Ionospheric error

The ionosphere is like a shell that is electrically charged and acts as a dispersive medium in which waves of different frequencies travel at different velocities. The ionosphere stretches from a height of 50 km to 1000 km, and the GNSS signal ionospheric refraction depends on its frequencies and the Total Electron Content (TEC). It is generally assumed that the electron density of the atmosphere is concentrated at an average height of 350 km (Schaer et al., 1999). The presence of electrons and ions within the ionosphere significantly influences the propagation of satellite signals, leading to discrepancies in the satellite ranging from the order of a few metres to tens of metres. The ionospheric effect affects pseudorange and carrier-phase measurements differently (Hofmann-Wellenhof et al., 2012; Leick et al., 2015).

Ionospheric errors can be mitigated for short-baseline RTK or ionosphere-free dual-frequency signal combination techniques. In Undifferenced and Uncombined (UDUC) mode, the slant ionospheric delay on the first frequency is estimated as an additional parameter and absorbs the receiver and satellite biases. Additionally, ionospheric delays can be approximated using the Klobuchar model and Global Ionospheric Maps (GIM) (Klobuchar, 1975; Jee et al., 2010). For PPP-RTK applications requiring higher accuracy, Regional Ionospheric Maps (RIM) are necessary (Li et al., 2022a).

Tropospheric wet delay

Unlike the ZHD, which contributes to approximately 90% of tropospheric errors and is relatively stable, the troposphere zenith wet delay (ZWD) accounts for around 10% of the errors and is highly variable due to fluctuations in water vapor pressure and temperature, which are difficult to predict. In short-baseline RTK positioning, ZWD is addressed using the double-differencing method, while ZWD is estimated as an additional parameter in PPP processing.

Carrier-phase ambiguity

Carrier-phase measurements are significantly more precise compared to code pseudorange measurements. However, they contain an ambiguity, which refers to the unknown integer number of carrier wavelengths between the satellite and the receiver. The initial estimation of the real ambiguities primarily relies on the code measurements, which have low accuracy due to their metre-level ranging precision (Hofmann-Wellenhof et al., 2012). The estimated float ambiguity is coupled with various biases, resolving the ambiguities to integers can notably enhance positioning quality, particularly for the east component.

Receiver hardware delays

Receiver hardware delays, akin to satellite hardware delays, affect both code and phase measurements. Generally, these delays are unknown to users and are either absorbed in the estimated receiver clock offset or mitigated through the between-satellite single-differencing method.

Noise

The unmodelled pseudorange and phase noises primarily originate from receiver noise and multipath effects. Receiver noise arises from imperfections in the signal processing chain, including components such as the antenna, cables, connectors, and the receiver itself (Teunissen and Montenbruck, 2017). Multipath effects occur when signals transmitted from satellites to receivers follow multiple paths, including indirect routes where the signal is reflected, diffracted, or scattered off surfaces like buildings, trees, or the ground before reaching the receiver. Both receiver noise and multipath effects are grouped in ε_P and ε_L in Equation (2.1).

2.2 GNSS techniques

This section provides an overview of three widely adopted GNSS techniques utilized in mass-market applications to improve accuracy and reliability: Single Point Positioning (SPP), relative GNSS, and Precise Point Positioning (PPP). Further information on each of these GNSS processing modes, including RTK, DGNSS, and SPP, can be found in (Hofmann-Wellenhof et al., 2012; Leick et al., 2015).

2.2.1 Single Point Positioning

SPP is a fundamental technique used in GNSS to determine the position of a receiver using signals from satellites. In SPP, a receiver calculates its position using measurements of the pseudorange (code) measurements transmitted by satellites in view. This technique is called "single point" because it relies solely on the measurements made at the receiver itself, without any assistance or augmentation from additional reference stations or network corrections. Trilateration involves determining the receiver's position by intersecting spheres centred on each satellite, with radii equal to the measured pseudorange between the receiver and

each satellite, and the basic observed pseudorange equation can be simplified as:

$$\begin{aligned}
P_r^1(t) &= \sqrt{(X^1 - X_r)^2 + (Y^1 - Y_r)^2 + (Z^1 - Z_r)^2} + cdt_r + \varepsilon_{P_1} \\
P_r^2(t) &= \sqrt{(X^2 - X_r)^2 + (Y^2 - Y_r)^2 + (Z^2 - Z_r)^2} + cdt_r + \varepsilon_{P_2} \\
P_r^3(t) &= \sqrt{(X^3 - X_r)^2 + (Y^3 - Y_r)^2 + (Z^3 - Z_r)^2} + cdt_r + \varepsilon_{P_3} \\
P_r^4(t) &= \sqrt{(X^4 - X_r)^2 + (Y^4 - Y_r)^2 + (Z^4 - Z_r)^2} + cdt_r + \varepsilon_{P_4}
\end{aligned} \tag{2.2}$$

where $P_r^j(t)$ is the measured pseudorange between the receiver r and the satellite j . By knowing the exact positions of the satellites (X^j, Y^j, Z^j) at epoch t , the receiver can calculate its own position X_r, Y_r, Z_r . Multilateration extends this concept to include signals from more than four satellites, enabling a more precise determination of the receiver's position.

SPP is frequently employed in applications like personal navigation devices, smartphones, and recreational GPS receivers, catering to users needing reasonably accurate positioning data for tasks such as navigation, mapping, and geotagging. However, SPP often offers lower accuracy, typically within a range of 10 m at the 95% confidence level in smartphone processing (Shinghal, 2020), due to several factors: 1) inaccurate code measurements, 2) errors in broadcast ephemeris products, and 3) atmospheric delays.

2.2.2 Relative GNSS

Relative GNSS is a technique employed to enhance positioning accuracy by leveraging Observation State Representation (OSR) corrections from a base (reference) station with known coordinates to refine the position calculations. This result is achieved by computing the differences between the observed code and phase measurements obtained by the receiver and those made at a nearby pre-surveyed location. By doing so, errors inherent in GNSS, such as satellite orbit and clock errors, as well as atmospheric delays, can be minimized or calibrated.

Between receiver differencing (single-differencing)

Assume that the same satellite is observed simultaneously from two different receivers. The measured pseudorange and carrier-phase measurements from the base station (b) and user

station (u) can be expressed as (Noureldin et al., 2012; Teunissen and Montenbruck, 2017):

$$\begin{aligned}
P_{b,i}^s &= \rho_b^s + c(dt_{rb} - dt^s) + \gamma_i I_{rb,1}^s + M_{rb}^s T_{rb} + b_{rb,P_i} - b_{P_i}^s + \varepsilon_{P_{b,i}} \\
P_{u,i}^s &= \rho_u^s + c(dt_{ru} - dt^s) + \gamma_i I_{ru,1}^s + M_{ru}^s T_{ru} + b_{ru,P_i} - b_{P_i}^s + \varepsilon_{P_{u,i}} \\
L_{b,i}^s &= \rho_b^s + c(dt_{rb} - dt^s) - \gamma_i I_{rb,1}^s + M_{rb}^s T_{rb} + \lambda_i (N_{b,i}^s + b_{rb,L_i} - b_{L_i}^s) + \varepsilon_{L_{b,i}} \\
L_{u,i}^s &= \rho_u^s + c(dt_{ru} - dt^s) - \gamma_i I_{ru,1}^s + M_{ru}^s T_{ru} + \lambda_i (N_{u,i}^s + b_{ru,L_i} - b_{L_i}^s) + \varepsilon_{L_{u,i}}
\end{aligned} \tag{2.3}$$

By differencing the measurements from two receivers, the observation equations can be rewritten as:

$$\begin{aligned}
\Delta P_{bu,i}^s &= P_{b,i}^s - P_{u,i}^s = \Delta \rho_{bu}^s + c(\Delta dt_{r,bu}) + \gamma_i \Delta I_{bu,1}^s + \Delta T_{bu} + \Delta b_{r_{bu},P_i} + \Delta \varepsilon_{P_{bu,i}} \\
\Delta L_{bu,i}^s &= L_{b,i}^s - L_{u,i}^s = \Delta \rho_{bu}^s + c(\Delta dt_{r,bu}) - \gamma_i \Delta I_{bu,1}^s + \Delta T_{bu} + \lambda_i (\Delta N_{bu,i}^s + \Delta b_{r_{bu},L_i}) + \Delta \varepsilon_{L_{bu,i}}
\end{aligned} \tag{2.4}$$

where Δ stands for the single-differencing between base (b) and user (u) stations. Notably, this method can eliminate the satellite clock offset and satellite equipment biases.

Real-Time Kinematic

In a single-difference setup, the receiver clock offset and the ambiguity parameters are highly correlated. To accurately estimate the ambiguity, double-differencing observations can be formed by applying the between receiver differencing and between satellite difference methods by selecting a reference satellite (k). The double-differenced code and phase measurements can be expressed as (Noureldin et al., 2012; Teunissen and Montenbruck, 2017):

$$\begin{aligned}
\nabla \Delta P_{bu}^{jk} &= \nabla \Delta \rho_{bu}^{jk} + \gamma_i \nabla \Delta I_{bu,1}^{jk} + \nabla \Delta T_{bu}^{jk} + \nabla \Delta \varepsilon_{P_{i,bu}}^{jk} \\
\nabla \Delta L_{bu}^{jk} &= \nabla \Delta \rho_{bu}^{jk} - \gamma_i \nabla \Delta I_{bu,1}^{jk} + \nabla \Delta T_{bu}^{jk} + \lambda_i \nabla \Delta N_{bu,i}^{jk} + \nabla \Delta \varepsilon_{L_{i,bu}}^{jk}
\end{aligned} \tag{2.5}$$

where the subscript bu represents the single-differencing between the user rover and base station, and superscript jk represents the double-differencing between the reference satellite k and satellite j . By applying double-differencing to the observations, common errors such as satellite orbit and clock errors, receiver clock offset, and hardware delays are effectively

eliminated. Additionally, tropospheric and ionospheric errors can be neglected if the base station is within approximately 10 km from the rover. This results in more reliable, accurate, and robust positioning solutions, making the system less susceptible to common sources of error inherent in GNSS measurements. Real-Time Kinematic (RTK) is a positioning technique that can achieve decimetre-to-centimetre level positioning accuracy in real-time over moving baselines using the double-differencing method.

2.2.3 Precise Point Positioning

PPP is an advanced point positioning technique that utilizes pseudorange and carrier-phase observations, coupled with State Space Representation (SSR) corrections obtained from IGS analysis centres or GNSS satellites, to achieve positioning accuracy ranging from decimetres to sub-centimetres with geodetic receivers (Zumberge et al., 1997; Kouba and Héroux, 2001), and the convergence rate and time vary with the specific PPP methods used. Both static and kinematic data processing can be conducted using PPP, either in post-processing or real-time mode (Gao and Chen, 2004; Chen and Gao, 2005). Compared to SPP techniques, PPP utilizes precise satellite corrections to effectively mitigate errors induced by satellite orbit and clock errors present in broadcast ephemeris data. Additionally, PPP incorporates precise calibration and estimation of various error sources as mentioned in Section 2.1, including antenna corrections and site displacements, thereby further enhancing accuracy. Moreover, PPP leverages additional carrier-phase measurements in its processing, utilizing a sequential least squares filter to exploit previous information. These combined strategies elevate PPP solutions to a significantly higher level of accuracy compared to SPP techniques.

Both PPP and RTK or Network RTK (NRTK) techniques are advanced satellite positioning techniques and have been widely accepted in mass-market applications. Here are some advantages of PPP compared to RTK:

1. Service range: PPP can provide high-accuracy positioning globally without the need for a local reference station (network). This makes PPP particularly advantageous in remote or global applications where establishing and maintaining an RTK reference network may be impractical or costly.

2. Cost-effective: PPP typically requires only one receiver and access to precise correction products for positioning, making it a more economical choice compared to setting up and maintaining an RTK network, which can be costly, especially in remote or inaccessible areas.
3. Improved reliability and availability: PPP is less susceptible to outages, spoofing, and jamming caused by local factors such as signal blockages or interference from the base station.

However, PPP often requires a longer convergence time compared to RTK, especially when starting from a cold or ambiguous state, as it needs to estimate additional parameters such as ionospheric and tropospheric delays. Thus, it is important to note that both PPP and RTK have their own limitations and considerations. The choice between them depends on factors such as the specific requirements of the application, cost considerations, and the availability of infrastructure. The subsequent sections introduce the common combinations of measurements and adjustment models utilized in PPP processing. Figure 2.1 illustrates the simple working principles and differences between RTK and PPP.

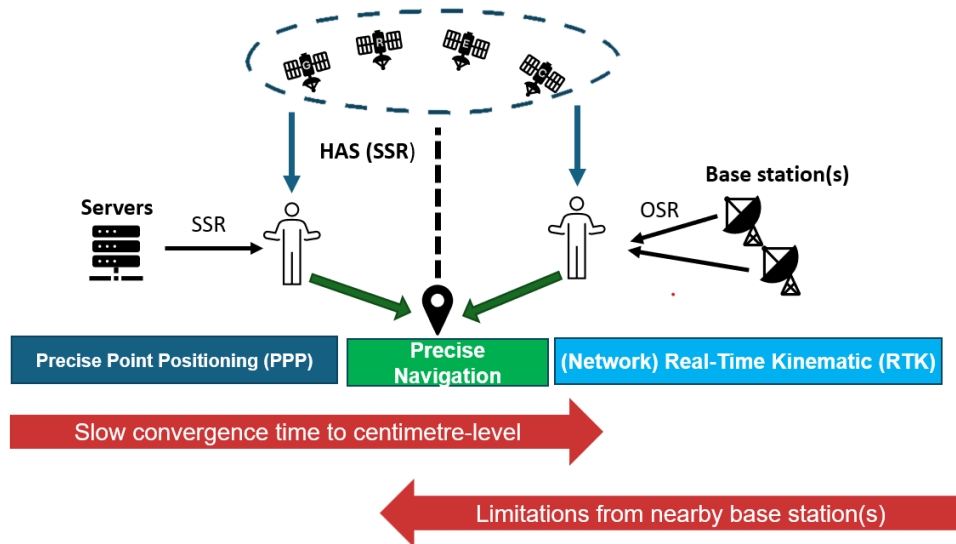


Figure 2.1: Comparison of PPP and RTK working principles

Ionospheric-free PPP

In traditional PPP, the ionospheric-free (IF) combination of observations is utilized to mitigate the first-order ionospheric effect, which constitutes a significant error source in PPP estimation (Bisnath et al., 2009; Cai et al., 2007). Consequently, the estimated receiver clock errors become intertwined with receiver equipment biases, as depicted in Equation (2.6).

$$\begin{aligned} P_{r,IF}^s &= \rho_r^s + (c dt_r + b_{r,P_{IF}}) - \left(dt^s + b_{P_{IF}}^s \right) + M_r^s T_r + \varepsilon_{P_{IF}} \\ L_{r,IF}^s &= \rho_r^s + (c dt_r + b_{r,P_{IF}}) - \left(dt^s + b_{P_{IF}}^s \right) + M_r^s T_r + \lambda_{IF} \overline{N_{IF}^s} + \varepsilon_{L_{IF}} \end{aligned} \quad (2.6)$$

where P_{IF} and L_{IF} represent the ionosphere-free code and carrier-phase measurements, respectively; the IF combination of receiver and satellite biases are denoted as $b_{r,IF}$ and b_{IF}^s . Additionally, $\overline{N_{IF}^s}$ signifies the biased ionosphere-free ambiguity, which includes the real IF ambiguity N_{IF}^s and the IF code and phase biases.

$$\begin{aligned} b_{r,IF} &= \frac{f_1^2}{f_1^2 - f_2^2} b_{r,1} - \frac{f_2^2}{f_1^2 - f_2^2} b_{r,2} \\ b_{IF}^s &= \frac{f_1^2}{f_1^2 - f_2^2} b_1^s - \frac{f_2^2}{f_1^2 - f_2^2} b_2^s \\ \overline{N_{IF}^s} &= N_{IF}^s + (b_{r,L_{IF}} - b_{r,P_{IF}}) - \left(b_{L_{IF}}^s - b_{P_{IF}}^s \right) \end{aligned} \quad (2.7)$$

Undifferenced and uncombined PPP with ionospheric-free products

PPP relies on the utilization of precise satellite corrections derived from a global network of stations, and most precise products from analysis centres such as Centre National d'Etudes Spatiales (CNES) and GeoForschungsZentrum (GFZ) are computed based on the ionospheric-free combination. To effectively leverage these precise products in PPP processing, the traditional UDUC PPP in dual-frequency mode can be reformulated as follows (Li

et al., 2013; Xiang et al., 2017):

$$\begin{aligned}
P_{r,1}^s &= \rho_r^s + (cdt_r + b_{r,P_{IF}}) - (dt^s + b_{P_{IF}}^s) + \gamma_1 \overline{I_{r,1}^s} + M_r^s T_r + \varepsilon_{P_i} \\
P_{r,2}^s &= \rho_r^s + (cdt_r + b_{r,P_{IF}}) - (dt^s + b_{P_{IF}}^s) + \gamma_2 \overline{I_{r,1}^s} + M_r^s T_r + \varepsilon_{P_i} \\
L_{r,1}^s &= \rho_r^s + (cdt_r + b_{r,P_{IF}}) - (dt^s + b_{P_{IF}}^s) - \gamma_1 \overline{I_{r,1}^s} + M_r^s T_r + \lambda_1 \overline{N_1^s} + \varepsilon_{P_i} \\
L_{r,2}^s &= \rho_r^s + (cdt_r + b_{r,P_{IF}}) - (dt^s + b_{P_{IF}}^s) - \gamma_2 \overline{I_{r,1}^s} + M_r^s T_r + \lambda_2 \overline{N_2^s} + \varepsilon_{P_i}
\end{aligned} \tag{2.8}$$

In Equation (2.8), the term $dt^s + b_{P_{IF}}^s$ denotes the precise satellite clock corrections derived from the ionospheric-free combination. It is worth noting that the estimated receiver clock offset is interrelated with the receiver biases in the IF combination. $\overline{I_{r,1}^s}$ represents the estimated ionospheric delay on the first frequency, and it is grouped with the receiver DCB for each satellite. The effect of the satellite DCB can be mitigated using the satellite code bias products.

$$\begin{aligned}
\overline{I_{r,1}^s} &= I_{r,1}^s - \frac{f_2^2}{f_1^2 - f_2^2} (b_{r,P_1} - b_{r,P_2}) + \frac{f_2^2}{f_1^2 - f_2^2} (b_{P_1}^s - b_{P_2}^s) \\
DCB_{P_1 P_2} &= b_{P_1} - b_{P_2}
\end{aligned} \tag{2.9}$$

Adjustment model

The sequential least-square filter is utilized in the PPP processing, and the iterated solution $\mathbf{x}$ is obtained from:

$$\mathbf{x} = \mathbf{x}_0 + \Delta \mathbf{x} \tag{2.10}$$

In Equation (2.10), $\mathbf{x}_0$ denotes the a priori estimate, and the correction vector $\Delta \mathbf{x}$ for unknown estimates can be represented as:

$$\Delta \mathbf{x} = [\delta \mathbf{P}_{x,y,z} \quad \delta dt_r \quad \delta T_r^s \quad \delta I_r^s \quad N^s] \tag{2.11}$$

where $\delta \mathbf{P}_{x,y,z}$ represent the position corrections in Earth-Centred Earth-Fixed (ECEF) coordinate system, such as the World Geodetic System 1984 (WGS84) and International Terrestrial Reference Frame (ITRF); δI_r^s and δT_r^s are corrections to the ionospheric and

tropospheric delays, respectively; δdt_r is the receiver clock offset correction and N^s comprise float ambiguities for each satellite. $\Delta \mathbf{x}$ can be estimated through:

$$\Delta \mathbf{x} = \left(\mathbf{H}^T \mathbf{P}_l \mathbf{H} + \mathbf{P}_x \right)^{-1} \mathbf{H}^T \mathbf{P}_l \mathbf{W} \quad (2.12)$$

where $\mathbf{H}$ is the design matrix and $\mathbf{W}$ is the misclosure matrix; $\mathbf{P}_l$ and $\mathbf{P}_x$ represent the observation weight matrix and process noise matrix, respectively. The design matrix $\mathbf{H}$ consists of the partial derivatives of the observation equations with respect to $\mathbf{x}$ and can be expressed as:

$$\mathbf{H} = \begin{bmatrix} e_x^1 & e_y^1 & e_z^1 & 1 & mf^1 & \gamma_1 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ e_x^1 & e_y^1 & e_z^1 & 1 & mf^1 & -\gamma_1 & \cdots & 0 & 0 & \cdots & 0 & \lambda_1 & \cdots & 0 \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ e_x^g & e_y^g & e_z^g & 1 & mf^g & 0 & \cdots & \gamma_5 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ e_x^g & e_y^g & e_z^g & 1 & mf^g & 0 & \cdots & -\gamma_5 & 0 & \cdots & 0 & 0 & \cdots & \lambda_5 \end{bmatrix} \quad (2.13)$$

e_x , e_y , and e_z are line-of-sight unit direction vectors from satellite to receiver. mf represents the wet mapping function of the tropospheric delay. The measurement precision ratio for phase to code is set to 100:1.

2.3 GNSS/IMU integration

An inertial measurement unit (IMU) is a device designed to measure and report specific force (acceleration) and angular rate (angular velocity) relative to a reference frame. Typically, an IMU comprises orthogonal triads of accelerometers and gyroscopes to measure 3-dimensional (3D) linear acceleration and angular velocity, respectively. This information enables the determination of the orientation, velocity, and position of a moving object within an inertial navigation system (INS). Unlike GNSS, an IMU operates independently of external signals, making it suitable for environments where satellite signals may be obstructed or unavailable, such as indoors, underground, or underwater. The high sampling rate (typically exceeding

100 Hz) and low latency of IMU measurements enable precise tracking of dynamic motions with high temporal resolution.

However, IMUs are prone to accumulated errors over time, primarily due to the drifts of accelerations and angular rates to determine orientation, velocity, and position. Those errors, including IMU biases, scale factor errors, cross-coupling errors, etc., can accumulate rapidly and lead to significant position errors over extended periods, especially in long-term navigation. Additionally, IMUs solely offer relative motion measurements, lacking absolute positioning information referenced to global coordinates. Therefore, GNSS/IMU integration has gained popularity for combining the strengths of both technologies to overcome their individual limitations. This integration leads to improved accuracy, robustness, and reliability of positioning solutions, making it the preferred choice for a wide range of navigation and positioning applications. This section primarily delves into the methodologies encompassing IMU initialization and alignment, IMU mechanization, and the tightly-coupled PPP/IMU model.

2.3.1 IMU initialization and alignment

An IMU computes its navigation result by integrating the measurements from its inertial sensors. In other words, each iteration for position, velocity, and attitude calculation relies on the previous navigation parameters as initial values. Therefore, the navigation solution must be initialized before the IMU device can operate effectively.

Position and velocity initialization

Inertial navigation is a self-contained navigation technique in which measurements provided by accelerometers and gyroscopes are utilized and integrated to calculate the relative navigation solutions. However, an IMU cannot offer the user the absolute position and velocity information which needs to be provided externally.

In the GNSS/IMU integration system, initial position information can be obtained from the GNSS receiver. Alternatively, if the IMU system is placed near a pre-surveyed point with known landmarks, its initial position can be determined. Velocity can be estimated

and initialized by assuming its value to be zero if the IMU is stationary with respect to the Earth. Additionally, another navigation system, such as GNSS, can be used as a reference for the initial velocity.

Attitude alignment

Though an IMU can determine its position and velocity using external information, the directions of its measured acceleration and angular rates in the navigation system cannot be determined initially. The process of initializing the attitude is called alignment, which involves aligning the attitude of the body navigation system (b-system) to the local navigation system (n-system) in the East-North-Up direction.

Typically, the self-alignment of the IMU can be performed when stationary. In essence, the gravity and Earth rotation vectors in the local coordinate system can be represented as (Groves, 2013):

$$\mathbf{g}^n = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}, \omega_{ie}^n = \begin{bmatrix} 0 \\ \omega \cos L \\ \omega \sin L \end{bmatrix} = \begin{bmatrix} 0 \\ \omega_N \\ \omega_U \end{bmatrix} \quad (2.14)$$

where g and ω represent the gravity and Earth's rotation rate, L represents the latitude. According to the specific force equation, the local frame acceleration can be estimated as:

$$\dot{\mathbf{v}}^n = \mathbf{C}_b^n \mathbf{f}^b - (2\omega_{ie}^n + \omega_{en}^n) \times \mathbf{v}^n + \mathbf{g}^n \quad (2.15)$$

since $(2\omega_{ie}^n + \omega_{en}^n) \times \mathbf{v}^n$ is 0 and can be neglected in the stationary mode, Equation (2.15) can be simplified as:

$$\mathbf{f}^b = \mathbf{C}_n^b (-\mathbf{g}^n) \quad (2.16)$$

where $\mathbf{f}^b$ is the specific force measured in the body coordinate system, and $\mathbf{C}_n^b$ represents the Direction Cosine Matrix that can convert the local navigation coordinate system to the

body system and can be defined as:

$$\mathbf{C}_n^b = \begin{bmatrix} \cos(\theta)\cos(\psi) & \cos(\theta)\sin(\psi) & -\sin(\theta) \\ -\cos(\phi)\sin(\psi) + \sin(\phi)\sin(\theta)\cos(\psi) & \cos(\phi)\cos(\psi) + \sin(\phi)\sin(\theta)\sin(\psi) & \sin(\phi)\cos(\theta) \\ \sin(\phi)\sin(\psi) + \cos(\phi)\sin(\theta)\cos(\psi) & -\sin(\phi)\cos(\psi) + \cos(\phi)\sin(\theta)\sin(\psi) & \cos(\theta)\cos(\phi) \end{bmatrix} \quad (2.17)$$

Obviously, $\mathbf{C}_n^b$ comprises three attitude components, namely heading (ψ), pitch (θ), and roll (ϕ). By combining Equation (2.16) and Equation (2.17), the pitch and roll can be resolved without knowledge of gravity as:

$$\mathbf{f}^b = \begin{bmatrix} f_x^b \\ f_y^b \\ f_z^b \end{bmatrix} = \begin{bmatrix} -\sin(\theta) \cdot g \\ \sin(\phi)\cos(\theta) \cdot g \\ \cos(\theta)\cos(\phi) \cdot g \end{bmatrix} \xrightarrow{\text{yields}} \begin{cases} \theta = -\arctan\left(f_x^b / \sqrt{f_y^{b^2} + f_z^{b^2}}\right) \\ \phi = \arctan\left(f_y^b / f_z^b\right) \end{cases} \quad (2.18)$$

Since self-alignment of heading requires at least the tactical or higher grade of IMUs to measure Earth's rotation rate, it is not possible to successfully achieve the gyro-compassing based on smartphone gyros and even the tactical grade of the MEMS IMUs. Therefore, these systems are most often equipped with magnetometers (compass) to determine a coarse magnetic heading. This research utilizes the GNSS trajectory to estimate and initialize the heading. Assuming the tested vehicle is traveling in a straight line and its travel direction coincides with the vehicle-body-frame x direction, the heading can be roughly determined as:

$$\psi = \arctan(v_E/v_N) \quad (2.19)$$

where the v_E and v_N are the GNSS estimated east and north velocities in the local coordinate system.

2.3.2 IMU mechanization

IMUs provide accelerometer and gyroscope readings in terms of specific force vector $\mathbf{f}_{ib}^b$ and angular rate vector ω_{ib}^b between timestamp t_0 and $t_0 + \tau_i$. Here, the index i denotes that the IMU measurements are made relative to the inertial frame of the object, where an object

is always stationary in its own inertial frame. In this context, the inertial frame refers to the Earth-centered inertial (ECI) frame. In the ECI frame, the origin is positioned at the Earth's center. The x-axis aligns with the vernal equinox, the y-axis lies within the plane formed by the vernal equinox and the Earth's orbit, and the z-axis completes the right-handed coordinate system. It is worth mentioning that most of the equations used in this section are referenced from Noureldin et al. (2012) and Groves (2013).

The specific force in the inertial frame is given as:

$$\mathbf{f}^i = \mathbf{a}^i - \bar{\mathbf{g}}^i \quad (2.20)$$

where $\mathbf{f}$ is the specific force vector, $\mathbf{a}$ is the acceleration vector of the body, and $\bar{\mathbf{g}}$ represents the gravity accelerations, and the superscript i denotes the inertial frame. Since $\mathbf{a}$ can be represented as the second derivative of the position vector $\mathbf{r}$ measured from the origin of the inertial frame to the moving platform, the second-order differential equations can be transformed into first-order differential equations as follows:

$$\begin{aligned} \dot{\mathbf{r}}^i &= \mathbf{v}^i \\ \dot{\mathbf{v}}^i &= \mathbf{f}^i + \bar{\mathbf{g}}^i \end{aligned} \quad (2.21)$$

Assuming that the body frame coincides with the sensor triad frame (which is usually the case to eliminate further transformation), these measurements can be transformed into the inertial frame using the transformation matrix $\mathbf{C}_b^i$ between the body frame and the inertial frame:

$$\mathbf{f}^i = \mathbf{C}_b^i \mathbf{f}^b \quad (2.22)$$

with the rate of change of a transformation matrix:

$$\dot{\mathbf{C}}_b^i = \mathbf{C}_b^i \boldsymbol{\Omega}_{ib}^b \quad (2.23)$$

where $\mathbf{\Omega}_{ib}^b$ represents the skew-symmetric matrix form of the vector of angular velocities, ω_{ib}^b is the angular rate of the body frame relative to the inertial frame, expressed in the body frame, directly obtained from gyroscopes. $\mathbf{\Omega}_{ib}^b$ can be mathematically represented as:

$$\omega_{ib}^b = \begin{bmatrix} \omega^x \\ \omega^y \\ \omega^z \end{bmatrix} \xrightarrow{\text{yields}} \mathbf{\Omega}_{ib}^b = \begin{bmatrix} 0 & -\omega^z & \omega^y \\ \omega^z & 0 & -\omega^x \\ -\omega^y & \omega^x & 0 \end{bmatrix} \quad (2.24)$$

To sum up, the mechanization equation for the i-frame can therefore be summarized as:

$$\begin{bmatrix} \dot{\mathbf{r}}^i \\ \dot{\mathbf{v}}^i \\ \dot{\mathbf{C}}_b^i \end{bmatrix} = \begin{bmatrix} \mathbf{v}^i \\ \mathbf{C}_b^i \mathbf{f}^b + \mathbf{C}_e^i \bar{\mathbf{g}}^e \\ \mathbf{C}_b^i \mathbf{\Omega}_{ib}^b \end{bmatrix} \quad (2.25)$$

Correspondingly, the mechanization equation in the e-frame (ECEF) can be represented as:

$$\begin{bmatrix} \dot{\mathbf{r}}^e \\ \dot{\mathbf{v}}^e \\ \dot{\mathbf{C}}_b^e \end{bmatrix} = \begin{bmatrix} \mathbf{v}^e \\ \mathbf{C}_b^e \mathbf{f}^b - 2\mathbf{\Omega}_{ie}^e \mathbf{v}^e + \bar{\mathbf{g}}^e \\ \mathbf{C}_b^e (\mathbf{\Omega}_{ie}^b + \mathbf{\Omega}_{ib}^b) \end{bmatrix} \quad (2.26)$$

Mechanization solutions consist of a series of physical equations used to derive relative attitude, velocity, and position changes based on IMU readings, also known as the IMU dead reckoning solution. In GNSS/IMU tightly-coupled integration, these equations provide solutions between GNSS observations and serve as state predictions. Generally, IMU mechanization can be divided into four steps: 1) attitude update, 2) specific force resolving, 3) velocity update, and 4) position update. These steps are briefly discussed in the following sections.

Attitude update

Attitude update matrices are represented as $\mathbf{C}_{b+}^{b-}$ to represent the change in body frame between consecutive epochs from the end of the navigation processing epoch (+) to the

beginning of the navigation epoch (-). $\mathbf{C}_{b+}^{b-}$ can be computed by assuming a constant angular rate between each epoch, in terms of attitude increment $\boldsymbol{\alpha}_{ib}^b \approx \boldsymbol{\omega}_{ib}^b \tau_{imu}$:

$$\mathbf{C}_{b+}^{b-} = \mathbf{C}_{b(t+\tau_{imu})}^{b(t)} = \exp \left[\begin{bmatrix} \boldsymbol{\alpha}_{ib}^b & \times \end{bmatrix} \right] = \frac{\sum_{r=0}^{\infty} \left[\boldsymbol{\alpha}_{ib}^b \times \right]^r}{r!} \quad (2.27)$$

where $\times$ represents the cross product. Expanding the equation by using skew-symmetric identity gives:

$$\mathbf{C}_{b+}^{b-} = \mathbf{I}_3 + \frac{\sin |\boldsymbol{\alpha}_{ib}^b|}{|\boldsymbol{\alpha}_{ib}^b|} \left[\boldsymbol{\alpha}_{ib}^b \times \right] + \frac{1 - \cos |\boldsymbol{\alpha}_{ib}^b|}{|\boldsymbol{\alpha}_{ib}^b|^2} \left[\boldsymbol{\alpha}_{ib}^b \times \right]^2 \quad (2.28)$$

In practical scenarios, $\boldsymbol{\alpha}_{ib}^b$ may sometimes become very small, leading to potential division-by-zero errors. To mitigate this issue, a fourth-order approximation is employed if the program detects that $\boldsymbol{\alpha}_{ib}^b$ is too small in magnitude:

$$\mathbf{C}_{b+}^{b-} \approx \mathbf{I}_3 + \left(1 - \frac{|\boldsymbol{\alpha}_{ib}^b|^2}{6} \right) \left[\boldsymbol{\alpha}_{ib}^b \times \right] + \left(\frac{1}{2} - \frac{|\boldsymbol{\alpha}_{ib}^b|^2}{24} \right) \left[\boldsymbol{\alpha}_{ib}^b \times \right]^2. \quad (2.29)$$

Therefore, the updated attitude $\mathbf{C}_b^e(+)$ is given by subtracting the computed attitude change against earth rotation rate $\boldsymbol{\omega}_{ie}^e$:

$$\mathbf{C}_b^e(+) = \begin{pmatrix} \cos \omega_{ie} \tau_{imu} & \sin \omega_{ie} \tau_{imu} & 0 \\ -\sin \omega_{ie} \tau_{imu} & -\cos \omega_{ie} \tau_{imu} & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{C}_b^e(-) \mathbf{C}_{b+}^{b-} \quad (2.30)$$

Specific-force transformation

Since the IMU provides an average of the specific force measured between epochs, the measured specific force $\mathbf{f}_{ib}^b$ can be transformed into accelerations in ECEF coordinates by assuming an average transformation matrix $\overline{\mathbf{C}}_b^e$ during the time interval:

$$\mathbf{f}_{ib}^e = \overline{\mathbf{C}}_b^e \mathbf{f}_{ib}^b, \quad (2.31)$$

with

$$\overline{\mathbf{C}}_b^e = \mathbf{C}_b^e(-) \mathbf{C}_b^{b-} - \frac{1}{2} \boldsymbol{\Omega}_{ie}^e \mathbf{C}_b^e(-) \tau_{imu} \approx \mathbf{C}_b^e(-) - \frac{1}{2} \boldsymbol{\Omega}_{ie}^e \mathbf{C}_b^e(-) \tau_{imu} \quad (2.32)$$

where $\mathbf{C}_b^{b-}$ represent the average attitude update matrix in the b-frame, expressed as:

$$\mathbf{C}_b^{b-} = \mathbf{I}_3 + \frac{1 - \cos |\boldsymbol{\alpha}_{ib}^b|}{|\boldsymbol{\alpha}_{ib}^b|^2} [\boldsymbol{\alpha}_{ib}^b \times] + \frac{1}{|\boldsymbol{\alpha}_{ib}^b|^2} \left(1 - \frac{\sin |\boldsymbol{\alpha}_{ib}^b|}{|\boldsymbol{\alpha}_{ib}^b|} \right) [\boldsymbol{\alpha}_{ib}^b \times]^2. \quad (2.33)$$

Velocity update

As described in equation 2.26, the time derivative of ECEF velocity $\mathbf{v}_{eb}^e$ can be expressed as:

$$\dot{\mathbf{v}}_{eb}^e = \mathbf{f}_{ib}^e - 2\boldsymbol{\Omega}_{ie}^e \mathbf{v}_{eb}^e + \mathbf{g}^e. \quad (2.34)$$

where $\mathbf{f}_{ib}^e$ is obtained from specific-force transformation, and $\mathbf{g}^e$ is the local gravity vector in ECEF frame. The integrated velocity $\mathbf{v}_{eb}^e(+)$ is expressed as:

$$\mathbf{v}_{eb}^e(+) \approx \mathbf{v}_{eb}^e(-) + (\mathbf{f}_{ib}^e + \mathbf{g}^e - 2\boldsymbol{\Omega}_{ie}^e \mathbf{v}_{eb}^e(-)) \tau_{imu} \quad (2.35)$$

Position update

After updating the velocity vector $\mathbf{v}_{eb}^e(+)$, the position estimated by the IMU mechanization $\mathbf{P}_{eb}^e(+)$ can be obtained through simple integration of a constant acceleration model, leveraging the high update rate of IMU measurements. This can be represented as:

$$\mathbf{P}_{eb}^e(+) = \mathbf{P}_{eb}^e(-) + \frac{1}{2} [\mathbf{v}_{eb}^e(-) + \mathbf{v}_{eb}^e(+)] \tau_{imu} \quad (2.36)$$

The IMU mechanization process is illustrated in Figure 2.2. The specific force $\mathbf{f}^i$ from the accelerometers and the angular rate ω_{ib}^b from the gyroscopes are initially provided in the b-frame. These measurements are then transformed into the e-frame using the transformation matrix $\mathbf{C}_b^e$. Following gravity compensation (WGS84 model with sub-microGal precision), Earth rotation computation, and bias corrections, the time integration steps, as described in

the preceding sections, yield the current position, velocity, and attitude based on previous information.

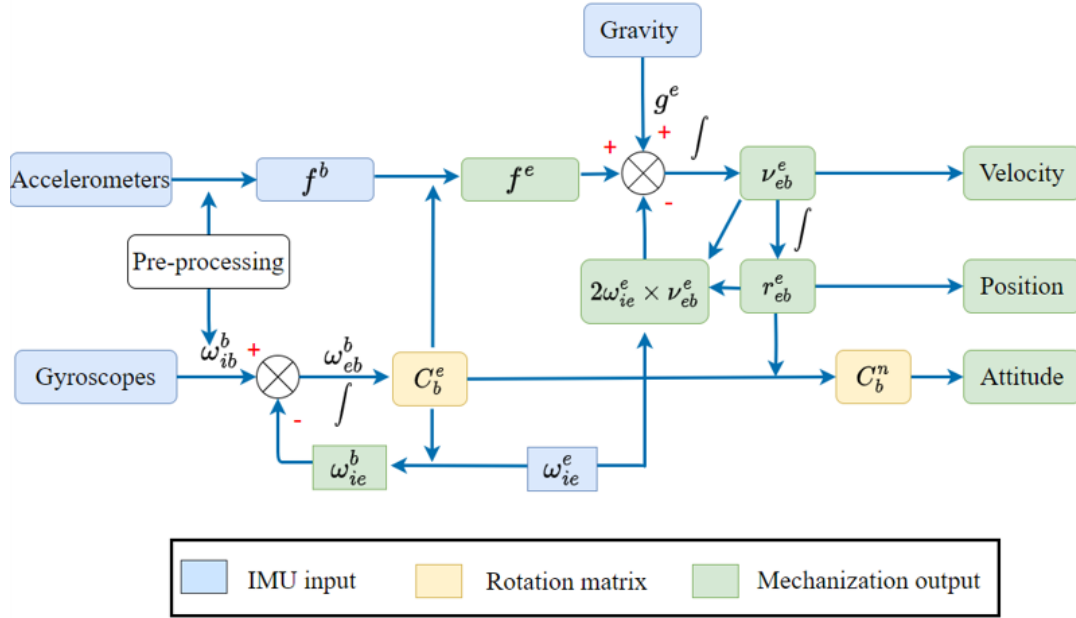


Figure 2.2: Working principles of IMU mechanization

2.3.3 Tightly-coupled PPP/IMU integration

The tightly-coupled PPP/IMU integration solution is obtained using the EKF. The EKF is a non-linear version of the Kalman Filter, which is capable of handling non-linear state estimation problems. The system and measurement models of a discrete EKF system are given as:

$$\begin{aligned} \mathbf{x}_{k|k-1} &= \mathbf{F}_k \hat{\mathbf{x}}_{k-1|k-1} + \mathbf{w}_{k-1} \\ \mathbf{z}_k &= \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k \end{aligned} \quad (2.37)$$

The state vector $\mathbf{x}_{k|k-1}$ at current epoch k can be predicted based on the optimal update state vector $\hat{\mathbf{x}}_{k-1|k-1}$ and transition matrix $\mathbf{F}$ at the previous epoch $k-1$. $\mathbf{w}$ and $\mathbf{v}$ denote the process noise and measurement noise, respectively, assumed to follow a Gaussian distribution with corresponding covariance matrix $\mathbf{Q}$ and $\mathbf{R}$. $\mathbf{H}$ is the design matrix described in Equation (2.13), and error measurement vector $\mathbf{z}$ is defined as the differences between the

observed GNSS pseudorange/carrier-phase measurements and the IMU modelled values.:

$$\mathbf{z} = \begin{bmatrix} \rho^i_{\text{GNSS}} - \rho^i_{\text{IMU}} \\ L^i_{\text{GNSS}} - L^i_{\text{IMU}} \\ \vdots \end{bmatrix} \quad (2.38)$$

Compared to the PPP error state vector listed in Equation (2.11), the PPP/IMU state vector is expanded as:

$$x_{\text{PPP/IMU}} = [\delta \mathbf{P}_{x,y,z} \quad \delta \mathbf{V}_{x,y,z} \quad \delta \mathbf{A}_{x,y,z} \quad \delta dt_r \quad \delta T_r^s \quad \delta I_r^s \quad N^s \quad \mathbf{B}_{x,y,z}^a \quad \mathbf{B}_{x,y,z}^g] \quad (2.39)$$

where $\delta \mathbf{V}_{x,y,z}$ and $\delta \mathbf{A}_{x,y,z}$ are velocity and attitude error vectors, respectively. $\mathbf{B}^a$ and $\mathbf{B}^g$ are the accelerometer bias and the gyroscope bias vectors, respectively. Other IMU-related errors such as accelerometer scale factor and cross-coupling errors are not estimated in the EKF states since their impact on smartphone applications is limited with appropriate bias estimation (Groves, 2013). Given that the previous sections have comprehensively introduced GNSS-related states, this section primarily focuses on the IMU-related state vector, which can be expressed as:

$$x_{\text{IMU}} = [\delta \mathbf{P}_{x,y,z} \quad \delta \mathbf{V}_{x,y,z} \quad \delta \mathbf{A}_{x,y,z} \quad \mathbf{B}_{x,y,z}^a \quad \mathbf{B}_{x,y,z}^g] \quad (2.40)$$

Correspondingly, the transition matrix $\mathbf{F}$ for Δx_{IMU} in ECEF-frame is:

$$\mathbf{F}_{\text{IMU}}^e \approx \begin{bmatrix} \mathbf{I}_3 & \mathbf{I}_3 \tau_{\text{GNSS}} & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{F}_{23}^e \tau_{\text{GNSS}} & \mathbf{I}_3 - 2\mathbf{\Omega}_{ie}^e \tau_{\text{GNSS}} & \mathbf{F}_{21}^e \tau_{\text{GNSS}} & \mathbf{C}_b^e \tau_{\text{GNSS}} & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{I}_3 - \mathbf{\Omega}_{ie}^e \tau_{\text{GNSS}} & \mathbf{0}_3 & \mathbf{C}_b^e \tau_{\text{GNSS}} \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{I}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{I}_3 \end{bmatrix} \quad (2.41)$$

where τ_{GNSS} refers to the GNSS time interval, and parameters $\mathbf{F}_{21}$ and $\mathbf{F}_{23}$ are IMU-related parameters explained in (Groves, 2013).

Subsequently, the predicted state covariance matrix $\mathbf{P}_{k|k-1}$ can be expressed as:

$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^\top + \mathbf{Q}_k \quad (2.42)$$

where $\mathbf{P}_{k-1|k-1}$ stands for the updated state covariance matrix from epoch $k-1$, the IMU-related process noise is assumed to be white noise, following a random process with a time-constant and frequency-independent spectral density. The autocorrelation function of a white noise process $\mathbf{w}$ can be expressed as (Groves, 2013):

$$\mathbf{R}_w(t_1, t_2) = \mathbb{E}[\mathbf{w}(t_1) \mathbf{w}(t_2)] = \text{PSD}_w^{f1} \delta(t_1 - t_2), \quad (2.43)$$

where PSD_w^{f1} is the single-sided Power Spectral Density (PSD). $\delta(t)$ is a Dirac delta function, and $\mathbf{R}_w$ is zero when $t_1 \neq t_2$. Considering that the IMU-related covariance matrix $\mathbf{Q}_{IMU}$ is obtained by integrating the PSD of these noise sources over the propagation time interval, the detailed explanations can be seen in Groves (2013) and $\mathbf{Q}_{IMU}$ can be approximated to:

$$\mathbf{Q}_{IMU} \approx \begin{pmatrix} \mathbf{P}_{x,p} & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \text{PSD}_{a,n} \mathbf{I}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \text{PSD}_{g,n} \mathbf{I}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \text{PSD}_{a,db} \mathbf{I}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \text{PSD}_{g,db} \mathbf{I}_3 \end{pmatrix} \tau_{GNSS} \quad (2.44)$$

where $\mathbf{P}_{x,p}$ is the process noise for the position vector; $\text{PSD}_{a,n}$ and $\text{PSD}_{g,n}$ represent the PSD for accelerometer and gyro noises, respectively; $\text{PSD}_{a,db}$ and $\text{PSD}_{g,db}$ represent the PSD for accelerometer and gyro dynamic biases, respectively.

Then, the innovation (pre-fit residual) covariance $\mathbf{S}$ and optimal Kalman gain $\mathbf{K}$ can be expressed as:

$$\begin{aligned} \mathbf{S}_k &= \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^\top + \mathbf{R}_k \\ \mathbf{K}_k &= \mathbf{P}_{k|k-1} \mathbf{H}_k^\top \mathbf{S}_k^{-1} \end{aligned} \quad (2.45)$$

Finally, the updated state vector $\hat{\mathbf{x}}_{k|k}$ and its covariance $\mathbf{P}_{k|k}$ can be calculated as:

$$\begin{aligned}\hat{\mathbf{x}}_{k|k} &= \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k(\mathbf{z}_k - \mathbf{H}_k\hat{\mathbf{x}}_{k|k-1}) \\ \mathbf{P}_{k|k} &= (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_{k|k-1}\end{aligned}\tag{2.46}$$

To sum up, Figure 2.3 demonstrates the design of the tightly-coupled PPP/IMU EKF based on a conventional closed-loop model. The raw measurements conditioned through pre-processing are fed into the mechanization equations to produce an IMU-only solution. Once the next GNSS observation set becomes available, the GNSS-PPP/IMU EKF runs to produce a solution. Finally, the bias estimates for accelerometers and gyroscopes are fed back to the IMU data reading in the closed-loop feedback approach for the next epoch. The feedback is additive to the raw $\mathbf{f}_{ib}^b$ and ω_{ib}^b inputs.

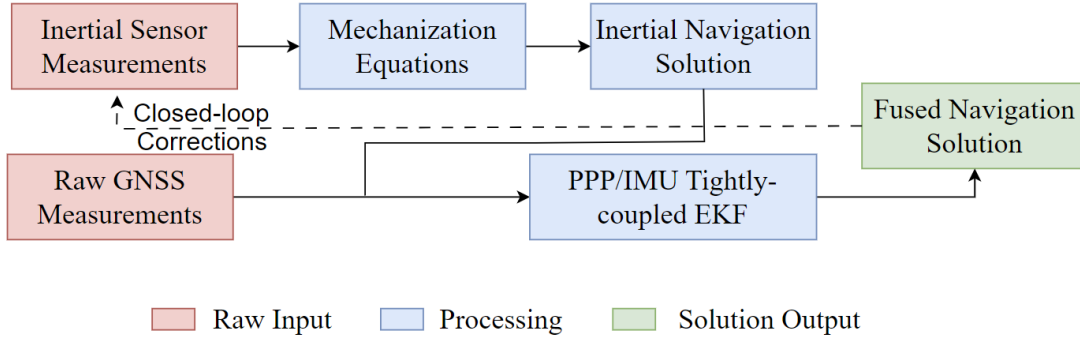


Figure 2.3: Schematic of the tightly-coupled EKF in processing engine

2.4 Overview of smartphone processing software

To enable sensor fusion processing for GNSS and IMU sensors on smartphones in real-world driving scenarios, a wider research project with the York GNSS Laboratory developed a smartphone-specific, scalable, and modular GNSS/IMU C++ processor called "Jaquen". The aim is for this software to exhibit the resilience and reliability of the character Jaquen H'ghar from the novel series "A Song of Ice and Fire", providing users with dependable and accurate positioning solutions. Jaquen can be debugged and released in Integrated Development Environments such as Windows Visual Studio and Linux Visual Studio Code. Our software

implementation strives to showcase high position accuracy, ensuring that smartphone positioning solutions achieve a success rate of 100% in open sky areas, with 95% of points having an accuracy within 1.5 m horizontally. In moderately urban areas, Jaqen targets more than 80% of points having an accuracy within 1.5 m horizontally.

The research scope includes the development and evaluation of two software packages within Jaqen: fused RTK GNSS package + IMU and fused PPP GNSS + IMU package. These packages feature robust algorithms for GNSS and IMU measurement pre- and post-processing, GNSS and IMU bias estimation and mitigation, GNSS carrier-phase cycle-slip detection, and multipath mitigation. Extensive testing has been conducted in laboratory settings and urban environments using various smartphones, GNSS chipsets, IMU chipsets, and antennas. These tests evaluate standard positioning performance metrics and compare them to other smartphones, professional-grade antennas, GNSS receivers, and IMUs, as well as low-cost alternatives. The basic structure of Jaqen’s implementation was completed over two years, from 2021 to 2023. The implementation plan was structured into four phases, each designed to achieve specific milestones and objectives, as illustrated in Figure 2.4.

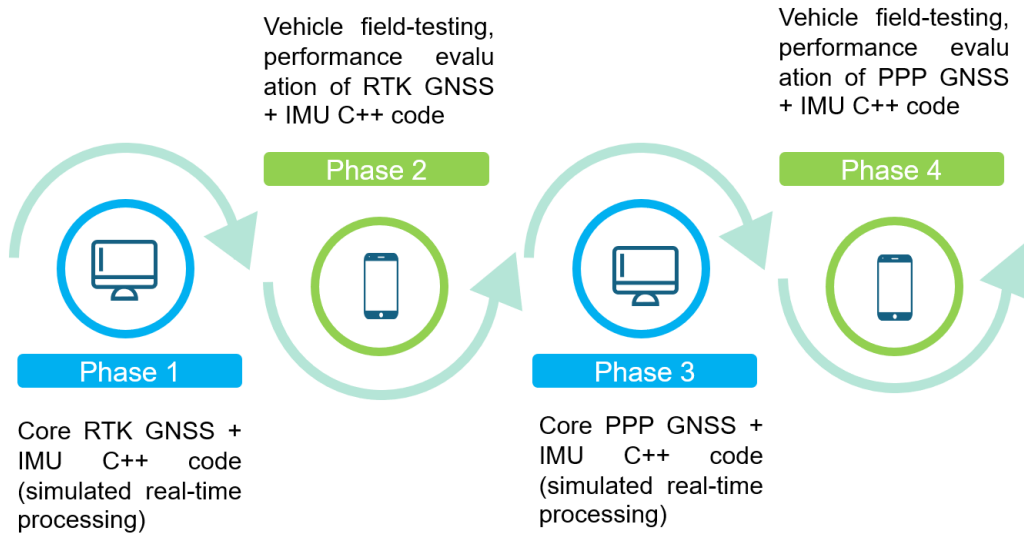


Figure 2.4: Jaqen implementation roadmap

The description and objectives of each phase can be summarized as follows:

Phase 1: Core RTK GNSS + IMU C++ code (simulated real-time processing).

- **Objective:** Develop core multi-GNSS single baseline float, and when possible, fixed RTK processing engine with the Sequential Least Squares (SLS) and Extended Kalman Filter (EKF) estimators. Expand to fuse IMU sensors in a tightly-coupled EKF architecture. The equations of the SLS estimator are the same as the EKF when the state transition matrix is identity and the process noise covariance matrix is zero. Therefore, the key difference between SLS and EKF is how measurements are processed with their a prior information, and SLS can be seen as a particular case of the EKF without state prediction.

Phase 2: Vehicle field-testing, parameters tuning, and performance evaluation of RTK GNSS + IMU C++ code.

- **Objective:** Evaluate the RTK GNSS + IMU integrated solution using real-world smartphone data collected from rural, suburban, and urban environments. Compare these results with other smartphone GNSS measurements and assess the performance against both professional-grade and low-cost hardware. Customize the pre- and post-processing algorithms and optimize the estimator software for the smartphone GNSS and IMU data. Conduct a comprehensive analysis to fully characterize the positioning performance.

Phase 3: Core PPP GNSS + IMU C++ code (simulated real-time processing).

- **Objective:** Develop a core multi-GNSS PPP processing engine utilizing the SLS and EKF estimators, leveraging the existing RTK code structure as a foundation. Subsequently, extend this framework to incorporate IMU sensor fusion capabilities.

Phase 4: Vehicle field-testing, parameters tuning, and performance evaluation of PPP GNSS + IMU C++ code.

- **Objective:** Phase 4 mirrors Phase 2 in structure, focusing on evaluating the PPP GNSS + IMU integrated solution using real-world smartphone data across diverse

multipath profiles. Additionally, it involves research and development of advanced algorithms aimed at enhancing traditional PPP algorithms for greater robustness and accuracy for smartphone applications.

Regarding the Jaqen coding domain, it is a simulated real-time smartphone processing toolkit comprising over 17,000 valid lines of C++ code, along with a corresponding Python analyst tool for solution assessment and performance visualization. Jaqen references from software processors such as York GNSS PPP engine, Natural Resources Canada (NRCan) PPP online service, RTKlib, and goGPS (Takasu et al., 2013; Aggrey, 2014; Herrera et al., 2016), a total of 14 solution projects are integrated into Jaqen, incorporating five main processing modules that contribute to the final navigation solutions. As illustrated in Figure 2.5, Jaqen utilizes the Configuration Controller to manage all input files, including GNSS and IMU measurements, precise satellite products, International GNSS Service (IGS) antenna correction files, as well as filter settings such as positioning modes, weighting schemes, rejection thresholds for pre- and post-fit residuals, and more. These configurations are then input into the Jaqen processing hub, which comprises three processing modules: the GNSS Handler, Correction Controller, and IMU Handler.

Following this, Jaqen automatically identifies the processing modes and processed constellations to apply error corrections. For RTK, this involves techniques like double-differencing, while for PPP, it includes modelling errors such as tropospheric hydrostatic delay, relativistic effects, site displacement, among others. Once the corresponding matrices, such as the design matrix, weight matrix, misclosure matrix, etc., are configured for both GNSS and IMU components, the processing hub automatically employs SLS or EKF to generate positioning solutions. Once the filtering quality checks, such as the IMU mechanization sanity check and pre- and post-fit residual rejections, are successfully completed, the Jaqen Output Handler produces output files. These files contain estimated parameters, processing parameters, solution quality indexes, as well as GNSS and IMU information, which are then provided to the user end.

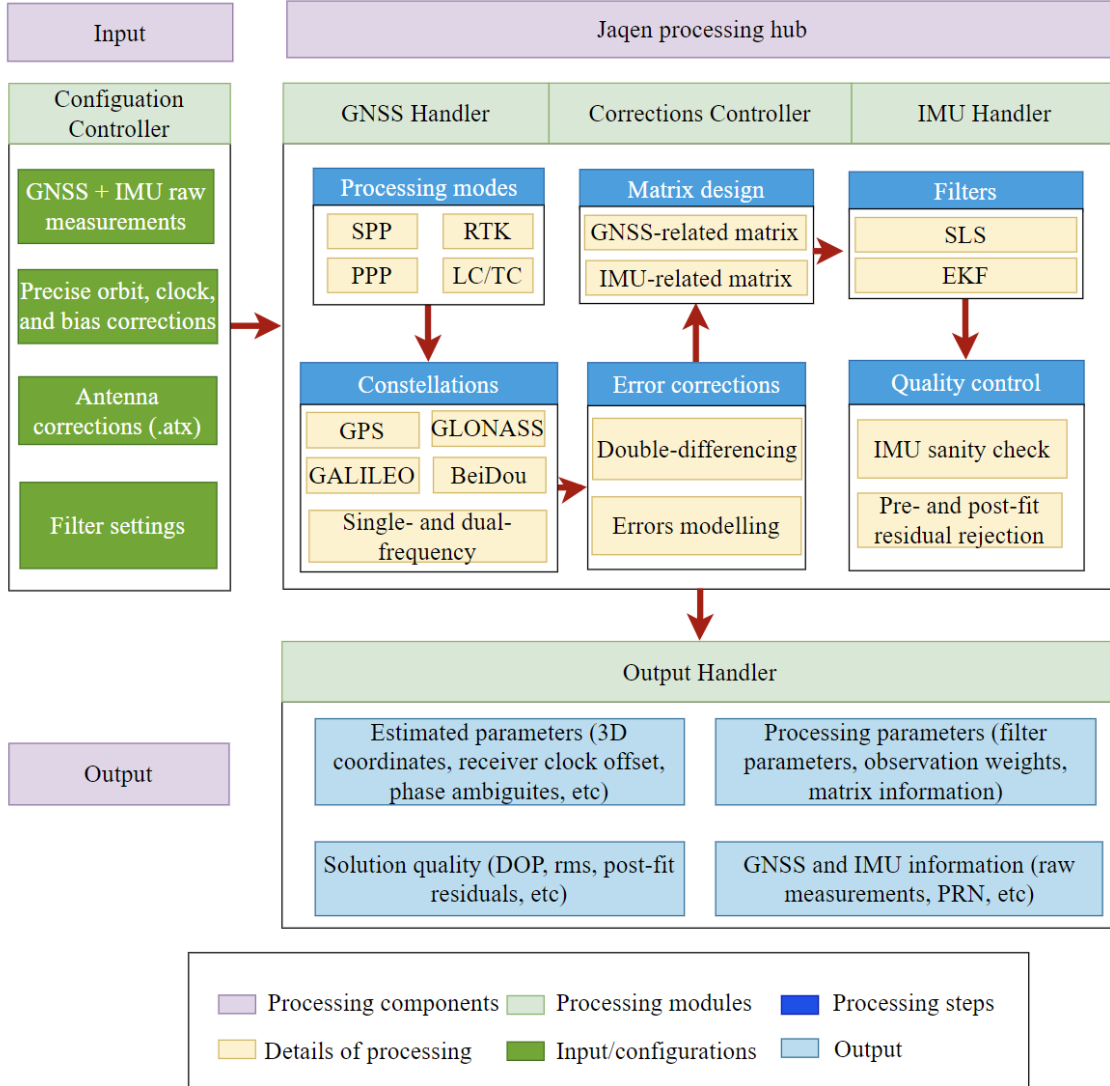


Figure 2.5: Architecture of Jaqen smartphone processing

2.5 Experimental setup and measurement campaigns

To evaluate the performance of the Jaqen processing toolkit and explore the proposed dissertation algorithms for smartphone positioning, this research utilized various grades of GNSS and IMU hardware during the measurement campaign. As shown in Figure 2.6, this research selected the NovAtel OEM7 as a representative of geodetic-grade GNSS receivers, while the SwiftNav Piksi and Septentrio Mosaic X5 were chosen as representatives for low-cost GNSS receivers. Similar experimental setups and measurement campaigns have been

explained in previous studies (Vana et al., 2019; Aggrey et al., 2020; Shinghal and Bisnath, 2021).

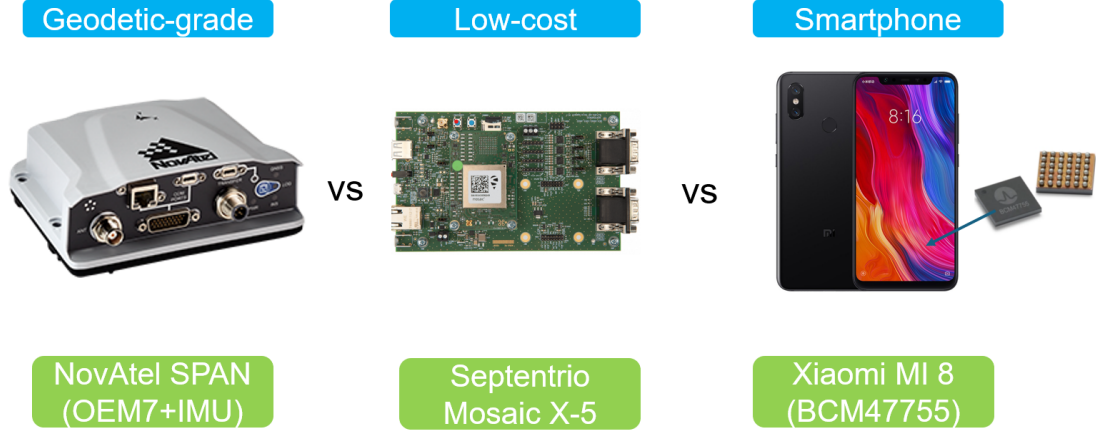


Figure 2.6: Three grades of experimental hardware

Additionally, smartphones equipped with BCM47755 GNSS chipsets, such as the Xiaomi MI 8 and Samsung Galaxy S21+, were employed for experiments. Both smartphones support tracking of GPS, GLONASS, Galileo, and BeiDou (GREC) constellations, in addition to GPS L5 and Galileo E5a signals. The relevant characteristics of these receivers are highlighted in Table 2.1.

Table 2.1: Comparison between different GNSS receivers

Receivers	Price (USD)	Tracked GNSS	Dual-frequency	Triple-frequency
NovAtel OEM7	~10,000	GREC	Yes	Yes
Septentrio Mosaic X5	~1,000	GREC	Yes	No
Xiaomi MI 8 (BCM47755)	~10	GREC	Yes (GE only)	No
<i>G: GPS, R: GLONASS, E: GALILEO, C: BEIDOU</i>				

The smartphones are equipped with consumer-grade inertial sensors. Their factory specifications are detailed in Table 2.2 in comparison with the automotive/industrial-grade Xsens MTi-7 example used alongside. The former class of inertial sensors usually cost less than 10 USD, whereas the latter class can cost up to 1000 USD. The specifications show a generally weaker performance of smartphone IMUs. Traditionally, it is expected that GNSS-denied navigation outages using an automotive/industrial grade IMU will be less than one minute.

By extension, smartphone IMUs should fill shorter GNSS-denied navigation outage periods. For the IMU model in Galaxy S21+, in-run bias is not found in the factory datasheet or in the literature.

Table 2.2: Factory specifications of the inertial sensors used in experiment (Yan et al., 2020)

Phone Models	Xiaomi MI 8	Samsung Galaxy S21+	Xsens MTi-7
IMU Model	InvenSense ICM-20690	ST-Microelectronics LSM6DSO	N/A
Gyroscope			
In-run bias stability ($^{\circ}/h$)	>1000	-	10
Noise density ($^{\circ}/s/\sqrt{Hz}$)	0.004	0.004	0.003
Standard full range ($\pm^{\circ}/s$)	2000	2000	2000
Accelerometer			
In-run bias stability (mg)	40	-	0.03
Noise density ($\mu g/\sqrt{Hz}$)	100	110	70
Standard full range ($\pm g$)	16	16	16

2.5.1 Static experiments

The static experiment was carried out on the roof of the Petrie Science and Engineering building at York University Toronto, Canada with the smartphone placed on a brick for 3 hours (see Figure 2.7).

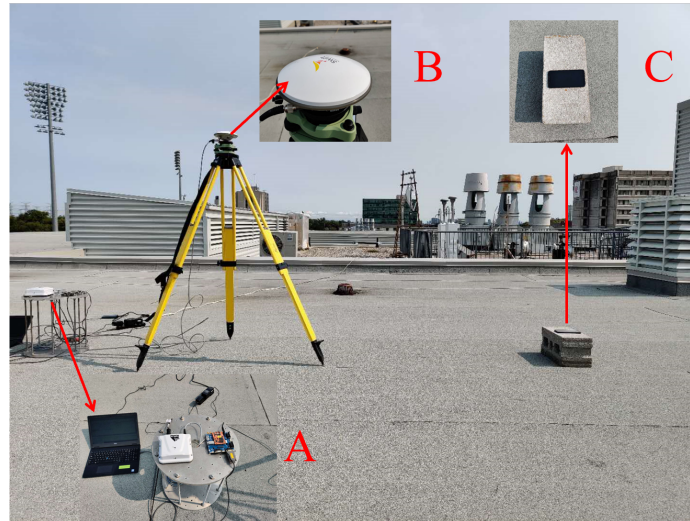


Figure 2.7: Static experiment setup with (A) NovAtel OEM7 and SwiftNav Piksi receivers, (B) SwiftNav antenna, and (C) Xiaomi MI 8

The NovAtel OEM7 and SwiftNav Piksi receivers (A) were set up and connected to the same geodetic antenna (B) through an antenna splitter. The NovAtel OEM7 few metre baseline RTK result processed by the commercial software Inertial Explorer represents the ground truth, and the horizontal and vertical distance between the SwiftNav antenna phase centre and Xiaomi MI 8 (C) had been taken into consideration.

2.5.2 Kinematic experiments

In assessing the performance of proposed algorithms and the processing toolkit for smartphone positioning in real-world driving environments, a series of vehicle experiments were conducted around York University, Toronto, Canada. The term "real-world driving environments" refers to conducting vehicle experiments in realistic settings, with smartphones mounted on the vehicle dashboard to replicate typical driving conditions and differentiate this work from previous studies, many of which have focused on walking experiments, simulated vehicle scenarios, or experiments involving smartphones mounted on vehicle rooftops. The experimental setup is illustrated in Figure 2.8.

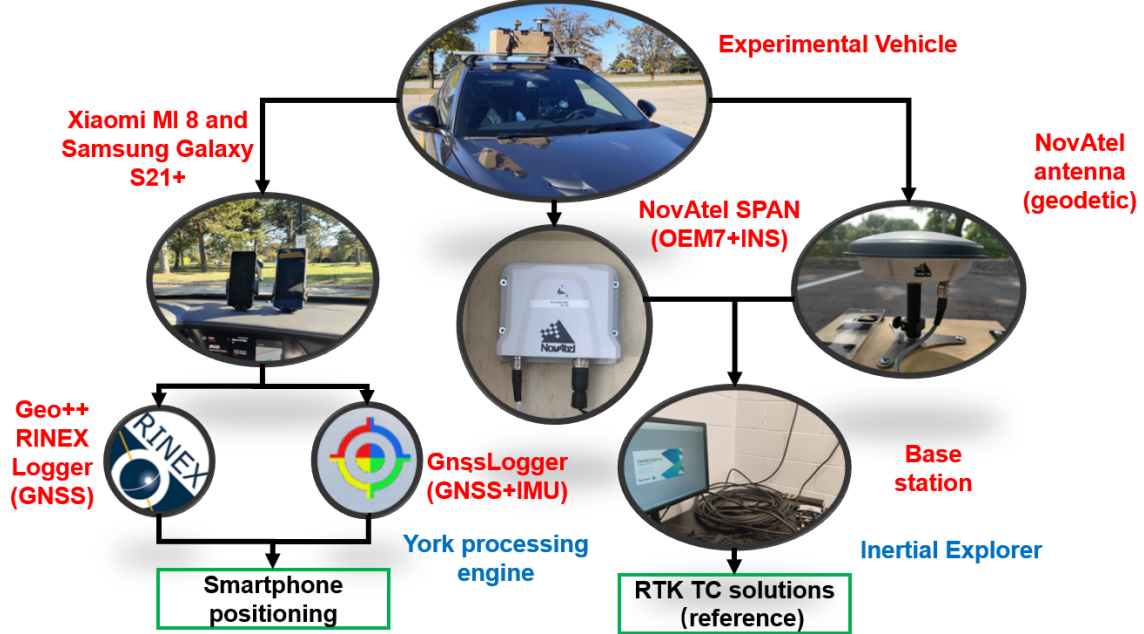


Figure 2.8: Experimental setup with smartphones and geodetic receivers: smartphone GNSS observations are retrieved from Geo++ RINEX Logger and processed with YorkPPP engine; NovAtel SPAN RTK/TC solutions are served as the reference

Figure 2.9 highlights the street view of one of the vehicle trajectories, with yellow arrows indicating the direction of movement. Additionally, a geofencing map is created to evaluate the positioning performance and analyze the raw measurements under different multipath profiles. The driving environments are classified into four categories: open-sky highways (A), vegetation (B), suburban roads (C), and overpasses (D). These categories are represented by purple, green, yellow, and pink polygons, respectively. It is also worth noting that the overpasses consist of two distinct areas, D1 corresponds to a short road passing under a railway bridge that spans approximately 10 m, while D2 represents an arterial road located beneath a highway viaduct, which is a more challenging area with a complete GNSS outage that extends over 100 m.

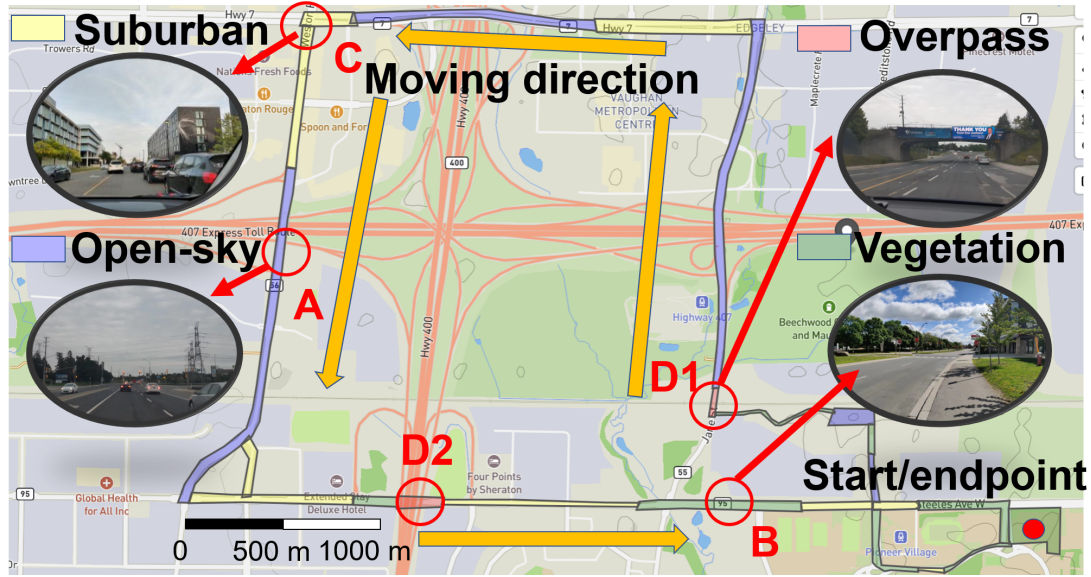


Figure 2.9: Street views of geofencing map with A) open-sky highways, B) vegetation, C) suburban roads, and D) overpasses

A combination of geodetic and automotive-grade GNSS and IMU sensors is employed to generate a reference positioning solution and compare raw measurements. These sensors are mounted on and inside a box placed on top of the vehicle. A NovAtel SPAN geodetic receiver with an IMU combination and a geodetic antenna are utilized, while a NovAtel base station is positioned on an open rooftop within a 5 km baseline to produce a post-processed, smoothed RTK/IMU tightly-coupled reference solution processed by the Inertial Explorer

software. Additionally, an Xsens MTi-7 automotive-grade IMU is used to offer a low-cost inertial sensor alternative to smartphone IMUs.

Smartphones Xiaomi Mi 8 and Samsung Galaxy S21+ were utilized in the kinematic experiments as well. The GnsLogger (Google) and Geo++ Rinex Logger (Geo++) apps are used concurrently to log raw IMU measurements and GNSS observations, respectively. In the initial stages of the research (2022 to 2021), the smartphones were affixed to the roof of the vehicle to ensure relatively strong GNSS signal reception with fewer blockages. As the research progressed (from 2022 to the present), this research transitioned to mounting the phones on the dashboard instead of the vehicle roof. This adjustment aimed to replicate real-world driving conditions, which present more challenging environments for smartphone positioning. Within the vehicle, GNSS signals experience significant refraction and blockages, imposing greater demands on our positioning algorithm.

Chapter 3

Ionospheric constraints in geodetic, low-cost, and smartphone PPP processing

The proliferation of low-cost GNSS hardware smartphones providing multi-constellation, multi-frequency carrier-phase and pseudorange measurements for the Android platform offers an avenue for GNSS PPP use in recent years. However, smartphone PPP is restricted due to high multipath effects and signal loss leading to metre-level accuracy and longer convergence time compared to professional-grade GNSS receivers. Proven by previous research (Zhang et al., 2013; Lou et al., 2016; Xiang et al., 2017), the combination of ionospheric constraints and receiver DCB estimation is a promising and efficient approach to improve PPP solution initialization. Therefore, this section analyzes the performance improvements by applying ionospheric constraints (IC) to PPP in comparison to conventional PPP solutions for different grades of GNSS receivers (geodetic, low-cost, and smartphone hardware) in open-sky and suburban environments. It is important to acknowledge that the content of this chapter draws upon the author's publication "Effects of ionospheric constraints in Precise Point Positioning processing of geodetic, low-cost, and smartphone GNSS measurements", which was published in *Measurement* journal (Yi et al., 2021).

3.1 Introduction and motivation

Over the past decades, the ionosphere-free model has been widely applied for PPP data processing given its ability to eliminate ionospheric delays through the dual-frequency linear combination (Cai et al., 2007; Ge et al., 2012). However, the drawbacks of this combination method cannot be ignored due mainly to the noise amplification and loss of ionospheric delay information, the latter of which can play an essential role in reducing PPP convergence time and initialization errors (Odijk, 2002; Tu et al., 2013; Banville et al., 2014). Therefore, there is a growing trend to utilize the uncombined PPP and with ionospheric constraints (PPP-IC) as an alternate approach in the precise positioning community. Shi et al. (2012) employed a prior ionospheric model to improve the single-frequency PPP performance based on GNSS raw measurements. A subsequent contribution from Zhang et al. (2013) demonstrated that PPP-IC convergence time is correlated with the quality of ionospheric maps used and receiver DCB through static experiments. Furthermore, not limited to single-frequency, PPP-IC showed a significant improvement for dual-frequency, multi-GNSS PPP solutions as well (Lou et al., 2016), and similar improvements can be also seen from recent studies (Xiang et al., 2017; Aggrey and Bisnath, 2019).

In response to market demands, a breakthrough happened in 2018 when the dual-frequency Xiaomi MI 8 was released, enabling ionospheric delays to be managed. These changes have motivated researchers to investigate whether there is the possibility of replacing geodetic-grade receivers with low-cost smartphones with PPP augmentation for particular mass-market applications. A detailed study on this topic compared the PPP performance of four newly released smartphones against geodetic-grade receivers, and results showed that it is feasible to achieve 0.4 m horizontal rms in static scenarios (Aggrey et al., 2020). More recently, PPP-AR was successfully performed using Xiaomi MI 8 with an external geodetic antenna. By fixing GPS and Galileo ambiguities, solutions can achieve centimetre-level accuracy under ideal static environments (Wen et al., 2020). Similarly, it is confirmed that centimetre-level static smartphone positioning is achievable through single-

baseline RTK processing (Dabove and Di Pietra, 2019a), and short-baseline RTK ambiguity resolution with the smartphone antenna (Gao et al., 2021).

Despite this remarkable progress, the main limitations of geodetic and smartphone PPP lie in its unstable positioning performance in challenging scenarios and long convergence times (Aggrey et al., 2020; Wu et al., 2019). Meanwhile, few recent studies focus on PPP-IC for smartphone measurements. The impact of different ionosphere modelling approaches for static single-frequency smartphone PPP solutions was analyzed in Banville et al. (2019). In this context, single- and dual-frequency smartphone PPP performance versus differential GNSS (DGNSS) in static and kinematic environments was assessed, but a Global Ionosphere Map (GIM) is only applied for single-frequency measurements (Lachapelle and Gratton, 2019). In view of all that has been mentioned so far, the significance of PPP-IC for the spectrum of next-generation GNSS hardware is unclear. Therefore, this research delves into the effects of ionospheric constraints on PPP processing of geodetic, low-cost, and smartphone GNSS measurements, aiming to enhance positioning accuracy and convergence time for various grades of GNSS receivers.

3.2 Mathematical models and data processing strategies

To have a better understanding of the methodology adopted in this research, this section begins with the basic equations of PPP-IC with receiver DCB estimation. Then, details on applying the ionospheric constraints and specific PPP settings are introduced.

3.2.1 Ionospheric-constrained PPP model with receiver DCB estimation

As outlined in Section 2.2.3, once the satellite DCB and clock offset are corrected, the UDUC PPP equations can be reformulated as Equation (3.1) for both L1 and L2 frequencies.

$$\begin{aligned}
P_{r,1}^s &= \rho_r^s + (cdt_r + b_{r,P_{IF}}) + M_r^s T_r + \bar{I}_{r,1}^s + \varepsilon_{P_1} \\
P_{r,2}^s &= \rho_r^s + (cdt_r + b_{r,P_{IF}}) + M_r^s T_r + \gamma_2 \bar{I}_{r,1}^s + \varepsilon_{P_2} \\
L_{r,1}^s &= \rho_r^s + (cdt_r + b_{r,P_{IF}}) + M_r^s T_r - \bar{I}_{r,1}^s + \lambda_1 \tilde{N}_{r,1}^s + \varepsilon_{L_1} \\
L_{r,2}^s &= \rho_r^s + (cdt_r + b_{r,P_{IF}}) + M_r^s T_r - \gamma_2 \bar{I}_{r,1}^s + \lambda_2 \tilde{N}_{r,2}^s + \varepsilon_{L_2}
\end{aligned} \tag{3.1}$$

where $\gamma_2 = \frac{f_1^2}{f_2^2}$ is the ratio of the frequencies, and the estimated ionospheric delay $\bar{I}_{r,1}^s$ is grouped with the real ionospheric delay $I_{r,1}^s$ the receiver DCB (Xiang et al., 2017; Xiang and Gao, 2017):

$$\bar{I}_{r,1}^s = I_{r,1}^s - \frac{1}{\gamma_2 - 1} DCB_r \quad (3.2)$$

Broadly speaking, treating the ionospheric information from a GIM as pseudo-observations is a popular approach and has been widely used in PPP-IC filter (Collins et al., 2012; Banville et al., 2014), the corresponding misclosure matrix can be expanded to:

$$\mathbf{W} = \begin{bmatrix} P_{r,1}^{s,estimated} - P_{r,1}^{s,observed} \\ L_{r,1}^{s,estimated} - L_{r,1}^{s,observed} \\ I_{r,1}^{s,estimated} - I_{r,1}^{s,GIM} + \frac{1}{\gamma_2 - 1} DCB_r \\ \vdots \end{bmatrix} \quad (3.3)$$

However, since more observations are introduced into the filter, the design, weighting, as well as variance-covariance matrix should be amplified in the sequential least-squares filter correspondingly, decreasing the computational efficiency. Meanwhile, the standard deviation and weighting of ionospheric delays need to be taken into consideration and tuned well. In response to this, a simpler approach is utilized to directly replace the estimated ionospheric delays with the ionospheric information provided by a GIM as a constraint for PPP solution and, consequently, the covariance of ionospheric errors needs to be adjusted in correspondence with the standard deviations of the GIM, indicating that the GIM would have more impact on the estimated ionospheric delays. To avoid adverse effects on the final solutions, it is crucial to properly adjust and tune covariance matrices. These two different PPP-IC approaches are compared later for smartphones in kinematic scenarios.

3.2.2 Stochastic weighting models

It is well known that measurement weighting plays a vital role in precise estimation. For geodetic and low-cost receivers, the elevation angle dependent weighting scheme is often

utilized and the standard deviation of the code observation σ is shown in Equation (3.4):

$$\sigma = \frac{\sigma_{90}}{a + b \sin(E)} \quad (3.4)$$

where σ_{90} is the standard deviation of the observation at zenith, E refers to the elevation angle, and a and b are constant coefficients that can be derived empirically. However, suggested by previous studies, the satellite elevation-based weighting scheme is inappropriate for smartphone processing, since the smartphone antenna picks up reflected signals contaminated with significant multipath from all directions and those multipath-affected signals have lower carrier-to-noise ratios (C/N_0), irrespective of elevation angle. Therefore, a C/N_0 -based weighting scheme is employed for Xiaomi MI 8 measurement processing (Banville et al., 2019; Zhang et al., 2018b; Shinghal and Bisnath, 2021). Correspondingly, the standard deviation of the code observation σ can be estimated as (Banville et al., 2019; Shinghal and Bisnath, 2021):

$$\sigma = a + b * 10^{-\frac{1}{2} * \frac{C/N_0}{10}} \quad (3.5)$$

where coefficient a is 4 m and 6 cm for pseudorange and carrier-phase observations, respectively. These values are empirically derived from the residuals. And b is the pseudorange chipping length and carrier-phase wavelength for pseudorange and carrier-phase measurements, respectively (Shinghal and Bisnath, 2021).

3.2.3 Smartphone PPP processing settings

Highlighted in Table 3.1 are the processing strategies adopted for different parameter estimation and precise product usage in subsequent results. The aforementioned elevation angle and C/N_0 -based weighting schemes are adopted for standalone receivers (geodetic and low-cost) and smartphones, respectively. The PPP engine is configured to automatically filter out satellites with post-fit residuals (residuals between the observed and the computed values after the SLS/EKF has been performed) exceeding ten times the empirical standard deviations of the measurements in order to mitigate the impact of outliers. Therefore, different standard deviations of pseudorange and carrier-phase measurements are

derived empirically for each receiver as shown in Table 3.1. Similar processing settings can be also seen from previous research (Geng et al., 2019; Banville et al., 2019; Wang et al., 2021). Furthermore, it should be noted that this research utilized the combined final IGSG GIM products in the data processing. Since the accuracy and reliability of the GIM standard deviations are varied from different IAACs (Zhao et al., 2021), the covariance setting for the GIM should be amplified to accommodate the GIM errors (Aggrey and Bisnath, 2019). Meanwhile, owing to the estimated ionospheric delays are inaccurate and unreliable during the PPP initialization stage for smartphone, a certain degree of deviation of the GIM standard deviations can be tolerated.

Table 3.1: Processing engine settings for NovAtel OEM7, SwiftNav Piksi, and Xiaomi MI 8

PPP-IC settings	Smartphone	Piksi and OEM7
Precise corrections	GFZ rapid products	GFZ rapid products
Tropospheric delays	Hydrostatic: GMF modelling Wet: estimated	Hydrostatic: GMF modelling Wet: estimated
Ionospheric delays	Constrained by GIM	Constrained by GIM
Covariance of ionospheric errors	Amplified GIM STD	Amplified GIM STD
Weighting schemes	Carrier-to-noise based	Satellite elevation based
STD of pseudorange (m)	4	Piksi: 0.4 OEM7: 0.1
STD of carrier-phase (m)	0.06	Piksi: 0.004 OEM7: 0.001

3.3 PPP-IC performance assessment

To investigate the impact of PPP solutions with ionospheric constraints for different GNSS hardware categories, the NovAtel OEM7 and SwiftNav Piksi were selected as representatives of geodetic-grade and low-cost GNSS receivers, respectively. A Xiaomi MI 8 phone embedded with a BCM47755 chip was employed for experiments. The relevant characteristics of these receivers are highlighted in Table 2.1. For consistency, only GPS and Galileo dual-frequency signals are processed considering the limited tracking capability of the Xiaomi MI 8. Furthermore, it must be pointed that both static and kinematic experiments were conducted in September 2020, with 2.7 Kp and 71.7 solar flux units F10.7 indexes on average (available from <https://omniweb.gsfc.nasa.gov/form/dx1.html>), indicating that

ionospheric and geomagnetic activities were undergoing a quiet period (Liu et al., 2018). Take the static experiment for example, Figure 3.1 shows the GIM standard deviations of GPS and Galileo satellites utilized by smartphone processing on Day of Year (DOY) 259, 2020, ranging from 0.1 to 0.35 m.

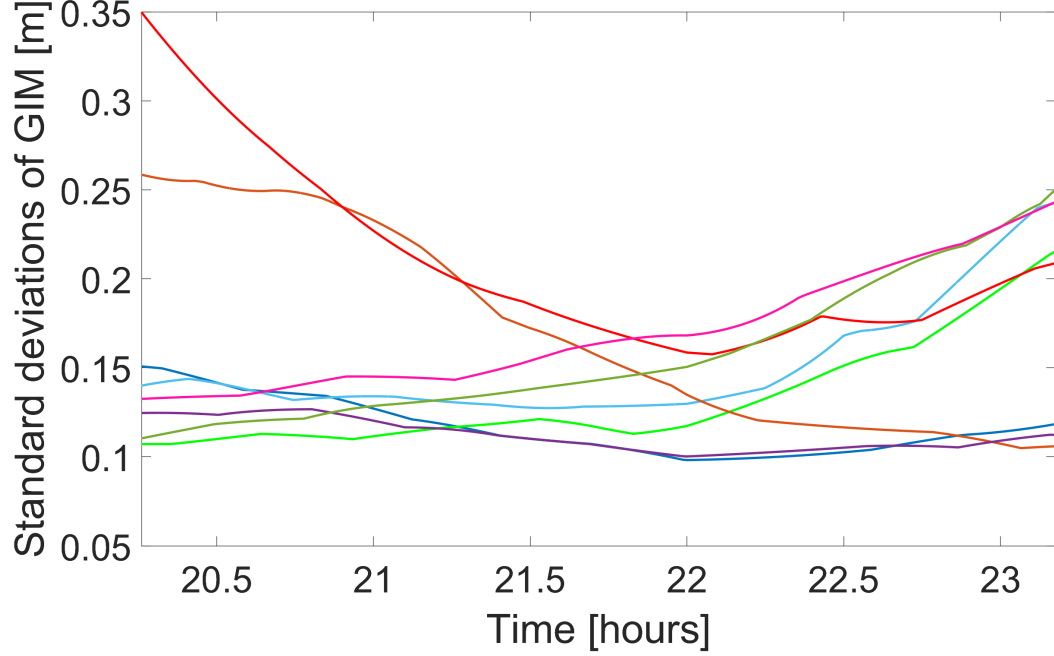


Figure 3.1: Time series of GIM standard deviations of GPS and Galileo satellites utilized by smartphone processing on DOY 259, 2020 (GPS time)

3.3.1 Static PPP-IC results

Data collected under the open-sky scenario on DOY 259 for 3 hours were chosen for static processing. The horizontal PPP performance comparison of these three receivers processed with and without IC is shown in Figure 3.2, the overall solutions of Xiaomi MI 8, SwiftNav Piksi, and NovAtel OEM7 are presented in different colours. To distinguish the performance of conventional PPP and PPP-IC, solid and dashed lines were drawn for better clarification to represent the positioning errors of PPP and PPP-IC, respectively. Table 3.2 highlights the horizontal rms after convergence and convergence time for different GNSS receivers.

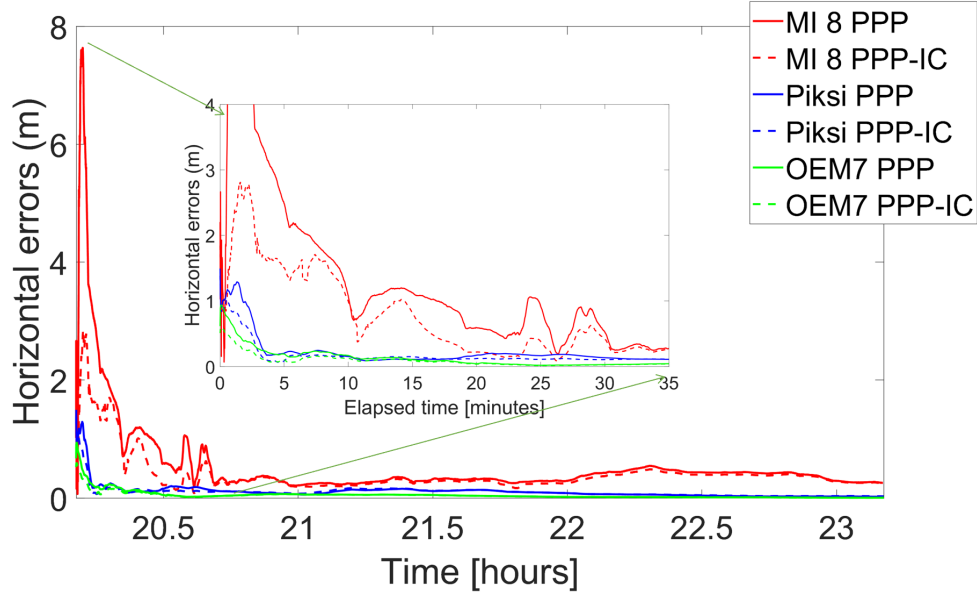


Figure 3.2: Horizontal performance comparison with conventional PPP and PPP-IC processing strategies for Xiaomi MI 8, SwiftNav Piksi, and NovAtel OEM7 in the static scenario

Table 3.2: Horizontal rms after convergence and convergence time for Xiaomi MI 8, SwiftNav Piksi, and NovAtel OEM7

Receivers	Horizontal rms after convergence (cm)		*Convergence time (mins)	
	Conventional PPP	PPP-IC	Conventional PPP	PPP-IC
Xiaomi MI 8	37	33	25	14
SwiftNav Piksi	9	9	33	33
NovAtel OEM7	3	3	15	15
* Convergence threshold is 10 cm for OEM7 and Piksi, 1 m for MI 8				

It can be seen from Figure 3.2 and Table 3.2 that the horizontal rms after convergence of NovAtel OEM7 (3 cm) outperformed the SwiftNav Piksi (9 cm) and the Xiaomi MI 8 (37 cm) in the conventional PPP mode, owing to its better hardware. Meanwhile, there are some error fluctuations or jumps after initialization that can be observed for smartphone solutions on account of its noisy measurements. In terms of the convergence time, it is imperative to note that 10 cm is too rigorous considering the decimetre- to metre-level positioning capability for the smartphone, thereby the convergence threshold is increased from 10 cm to 1 m for the smartphone dataset. Interestingly, no obvious improvement can be found for OEM7 and Piksi to reach convergence (10 cm) compared to conventional PPP,

which was due mainly to 2-8 TECU (32-130 cm difference on L1 frequency) inherited errors from the GIM (Hernández-Pajares et al., 2009). However, PPP-IC provides a significant improvement regarding smartphone, expressed as the convergence time is shorten from 25 minutes to 14 minutes to attain a 1 m level of horizontal accuracy.

Given that the benefits of ionospheric constraints are most noticeable within the initialization stage of the PPP processing, the investigation focuses on horizontal positioning performance in the first 20 minutes. As shown in Table 3.3, it is clear that there is an obvious trend of reduction of horizontal errors at the first epoch, as well as the horizontal rms for all of the receivers when applying PPP-IC, which are expected. Surprisingly, what stands out in the statistics is the improvement of PPP-IC for the smartphone, leading to 30% and 57% reduction concerning the horizontal errors at the first epoch and first 20 minutes horizontal rms, respectively, which are greater than that of the OEM7 and Piksi. In other words, this research finds that PPP-IC is capable of providing more corrections or constraining for low-quality measurements before solutions reach convergence.

Table 3.3: Horizontal error at first epoch and horizontal rms within first 20 minutes for Xiaomi MI 8, SwiftNav Piksi, and NovAtel OEM7

Receivers	Horizontal error at first epoch (m)			Horizontal rms within first 20 minutes (m)		
	PPP	PPP-IC	Improvement	PPP	PPP-IC	Improvement
MI 8	2.6	1.8	30%	2.6	1.1	57%
Piksi	1.5	1.2	21%	0.4	0.3	26%
OEM7	0.6	0.5	18%	0.3	0.2	23%

To better appreciate the impact of ionospheric constraints on the precision from GNSS filters, Figure 3.3 displays the horizontal covariance time series with and without the GIM information for these aforementioned receivers. In this context, the horizontal covariance refers to the updated user position in the local coordinate system for each epoch, it can be seen as the internal precision obtained from the filter. Similarly, it appears that PPP-IC is able to reduce horizontal covariance significantly for smartphone solutions in the first 5 minutes. In contrast, PPP-IC has little success in influencing horizontal covariance for low-cost and geodetic receivers.

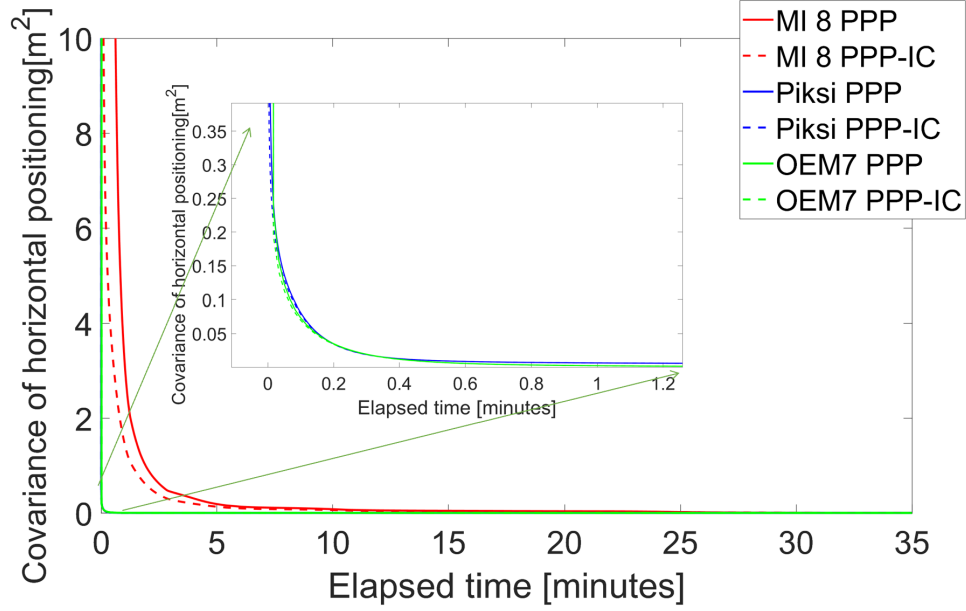


Figure 3.3: Horizontal performance comparison with conventional PPP and PPP-IC processing strategies for Xiaomi MI 8, SwiftNav Piksi, and NovAtel OEM7 in the static scenario

Note that the constrained time plays a critical role in PPP-IC solutions, the ionospheric constraints are applied only in the first epoch for geodetic and low-cost receivers due to errors from GIM corrections cannot be ignored when reaching convergence. To determine the impact of different constraining time for smartphone measurements, Figure 3.4 displays three different processing strategies for smartphone positioning, in which the red line represents the traditional PPP solution, and the blue and green lines represent solutions processed by PPP-IC with different constrained times. Several conclusions can be drawn from this plot. First, it is expected that the blue line takes less time to become identical to PPP solution (red) without any constraints given the ionospheric constraints are applied only for the first epoch. Meanwhile, it is interesting to find that by extending the constrained time until solution convergence, the green line enjoys better positioning performance during the initialization stage, mitigating the horizontal rms from 2.6 m (red) and 1.5 m (blue), to 1.1 m within the first 20 minutes. The most likely cause of such an improvement is that estimating ionospheric errors correctly is more difficult when it comes to the smartphone PPP owing to its poor positioning capability (decimetre to metre level) and frequent loss

of observation measurements. Based on this analysis, applying external GIM corrections continuously until the PPP solution converges is selected as the processing strategy for smartphone PPP-IC.

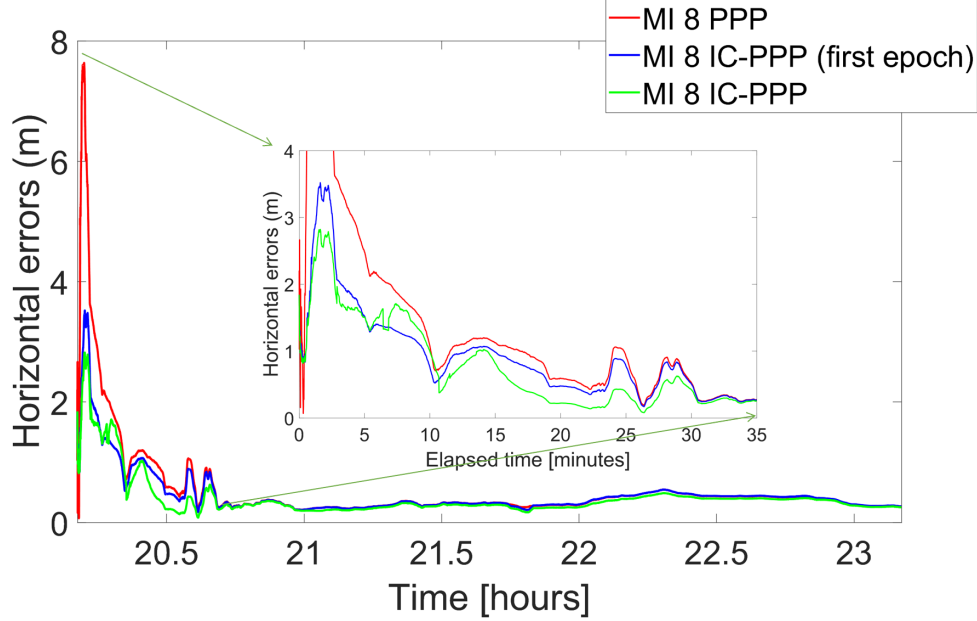


Figure 3.4: Horizontal performance comparison with conventional PPP and PPP-IC with different constraining time for Xiaomi MI 8

To investigate the validity and repeatability of the experimental results, the static smartphone dataset was split into 9 segments, meaning the processing engine resets and continues to process the data every 20 minutes. Corresponding results are presented in Figure 3.5, where the solid and dashed lines stand for the smartphone dataset processed by conventional PPP and PPP-IC, respectively. The detailed statistics at the first epoch and horizontal rms over each reset time are provided in Table 3.4 and Table 3.5, respectively. It is worth noting that there are fluctuations in smartphone PPP solutions during the initialization stage, which are due mainly to noisy smartphone measurements. Considering that, it is difficult to estimate ionospheric delays, receiver clock offset, as well as carrier-phase float ambiguities correctly at the outset without any constraint. For instance, the horizontal error is about 8.4 m when the elapsed time is 60 minutes, but increasing to 17.2 m in the next epoch. In short, PPP-IC is able to provide significant improvements for the MI 8 in the first 20 min-

utes, resulting in the horizontal errors at the first epoch for each reset time being reduced from 6.9 m, on average, to 1.3 m, and the horizontal rms is reduced from 1.7 m to 0.9 m. In terms of Piksi and OEM7 receivers, the same reset experiments were carried out as well, and most of the solutions follow the same pattern as shown in Figure 3.2.

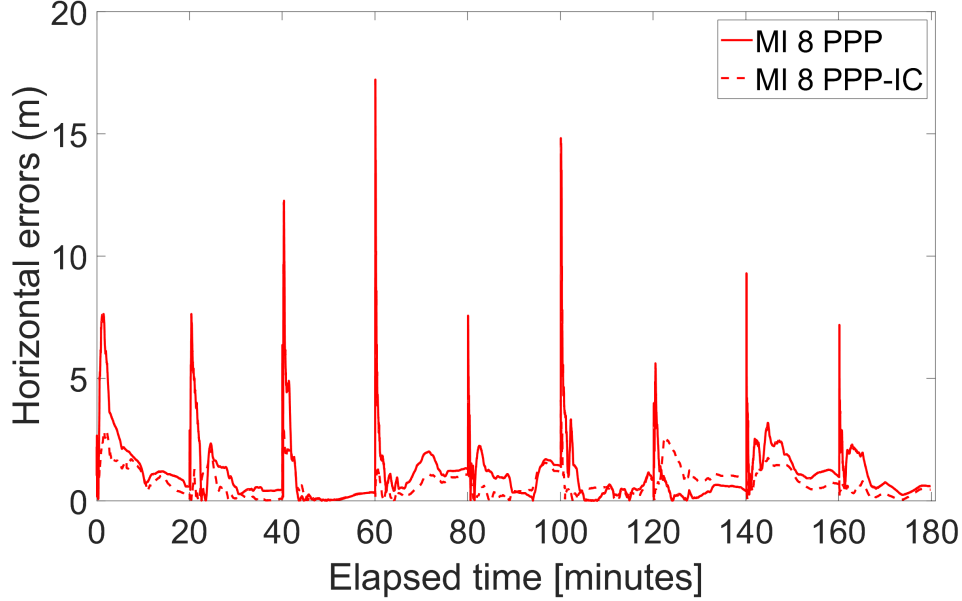


Figure 3.5: Horizontal errors comparison for smartphone reset experiment

However, the occurrence of an outlier cannot be neglected at a period of 120-140 minutes (see Figure 3.5). During this interval, there is an evident approximately 10% degradation in horizontal positioning accuracy when applying PPP-IC. Theoretically, ionospheric delays from a GIM are more accurate and reliable than initial estimates from the PPP filter, accounting for the improvements from PPP-IC. However, it cannot be neglected that in some cases, the information provided by the GIM is not as accurate as the ionospheric estimates from the PPP engine for certain epochs, resulting in increases of post-fit residuals and consequently, some measurements would be regarded as outliers and mistakenly rejected. Thus, the solution is negatively impacted by the PPP-IC owing to the GIM over-constrained measurements. Admittedly, it is difficult to distinguish whether the residual is caused by GIM constraints or measurement noise, which will be the subject of further testing.

Table 3.4: Horizontal error at the first epoch for Xiaomi MI 8 measurements with and without ionospheric constraints (reset every 20 minutes)

Elapsed time (mins)	0-20	20-40	40-60	60-80	80-100	100-120	120-140	140-160	160-180
PPP (m)	2.6	2.9	6.4	8.4	7.6	14.8	3.1	9.3	7.2
PPP-IC (m)	1.8	0.3	3.0	0.5	1.6	4.1	0.1	0.2	0.4
Improvement	30%	89%	53%	94%	79%	72%	97%	97%	95%

Table 3.5: Horizontal rms over first 20 minutes for Xiaomi MI 8 measurements with and without ionospheric constraints (reset every 20 minutes)

Elapsed time (mins)	0-20	20-40	40-60	60-80	80-100	100-120	120-140	140-160	160-180
PPP (m)	2.6	1.7	1.8	2.0	1.3	1.9	0.9	1.8	1.2
PPP-IC (m)	1.1	1.1	0.8	0.9	0.8	0.7	1.0	1.1	0.9
Improvement	57%	36%	57%	56%	40%	65%	-10%	40%	23%

3.3.2 Kinematic PPP-IC results

A vehicle experiment was carried out on DOY 252, 2020 around York University, Toronto, Canada for 30 minutes. Figure 3.6 shows the time series of horizontal errors for these three receivers with and without ionospheric constraints, and the NovAtel OEM7 RTK result processed by the NovAtel Inertial Explorer represents the ground truth.

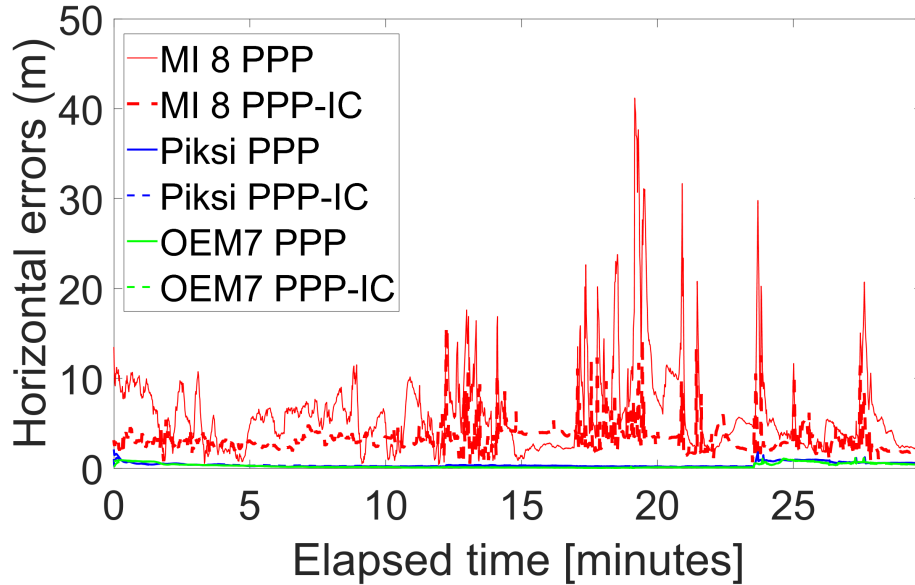
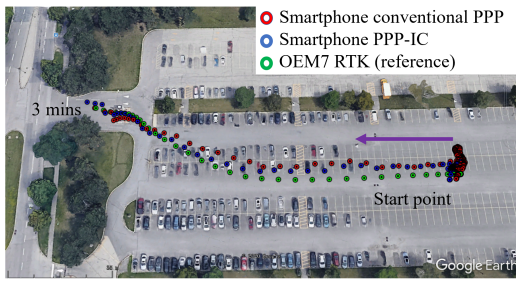
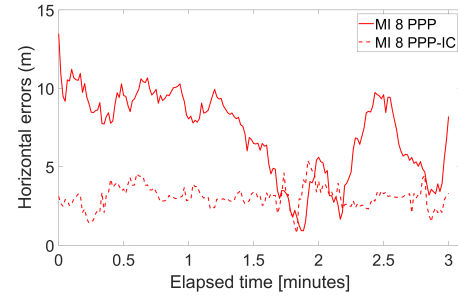


Figure 3.6: Horizontal performance comparison with conventional PPP and PPP-IC processing strategies for Xiaomi MI 8, SwiftNav Piksi, and NovAtel OEM7 in the kinematic scenario

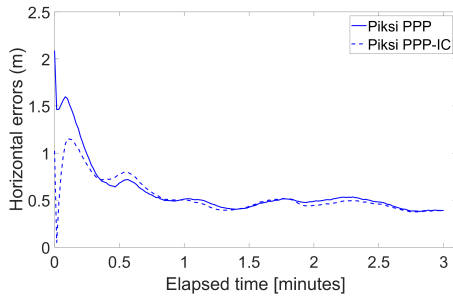
Two specific drive portions were selected in the following PPP kinematic analysis, namely scenarios in a parking lot (A) and suburban environment (B). Figure 3.7a illustrates three trajectories in a parking lot scenario, where smartphone conventional PPP and PPP-IC trajectories are represented by red and blue dots, respectively, and referenced NovAtel OEM7 RTK solutions and moving direction are marked by green dots and purple arrow, respectively. It is worth noting that the SwiftNav antenna was mounted on the vehicle roof resulting in good signal availability, while the smartphone was set up on the vehicle dashboard to match realistic usage and susceptible to signal blockage, carrier-phase measurements missing and multipath effects. Compared to the OEM7 RTK results (reference), it is anticipated that the horizontal rms in the parking lot (0-3 minutes) of the smartphone (7.6 m) is not comparable to that of Piksi (0.7 m) and OEM7 (0.6 m) in conventional PPP processing mode.



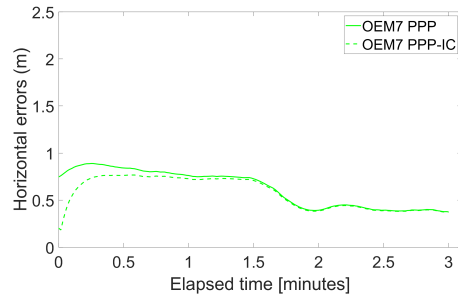
(a) Google Earth view of smartphone positioning with conventional PPP and PPP-IC



(b) Horizontal errors comparison between PPP and PPP-IC with Xiaomi MI 8 measurement



(c) Horizontal errors comparison between PPP and PPP-IC with SwiftNav Piksi measurement



(d) Horizontal errors comparison between PPP and PPP-IC with NovAtel OEM7 measurement

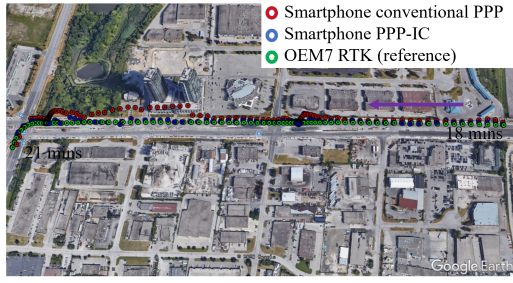
Figure 3.7: Horizontal performance comparison with conventional PPP and PPP-IC processing strategies for Xiaomi MI 8, SwiftNav Piksi, and NovAtel OEM7 at a parking lot in the kinematic scenario

As highlighted in Table 3.6, some significant improvements draw our attention to the smartphone PPP-IC solutions in the kinematic scenario. 77% and 59% improvements are observed for MI 8 in terms of first epoch horizontal errors and horizontal rms in the first 3 minutes, respectively, which are greater than that of Piksi and OEM7. Meanwhile, it is important to note that the horizontal improvement for OEM7 in the first epoch is 74% though, there are more corrections for Piksi and smartphone solutions in magnitude can be found when ionospheric constraints are applied, implying that PPP-IC has greater impact for such low-quality noisy measurements.

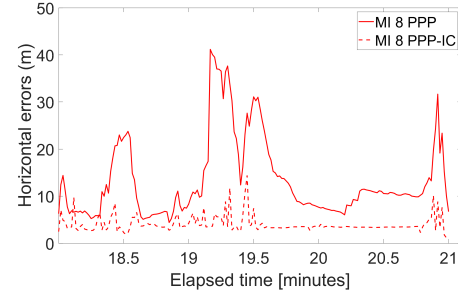
Table 3.6: Horizontal error at the first epoch and horizontal rms at the parking lot (3 minutes) for NovAtel OEM7, SwiftNav Piksi, and Xiaomi MI 8

Receivers	Horizontal error at first epoch (m)			Horizontal rms (m)		
	PPP	PPP-IC	Improvement	PPP	PPP-IC	Improvement
MI 8	13.5	3.1	77%	7.6	3.1	59%
Piksi	2.1	1.0	51%	0.7	0.6	12%
OEM7	0.7	0.2	74%	0.6	0.5	8%

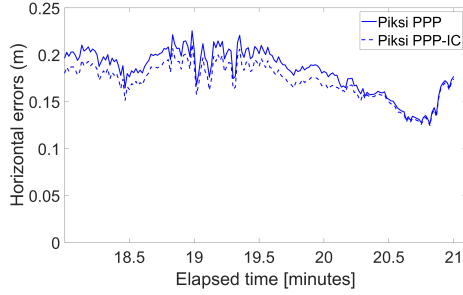
Another portion of the trajectory in a suburban environment (18-21 minutes) was selected for the kinematic analysis. As shown in Figure 3.8a, some positioning errors occurred owing to multipath errors when driving close to high-rise buildings, resulting in part of the conventional PPP trajectory (red dots) is overlapped with roadside construction. Interestingly, a significant level of improvement is noticed in Figure 3.8b and Table 3.7, PPP-IC smartphone solutions show 72% horizontal improvement as compared to the conventional PPP solutions. Furthermore, there is a significant decrease can be observed for smartphone solutions with the standard deviation reduces from 8.5 m to 1.8 m, indicating how accurate and smooth the smartphone PPP-IC solutions can realize even in the suburban environment. As for the Piksi and OEM7 results, PPP-IC results in almost no improvements after convergence. These results provide important insights into how PPP-IC impacts different grades of GNSS receivers: as the quality of collected measurements decreases, PPP-IC tends to provide more corrections for positioning solutions.



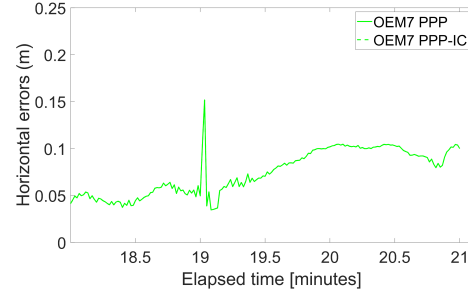
(a) Google Earth view of smartphone positioning between conventional PPP and PPP-IC



(b) Horizontal errors comparison between PPP and PPP-IC with Xiaomi MI 8 measurement



(c) Horizontal errors comparison between PPP and PPP-IC with SwiftNav Piksi measurement



(d) Horizontal errors comparison between PPP and PPP-IC with NovAtel OEM7 measurement

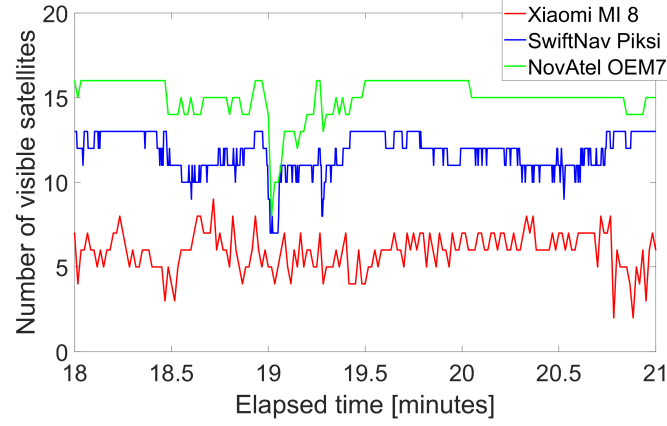
Figure 3.8: Horizontal performance comparison with conventional PPP and PPP-IC processing strategies for Xiaomi MI 8, SwiftNav Piksi, and NovAtel OEM7 in a suburban environment for the kinematic scenario

Table 3.7: Horizontal rms in a suburban environment (3 minutes) for NovAtel OEM7, SwiftNav Piksi, and Xiaomi MI 8

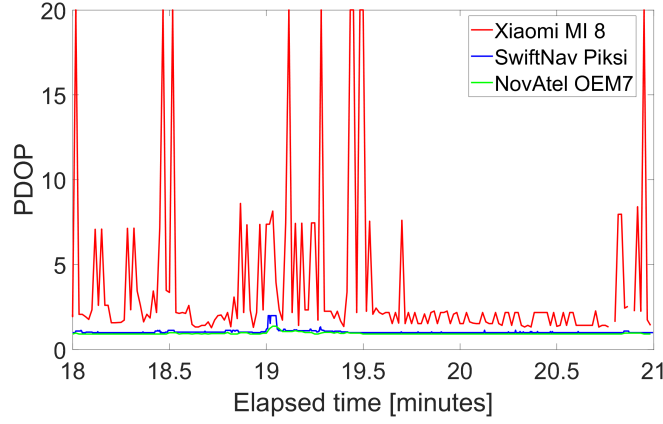
Receivers	Horizontal rms (m)		
	PPP	PPP-IC	Improvement
MI 8	15.9	4.4	72%
Piksi	0.2 (0.18)	0.2 (0.17)	5%
OEM7	0.1	0.1	0%

To assess the impact of data collection in a suburban environment for different GNSS receivers, Figure 3.9 presents the number of satellites (GPS and Galileo) and Position Dilution Of Precision (PDOP) for the three chosen GNSS receivers. As shown in Figure 3.9a, it is clear that more satellites were tracked by the Piksi and OEM7 than the MI 8, which is expected given the hardware discrepancy. The average number of satellites for the MI 8,

Piksi, and OEM7 are 5.9, 11.7, and 15.0, respectively, leading to the considerable PDOP differences shown in Figure 3.9b.



(a) Number of satellites comparison



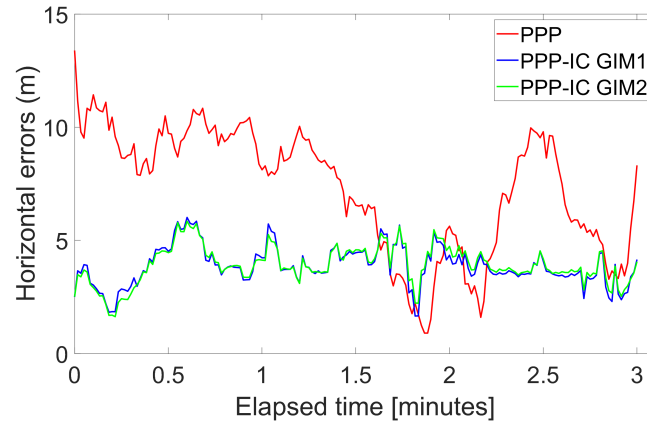
(b) PDOP comparison

Figure 3.9: Number of satellites and PDOP comparison for Xiaomi MI 8, SwiftNav Piksi, and NovAtel OEM7 at the suburban scenario

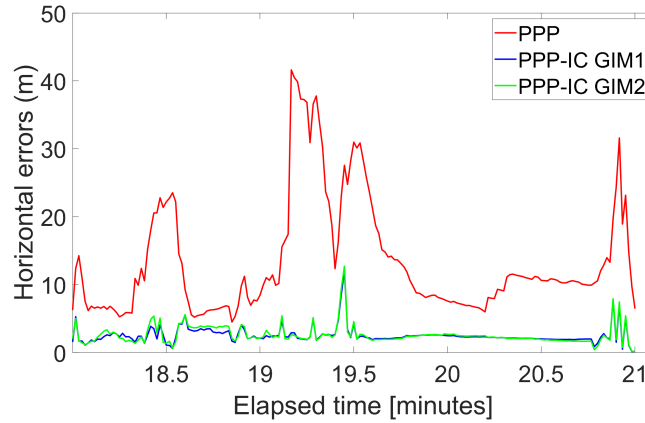
Furthermore, by comparing Figure 3.8b and Figure 3.9b, a positive correlation can be found between PDOP and horizontal error for smartphone solutions, especially when high-rise buildings block satellite signals, e.g., the elapsed time at 18.5, 19.5, and 21 minutes. Similarly, the variation of visible satellites explains small biases for Piksi and OEM7 horizontal solutions from Figure 3.8. Taken together, these results suggest that, on the one hand, Piksi and OEM7 PPP solutions are more resilient to the signal blockage prone environments, thereby no significant improvement can be observed when applying ionospheric

constraints in the suburban environment. On the other hand, in the case of a high PDOP scenario for the smartphone, PPP-IC is capable of providing significant corrections and stable solutions.

Figure 3.10 shows the comparison of two PPP-IC approaches mentioned in Section 3.2.1, "GIM 1" presented by blue lines refer to PPP-IC solutions which apply GIM corrections directly, and "GIM 2" presented by green lines stand for PPP-IC solutions treating the GIM information as pseudo-observations.



(a) Two PPP-IC approaches comparison for MI 8 at the parking lot



(b) Two PPP-IC approaches comparison for MI 8 at the suburban scenario

Figure 3.10: The comparison of two different PPP-IC approaches for the MI 8 in the kinematic scenarios

Apparently, "GIM 1" provides results almost identical to "GIM 2" and both of these two approaches have significant improvements for smartphone PPP solutions (red lines). Such performances are possible owing to the embedded linearly low-cost antenna, leading to frequent satellites missing and filter re-initialization in smartphone PPP processing. Given that, it is expected that the ionospheric delays from conventional PPP are inaccurate and unstable under kinematic environments, thereby the filter would greatly "trust" the external corrections from GIM rather than the filter estimated ionospheric errors, irrespective of the PPP-IC methods.

3.4 Effect of ionospheric constraints on PPP states

The previous results have shown that PPP-IC has significant improvements for smartphone positioning performance in suburban environments where the satellite geometry is considered to be weak. To better understand the impact of ionospheric constraints on PPP states, i.e., estimated ionospheric delays, tropospheric delays, receiver clock offset, and float ambiguities, these states are compared between conventional PPP and PPP-IC. The kinematic datasets collected on DOY 252, 2020 were used for the analysis.

Figure 3.11a and Figure 3.11b illustrate the estimated slant ionospheric delays and their difference for satellite E27 on the E1 frequency. Owing to the signal blockage in the suburban area, frequent loss of E27 tracking led to the missing measurements (gaps) and re-initialization errors, and this is why ionospheric delays estimated from conventional PPP (red line) did not settle down with time in Figure 3.11a. By contrast, it is noticeable that the curve smoothness of estimated ionospheric delays from PPP-IC (blue line), indicating that by constraining the ionospheric delay for a smartphone until the PPP solution converges, the GIM is capable of providing continuous and stable corrections for estimated ionospheric errors even when there are measurement gaps. On the other hand, it appears that the GIM constraint plays a critical role in the initialization and re-initialization epochs (Paziewski, 2020b). Accordingly, there are numerous jumps can be observed in the difference curve (time series of differenced value with/without applying PPP-IC) in Figure 3.11b. Similarly, Figure

3.11c and Figure 3.11d display the comparison and differences of tropospheric estimations with and without ionospheric constraints. Unlike the behaviour of ionospheric estimates, the value of tropospheric estimate differences are almost zero at the first epoch and ranging within 6 cm, implying the PPP-IC has little effect on the tropospheric estimations.

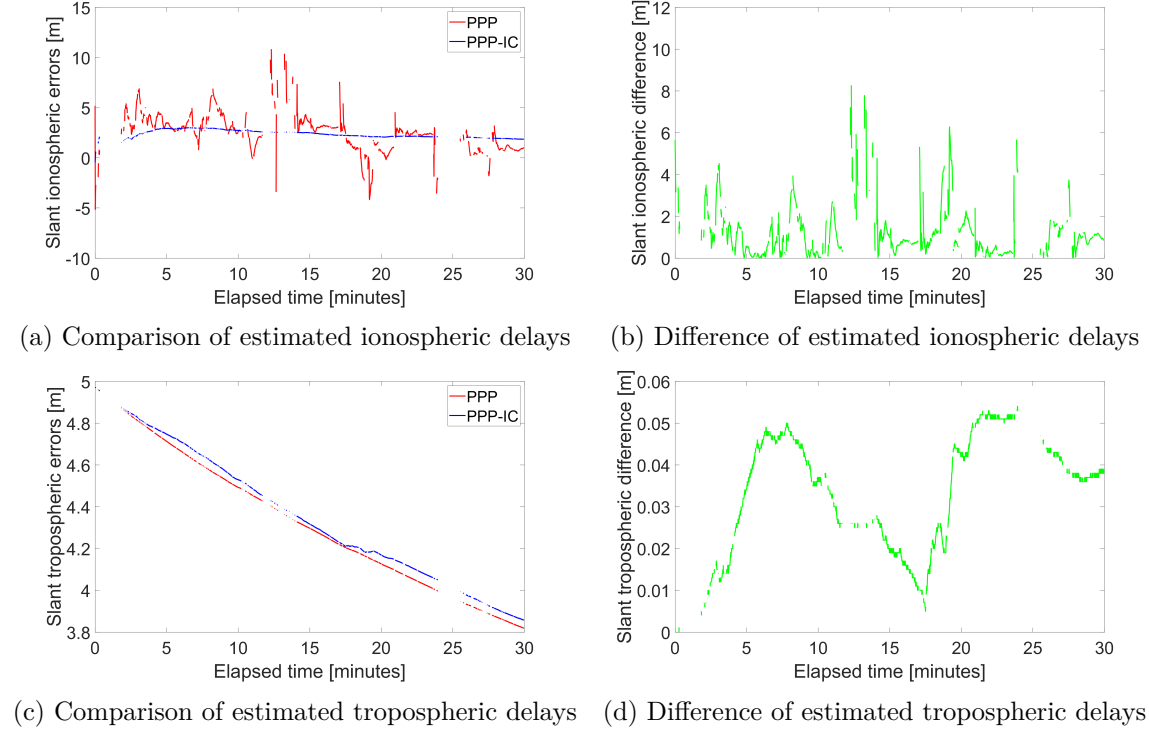


Figure 3.11: Estimated ionospheric and tropospheric delays comparison with and without applying ionospheric constraints for satellite E27

Two other critical parameters estimated in the PPP solution are the receiver clock offset and carrier-phase ambiguities. Figures 3.12a-3.12d highlight the comparison and differences of estimated Galileo receiver clock offset and E1 float ambiguities of satellite E27, respectively, and there are some interesting characteristics worth noting. First, it is clear that both conventional PPP and PPP-IC estimations are not identical to each other from the beginning, i.e., up to 13 m and 15 m differences in the first epoch can be observed for the Galileo receiver clock offset and E1 float ambiguities respectively, indicating that ionospheric constraints affect these states from the first epoch. Secondly, the ionospheric constraints aid in stabilizing the estimated E1 float ambiguity estimates, reducing the standard deviation

from 3.1 m to 0.6 m. Such performance is possible thanks to the extra information from the GIM, which reduces the errors in the estimated ionospheric delays and, consequently, these more accurate ionospheric delays and scaling ionospheric variance provide a positive impact on receiver clock offset and float ambiguities estimation.

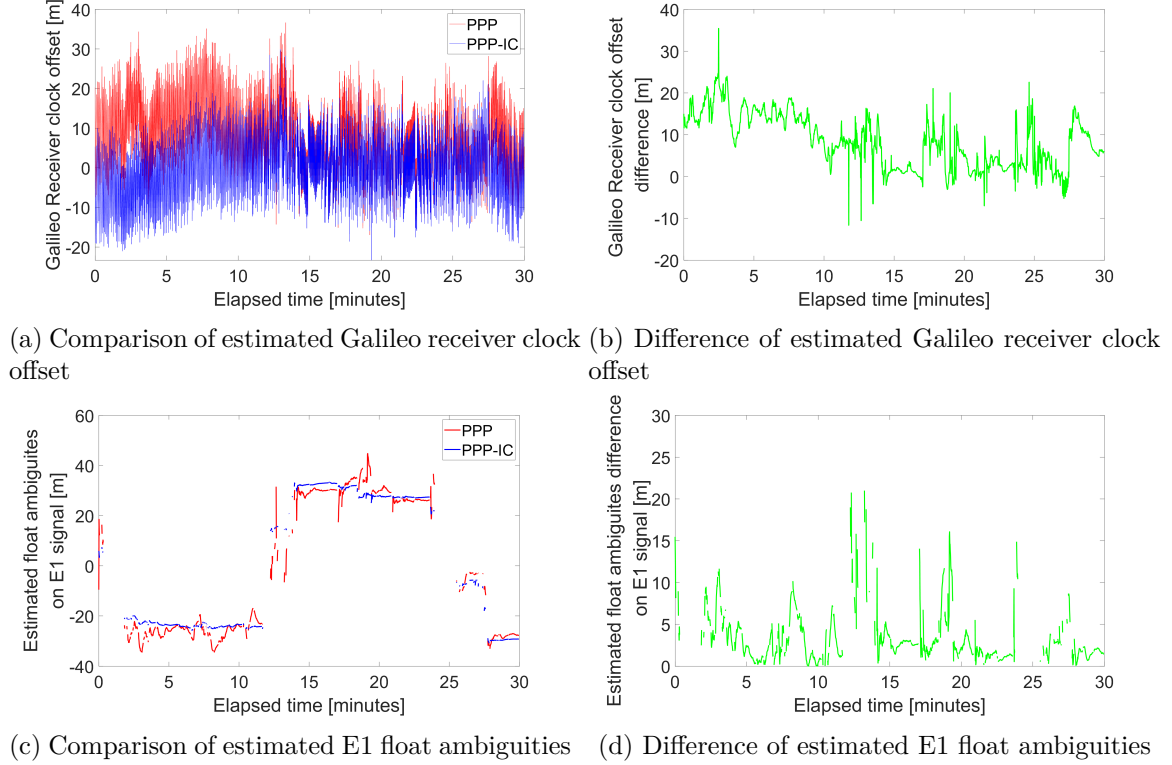


Figure 3.12: Estimated receiver clock offset and E1 float ambiguities comparison with and without applying ionospheric constraints for satellite E27

3.5 Summary

Ionospheric refraction is recognized as a significant error source in GNSS signal propagation, posing challenges to accurate positioning solutions. To address this issue, this research focused on mitigating ionospheric errors in PPP processing. Specifically, the research explored applying ionospheric constraints for three different grades of GNSS receivers: geodetic NovAtel OEM7, low-cost SwiftNav Piksi, and smartphone MI 8. Through rigorous validation

via static and kinematic tests, the contributions and novelty of this work are outlined as follows:

The static results highlighted the substantial benefits of incorporating ionospheric constraints into smartphone dual-frequency positioning, a novel aspect that had not been previously explored. Notably, there was a remarkable 30% improvement in horizontal rms for the initial epoch positioning solution and a substantial 57% reduction in horizontal rms within the first 20 minutes compared to traditional dual-frequency PPP solutions. This enhancement resulted in a reduction of the overall rms from 2.6 m to 1.5 m. Furthermore, smartphone PPP-IC took 14 minutes to reach the convergence threshold of 1 metre, which was 11 minutes faster than conventional PPP. Such substantial improvements can be attributed to the difficulty in accurately estimating ionospheric errors in smartphone PPP due to its noisy measurements and frequent loss of observations.

Similarly, the kinematic vehicle experiments yielded comparable findings. The incorporation of PPP-IC in smartphone solutions demonstrated a remarkable 77% and 59% improvements for the initialization error and horizontal rms over the first three minutes. Conversely, for Piksi and OEM7 receivers, it is expected that PPP-IC did not lead to significant improvements before convergence owing to the limited accuracy from the GIM (2-8 TECU). These findings underscored the efficacy of ionospheric constraints in improving the precision and reliability of kinematic GNSS positioning solutions, particularly for smartphone users in dynamic environments.

To investigate the reasons behind the significant improvement in smartphone solutions with PPP-IC, this research conducted a comparison of PPP estimates with and without ionospheric constraints. Interestingly, it was found that not only the estimated slant ionospheric delays, but also tropospheric delays, receiver clock offset, and float ambiguities estimation were affected by ionospheric constraints. For instance, the utilization of ionospheric constraints contributed to stabilizing the estimated float ambiguity estimates, leading to a reduction in the standard deviation from 3.1 m to 0.6 m. This enhanced performance can

be attributed to the additional information provided by the GIM, which aids in reducing errors in the estimated ionospheric delays.

In conclusion, this research made several significant contributions to the application of PPP-IC technology across three different grades of GNSS receivers. Leveraging GIM information and DCB estimation, the static and kinematic solutions verified that PPP-IC is a promising approach and has a greater impact on smartphones compared to low-cost and geodetic receivers, a novel finding not previously reported in the literature. Furthermore, in comparison to other studies conducted under similar kinematic conditions, such as Wu et al. (2019) utilized MI 8 dual-frequency observations and results showed significant fluctuations with 4-5 m, Aggrey et al. (2020) employed multiple dual-frequency smartphones and achieved kinematic solutions with rms around 5-7 m, the proposed PPP-IC method consistently achieved superior performance with horizontal rms performance of less than 3 m even in urban driving environments. The outcomes suggested that PPP-IC holds the potential to facilitate rapid initialization and precise positioning within a short time for autonomous vehicles, precision agriculture, and smartphone-based location services.

Chapter 4

Improving PPP smartphone processing with adaptive quality control method in obstructed environments when carrier-phase measurements are missing

The high level of signal noise and multipath effect errors cannot be neglected when it comes to smartphone-grade GNSS receivers and antennas and, along with frequent carrier-phase measurement discontinuities and losses, pose a challenge for advanced GNSS positioning techniques. To effectively utilize all satellite measurements in the absence of phase measurements, this chapter proposes a pseudorange-only measurement enhanced PPP method with single- and dual-frequency combinations. In other words, the enhanced PPP utilizes the satellites with pseudorange-only observations that are typically excluded in traditional PPP processing with precise corrections. The content of this chapter is primarily drawn from the author's publication entitled "Improving PPP smartphone processing with adaptive quality control method in obstructed environments when carrier-phase measurements are missing", published in *GPS Solutions* (Yi et al., 2024).

4.1 Introduction and motivation

While significant efforts, such as the PPP-IC method discussed in Chapter 3, have been dedicated to further enhancing precise positioning for smartphones, there are still significant challenges to achieve dm- or cm-level accuracy for smartphone navigation in real-world driving environments. Continuing the research, this chapter focuses on further enhancing smartphone positioning performance specifically to address the challenges: (1) insufficient observations owing to hardware limitations and obstructed environments, and (2) noisy measurement and multipath effects.

Regarding the first category of challenges, numerous endeavours have been made to enhance GNSS solutions with additional sensors, such as IMUs (Gikas and Perakis, 2016). A recent study showed that tightly-coupled PPP/IMU solutions have the capacity to bridge gaps in positioning solutions and markedly enhance overall positioning performance (Yang et al., 2023). However, dead-reckoning solutions with smartphone-grade IMUs cannot continuously and precisely track a vehicle over a long time, mainly due to its drifting nature. Consequently, how best to use the available raw measurements to improve smartphone GNSS performance remains a core challenge. In this context, Nie et al. (2020) proposed a PPP algorithm that combines single-frequency, ionospheric-corrected pseudorange measurements with dual-frequency ionospheric-free pseudorange and phase measurements. Continuing this research, Zangenehnejad and Gao (2021) introduced the single-frequency phase observable in smartphone processing with dual-frequency observations.

In terms of the second category of challenges, due to hardware disparities as compared with, e.g., geodetic-grade equipment, smartphones equipped with linear polarized antennas are more vulnerable to multipath effects, signal attenuation, and reduced reception capability (Pesyna et al., 2014). Prior studies have also demonstrated that GNSS observations suffer from severe signal suppression and interference when smartphones are placed on a vehicle dashboard (Li et al., 2022b), and the orientation of the smartphone can also affect the positioning performance (Yong et al., 2021). More recently, Hu et al. (2023) assessed smartphone multipath effects through the actual range errors, and the results suggest that

smartphone range errors characterization differs for different constellations and frequencies. To identify and manage these coarse measurements in GNSS processing, numerous quality control methods have been proposed, which can be realized through 1) data quality monitoring using raw measurement information, such as satellite elevation angle and carrier-to-noise density ratio (C/N_0), and 2) pre-fit and post-fit residual analysis to detect outliers (Lewen et al., 2018; Blewitt, 1990). Despite these advancements, precisely detecting measurement outliers remains an arduous task. Most studies and open-source software, such as RTKlib and GAMP, often utilize an empirical and fixed residual threshold to reject noisy measurements and satellites for geodetic receivers (Zhou et al., 2018; Everett et al., 2022). Due to the poor quality of data and limited tracking capability of GNSS signals, achieving accurate kinematic smartphone positioning necessitates higher demands on the quality control approach to filter out noisy measurements, whilst keeping as many quality measurements as possible.

While the pursuit of precise smartphone navigation using PPP processing has been ongoing for several years, it is important to highlight that previous studies have not made use of all pseudorange and phase measurements for smartphone PPP processing. Especially in situations where a smartphone chipset occasionally records only the pseudorange measurements from satellites, while omitting the corresponding phase measurements. This research will refer to these satellites as “pseudorange-only satellites” throughout this chapter. To address the first challenges stemming from inadequate measurements, this research proposes a pseudorange-only measurements enhanced PPP approach to leverage the full potential of all available GNSS observations for smartphone processing. Furthermore, an adaptive post-fit residual rejection method is proposed to mitigate the effects of noisy measurements and multipath effects on smartphone processing for the second set of challenges. This adaptive method can be realized with the aid of range errors and geofencing analysis to prevent over- and under-rejection of measurements based on different multipath profiles.

4.2 Enhanced smartphone PPP model

This section presents the mathematical models used in the enhanced PPP algorithm, as well as the models for range errors and the adaptive post-fit residual rejection threshold, followed by processing settings and configurations employed in smartphone processing.

4.2.1 A pseudorange-only measurement enhanced PPP method

When a smartphone is recording both pseudorange and phase measurements from the number of g satellites, the measurement matrix $\mathbf{z}$ is:

$$\mathbf{z} = [P_1^1, L_1^1, P_5^1, L_5^1, \dots, P_1^g, L_1^g, P_5^g, L_5^g] \quad (4.1)$$

With the inclusion of pseudorange-only measurements, and assume that there are m pseudorange-only measurements available for a specific epoch, the corresponding measurement matrix must be expanded to:

$$\mathbf{z}_{\text{updated}} = [P_1^1, L_1^1, P_5^1, L_5^1, \dots, P_1^g, L_1^g, P_5^g, L_5^g, \underbrace{P_1^{g+1}, P_5^{g+1}, \dots, P_1^{g+m}, P_5^{g+m}}_{\text{expanded}}] \quad (4.2)$$

and the formation of the design matrix A can then be represented as:

$$\mathbf{H} = \begin{bmatrix} e_x^1 & e_y^1 & e_z^1 & 1 & mf^1 & \gamma_1 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ e_x^1 & e_y^1 & e_z^1 & 1 & mf^1 & -\gamma_1 & \cdots & 0 & 0 & \cdots & 0 & \lambda_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ e_x^g & e_y^g & e_z^g & 1 & mf^g & 0 & \cdots & \gamma_5 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ e_x^g & e_y^g & e_z^g & 1 & mf^g & 0 & \cdots & -\gamma_5 & 0 & \cdots & 0 & 0 & \cdots & \lambda_5 \\ e_x^{g+1} & e_y^{g+1} & e_z^{g+1} & 1 & mf^{g+1} & 0 & \cdots & 0 & \gamma_1 & \cdots & 0 & 0 & \cdots & 0 \\ e_x^{g+m} & e_y^{g+m} & e_z^{g+m} & 1 & mf^{g+m} & 0 & \cdots & 0 & 0 & \cdots & \gamma_5 & 0 & \cdots & 0 \end{bmatrix} \quad (4.3)$$

where e_x , e_y , and e_z are line-of-sight unit direction vectors from satellite to receiver. mf represents the wet mapping function of the tropospheric delay, other parameters can be

referred to above equations. The measurement precision ratio for phase and code are set to 100:1, and after applying C/N_0 -based weighting scheme (σ) to different measurements, the final variance matrix R can be given as:

$$R = \text{diag}(\sigma_{P1}^1, \sigma_{L1}^1, \sigma_{P5}^1, \sigma_{L5}^1, \dots, \sigma_{P1}^g, \sigma_{L1}^g, \sigma_{P5}^g, \sigma_{L5}^g, \underbrace{\sigma_{P1}^{g+1}, \sigma_{P5}^{g+1}, \dots, \sigma_{P1}^{g+m}, \sigma_{P5}^{g+m}}_{\text{expanded}}) \quad (4.4)$$

Range errors

To analyze the smartphone observation quality and multipath effects, the range error derivation algorithm is adopted in this research. This method involves forming measurement-differencing pseudorange observations using both the smartphone and collocated geodetic-grade receiver data. The lever arm between the smartphone and geodetic receiver is pre-measured as (x_0, y_0, z_0) in the vehicle body coordinate system. The following equation shows the derivation of range errors of satellite n (Hu et al., 2023):

$$\text{RangeError} = \nabla \Delta P - \left((e_x^q \cdot x_0 + e_y^q \cdot y_0 + e_z^q \cdot z_0) - (e_x^n \cdot x_0 + e_y^n \cdot y_0 + e_z^n \cdot z_0) \right) \quad (4.5)$$

4.2.2 Adaptive post-fit residual threshold method

Traditional post-fit residual methods typically employ fixed empirical values to detect and filter out noisy measurements or satellites that exceed these thresholds, and there is often a tradeoff between retaining a higher number of observations and filtering out noisy measurements. This tradeoff can result in situations where the use of a tight threshold leads to the excessive rejection of satellites, ultimately causing solution gaps. To address this issue and ensure that measurement outliers are effectively identified and removed from the solution, this research proposes an adaptive post-fit residual rejection threshold to dynamically adjust the threshold for outlier detection. The adaptive threshold is shown as:

$$\text{Threshold}_{\text{adaptive}} = \alpha * \text{Threshold}_{\text{original}} + c_2 \quad (4.6)$$

where α is an adaptive factor and c_2 is a constant representing the minimum post-fit threshold for pseudorange residuals. The adaptive threshold is equal to the traditional post-fit threshold when the adaptive factor is 1.

As the adaptive factor significantly impacts the pseudorange threshold and positioning solution, it is crucial to determine it according to a learning statistic (LS), which can reflect the actual traffic conditions and is varied with different driving environments. This work utilizes the ratio of visible satellites over pseudorange-only satellites as the learning statistic, which can be also determined by the PDOP or other indicators that can reflect the GNSS conditions. The form of the adaptive factor based on a three-segment function is given as (Yang et al., 2001):

$$\alpha = \begin{cases} 1 & LS \leq c_0 \\ \frac{c_0}{LS} \left(\frac{c_1 - LS}{c_1 - c_0} \right)^2 & c_0 < LS \leq c_1 \\ 0 & LS > c_1 \end{cases} \quad (4.7)$$

with

$$LS = \begin{cases} c_0 & \text{Challenging} \\ \frac{PS}{POS} & \text{Moderate and POS} \neq 0 \\ c_1 & \text{Moderate and POS} = 0 \end{cases} \quad (4.8)$$

where c_0 and c_1 correspond to the minimum and maximum values of the learning statistic, respectively. The abbreviations PS and POS refer to the number of processed satellites and the number of pseudorange-only satellites, respectively. As shown in Equation (4.8), the learning statistic is set to c_0 in GNSS-challenging environments, resulting in a loosened post-fit residual threshold. This process aims to maintain enough satellites for reliable positioning, even if some signals exhibit a higher level of noise. On the other hand, when there are enough observations available and the learning statistic exceeds c_0 , the adaptive threshold adopts a more rigorous rejection stance to filter out noisy measurements and enhance the overall positioning accuracy. The distinguishing of different GNSS conditions can be based on geofencing maps, the number of visible satellites, and PDOP. The effectiveness of the adaptive post-fit residual threshold algorithm relies heavily on fine-tuning the constants

c_0 , c_1 , and c_2 , which can be initialized with empirically determined parameters and tuned adaptively according to different phone models and traffic conditions. Also, the post-fit threshold rejection iterations and overall processing time required are typically faster than the smartphone GNSS sampling time, demonstrating that the proposed adaptive post-fit threshold algorithm is feasible and suitable for real-time applications.

Figure 4.1 shows how the adaptive post-fit rejection threshold varies with the learning statistic and elapsed time for dataset 4. The left subplot illustrates that the adaptive post-fit threshold decreases as the learning statistic grows with predefined empirical constants c_0 , c_1 , and c_2 , which are set to 1, 21, and 6, respectively. These values are derived empirically based on the ratio of visible satellites over pseudorange-only satellites. In other words, the post-fit threshold would be adjusted adaptively according to different driving conditions as in the right subplot: in harsh GNSS environments, where the quality of satellite measurements may be weaker, the post-fit threshold for satellite rejection is set higher. Conversely, in less challenging GNSS environments, the threshold is set lower. This adaptive adjustment allows for improved customization of the measurement rejecting criterion to suit different driving conditions and improve the PPP solutions accordingly.

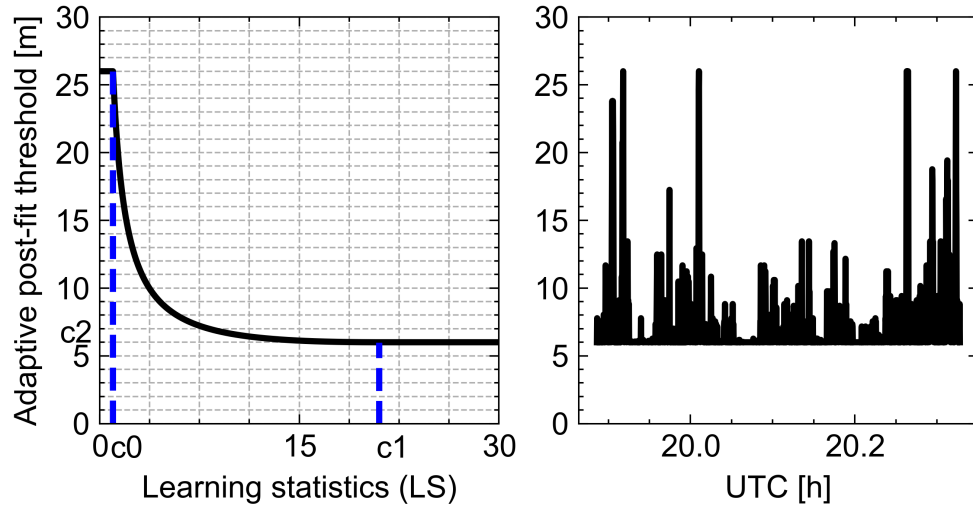


Figure 4.1: Left subplot: variation of adaptive post-fit rejection threshold with respect to learning statistic (LS). Right subplot: time series depicting the adaptive post-fit rejection threshold for dataset 4

Smartphone processing engine setting

Table 4.1 presents the processing strategy adopted in this research. Considering GIM products cannot be assessed in real-time and typically exhibit errors ranging from 2-8 TECU (approximately 16-130 cm on GPS L1 frequency) (Banville et al., 2019), the ionospheric effect is initially constrained using the Klobuchar model, and the remaining residuals are estimated with a small variance. This approach is specifically designed to address the rank deficiency issue that arises when estimating both the ambiguity and ionosphere delay simultaneously where only single-frequency observations are available. Satellite DCBs are corrected with Chinese Academy of Sciences (CAS) products. The float carrier-phase ambiguities are treated as constant terms in the estimation as long as cycle slips are not detected, and only float solutions are estimated due to the noisy measurements with the internal smartphone antenna. Conventional cycle slip detection methods are applied, specifically, loss of lock indicator, geometry-free (GF) combination (Blewitt, 1990), Melbourne-Wübbena (MW) combination (Melbourne, 1985), and Doppler cycle slip detection. The thresholds are 0.1 m for GF, 3 m for MW, and 10 for Doppler cycle slip detection, respectively. If a cycle slip is detected, the ambiguity parameter variance in the filter is reset. Furthermore, due to the constraints imposed by tracked satellites/observations, the elevation cut-off angle is set to a low threshold of 10 degrees. This adjustment helps in effectively filtering out noisy measurements, while maximizing the inclusion of much needed satellites/measurements for smartphone processing. Meanwhile, a C/N_0 -based weighting scheme is utilized in smartphone processing, and the measurement standard deviation σ is estimated from coefficients a and b , which are derived from filter residuals and frequency information (Banville et al., 2019; Shinghal and Bisnath, 2021).

To ensure the replicability and reliability of this research, a total of ten road tests were performed on different days, following the identical routes. Table 4.2 provides comprehensive details of these datasets. Notably, road tests 3-6 were conducted during rush hour in Toronto, leading to more challenging traffic conditions compared to the other tests.

Table 4.1: Processing strategies of pseudorange-enhanced PPP

Items	Strategy
Coordinates	White noise
Receiver clock	White noise
Ionospheric effect	Constrained with Klobuchar model
ZWD	Estimated
ZHD	Saastamoinen model
Ambiguity	Constant if no cycle slip detected
Troposphere mapping function	NMF
Satellite orbit/clock corrections	GFZ rapid products
Satellite DCB	CAS products
Processed constellations	GPS/GLONASS/Galileo/BDS
Processed frequencies	Single- and dual-frequency
Elevation cut-off angle	10°
Weighting scheme	C/N_0 -based

Table 4.2: Summary of ten road tests

Test #	Phone model	Collection Time (Local time)	Traffic conditions
1	MI 8	Oct. 8, 2022, 22:03-22:24	Good
2	S21+	Oct. 8, 2022, 22:03-22:24	Good
3	MI 8	Oct. 10, 2022, 15:53-16:20	Harsh
4	S21+	Oct. 10, 2022, 15:53-16:20	Harsh
5	MI 8	Oct. 10, 2022, 16:21-16:44	Harsh
6	S21+	Oct. 10, 2022, 16:21-16:44	Harsh
7	MI 8	Aug. 7, 2021, 21:52-20:18	Good
8	S21+	Aug. 7, 2021, 21:52-20:18	Good
9	MI 8	Jul. 29, 2021, 20:48-21:13	Good
10	S21+	Nov. 7, 2022, 21:17-22:41	Good

4.3 Raw measurement assessment and smartphone enhanced PPP performance evaluation

To better appreciate the smartphone raw measurements behaviour under different multipath profiles, the first four datasets listed from Table 4.2 were taken as examples and analyzed. Smartphones are usually employed for applications such as pedestrian navigation and vehicle navigation, where the primary focus lies in assessing user horizontal positioning errors. In alignment with this perspective, notable competitions such as the Google Smartphone Decimeter Challenge in 2021, 2022, and 2023 have adopted horizontal positioning error as a key evaluation criterion (Fu et al., 2020). Therefore, in this section, the horizontal solutions

together with the statistics from all ten datasets are discussed and compared with different PPP processing modes.

4.3.1 Raw GNSS measurements analysis under different driving environments

Figure 4.2 shows a comparison of the observed number of pseudorange and phase measurements between smartphone MI 8 (dataset 1) and a geodetic OEM7 GNSS receiver. For OEM7 measurements, the absence of a blue line is due to the complete overlap between the green and blue lines, indicating that the number of code and phase measurements recorded by the geodetic receiver is always the same. For the MI 8 phone, the average number of pseudorange and phase measurements are 33 and 27, respectively. These quantities indicate that, on average, 6 phase measurements were lost during the data collection period, while the corresponding pseudorange measurements remained available. Conversely, no phase measurement loss is observed when the code measurements are available in the geodetic data collected by the OEM7 receiver. This comparison highlights the performance discrepancies that the smartphone experiences more frequent phase measurement losses in challenging environments.

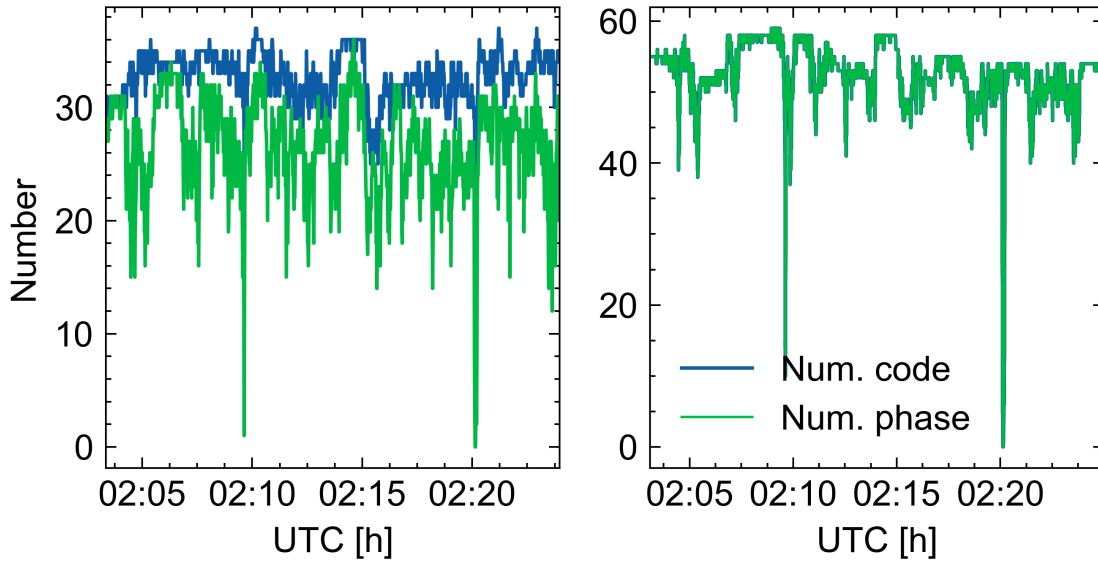


Figure 4.2: Number of pseudorange and phase measurements (GREC) for smartphone (left) and geodetic receiver (right)

In Figure 4.3, a band-by-band, constellation-by-constellation analysis is performed to provide a comprehensive view of the phase unlock situation. The blue dots represent the difference between the number of pseudorange and phase measurements, while the red lines indicate the phase unlock ratio. From Figure 4.3, it can be observed that no significant constellation- or band-specific trends are found in these phase unlock cases, suggesting that the phase losses issue is a general problem in smartphone raw observations and affects different frequencies and constellations in a similar manner.

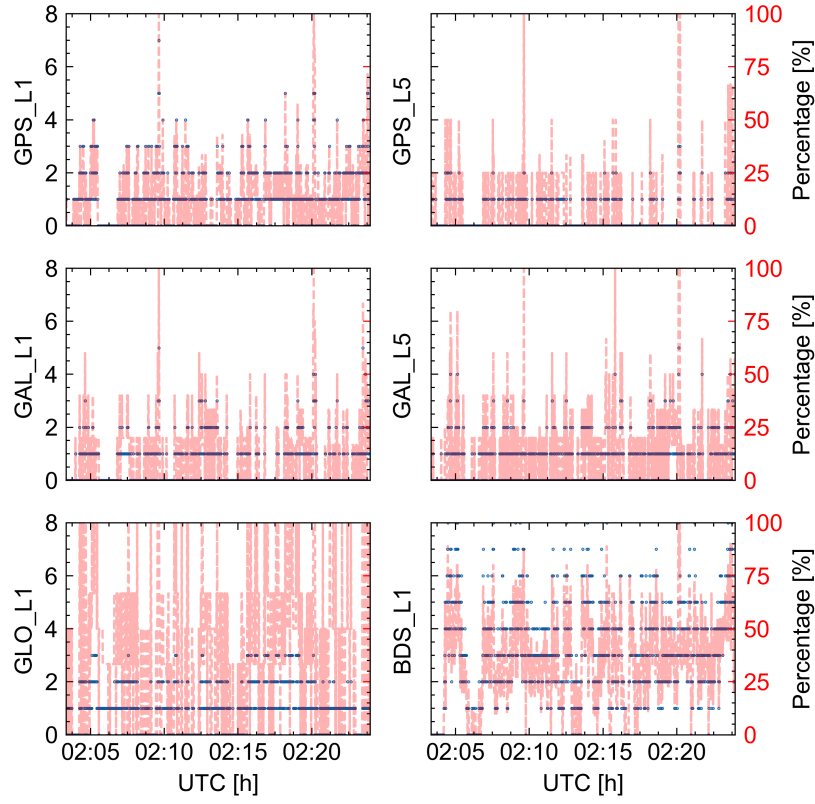


Figure 4.3: Difference between the number of pseudorange and phase measurements (blue dots) for different constellations and the corresponding phase unlock ratios (red lines)

Figure 4.4 presents the number of visible satellites and pseudorange-only satellites, depicted as blue and green bars, respectively, under varying multipath conditions. Two key observations can be made from the plot. First, GNSS geometry remains relatively consistent across most driving environments irrespective of multipath effects. However, it is observed that multipath effects can vary significantly depending on the specific driving environments.

Take satellite G10 as an illustrative example. The L1 code-minus-carrier (CMC) residuals for G10 differ across environments, measuring 2.0 m for open-sky conditions, 3.6 m in vegetation roads, 3.3 m in suburban roads, and 5.4 m in overpasses areas. Second, it should be noted that the overpass areas encompass not only the immediate overpass structures but also the nearby roads when geofencing was created, and as the vehicle approaches and traverses overpasses, the number of pseudorange-only satellites increases (on-average to 7.7), while the availability of visible satellites decreases (on-average to 15.2). This finding suggests that the smartphone tracking capability of phase measurements is significantly susceptible to obstructed environments.

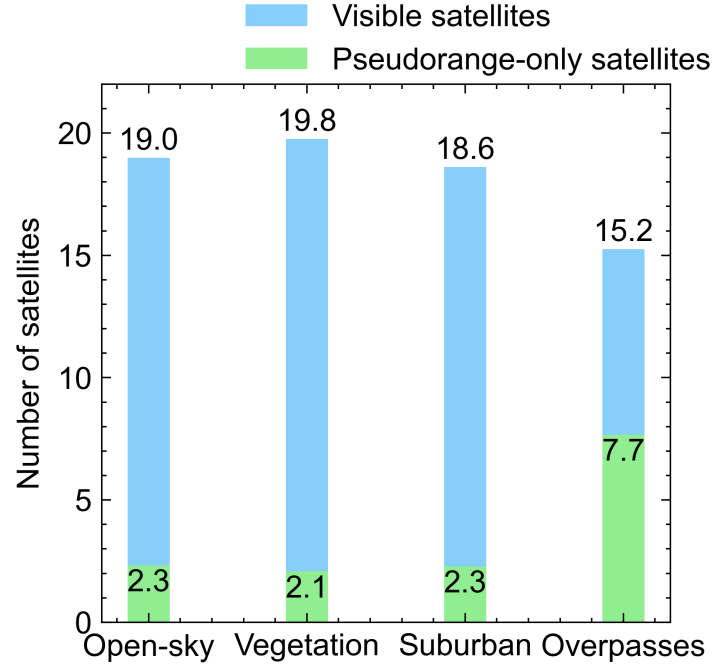


Figure 4.4: Number of visible and pseudorange-only satellites under different driving environments

Figure 4.5 illustrates a comparative analysis of L1 time-differenced code-minus-carrier (TDCMC) residuals between road tests 1 and 3, offering insights into the smartphone observation quality influenced by various factors, including multipath effects and other noises, and different colours stand for CMC residuals from different satellites, details of the TDCMC method are explained in Caamano et al. (2020). As stated previously, test 3 experienced

more challenging traffic conditions compared to test 1. As anticipated, the average L1 CMC residual for test 3 was 7.7 m, indicating a higher level of noise compared to test 1, which had an average residual of 5.5 m. The increased noise can be mainly attributed to more prominent multipath effects and signal interference caused by passing vehicles.

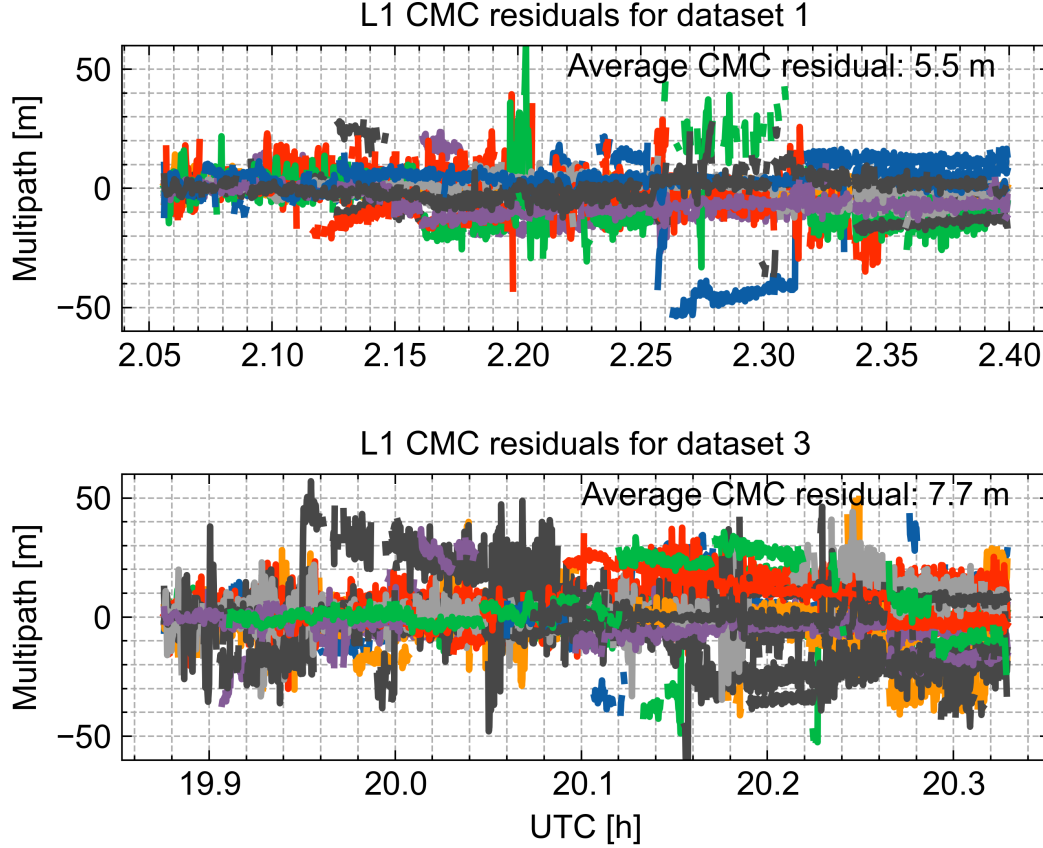


Figure 4.5: Overall satellite CMC L1 residuals for datasets 1 (lower traffic) and 3 (higher traffic)

4.3.2 Pseudorange-enhanced PPP vs PPP in GNSS harsh environments

Figure 4.6 shows the time series of smartphone PPP horizontal errors for datasets 1 to 4. The conventional GNSS PPP solution is represented by the red line, while the enhanced GNSS PPP solution is represented by the blue line. It is evident that the smartphone pure GNSS-PPP solution experiences significant degradation during GNSS outages. Additionally, although vehicle datasets 1-4 followed the same route, their PPP traditional solutions

exhibited significant variations. Notably, datasets 3 and 4 display markedly inferior PPP solutions compared to datasets 1 and 2. This discrepancy can be attributed to the challenging traffic conditions during rush hours that are highlighted in Table 4.2, which results in frequent signal blockages and carrier-phase measurements loss.

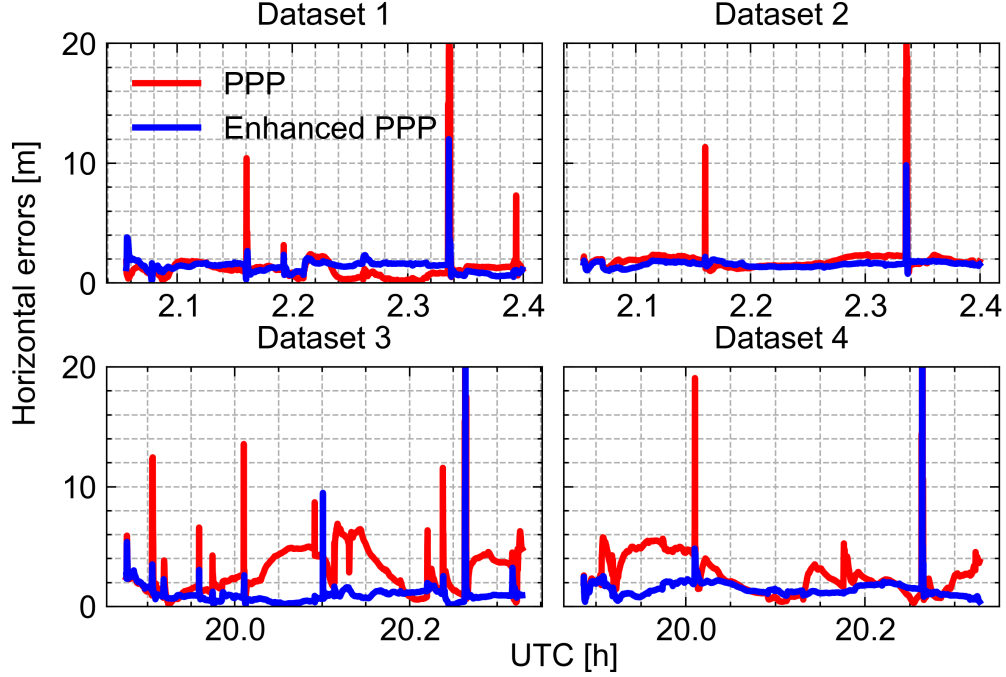


Figure 4.6: Horizontal error with respect to the reference trajectory processed by PPP and enhanced PPP solutions with pseudorange-only measurements, and the PPP maximum errors are 25.9 m, 26.5 m, 25.1 m, 45.9 m for road tests 1-4, respectively

In contrast, the proposed enhanced PPP strategy, which incorporates additional pseudorange observations, provides more stable and resilient solutions in those GNSS-challenging environments. The improvement is particularly evident in datasets 1 and 2, where the maximum horizontal errors are reduced from 25.9 m and 26.5 m to 12.0 m and 9.8 m, respectively. Figure 4.7 summarizes the overall horizontal statistics for road tests 1 to 4, and corresponding metrics 95th percentile positioning error, 68th percentile positioning error, and overall rms are represented as red, blue, and green bars, respectively. The enhanced PPP solutions are depicted in lighter colours. It can be observed that the positioning performance is improved for all four datasets. For example, in dataset 4, the horizontal positioning accuracy

is improved from 5.4 m, 3.1 m, and 3.4 m to 2.2 m, 1.8 m, and 1.9 m for the 95th percentile error, 68th percentile error, and rms, respectively.

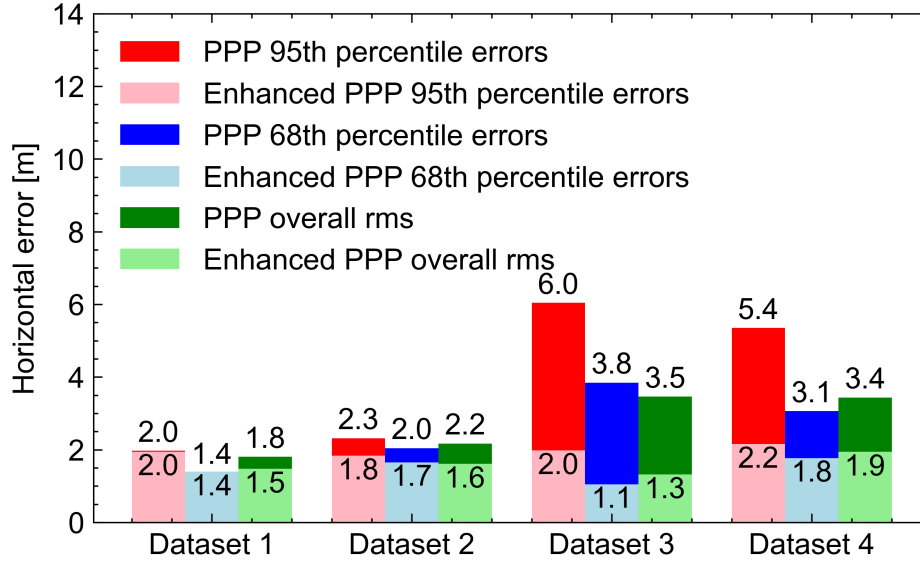


Figure 4.7: Overall horizontal statistics for road tests 1 to 4 processed by PPP and pseudorange-enhanced PPP strategies

To investigate the reasons behind the improved solutions with enhanced PPP strategy, Figure 4.8 shows the comparison between PDOP with/without the inclusion of pseudorange-only satellites. The pink dashed lines (background lines) represent the PDOP improvement achieved by including pseudorange-only satellites for test 4. It is observed that the pseudorange-enhanced PPP strategy can improve PDOP by up to 60% compared to conventional PPP processing. Additionally, the 95th percentile positioning often reflects performance in challenging environments, such as those near overpasses or high-rise buildings where the phase measurements may experience significant signal loss. For instance, there could be situations with 4 satellites resulting in both pseudorange and carrier-phase measurements and 4 satellites resulting in only pseudorange observations. Including these 4 additional satellites/observations can lead to a substantial improvement in PDOP. Consequently, a noticeable reduction of approximately 0.2 in PDOP can be observed when pseudorange-only satellites are included in the data processing. Such improvements play

a crucial role in mitigating GNSS outliers, especially when there are only a few satellites available.

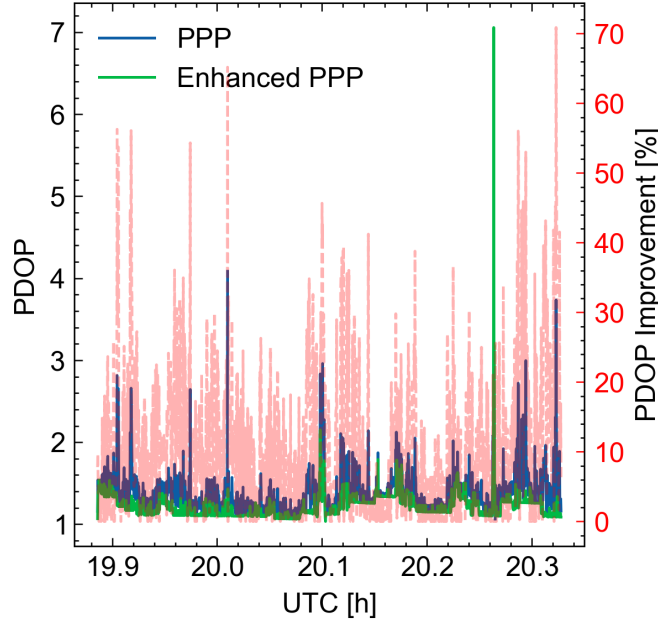


Figure 4.8: PDOP comparison for road test 4 with/without pseudorange-only satellites. Blue line denotes PDOP for PPP solution, while green line is the PDOP for enhanced PPP. Pink line is the percentage of PDOP improvement for these two solutions

4.3.3 Pseudorange-enhanced PPP with adaptive post-fit residual threshold

The inclusion of pseudorange-only measurements provides significant improvement over conventional PPP performance in harsh environments, and based on the range error analysis, the average measured range error for pseudorange-only observations is 2.4 m. This value is consistently compared to regular pseudorange observations, which exhibit an average range error of 2.3 m when phase measurements are available. However, the involvement of unexpected noisy pseudorange-only observations can also be detrimental to solutions. For instance, during the initialization time for road test 1, there are noisy measurements related to G09 pseudorange-only observations with range errors exceeding 20 m. Nevertheless, the post-fit residual for G09 observations was calculated at 7.4 m, which remained undetected in the traditional fixed post-fit threshold rejection function. To mitigate this problem, a range

error rejection function is applied to the enhanced PPP solutions. Figure 4.9 indicates that the pseudorange-enhanced PPP with range errors rejection function (green) can further improve the positioning solutions for dataset 1.

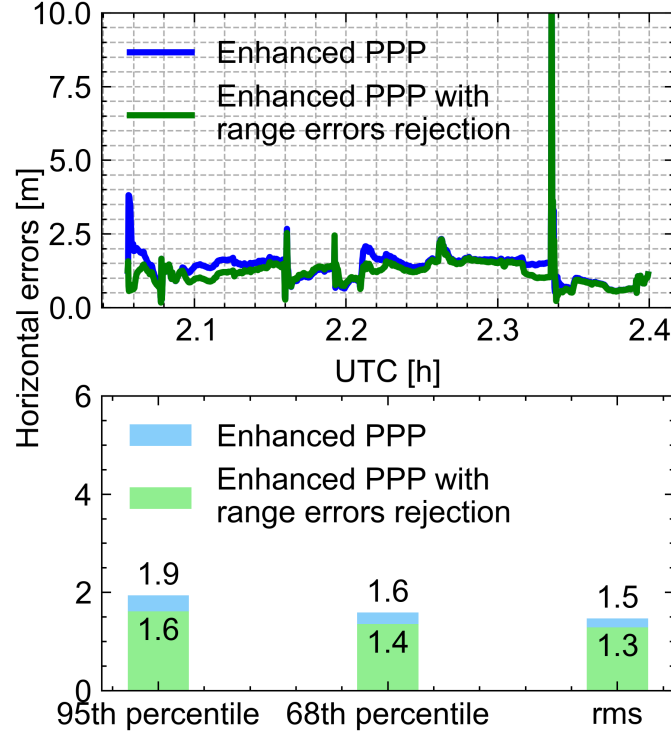


Figure 4.9: Time series of horizontal error and corresponding statistics for dataset 1 processed by pseudorange-enhanced PPP (blue) and pseudorange-enhanced PPP with range errors rejection function (green)

Although having a range errors rejection function is beneficial for solutions, it is impractical and cost-inefficient for setting a geodetic rover adjacent to the smart device throughout real-time applications. Meanwhile, CMC assessment necessitates access to carrier-phase measurements, which are unavailable for pseudorange-only observations. Therefore, how to optimize the quality control methods and remove measurement outliers based on filter-inherited information are vital to smartphone positioning. Figure 4.10 describes the scatter diagrams and correlation analyses between smartphone pseudorange range errors and several parameters: pre-fit residuals, post-fit residuals, signal strength, and satellite elevation. These parameters are available in real-time PPP processing and have been commonly used

for stochastic modelling and measurement optimization in smartphone positioning. It can be observed that the post-fit residuals exhibit the strongest correlation (0.67 on L1 frequency and 0.55 on L5 frequency) with range errors among the examined parameters, suggesting that utilizing post-fit residuals as a criterion for distinguishing measurement quality is more suitable and effective in smartphone positioning amongst other indices.

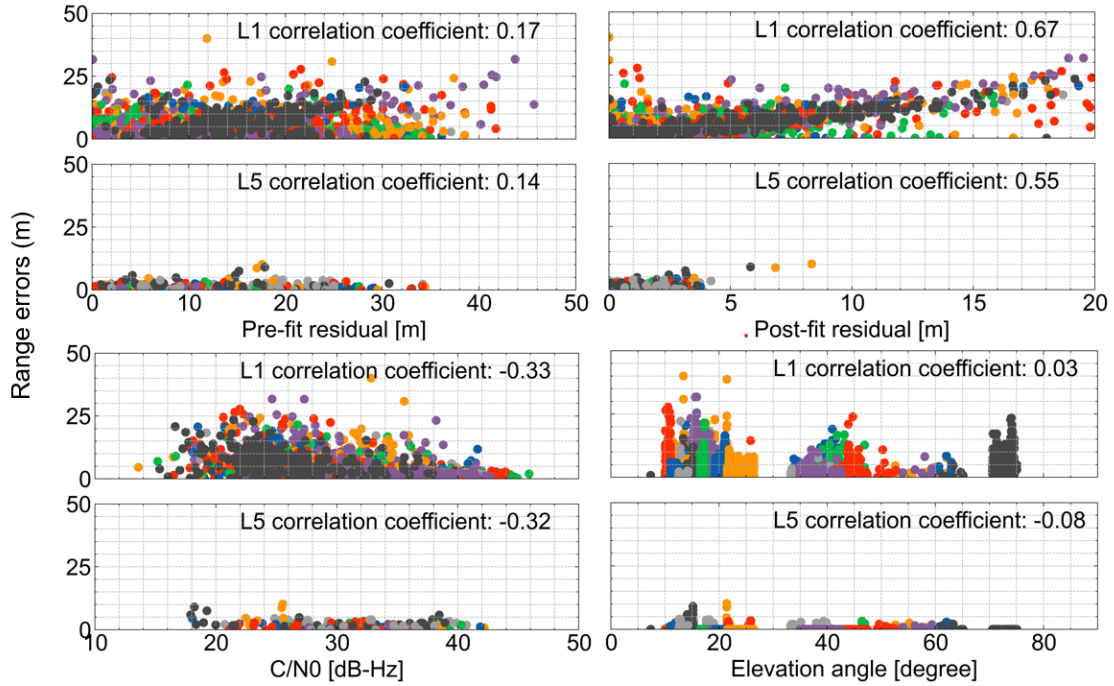


Figure 4.10: Correlation analysis of smartphone range errors with pre-fit residuals, post-fit residuals, signal strength, and elevation angle

Considering that the smartphone measurement quality is not only limited by the low-cost GNSS chipset and antenna but also the environment, it is unrealistic to assume a constant post-fit residual rejection threshold throughout smartphone processing. Figure 4.11 presents the time series of horizontal errors for datasets 1 to 4, where the enhanced PPP with the traditional fixed post-fit residual threshold (blue lines) and enhanced PPP with adaptive post-fit residual threshold (green lines) are compared. The first subplot of Figure 4.11 demonstrates that the adaptive post-fit residual threshold can detect and mitigate noisy measurements in the initialization stage for dataset 1. Although there are still performance discrepancies compared to the enhanced PPP with the range error rejection approach, the

adaptive threshold approach shows its potential to reduce the maximum error from 12.0 m to 4.2 m. Furthermore, the last subplot of Figure 4.11 exemplifies the importance of the proposed adaptive algorithm for dataset 4. Enhanced PPP solutions with the adaptive post-fit residual threshold exhibit improved positioning performance, with 95th percentile error, 68th percentile error, and overall rms reduced to 1.5 m, 1.0 m, and 1.6 m, respectively. Although the adaptive post-fit residual threshold provides little improvement for datasets 2 and 3, the proposed algorithm does not harm the enhanced PPP solutions with good GNSS geometry and data quality. Meanwhile, owing to the harsher driving conditions, the maximum horizontal positioning errors for datasets 3 and 4 can reach as high as 24.7 m and 44.9 m at the overpass D2, respectively. Given the limited availability of GNSS observations during these instances, neither the proposed methods could effectively mitigate these gross errors. Consequently, it is anticipated that the inclusion of an IMU holds the potential to significantly reduce such large errors, which deserves future research.

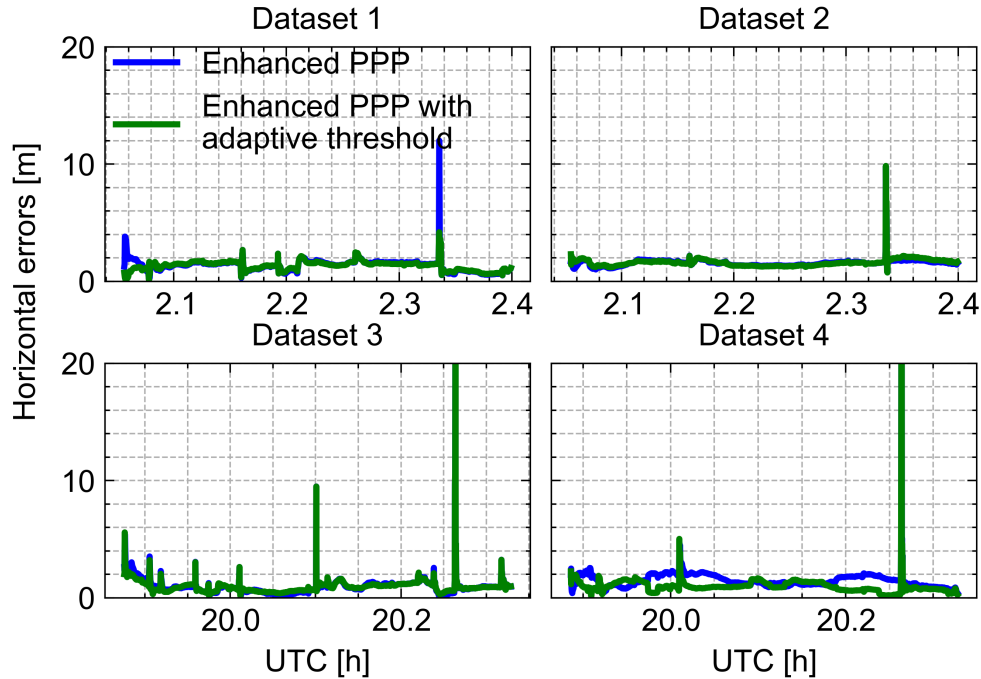


Figure 4.11: Horizontal error with respect to the reference trajectory for road tests 1 to 4 processed by pseudorange-enhanced PPP and pseudorange-enhanced PPP with adaptive post-fit thresholds, and maximum errors are 24.7 m, 44.9 m for road tests 3 and 4, respectively

Figure 4.12 highlights the overall horizontal error Probability Density Function (PDF) and Cumulative Distribution Function (CDF) for ten tested datasets. The solutions processed by PPP, enhanced PPP, and enhanced PPP with adaptive post-fit residual threshold are represented by red, blue, and green colours, respectively. It is evident that the inclusion of pseudorange-only measurements significantly enhances the accuracy of PPP positioning, as it effectively reduces the occurrence of horizontal errors exceeding 2.5 m. Furthermore, as highlighted in Table 4.3, the proposed adaptive post-fit residual rejection function further improves overall solutions, resulting in improvements of approximately 0.3 m, 0.2 m, and 0.3 m for overall rms, mean, and 95th percentile errors, respectively, when compared to employing a traditional fixed post-fit threshold across ten datasets. Overall, when compared to conventional PPP strategy, the horizontal rms, 95th percentile error, and 68th percentile error are reduced from 2.8 m, 5.5 m, and 2.2 m to 1.4 m, 1.8 m, and 1.3 m, respectively.

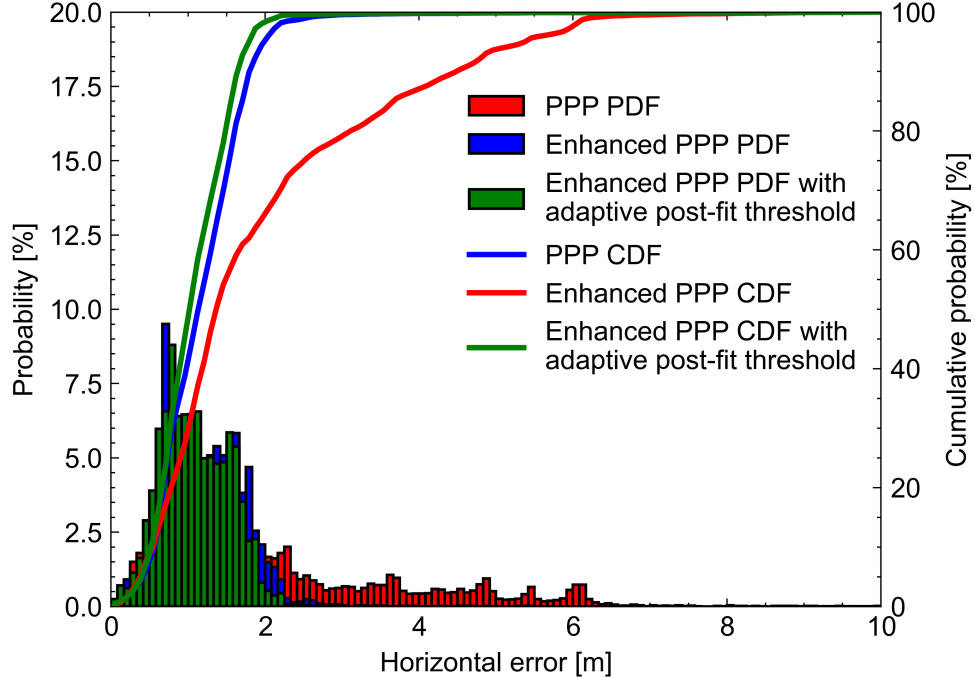


Figure 4.12: Overall horizontal errors Probability Density Function (PDF) and Cumulative Distribution Function (CDF) for all tested ten datasets processed by PPP (red), pseudorange-enhanced PPP (blue), and pseudorange-enhanced PPP with adaptive post-fit residual threshold rejection function (green)

Table 4.3: Overall horizontal statistics for all tested ten datasets processed by PPP, pseudorange-enhanced PPP, and pseudorange-enhanced PPP with adaptive post-fit residual threshold rejection function (unit: m)

Processing strategy	RMS	STD	Mean	95 th errors
PPP	2.8	1.9	2.1	5.5
Enhanced PPP	1.7	0.8	1.3	2.1
Enhanced adaptive PPP	1.4	0.7	1.1	1.8

4.4 Summary

Since every satellite measurement matters in GNSS processing, this research uniquely proposed a new PPP algorithm by enhancing the single- and dual-frequency PPP with pseudorange-only measurements to improve the satellite geometry and measurement redundancy. Additionally, a new adaptive post-fit residual rejection threshold was adopted to improve the measurement quality control strategy for different driving conditions. Through extensive validation tests involving ten vehicle experiments across various driving conditions, the research demonstrated the superior performance of the enhanced PPP approach compared to traditional methods.

By incorporating pseudorange-only measurements into single- and dual-frequency PPP processing, significant enhancements in mitigating positioning errors have been observed. Specifically, there were improvements of 64%, 23%, and 46% observed in the 95th percentile positioning error, 68th percentile positioning error, and overall rms statistics, respectively. This notable improvement can be attributed to a 60% enhancement in PDOP compared to conventional PPP processing, which plays a critical role in mitigating GNSS outliers, particularly in scenarios where only a few satellites are available for smartphone positioning.

Additionally, the range error correlation analysis explicitly revealed that post-fit residuals exhibit the strongest positive correlation with receiver noise and multipath effects, among the examined parameters including pre-fit residuals, C/N_0 values, and elevation angles. Building upon this insight, the proposed adaptive post-fit residual rejection function further refined the measurement quality control scheme, resulting in improved solutions with a 95th percentile positioning error of 1.8 m and an overall rms error of 1.4 m for the

horizontal component. These results demonstrated a notable improvement in smartphone PPP performance compared to previous studies conducted in harsh GNSS environments (Wu et al., 2019; Aggrey et al., 2020; Yi et al., 2021).

To sum up, this research introduced a novel enhanced PPP algorithm that utilized all available smartphone observations and incorporated an adaptive quality control method to mitigate the negative effects of noisy measurements and multipath effects. The results showed that this enhanced adaptive PPP algorithm can reduce the smartphone horizontal rms and 95th percentile errors from 2.8 m and 5.5 m to 1.4 m and 1.8 m. These findings not only underscored the importance of utilizing all available satellite observations for enhanced accuracy but also highlighted the effectiveness of adaptive techniques in improving smartphone positioning performance.

Chapter 5

Hybridization of smartphone GNSS PPP/RTK with internal IMU

The proliferation of GNSS-enabled smartphones has increased the demands for Location-Based Services (LBS). With the deployment of multi-constellation, dual-frequency chipsets and the availability of GNSS raw measurements from the Android operating system, it is now possible to utilize advanced GNSS positioning techniques such as RTK and PPP for smart devices, which brings low-cost mobile positioning accuracy to the next level. However, smartphone GNSS observations are vulnerable to multipath effects and poor signal reception. Consequently, GNSS-only smartphone positioning performance tends to degrade significantly in harsh environments. To tackle these issues, this chapter comprehensively analyzes the performance improvements by integrating the IMU measurements retrieved from smartphones. Additionally, this chapter proposes a novel PPP/RTK hybridization algorithm, combined with the single- and dual-frequency enhanced PPP strategy for smartphone processing. The contents of this chapter draws upon the author's publications "Native Smartphone Single- and Dual-Frequency GNSS-PPP/IMU Solution in Real-World Driving Scenarios" and "Hybridization of smartphone GNSS PPP/RTK with native IMU in realistic driving scenarios," which were published in the *Remote Sensing* journal and presented at the *ION GNSS+ 2023* conference, respectively (Yi et al., 2022; Yi and Yang, 2023).

5.1 Introduction and motivation

The smartphone precise positioning market is dynamic and driven by emerging applications and technological innovations. How to provide accurate, continuous, and resilient positioning, navigation, and timing (PNT) solutions to the mass market with low-cost devices has been a topic and major challenge for industry and research communities. A breakthrough occurred in 2016 when Google released the Android 7.0 platform that allows both users and developers to retrieve raw measurements including pseudorange and carrier-phase observations directly from the GNSS chipsets free of charge (Malkos, 2016), and this progression made the precise smartphone navigation possible to utilize RTK and PPP technologies. As a pioneer, Pesyna et al. (2014) proved that centimetre-level accuracy is achievable with smartphone-grade antenna and single-frequency GNSS technology in an ideal environment. Following this contribution, a host of studies explored smartphone RTK/NRTK/PPP positioning performance, and solutions demonstrated that the decimetre- to centimetre-level positioning accuracy can be reached for single-frequency smartphones under open-sky static environments (Realini et al., 2017; Gill et al., 2017; Dabove and Di Pietra, 2019b; Odolinski and Teunissen, 2019; Sun et al., 2020b).

For smartphone positioning, the necessity of improving solution performance in obstructed environments, such as urban canyons is well-recognized for user experience. Ultra-low-cost hardware and a user tendency to place smartphones in adverse positions for GNSS measurements. Fortunately, contemporary smartphones are equipped with motion sensors, and most motion sensors employed are in the form of micro-electromechanical system (MEMS) IMUs. MEMS-IMUs are low-cost inertial sensors that are commonly fused with various grades of GNSS receivers to achieve dead-reckoning solutions (Abd Rabbou and El-Rabbany, 2014; Vana and Bisnath, 2020). Following the research in the low-cost independent sensor domain, work has been conducted to investigate such sensor fusion using smartphone sensors (Gikas and Perakis, 2016; Yan et al., 2019, 2020). Most literature has been focused on basic Single Point Positioning (SPP), and few have demonstrated the potential of implementing precise techniques such as PPP or RTK for sensor fusion. The

authors were able to show the potential of improved accuracy to metre-level during GNSS outages in previous studies using RTK (Yang et al., 2021; Seepersad et al., 2021) or PPP (Seepersad et al., 2022). Kaiser et al. (2021) investigated the quality of GNSS and IMU measurements obtained from a Xiaomi MI 8 smartphone. Their findings indicated that the smartphone achieved a positioning performance of 5 m in open-sky areas.

In view of the published literature so far, the quest to produce RTK-like performance with a PPP configuration has been ongoing for decades as the former technique enjoys great benefits from instantaneous convergence and solution stability. Although RTK and PPP approaches are not novel conceptions, it should be noted that PPP is gradually narrowing the gap compared to RTK with atmospheric constraints and multi-frequency observations (Yi et al., 2021; Wang et al., 2020; Aggrey and Bisnath, 2019). However, most studies in the field of smartphone positioning have only focused on standalone GNSS-RTK or GNSS-PPP technique, and a few recent studies are concerned about the fusion technique with smartphone internal GNSS and IMU sensors (Yi et al., 2022; Yang et al., 2021). To assess smartphone RTK and PPP positioning performance discrepancies, this research uniquely adopts ionospheric constraints for PPP processing and compares the smartphone positioning performance with the combination of smartphone internal inertial sensors under different driving environments. Furthermore, a novel PPP/RTK or RTK/PPP hybrid algorithm is proposed to further enhance solutions in harsh urban environments.

5.2 Mathematical models and data processing strategies

This section begins with details regarding smartphone IMU raw data preprocessing and tightly-coupled GNSS/IMU integration, followed by the smartphone hybrid processing flowchart.

5.2.1 Smartphone IMU raw data preprocessing

Smartphone IMU measurements were gathered using the GnsLogger application released by Google. This application can log raw IMU and GNSS measurements into a text file for post-processing. To align the navigation engine with the output from the smartphone IMU, the IMU initialization and alignment procedures outlined in Section 2.3.1 were conducted,

allowing for the conversion of the IMU coordinate system to the vehicle body coordinate system, and details about the IMU specifications are highlighted in Table 2.2. Figure 5.1 presents the comparison of IMU raw measurements after alignment between the Xsens IMU (green) and Xiaomi Mi 8 IMU (blue). The Xsens MTi-7 low-cost IMU is used as a comparison. Though the low-cost IMU does not provide an ideal truth as a reference, the matching pattern overall measurements and axes for Mi 8 against Xsens MTi-7 showed promising performance of the integrated solution.

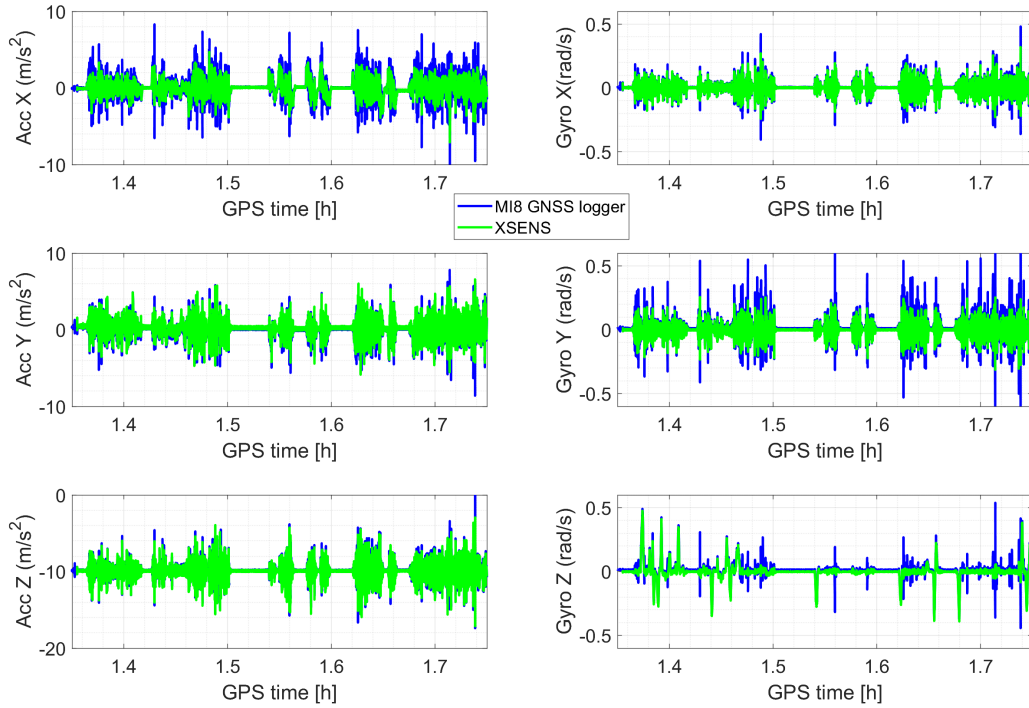


Figure 5.1: Raw IMU measurements: Mi 8 vs Xsens MTi-7

Furthermore, all raw IMU measurements were preprocessed by re-sampling to 100 Hz with the linear interpolation method. Three reasons led to the re-sampling decision: (a) as demonstrated in Figure 5.2, the accelerometer and gyroscope readings were not synchronized over the sampled 0.25 s, showing separate timestamps, in contrast to conventional IMUs; (b) Figure 5.3 shows that the IMU raw measurements were not spaced equally, because the time gaps between measurements were not constant; and (c) by setting up a uniform data

rate, the same integrated filter solution can be compared between the commercial low-cost Xsens and the smartphone without additional processing.

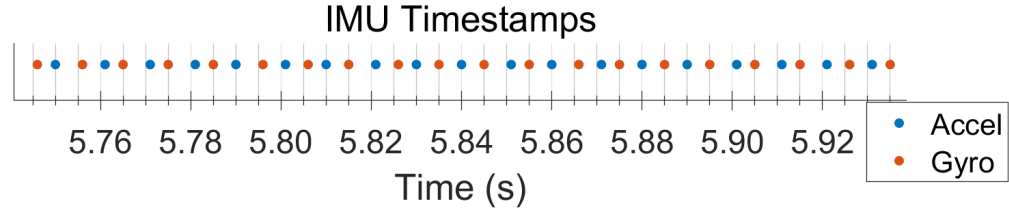


Figure 5.2: A snapshot of IMU timestamps over 0.25 s

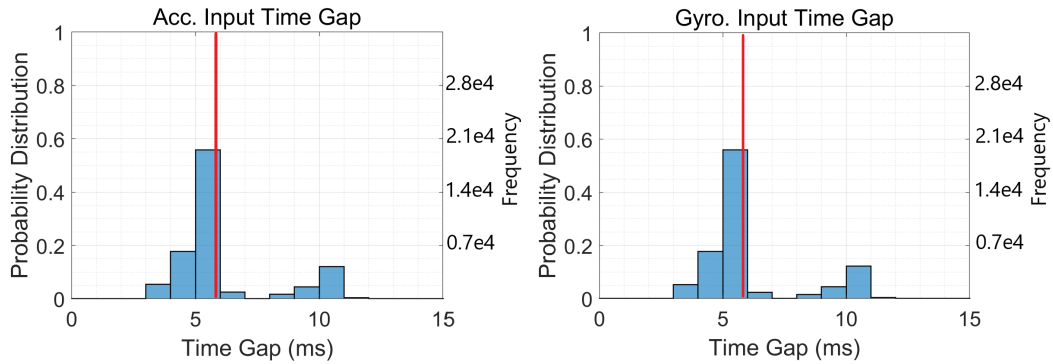


Figure 5.3: Histograms of time gaps between IMU accelerometer (up subplot) and gyroscope (down subplot) measurements; the red line indicates mean time gap of 5.8 ms

5.2.2 Tightly-coupled GNSS/IMU integration

The sensor fusion with IMUs is carried out using a tightly-coupled approach. In a tightly-coupled GNSS/IMU integration, a centralized filtering scheme is employed to update both IMU and GNSS estimated states. A solution-state vector is defined as a concatenated vector of IMU-specific states and GNSS-specific states. The first part of the solution vector is shared between PPP/IMU processing and RTK/IMU processing, while the second part is dependent on RTK or PPP the engine is processing. The PPP/IMU error measurement vector and error state vector have been introduced in Equation (2.38) and Equation (2.39) (Groves, 2013; Farrell and Barth, 1999; Seepersad et al., 2021). Similarly, RTK/IMU esti-

mated error state vector is represented as Equation (5.1):

$$\mathbf{x}_{RTK/IMU} = [\delta \mathbf{P}_{x,y,z} \quad \delta \mathbf{V}_{x,y,z} \quad \delta \mathbf{A}_{x,y,z} \quad \Delta \nabla T_r \quad \Delta \nabla I_r^s \quad \Delta \nabla N^s \quad \mathbf{B}_{x,y,z}^a \quad \mathbf{B}_{x,y,z}^g] \quad (5.1)$$

The same terms found in Equation (2.39) also apply. In cases where double-differencing is available, the receiver clock error is cancelled out. The measurement vectors take the pseudorange and carrier-phase measurements into account. In RTK/IMU integration, the error measurement vector is given as:

$$z_{k(RTK/IMU)} = \begin{bmatrix} \Delta \nabla P_{br,i,GNSS}^s - \Delta \nabla P_{br,i,IMU}^s \\ \Delta \nabla L_{br,i,GNSS}^s - \Delta \nabla L_{br,i,IMU}^s \end{bmatrix} \quad (5.2)$$

5.2.3 Smartphone hybrid RTK and PPP engine

Smartphone hybrid RTK and PPP engine is a well-established processing tool that is able to generate GNSS/IMU tightly-coupled solutions for geodetic, low-cost, as well as smartphone devices. As shown in Figure 5.4, the processing architecture comprises modules of the measurements/corrections sources, measurements/corrections, processing handlers, processing steps, and output, which are represented as red, green, blue, yellow, and purple boxes, respectively. The processing engine firstly stores all needed smartphone measurements retrieved from the Geo++ RINEX Logger (GNSS) and GnssLogger Apps, along with the Observation Space Representation (OSR) and State Space Representation (SSR) corrections for RTK and PPP processing, respectively. With time synchronization, the IMU handler is expected to re-sampling and align the raw IMU measurements, followed by initialization and biases compensation steps. The synchronization of GNSS and IMU data using GPS Pulse Per Second (PPS) was not incorporated in this study, which deserves future research. The states such as smartphone attitude, position, and velocity predicted from IMU mechanization then are fed back to the GNSS handler. Together with GNSS measurements, both of GNSS and IMU-related states are tightly-coupled and updated in the Kalman filter. The overall sanity check for GNSS/IMU solutions includes checks on various parameters such

as speed, accelerometer and gyroscope biases, pitch angles, covariance, positioning, and velocity differences to verify the quality and reliability of the TC solutions. After passing the sanity check, the processing engine steps into the next epoch. The output including RTK, PPP, their tightly-coupled GNSS/IMU solutions, as well as the corresponding state covariance matrices are generated concurrently to ensure the real-time and forward processing capability of the processed PPP/RTK approach.

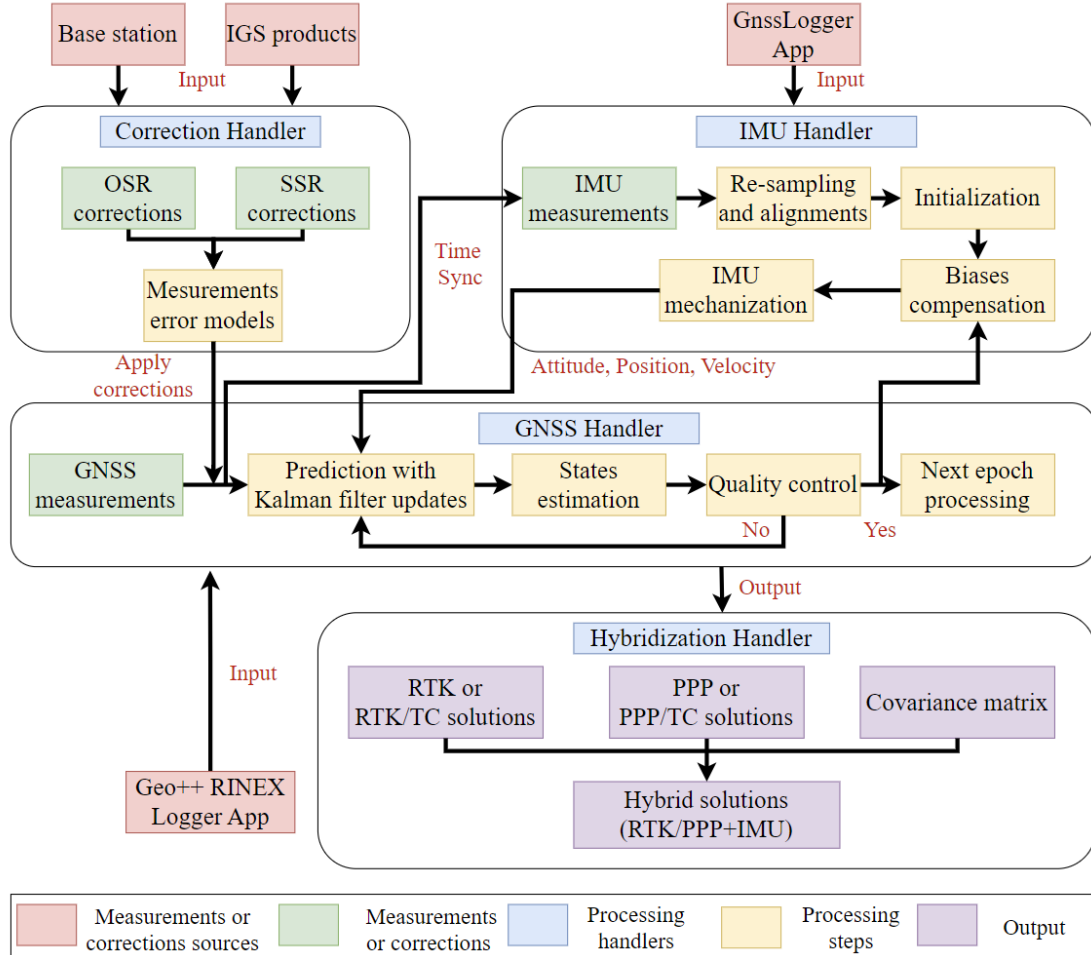


Figure 5.4: Processing architecture for smartphone hybrid RTK and PPP engine

In terms of PPP processing, this research uses the rapid satellite orbit and clock products from GFZ to simulate real-time positioning, CAS products are used for satellite DCB corrections. Furthermore, the tropospheric hydrostatic delay is corrected by GMF (Global mapping function) while its wet delay is estimated in the filter. The ionospheric delays are

constrained by the Klobuchar model. The hybrid algorithm uses the RTK and PPP solutions and the corresponding state covariance matrices to further enhance the overall positioning performance. The hybrid PPP/RTK solution processed with the GNSS measurements from the same rover can be given as Equation (5.3) (Delchambre, 2015):

$$X_{hybrid} = \frac{(W_{RTK}X_{RTK} + W_{PPP}X_{PPP})}{W_{RTK} + W_{PPP}} \quad (5.3)$$

where W_{RTK} and W_{PPP} are the estimated weights for RTK and PPP, respectively.

To mimic daily driving and correspond to consumer habits, the tested smartphone is mounted on the vehicle dashboard. Three smartphone datasets were collected on separate days on the same route, and Table 5.1 highlights the test numbers, collection dates, as well as time duration for these datasets. Multiple multipath profiles including (A) suburban arterial roads, (B) suburban roads, (C) local streets, as well as (D) overpasses, are taken into consideration to characterize smartphone precise positioning performance under different environments with different processing strategies.

Table 5.1: Summary of collected smartphone datasets

Test #	Collection Date (2021)	GPS time	Duration
1	30 November	02:04-02:28	24 mins
2	8 August	01:15-01:42	27 mins
3	12 October	00:20-00:43	23 mins

5.3 Smartphone GNSS/IMU performance assessment

The smartphone positioning performance processed with RTK, PPP, and hybrid PPP/RTK with tightly-coupled IMU integration are comprehensively compared in this section. All these aforementioned algorithms are verified through three kinematic vehicle experiments.

5.3.1 IMU dead reckoning assessment

This research first attempts to produce IMU-only solutions over two separate 60-second GNSS outages where the positioning solution is generated solely from IMU inputs. To

simulate such outages, outages A and B are extended by not including GNSS measurements during the 60 seconds. The solution states are first initialized using a reference solution produced by SPAN, with lever arm correction to the smartphone frame. The horizontal errors are compared against the SPAN reference solution in post-processed RTK/IMU with full access to all available measurements in Figures 5.5 and 5.6. The results show higher horizontal errors from Mi 8 (blue) than from Xsens (red) for the complete duration of Outage A and after 20 seconds for Outage B. The Xsens has a slower diverging rate, suggesting a less noisy raw IMU performance. In general, the result suggests robust ultra-low-cost IMU performance for an outage less than 10 seconds comparable to that of automotive-grade IMUs such as the Xsens.

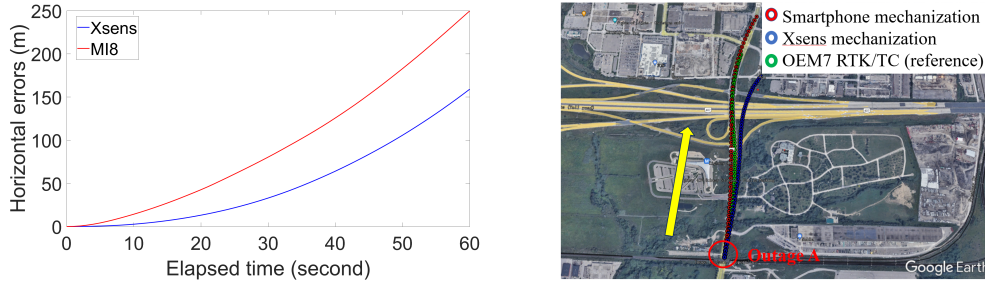


Figure 5.5: Position estimates based on dead-reckoning of Xsens and MI 8 IMUs only within 60 s. (a) IMU dead-reckoning horizontal errors within 60 s at Outage A. (b) Google Earth view of IMU mechanization trajectories at Outage A.

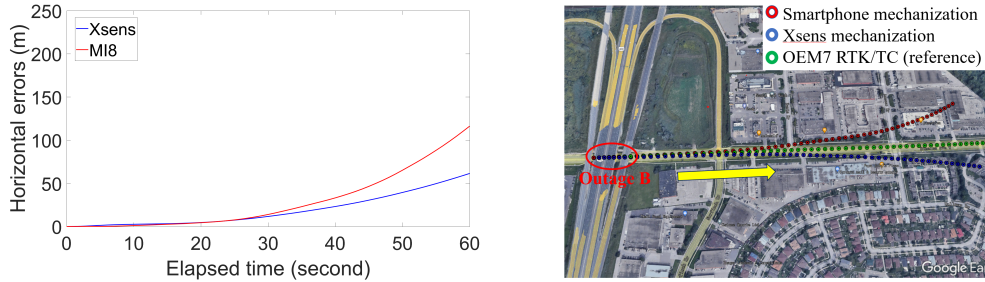


Figure 5.6: Position estimates based on dead-reckoning of Xsens and MI 8 IMUs only within 60 s. (a) IMU dead-reckoning horizontal errors within 60 s at Outage B. (b) Google Earth view of IMU mechanization trajectories at Outage B.

Another key observation is that the IMU-only solution shows better accuracy through Outage B compared to Outage A. Theoretically, IMU error propagation is complicated and

influenced by a range of factors including initialization errors, IMU timing discrepancy, finite iteration rates, and noises. Measurement biases, mainly from gyroscope also play a critical role in impacting IMU dead-reckoning solutions, and the positioning errors δX in the respective axis can be represented as Equation (5.4) (Groves, 2013):

$$\begin{aligned}\delta X_a &\approx \frac{1}{2}b_a t^2 \\ \delta X_g &\approx \frac{1}{6}b_g g t^3\end{aligned}\tag{5.4}$$

where δX_a and δX_g are the position errors caused by accelerometer and gyroscope biases (b_a and b_g), respectively; g is gravity and t is the IMU mechanization time. In other words, a 10^{-4} m/s² accelerometer bias and a 10^{-3} rad/s gyro bias theoretically produce position errors of 0.5 cm and 1.6 m, respectively, in 10 s.

Further analysis of these IMU biases is presented in Figure 5.7. The experiments were conducted in kinematic environments that started with a straight path. The initial biases are set as zero and it takes time for both of IMUs' biases to converge. The engine has run for 5 min prior to Outage A compared to 15 min prior to Outage B. From Figure 5.7, it can be observed that the bias states are better converged at the 15-min mark than the 5-min mark, potentially giving a better bias estimation. In addition, the Xsens IMU has more stable bias estimation which converges faster than for the smartphone IMU. For the accelerometers, the Mi 8 IMU does not show a clear converging pattern. For the gyroscopes, the Mi 8 IMU takes almost 10 min more to reach a stable state compared to the Xsens. This comparative performance is expected owing to the lower cost and performance of the smartphone IMU. As a consequence of these two reasons, the early estimated biases lead to the lower dead-reckoning performance at Outage A compared to Outage B.

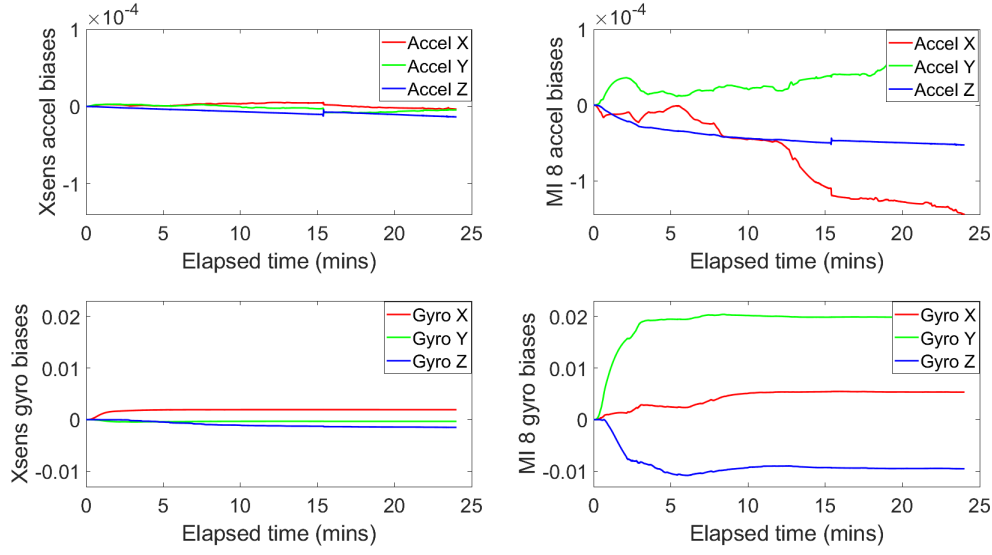


Figure 5.7: Time series of estimated IMU accelerometer and gyroscope biases with along three axes. The unit for acceleration bias is metres per second squared (m/s^2), and the unit for gyro bias is radians per second (rad/s)

5.3.2 Positioning performance assessment with RTK and PPP

To investigate smartphone positioning performance in realistic driving environments, three different GNSS configurations namely SPP, RTK, and PPP, are utilized for test 1. As a traditional GNSS positioning technique, SPP has been widely utilized for smartphones that can only receive code-only observations. Figure 5.8 illustrates the time series of horizontal errors (with respect to the reference trajectory) produced by SPP, RTK, as well as PPP, which are represented as red, green, and blue lines, respectively. It is anticipated that the RTK and PPP solutions will outperform the SPP solution, and the latter approach cannot either provide stable positioning solutions, owing to inaccurate pseudorange-only measurements and insufficient observations. Meanwhile, it is worth noting that horizontal errors increase significantly at the time 2.33 hours, when the vehicle traverses the underpass, which shows the limitation of these GNSS-only positioning techniques. Furthermore, in spite of the initial convergence time, it is interesting to observe that smartphone PPP and RTK have similar positioning performances throughout, indicating that PPP has the capability

of achieving similar accuracy as RTK in various complex environments, without double-differencing and base station setup.

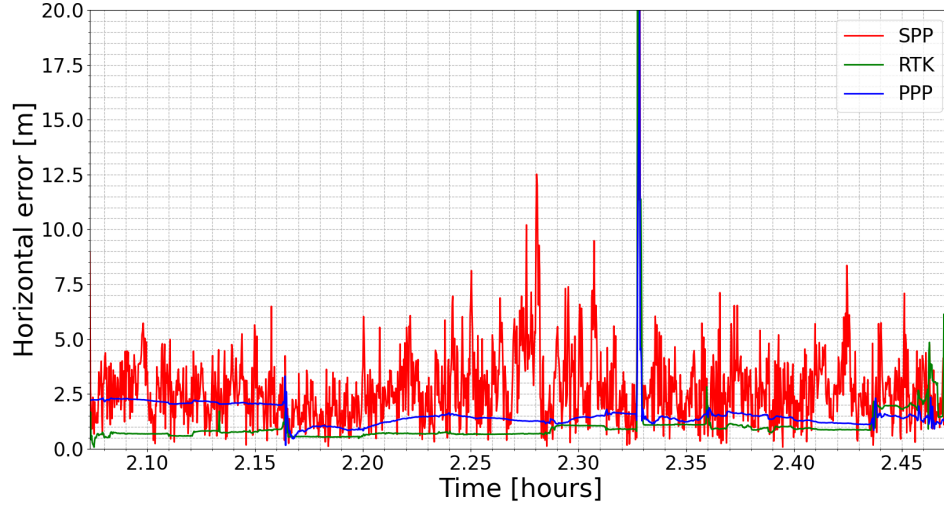


Figure 5.8: Horizontal error with respect to the reference trajectory comparison processed by SPP (red), RTK (green), and PPP (blue) for test 1

Table 5.2 highlights the horizontal error summary statistics from SPP, RTK, and PPP for test 1. As expected, the SPP solution has the worst positioning performance with 5.6 m, 3.1 m, and 1.8 m for 95th percentile error (2σ), 68th percentile error (1σ), and overall rms, respectively, indicating the importance of incorporating carrier-phase measurements in the processing. The 68th percentile error cannot be equated with the overall rms since the smartphone’s horizontal performance exhibits non-zero mean errors in this research. In comparison to PPP, RTK performs slightly better with a few decimetres in terms of 95th and 68th percentile errors, due mainly to the mitigation or elimination of atmospheric errors. However, it is surprising to note that the overall rms from PPP is only 1.8 m, which is better than the RTK overall rms (2.2 m), owing to the impacts of position outliers.

Table 5.2: Statistics of horizontal errors with different processing strategies (unit: m)

Processing approaches	95 th Percentile Error	68 th Percentile Error	RMS	Maximum Error
SPP	5.6	3.1	3.9	31.9
RTK	1.9	0.9	2.2	62.5
PPP	2.2	1.6	1.9	35.3

Correspondingly, Figures 5.9 and 5.10 show the time series of the number of processed satellites, observations, and PDOP processed by PPP/RTK strategies for test 1. Since the processing engine utilizes the single- and dual-frequency signals combination method, an average of 26.5 satellites with 58.5 observations (code+phase) are tracked and processed, which significantly enhances the GNSS positioning accuracy and robustness compared to single-frequency GNSS or dual-frequency GNSS. These two figures reflect the GNSS conditions around adjoining areas, and it is observed that the number of satellites/observations and PDOP change significantly when the observed time is 2.33 hours owing to signal blockage, resulting in large positioning errors. Therefore, mitigating the negative effects during GNSS outages is the key to enhancing smartphone precise positioning in challenging environments.

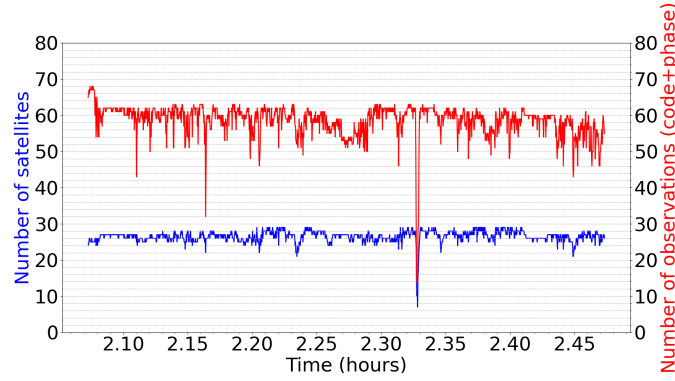


Figure 5.9: Time series of the number of processed satellites (blue) and observations (red, code+phase) for test 1

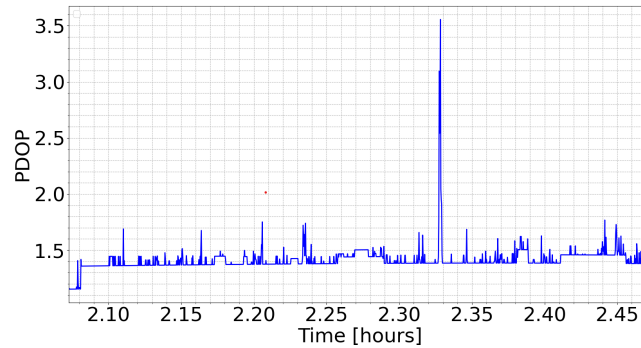


Figure 5.10: Time series of PDOP for test 1

Since different factors such as traffic conditions, satellite availability, and atmospheric activity significantly affect smartphone GNSS positioning performance, it is important to validate the processing algorithm and strengthen the experimental repeatability through multiple independent experiments. Considering that, this research analyzes two more datasets collected on separate days with identical route to better appreciate smartphone RTK and PPP performance. The following set of time series of horizontal errors for road tests 2 and 3 are illustrated in Figure 5.11 and Figure 5.12, respectively. Table 5.3 displays the corresponding horizontal statistics.

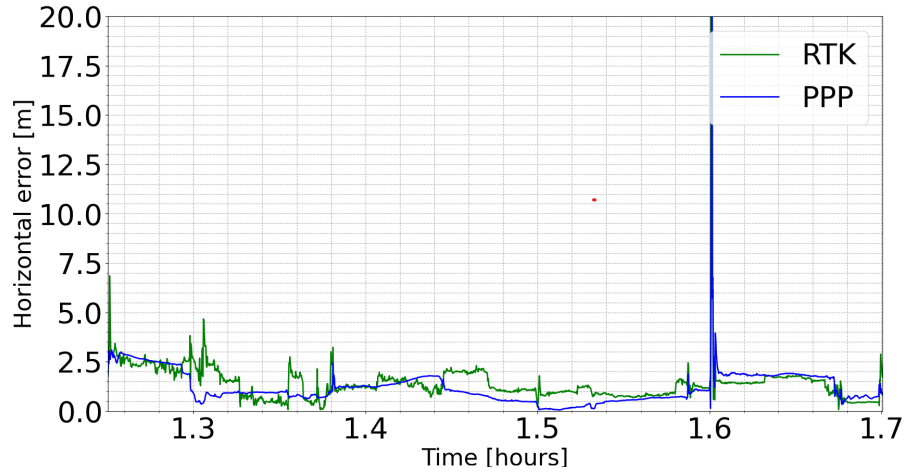


Figure 5.11: Time series of horizontal error for test 2

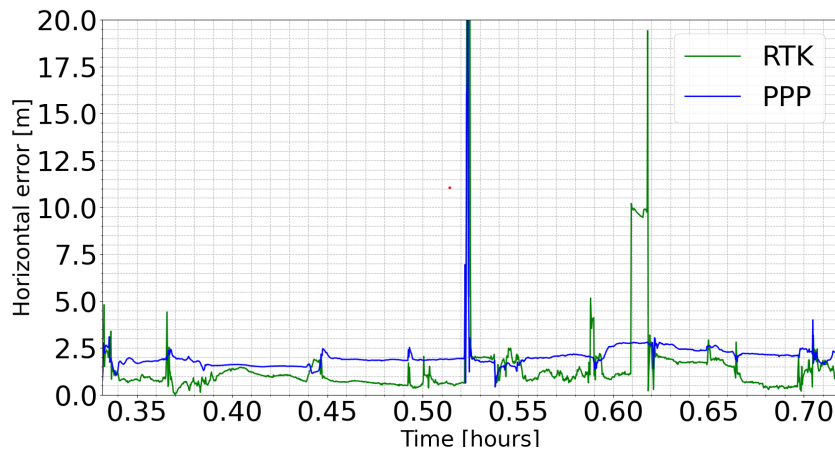


Figure 5.12: Time series of horizontal error for test 3

Together with test 1, it is concluded that RTK solutions have more accurate positioning performance compared to PPP GNSS-only solutions with regards to 95th and 68th percentile errors, due to fast initial convergence. Take test 3 as an example, RTK solution can provide 2.4 m and 1.3 m of 95th and 68th percentile errors which outperform the PPP solutions (2.7 m and 2.1 m). Similar levels of PPP or RTK positioning accuracy were also seen in recent publications (Fortunato et al., 2019a; Li et al., 2022b; Wang et al., 2021). However, it should be noted that none of the aforementioned studies achieve the smartphone precise positioning in both RTK and PPP modes with similar accuracy under suburban environments, and it is interesting to note that PPP is capable of providing the same level of positioning accuracy as the RTK after convergence, and owing to the negative impacts from the large positioning outliers during GNSS outages, PPP solutions can even provide more accurate overall rms in comparison to RTK solutions, and this finding has not been seen in previous studies. Meanwhile, it is important to note that both of these two GNSS-only positioning performances degrade significantly in GNSS-challenging environments, which restricts the use of smart devices for precise navigation applications in real driving scenarios.

Table 5.3: Statistics of horizontal errors with different processing strategies for road tests 2 and 3 (unit: m)

Processing approaches	95 th Percentile Error	68 th Percentile Error	RMS	Maximum Error
RTK (test2)	2.5	1.5	2.1	38.5
PPP (test2)	2.6	1.6	1.6	27.3
RTK (test3)	2.4	1.3	2.6	41.4
PPP (test3)	2.7	2.1	2.2	22.7

5.3.3 Positioning performance assessment with GNSS/IMU

To assess the performance discrepancy with and without smartphone native IMU tightly-coupled integration, Figures 5.13-5.14 display the time series of horizontal errors processed by RTK, RTK/TC, PPP, and PPP/TC for road tests 1 and 2, and these four aforementioned positioning solutions are marked as green, red, blue, and black, respectively.

In contrast to GNSS-only RTK and PPP, the tightly-coupled RTK/TC and PPP/TC approaches are capable of reducing positioning errors significantly, most notably during GNSS outages. As highlighted in Table 5.4, it can be observed that RTK/TC and PPP/TC

solutions provide little enhancement for 95th and 68th percentile errors, showing that the fusion method did no harm to the GNSS-only solutions for most epochs with good satellite geometry. On the other hand, the integration of the IMU brings significant benefits in mitigating the majority of error peaks. For example, the maximum errors observed at test 1 are reduced from 62.5 m and 35.3 m to 15.8 m and 18.9 m, respectively, for RTK/TC and PPP/TC solutions, resulting in better overall positioning performance. Similarly, with the inclusion of IMU, the maximum errors processed by RTK (38.5 m) and PPP (27.3 m) for test 2 are down to 13.3 m and 10.6 m, respectively. Consequently, overall rms for RTK and PPP are reduced from 2.1 m and 1.6 m to 1.5 m and 1.4 m, respectively. However, owing to the noisy measurements and IMU biases instability, smartphone GNSS-RTK/IMU or GNSS-PPP/IMU solutions cannot completely eliminate the positioning gross errors.

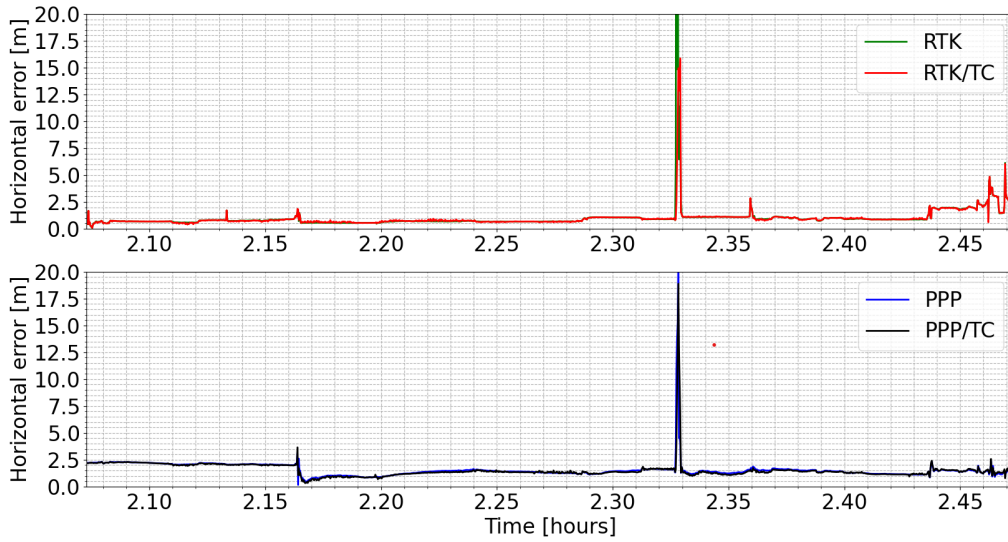


Figure 5.13: Horizontal error with respect to the reference trajectory comparison processed by RTK (green), RTK/TC (red), PPP (blue), and PPP/TC (black) for test 1

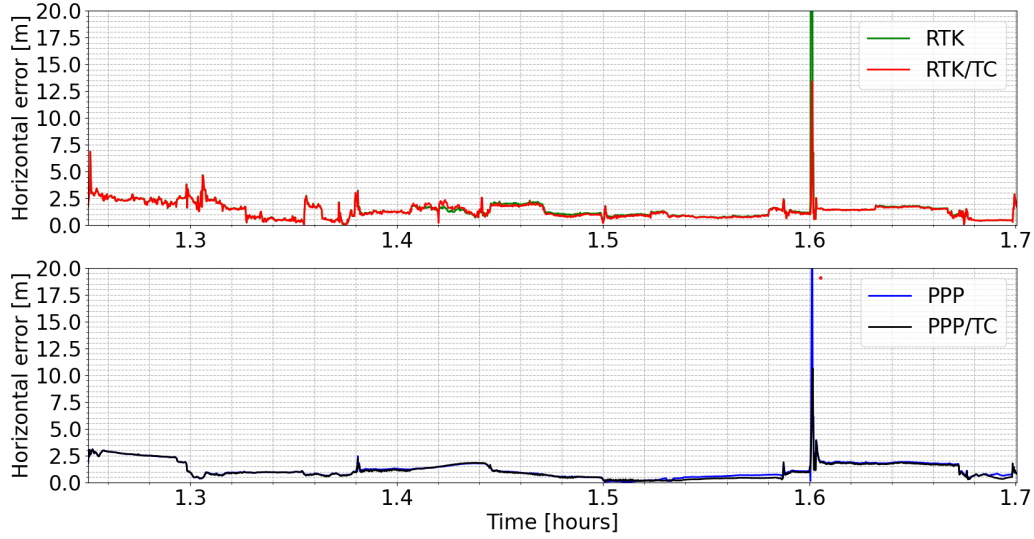


Figure 5.14: Horizontal error with respect to the reference trajectory comparison processed by RTK (green), RTK/TC (red), PPP (blue), and PPP/TC (black) for test 2

Table 5.4: Statistics of horizontal errors with different processing strategies for road tests 1-3 (unit: m)

Processing approaches	95 th Percentile Error	68 th Percentile Error	RMS	Maximum Error
RTK/TC (test1)	1.9	0.9	1.3	15.8
PPP/TC (test1)	2.2	1.6	1.7	18.9
RTK/TC (test2)	2.5	1.5	1.5	13.3
PPP/TC (test2)	2.6	1.6	1.4	10.6
RTK/TC (test3)	2.1	1.3	1.6	24.0
PPP/TC (test3)	2.6	2.1	1.9	14.8

The smartphone measurement quality and the number of satellite tracked is dependent on the environment in which the smartphone is operating, and the measurement quality and availability define the accuracy of the position solution. Thus, the effect of the environment on position solution accuracy is studied in the current research. As shown in Figure 5.15, the track of the smartphone data is divided into suburban arterial roads, local streets, and overpasses. Whilst the local streets are colour coded in green, and the overpass is coded in red. The position horizontal accuracy was analyzed for the environments of different navigation engines. Table 5.5 demonstrates the statistics of horizontal errors comparison with different processing strategies under varied multipath profiles.

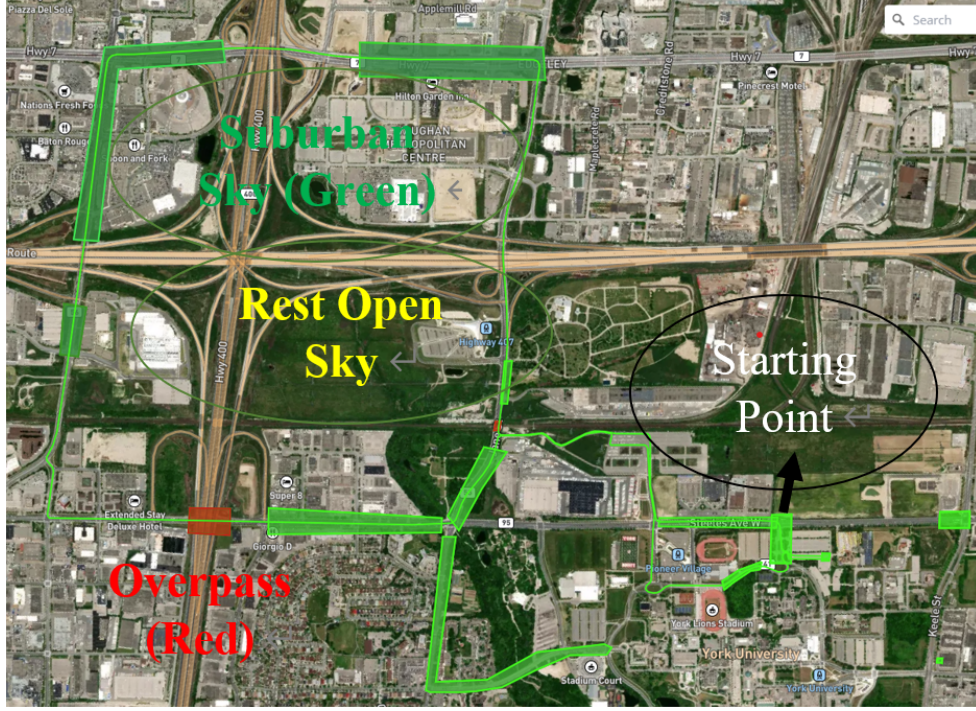


Figure 5.15: Geofencing of vehicle routes

Table 5.5: Statistics of horizontal errors with different processing strategies under different multipath profiles (unit: m)

Processing approaches	Suburban arterial roads		Local streets		Overpass	
	95 th Percentile	RMS	95 th Percentile	RMS	95 th Percentile	RMS
RTK	2.0	1.1	2.0	1.0	19.1	10.0
RTK/TC	2.0	1.1	2.0	1.0	15.2	7.7
PPP	2.2	1.5	2.2	1.6	21.5	9.0
PPP/TC	2.1	1.5	2.2	1.6	14.7	6.3

The result shows similar performance between suburban arterial roads and local streets. For example, both RTK and RTK/TC solutions provide 2.0 m percentile error and 1.1/1.0 m overall rms in these areas, which is somewhat in conflict with the expectation that suburban arterial roads should have performed a superior solution in a more open-sky scenario, the possible reason is that the heavier traffic and urban development along the arterial roads obscure the two types of environments. Furthermore, it is again proved that the integration of smartphone IMU is beneficial for improving the overall positioning performance, specifically in the GNSS obstruction scenarios. Correspondingly, the PPP 95th percentile error

and overall rms at the overpass are reduced from 21.5 m and 9.0 m to 14.7 m and 6.3 m, respectively.

5.3.4 Hybridization of RTK and PPP solutions

The hybridization of RTK and PPP is another focus of this work, and the hybrid solutions are fused from RTK and PPP solutions according to the estimated precision values from the filter covariance matrix. Figures 5.16-5.17 show the time series of horizontal error comparison for road tests 1 and 2, where RTK, PPP, and hybrid PPP/RTK including their TC solutions are represented as green, blue, and red lines, respectively. It is noticeable that the hybrid solutions inherit advantages from PPP and RTK and provide more accurate positioning for all tested datasets throughout.

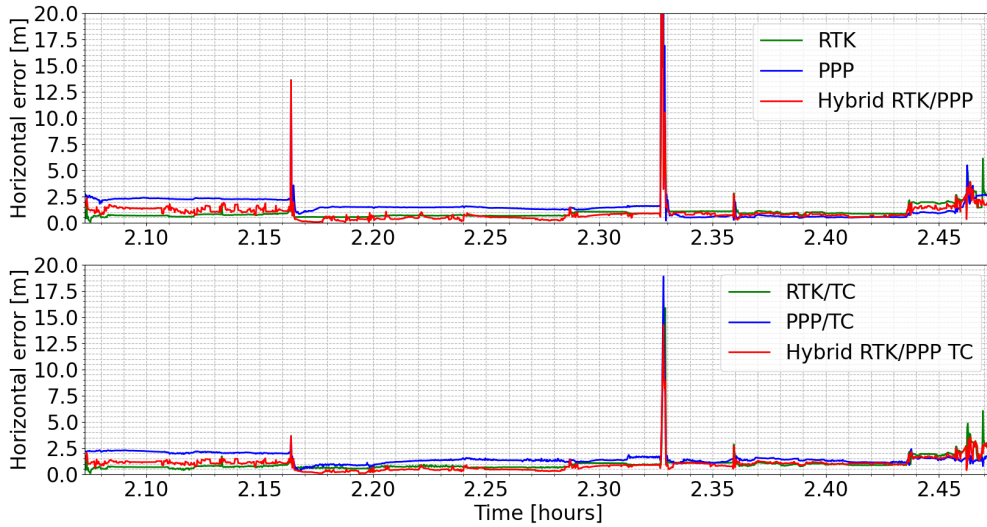


Figure 5.16: Horizontal error with respect to the reference trajectory for road tests 1. RTK, PPP, and PPP/RTK hybrid solutions are represented as green, blue, and red lines, respectively. Same for the tight-coupled solutions

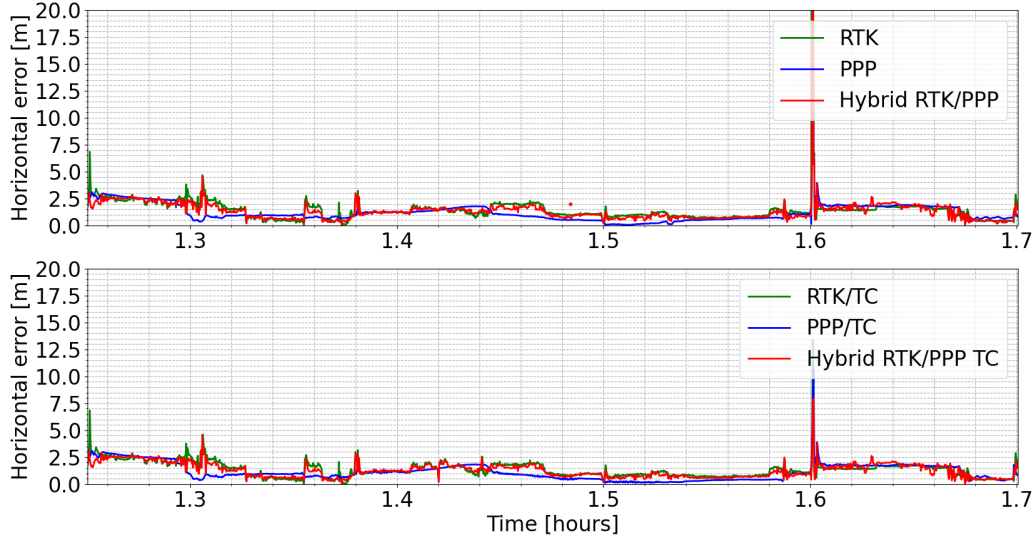


Figure 5.17: Horizontal error with respect to the reference trajectory for road tests 2. RTK, PPP, and PPP/RTK hybrid solutions are represented as green, blue, and red lines, respectively. Same for the tight-coupled solutions

Statistically, Figure 5.18 summarizes the overall horizontal positioning errors with six combinations of processing strategies through all three road tests 1-3. RTK, PPP, and hybrid PPP/RTK solutions are marked as green, blue, and red bars, respectively, and their TC solutions are represented in lighter colours. On the one hand, it is found that the PPP solution is capable of delivering comparable positioning accuracy as RTK solutions after the convergence time. On the other hand, the integration of IMU can significantly reduce the overall rms for the GNSS-only solutions. More importantly, the PPP/RTK hybridization technique enhances smartphone GNSS-only positioning overall performance, namely 2.3 m, 1.3 m, and 2.0 m of horizontal 95th percentile error, 68th percentile error, and rms can be observed, respectively, which are the best solutions among the RTK and PPP strategies. Furthermore, the integration of IMU can still contribute to the hybrid solution, which reduces the overall rms from 2.0 m to 1.4 m. Although similar smartphone positioning performances are achievable and seen in recent studies (Li et al., 2022b; Fortunato et al., 2019a; Wang et al., 2021), our research is the first to present a superior hybrid solution than either single processing mode of RTK or PPP, while achieving a robust solution accuracy in each mode alone.

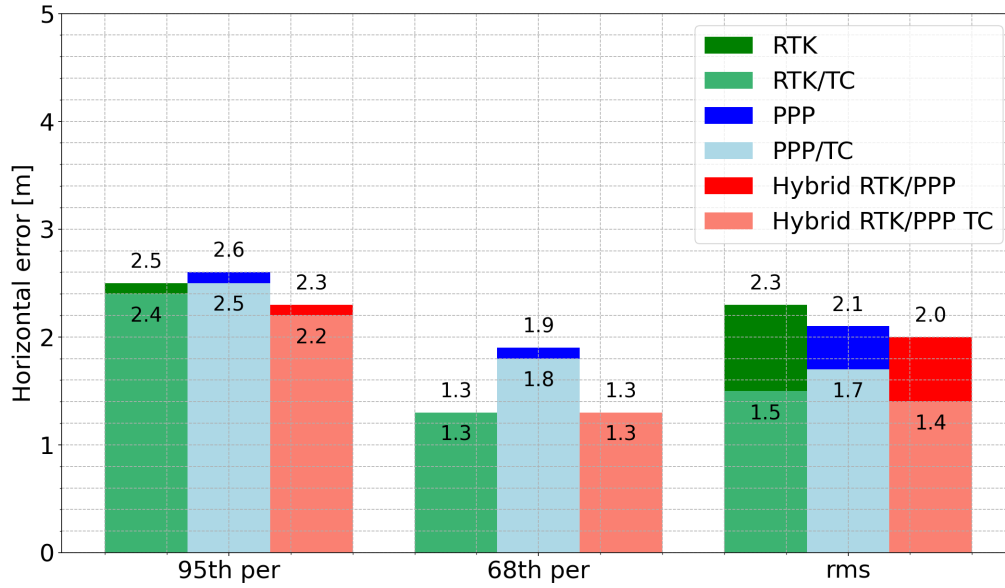


Figure 5.18: Overall horizontal positioning statistics with RTK, PPP, hybrid PPP/RTK, and their tightly-coupled solutions

5.3.5 Identifying scenarios where IMU integration benefit most

To specify the benefits of incorporating smartphone IMU measurements under different environments, Figure 5.19 presents the horizontal rms with the number of satellites used in single and dual-frequency PPP processing. On account of buildings and tunnels limiting portions of the sky, the minimum of 4 satellites are not always available for smartphone GNSS positioning in suburban driving. Observing both Table 5.6 and Figure 5.19, the smartphone IMU can bridge these positioning gaps and significantly mitigate positioning errors when visible satellites are insufficient, e.g., a significant level of 98% and 87% horizontal positioning improvements is observed with the aid of the smartphone IMU measurements when the number of satellites is reduced to 4 and 5, respectively. The smartphone PPP/IMU solution is capable of providing stable and continuous positioning solutions, and the sensor fusion solution maintains the same grade of accuracy performance even with fewer satellites. The possible reason is that smartphone PPP/IMU solutions not only rely on GNSS satel-

lite measurements, but also are relevant to predicted state covariance, previous estimated states, and IMU raw measurement quality.

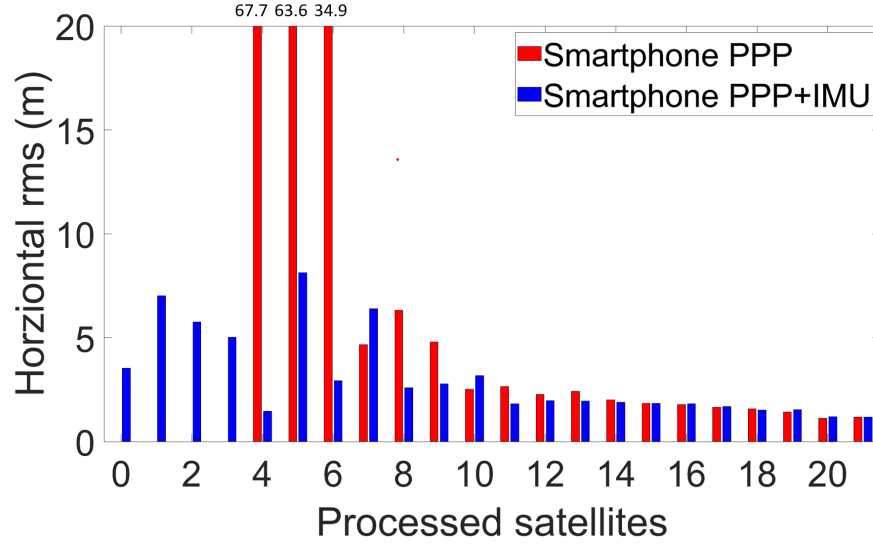


Figure 5.19: Correlation of the horizontal errors on the number of satellites.

Table 5.6: Statistics of horizontal errors with processed satellites for single- and dual-frequency processing (unit: m)

Processed satellites	0	1	2	3	4	5	6	7	8	9	>9
PPP	-	-	-	-	67.7	63.6	34.9	4.6	6.3	4.7	1.7
PPP/IMU	3.5	7.0	5.7	5.0	1.5	8.1	2.9	6.4	2.6	2.7	1.6

5.4 Summary

The findings from this research made several noteworthy contributions by integrating smartphone IMU with GNSS measurements. Smartphone precise positioning performance was comprehensively assessed and compared with advanced GNSS techniques such as RTK and PPP. Furthermore, this research proposed a novel hybridization approach by utilizing both the RTK and PPP solutions and covariance to improve smartphone positioning accuracy. To validate the effectiveness of the proposed algorithm and investigate the smartphone positioning performance with different processing configurations, a total of three datasets collected on separate days were analyzed, whilst the smartphone was mounted on the dash-

board to mimic realistic usage. Based on these investigations, the contributions and novelty of this chapter can be summarized as:

First, the dead-reckoning experiments demonstrated that the smartphone's IMU can achieve metre-level positioning accuracy during outages lasting less than 10 seconds, a performance comparable to the low-cost Xsens IMU. Furthermore, the theoretical analysis revealed that a 10^{-4} m/s² accelerometer bias and a 10^{-3} rad/s gyro bias could result in position errors of 0.5 cm and 1.6 m, respectively, over a span of 10 seconds. This underscores the critical role of bias compensation in producing IMU mechanization solutions.

In this context, the research proposed a PPP/RTK hybridization algorithm to address the inherent limitations of RTK and PPP for smartphone processing. By adjusting the solution weights based on estimated precision values, the hybrid algorithm achieved significant improvements, with 95th percentile error, 68th percentile error, and overall horizontal rms values of 2.3 m, 1.3 m, and 2.0 m, respectively. Compared to prior investigations employing standalone GNSS PPP or RTK strategies (Wu et al., 2019; Fortunato et al., 2019b; Yi et al., 2021), the integration of the smartphone IMU with this hybrid PPP/RTK algorithm led to better results, with 95th percentile error and overall rms reduced to 2.2 m and 1.4 m, respectively, representing an "optimal" solution compared to other GNSS/IMU strategies and previous studies.

Chapter 6

Galileo High Accuracy Service (HAS) with smartphone GNSS/IMU sensors

The Galileo High Accuracy Service (HAS) has undergone substantial development in recent years, offering users free access to GPS and Galileo satellite orbit, clock, and code bias corrections for user PPP on a global scale. This chapter researches the use of the currently available HAS corrections for smartphone positioning. Due to hardware disparities and limited tracking capabilities, smartphone processing with only two constellations struggles to ensure satisfactory satellite geometry and sufficient observations in realistic user environments. To fully harness all observed code and phase measurements and the orbit and clock information directly disseminated from satellites, this chapter introduces a new PPP algorithm combining HAS corrections and broadcast ephemerides (HASandBRDC PPP) for smartphone processing. This chapter is mainly referenced from the author's latest manuscript entitled "Precise positioning utilizing smartphone GNSS/IMU integration with the combination of Galileo High Accuracy Service (HAS) corrections and broadcast ephemerides", currently under review at *GPS Solutions*.

6.1 Introduction and motivation

A recent trend in positioning communities has witnessed the emergence of the direct broadcast of SSR corrections through GNSS constellations. Japan's Quasi-Zenith Satellite System

Centimeter-Level Augmentation Service (QZSS CLAS) is notable for being the first augmentation system to transmit Compact SSR (CSSR) corrections through the L6D signal. CLAS aims to provide a range of corrections, including satellite orbit, clock, code/phase biases, and ionospheric/tropospheric corrections, to GPS, Galileo, and QZSS satellites, covering the entirety of Japan (Saito et al., 2011; Miya et al., 2016). For wide usages, the Japan Aerospace Exploration Agency (JAXA) has introduced the Multi-GNSS Advanced Orbit and Clock Augmentation (MADOCA) system, which initiated the MADOCA-PPP trial service on September 30th, 2022, aiming to provide PPP services in the Asia-Oceania region by delivering satellite orbit, clock, code/phase corrections for GPS, Galileo, GLONASS, BDS, and QZSS satellites through the L6E signal (Kawate et al., 2023). Similarly, BDS-3 (BeiDou-3) offers customized CSSR corrections, including GPS and BDS satellite orbit/clock corrections and code biases through the B2b signal, all geared toward achieving high PPP performance for users in China and the surrounding regions (Tao et al., 2021; Xu et al., 2021).

Unlike regional services, the European Union is actively developing the Galileo HAS, poised to play a pioneering role by offering Signal-in-Space (SIS) corrections to global users free of charge. These corrections, generated by Galileo Sensors Stations and corresponding infrastructures, are transmitted within the Galileo C/NAV navigation message through the E6B signal (1278.75 MHz). In accordance with the specifications and provisions put forth by the European Commission, Galileo HAS Service Level 1 (SL1) has a primary goal of providing global GPS and Galileo satellite orbit, clock, and code bias corrections, conveyed at a data rate of 448 bits and structured in a format similar to CSSR. The aim is to facilitate PPP-based horizontal and vertical accuracies better than 20 cm and 40 cm, respectively, with a convergence time of 300 seconds and 99% data availability. Galileo HAS successfully declared its initial services operational starting from January 24th, 2023 (EUSPA, 2023), and these corrections have also been accessible through the HAS Internet Data Distribution (IDD) service since then. Moving forward, the Galileo HAS Service Level 2 (SL2) is designed to extend its offerings by providing atmospheric corrections to users within the European Coverage Area (ECA). This expansion makes PPP-RTK feasible, potentially

reducing the convergence time to less than 100 seconds. The full operational deployment of Galileo HAS is scheduled for 2024 and later, promising further advancements in satellite-based positioning accuracy and capabilities (EUSPA, 2020).

Recently, Galileo HAS has garnered considerable attention in the GNSS community, with a focus on exploring its accuracy and potential for mass-market applications. Pioneering this field, Fernandez-Hernandez et al. (2022) provided an overview of HAS architecture, corrections accuracy and user PPP performance in the HAS initial phase. Initial results demonstrated that HAS PPP performance can achieve 10 cm in both horizontal and vertical domains under static and open-sky conditions. Naciri et al. (2023) confirmed that HAS corrections can provide signal-in-space range errors (SISRE) of 10.6 cm and 11.8 cm for Galileo and GPS satellites, respectively. Additionally, HAS float PPP solutions can achieve targeted performance with 13.1 cm and 18.6 cm in horizontal and vertical domains, respectively. Similar analyses accessing HAS corrections in 2023 can also be found in recent studies (Parra et al., 2023; Yi et al., 2023). The introduction of Galileo HAS has also advanced PNT applications. Hauschild et al. (2022) successfully utilized HAS corrections for the precise orbit determination (POD) for low Earth orbit (LEO) satellites. Martini et al. (2024) characterized Galileo HAS corrections and improved the positioning performance with satellite anomaly mitigation in static and kinematic environments.

Meanwhile, the maturity of GNSS and its associated augmentation services has catalyzed unprecedented growth and evolution of low-cost, multi-frequency, multi-constellation GNSS receivers and antennas. This surge in technological advancement has given rise to cost-effective GNSS/IMU sensor integration, which has received considerable attention (Abd Rabbou and El-Rabbany, 2015; Vana and Bisnath, 2020; Yi et al., 2022). Although progress has been made, there has been no quantitative investigation on the low-cost sensors HAS solutions in realistic driving scenarios. Furthermore, there are still significant challenges to relying solely on HAS corrections for smartphone positioning. While it has been confirmed that sub-metre accuracy is possible with smartphone HAS PPP in an open-sky static environment (Glaner and Weber, 2023), it is crucial to note that achieving such ideal

conditions is not feasible in, e.g., real-world, urban driving environments. Complex road conditions, obstructions, and complex multipath profiles make it extremely challenging to ensure precise smartphone positioning with sufficient GPS and Galileo satellites (Aggrey et al., 2020; Yi et al., 2021). Additionally, geodetic versus smartphone hardware disparities, such as the use of linearly polarized antennas mounted on smartphones, makes smartphone positioning more vulnerable to multipath effects and signal attenuation compared to traditional geodetic and low-cost receivers (Pesyna et al., 2014; Zhang et al., 2018b; Håkansson, 2019).

To fully utilize the orbit and clock information directly disseminated by satellites, an augmented PPP algorithm for smartphone navigation is proposed in this chapter. This algorithm utilizes HAS corrections and broadcast ephemeris information, leveraging GPS, Galileo, GLONASS, and BeiDou single- and dual-frequency observations, along with pseudorange-only measurements in cases where carrier-phase data is missing (Zangenehnejad and Gao, 2021; Yi et al., 2024). This approach is specifically designed to increase the redundancy of observations and optimize satellite geometry, thereby improving smartphone positioning performance when utilizing HAS corrections. Additionally, smartphone IMU measurements are incorporated to enhance the PPP performance in challenging environments.

6.2 Galileo HAS overview and SISRE computation

As outlined in the Galileo HAS information note, the implementation of Galileo HAS is organized into three phases, as depicted in Figure 6.1 (EUSPA, 2020). Phase 0 involved the successful completion of HA testing and experimentation activities between 2019 and 2022, validating Galileo’s dissemination capabilities. Phase 1, referred to as HA Initial Service, was designed to provide an SL1 service with reduced performance, and this phase was officially begun in January 2023. HAS implementation will then advance to Phase 2, which is aimed at achieving the full implementation of Galileo HAS and meeting the performance objectives of SL1 and SL2. Current improvements being focused on include additional reference stations

and the introduction of Galileo and GPS satellite phase bias corrections, which will allow for faster and more accurate PPP-Ambiguity Resolution (AR) on the part of users.

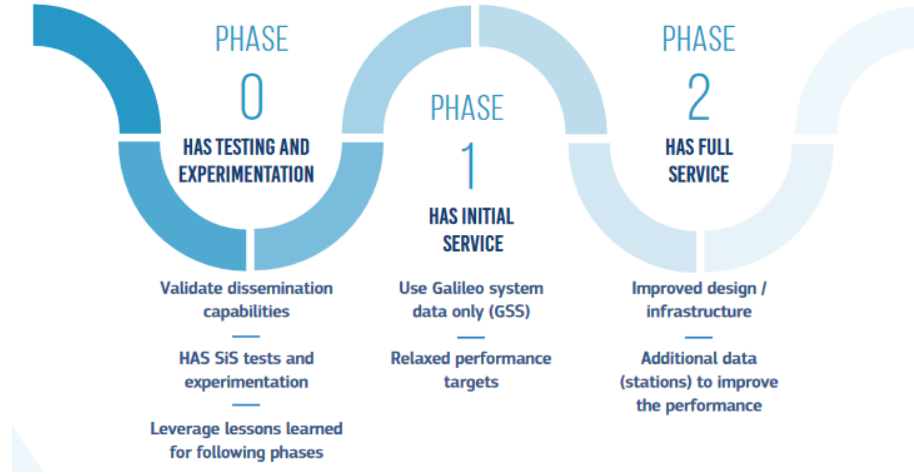


Figure 6.1: Galileo HAS implementation roadmap (EUSPA, 2020)

According to the Galileo HAS Interface Control Document (ICD) (European Union, 2022), HAS corrections are disseminated through the Galileo E6B signal, and these corrections correspond to GPS LNAV and Galileo I/NAV messages. The Center for Orbit Determination in Europe (CODE) products were chosen as the reference due to their high-quality data, and Table 6.1 illustrates the distinctions between HAS and CODE products. CODE products provide centre-of-mass (CoM) satellite coordinates, while HAS orbit corrections are to the ionosphere-free (IF) antenna phase centre (APC). The differences in utilizing the first primary signal and second primary signal between CODE and HAS affect the SISRE computation, which will be elaborated upon in subsequent sections.

Table 6.1: Comparison of HAS and CODE products in terms of orbit centre and the primary signals of the ionosphere-free (IF) combination that the satellite clocks refer to

Corrections	Orbit centre	First primary signal	Second primary signal
HAS Galileo	APC_IF	E1 (C1C)	E5b (C7Q)
CODE Galileo	CoM	E1 (C1C)	E5a (C5Q)
HAS GPS	APC_IF	L1 C/A (C1C)	L2P (C2P)
CODE GPS	CoM	L1 (C1W)	L2 (C2W)

6.2.1 Orbit corrections

The HAS orbit corrections are associated with GPS LNAV and Galileo I/NAV messages, referring to the ionosphere-free APC. These corrections consist of radial, along-track, and cross-track (RAC) corrections in the Satellite Coordinate System (SCS). Figure 6.2 illustrates the SCS system, where the along-track component represents the satellite velocity vector, the cross-track component is normal to the orbital plane, and the radial component is perpendicular to both the along-track and cross-track components (Fernandez-Hernandez et al., 2022).

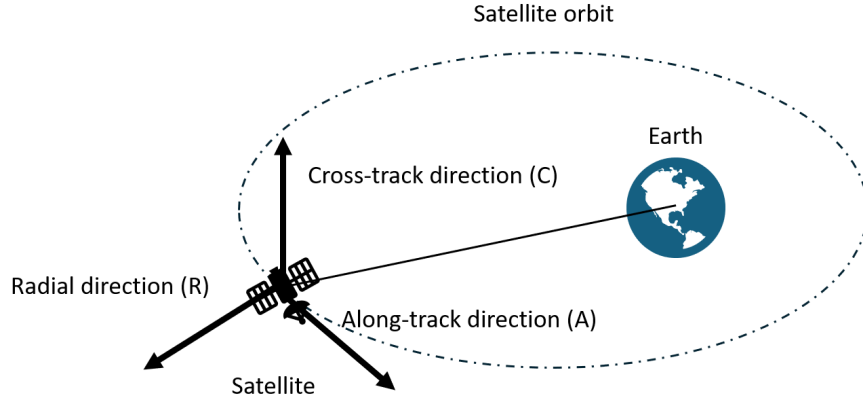


Figure 6.2: Satellite Coordinate System

HAS equations and application methods are references from HAS ICD (European Union, 2022), the Galileo HAS corrected orbit coordinates $\tilde{x}_{APC_{IF}}^s$ can be expressed as:

$$\begin{aligned}\tilde{x}_{APC_{IF}}^s &= x_{s_{IF}} + R_{RAC}^{ECEF} \delta R^s \\ R_{RAC}^{ECEF} &= \begin{bmatrix} \frac{\dot{x}^s}{|\dot{x}^s|} \times \frac{x^s \times \dot{x}^s}{|x^s \times \dot{x}^s|} & \frac{\dot{x}^s}{|\dot{x}^s|} & \frac{x^s \times \dot{x}^s}{|x^s \times \dot{x}^s|} \end{bmatrix}\end{aligned}\quad (6.1)$$

where $x_{s_{IF}}$ represents the broadcast satellite position, and δR^s represents the corresponding HAS orbit corrections, ranging from ± 10.2375 m, ± 16.376 m, and ± 16.376 m for the RAC directions, respectively. R_{RAC}^{ECEF} refers to the rotation matrix that converts RAC corrections to the broadcast ECEF frame, x^s and $\dot{x}^s$ represent the broadcast satellite position and velocity in the ECEF frame, respectively.

To evaluate the HAS orbit SISRE, the first step involves translating the HAS orbit coordinates from the APC $\tilde{x}_{APC}^s$ to the CoM $\tilde{x}_{CoM}^s$ using the ionospheric-free phase centre offsets (PCO) corrections obtained from the supplementary ANTEX files. Subsequently, the orbit errors in the radial (δR), along-track (δA), and cross-track (δC) directions can be calculated by taking the differences between $\tilde{x}_{CoM}^s$ from HAS and the referenced orbit coordinates x_{CoM}^s from CODE:

$$\begin{aligned} \tilde{x}_{CoM}^s &= \tilde{x}_{APC_{IF}}^s - PCO_{IF} \\ \begin{bmatrix} \Delta R & \Delta A & \Delta C \end{bmatrix} &= R_{ECEF}^{RAC} (\tilde{x}_{CoM}^s - x_{CoM}^s) \end{aligned} \quad (6.2)$$

And the orbit SISRE can be expressed as:

$$SISRE_{orbit} = \sqrt{w_R^2 \Delta R^2 + w_{A,C}^2 (\Delta A^2 + \Delta C^2)} \quad (6.3)$$

where the weight factors w_R is 0.98 and $w_{A,C}$ are 1/49 and 1/61 for GPS and Galileo, respectively (Montenbruck et al., 2015).

6.2.2 Clock corrections

The corrected HAS clock, denoted as $\tilde{dt}_{IF}^s$, can be expressed as (European Union, 2022):

$$\tilde{dt}_{IF}^s = dt_{IF}^s + \Delta t_r^s + \frac{\delta C^s}{c} \quad (6.4)$$

where dt_{IF}^s is the ionosphere-free broadcast satellite clock errors, Δt_r^s is the relativistic correction, and δC^s corresponds to the HAS clock corrections, ranging from ± 40.95 m. Considering that CODE and HAS clock products utilize different L1 and L2 signals for forming the ionosphere-free combination (as detailed in Table 6.1), it is essential to incorporate DCB corrections for assessing clock precision. Equation (6.5) describes the GPS satellite clock differences Δdt_{GPS} between HAS clock $\tilde{dt}_{IF}^{s,GPS}$ and CODE referenced clock $\overline{dt}_{IF}^{s,GPS}$, and relativistic correction should be removed as it is not applied in CODE clock

products.

$$\Delta dt_{GPS}^s = \tilde{dt}_{IF}^{s,GPS} - \overline{dt}_{IF}^{s,GPS} - \Delta t_r^s + \frac{f_1^2}{c(f_1^2 - f_2^2)} DCB_{C1C-C1W}^s \quad (6.5)$$

Considering that the constellation-specific systematic bias in clock products affects satellites uniformly, it is crucial to calculate the average clock differences $\overline{\Delta cdt}$ and eliminate it in the computation of total SISRE.

$$SISRE_{total}^s = \sqrt{\left(w_R \Delta R - (\Delta cdt - \overline{\Delta cdt})\right)^2 + w_{A,C}^2 (\Delta A^2 + \Delta C^2)} \quad (6.6)$$

6.2.3 Satellite biases

Unlike the satellite Differential Signal Bias (DSB) products, HAS satellite code biases are transmitted in Observable-specific Signal Biases (OSBs) format (Schaer, 2016). At present, satellite phase biases are not provided, and therefore PPP-AR is not feasible with the currently available HAS corrections. Table 6.2 outlines the specifications of HAS OSB corrections for both GPS and Galileo satellites,

Table 6.2: Specifications of HAS OSB corrections for GPS and Galileo

OSB types	Applied signals	Range (m)	Validity and update intervals (s)
HAS Galileo	E1 C, E5a Q, E5b Q, E6 C	± 20.46	3600 (50)
HAS GPS	L1 C/A, L2C, L2P	± 20.46	3600 (50)

The corrected pseudorange measurements can be expressed as follows:

$$\tilde{P}_{r,i}^s = P_{r,i}^s + OSB_{HAS} \quad (6.7)$$

6.3 PPP/IMU processing settings with the combination of HAS and broadcast information

Due to the restricted tracking capability of smartphone GNSS chipsets compared to geodetic hardware, each measurement is significant in smartphone PPP processing. To fully leverage GNSS observations from GREC (GPS-Russia-Europe-China) four constellations, this

chapter adopts an approach named "HASandBRDC PPP". Specifically, it proposes the utilization of HAS products for GPS and Galileo PPP processing and the use of broadcast products for GLONASS and BeiDou constellations for which HAS products are not available for. However, compared to HAS and precise products, broadcast ephemerides tend to exhibit significantly larger SISRE errors, which can adversely impact PPP performance.

To compensate the SISRE errors induced by broadcast products, a common approach is to incorporate the projected SISRE error as an additional estimated variable S^s into observation equations. The updated UDUC mode is expressed as (Gunning et al., 2019; Carlin et al., 2021):

$$\begin{aligned} P_{r,i}^s &= \rho_{r,i}^s + (cdt_r + b_{r,P_{IF}}) - (dt^s + b_{P_{IF}}^s) + \gamma_i \overline{I_{r,1}^s} + M_r T_r + S^s + \varepsilon_{P_i} \\ L_{r,i}^s &= \rho_{r,i}^s + (cdt_r + b_{r,P_{IF}}) - (dt^s + b_{P_{IF}}^s) + \gamma_i \overline{I_{r,1}^s} + M_r T_r + S^s + \lambda_i \overline{N_t^s} + \varepsilon_{p_i} \end{aligned} \quad (6.8)$$

It is important to note that the broadcast clock of BeiDou satellites is determined by two-way satellite time frequency signal (Tang et al., 2016), and the clock offset is coupled with the equipment bias from the B3I signal. Consequently, the total group delay (TGD) correction should be applied when using the B1I signal for smartphone positioning. The pseudorange observation of BDS satellites with broadcast products can be expressed as:

$$\begin{aligned} P_{r,B1I}^{s(BDS)} &= \rho_{r,B1I}^{s(BDS)} + (cdt_r + b_{r,P_{B3I}}) - (dt^s + b_{P_{B3I}}^s) + \gamma_i I_{r,B1I}^s + M_r T_r + S_{B1I}^s + \\ &\quad (b_{r,P_{B1I}} - b_{r,P_{B3I}}) - (b_{P_{B1I}}^s - b_{P_{B3I}}^s) + \varepsilon_{P_{B1I}} \end{aligned} \quad (6.9)$$

However, this SISRE estimation is not applicable in smartphone single-frequency processing due to rank deficiency issues. Consequently, this research introduces additional process noise to the carrier-phase ambiguities, under the assumption that unmodelled SISRE errors are partially absorbed in the estimated float ambiguities. It has been demonstrated that the aforementioned two approaches yield comparable performance in PPP processing with broadcast products (Carlin et al., 2021). For smartphone processing, the process noise

standard deviation σ_p for ambiguity needs to be amplified to 0.3 m to absorb the SISRE because of the noisy measurements.

To gain a deeper understanding of the distinctions resulting from various products and to highlight the advantages of the proposed "HASandBRDC PPP" approach, various positioning strategies are employed in this chapter. Table 6.3 provides details on the orbit, clock, and constellations information utilized for different smartphone positioning strategies.

Table 6.3: Orbit, clock, and constellations information for different smartphone positioning strategies

Processing strategies	Orbit and clock products	Satellite code biases	Constellations and observations
SPP	Broadcast ephemeris	N/A	GE (code-only)
HAS PPP	HAS corrections	HAS OSB (GE)	GE (code+phase)
BRDC PPP	Broadcast ephemeris	HAS OSB (GE)	GE (code+phase)
CNES PPP	CNES ultra-rapid products	CAS DCB (GREC)	GE, GREC (code+phase)
HASandBRDC PPP	HAS + broadcast ephemeris	HAS OSB (GE) + TGD (C)	GREC (code+phase)

6.4 Results and analysis

This section first offers a thorough evaluation of the HAS and broadcast products for both 2022 and 2023. Subsequently, it delves into the analysis of smartphone horizontal positioning performance employing the developed processing strategies. Table 6.4 summarizes the dataset details, including the identification, phone models used, collection date, and duration for each dataset.

Table 6.4: Summary of four road tests

Dataset identification	Phone used	Collection date	Time duration
1	S21+	Oct. 9, 2022	21 mins
2	MI 8	Oct. 10, 2022	28 mins
3	S21+	Jul. 26, 2023	36 mins
4	S21+	Jul. 31, 2023	25 mins

6.4.1 HAS and broadcast products assessment

The availability of HAS data was initially assessed for the years 2022 and 2023. The analysis period for the 2022 test extends over one week from October 5 to October 11, involving the timeframe of datasets 1-2. The analysis period for 2023 also spans one week from July 26 to August 1, covering the timeframe of datasets 3-4. Both the presence and validity times of HAS orbit and clock corrections are considered when calculating HAS availability.

Figure 6.3 depicts the availability of satellite-specific HAS corrections for both GPS and Galileo constellations, and the corrections availabilities for the years 2022 and 2023 are represented by orange and green bars, respectively. The results reveal that the HAS corrections availability for Galileo satellites is slightly higher than that for GPS satellites. Although G26 HAS corrections available in 2023 is less than 60% for unknown reasons, the availability of both GPS and Galileo HAS corrections shows a significant improvement in 2023 compared to 2022. Specifically, the average availability of GPS HAS corrections has increased from 82% to 92%, while the average availability of Galileo HAS corrections has seen an improvement from 91% to 99%, which meets the targeted performance (EUSPA, 2020). This enhancement may be attributed to the continued development and maturation of the HAS system as it transitioned from HA testing Phase 0 to Phase 1 in 2023.

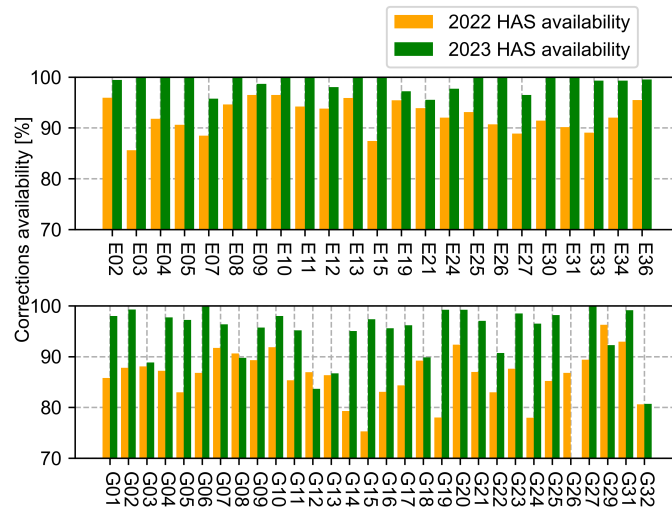


Figure 6.3: HAS corrections availability comparison for Galileo (upper plot) and GPS satellites (lower subplot) for years 2022 and 2023

In Figure 6.4 and Figure 6.5, the total SISRE analysis is presented for GPS and Galileo satellites in 2022 and 2023, respectively. The green and red bars represent the average total SISRE values for each satellite, calculated from broadcast ephemeris with and without HAS corrections, respectively. Two noteworthy observations can be made from the results. Firstly, it is confirmed that the GPS constellation exhibits significantly larger orbit and clock errors with LNAV ephemeris compared to the Galileo constellation with I/NAV ephemeris. This result is expected and can be attributed to the use of highly stable hydrogen maser clocks onboard Galileo satellites and the higher upload rate of Galileo broadcast information. Conversely, the utilization of HAS corrections proves to have a more substantial impact on GPS satellites, enabling the final SISRE of GPS satellites to exhibit comparable performance against Galileo satellites after applying HAS corrections. Secondly, it is observed that the HAS SISRE remains at the same level of accuracy in 2023 compared to 2022 for both GPS and Galileo constellations. This consistency can be attributed to the absence of any increase in the utilization of Galileo Sensor Stations, which plays a crucial role in generating HAS corrections.

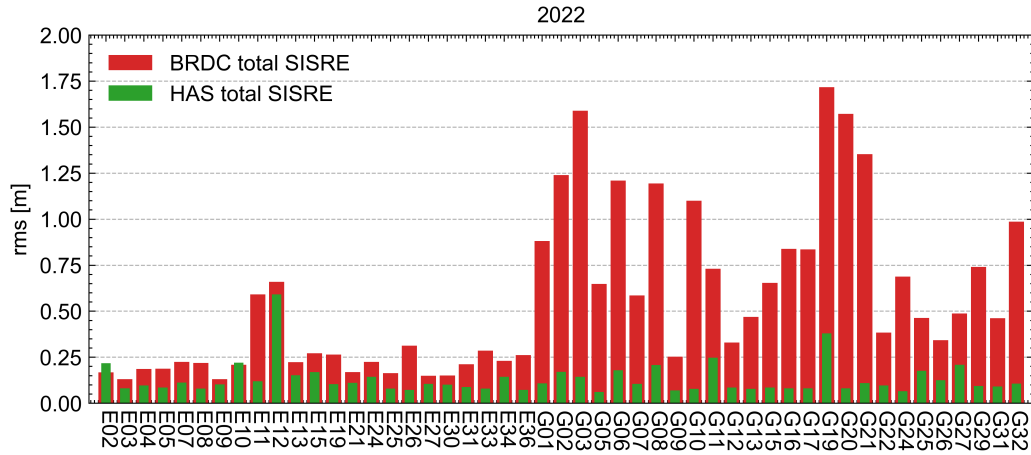


Figure 6.4: Total SISRE analysis for GPS and Galileo satellites from October 5 to October 11, 2022. The green and red bars represent the average total SISRE values for each satellite with and without HAS corrections, respectively

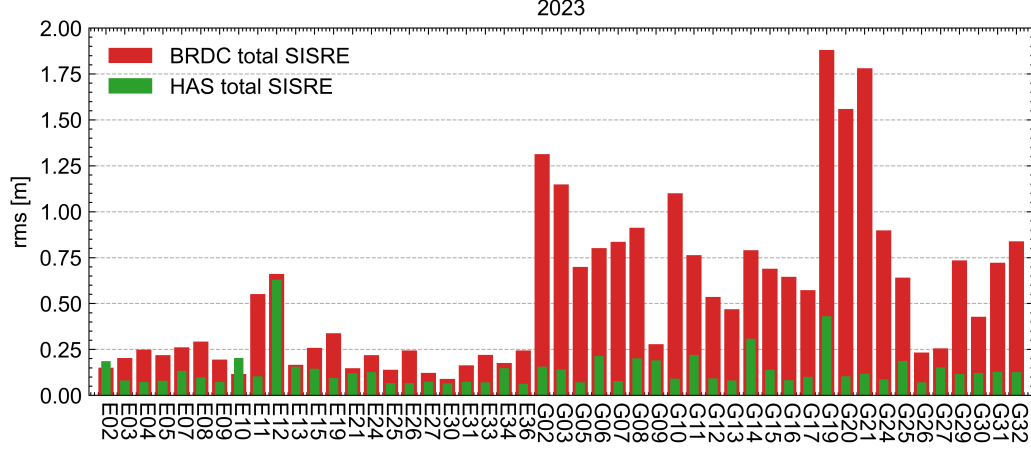


Figure 6.5: Total SISRE analysis for GPS and Galileo satellites from July 26 to August 1, 2023. The green and red bars represent the average total SISRE values for each satellite with and without HAS corrections, respectively

Correspondingly, Table 6.5 provides an overview of the overall statistics of orbit and clock errors with and without HAS corrections for a one-week duration in 2023. It is important to note that satellite E12 has been excluded from the statistics computation due to its HAS SISRE exhibiting comparatively large values compared to other satellites. This discrepancy may be associated with systematic biases generated from the nature of the HAS clock and requires further investigation. The results suggest that the utilization of HAS corrections proves to be beneficial in reducing errors for both GPS and Galileo broadcast ephemeris. Specifically, the GPS clock rms and total SISRE are reduced from 91.7 cm to 15.2 cm and 93.5 cm to 15.8 cm, respectively. Concurrently, the Galileo clock rms and total SISRE are reduced from 23.6 cm to 10.6 cm and 23.9 cm to 11.3 cm, respectively. These findings underscore the significant potential of HAS corrections in enhancing PPP performance.

Table 6.5: Statistics of orbit and clock errors with and without HAS corrections from July 26 to August 1, 2023 (unit: cm)

Products	Radial-track rms	Along-track rms	Cross-track rms	Clock rms	Total SISRE
GPS HAS	3.4	13.0	8.1	15.2	15.8
GPS LNAV	24.3	89.4	50.3	91.7	93.5
Galileo HAS	2.7	8.0	7.5	10.6	11.3
Galileo I/NAV	11.9	24.9	18.6	23.6	23.9

To better appreciate the distribution and inherent characteristics of HAS SISRE, Figure 6.6 displays the probability distribution function (PDF) and Kernel density estimation (KDE) of Galileo and GPS SISRE, and green and blue represent the SISRE with and without HAS corrections, respectively. The results indicate that, in comparison to broadcast ephemerides, the distribution of HAS products is closer to a Chi-squared distribution.

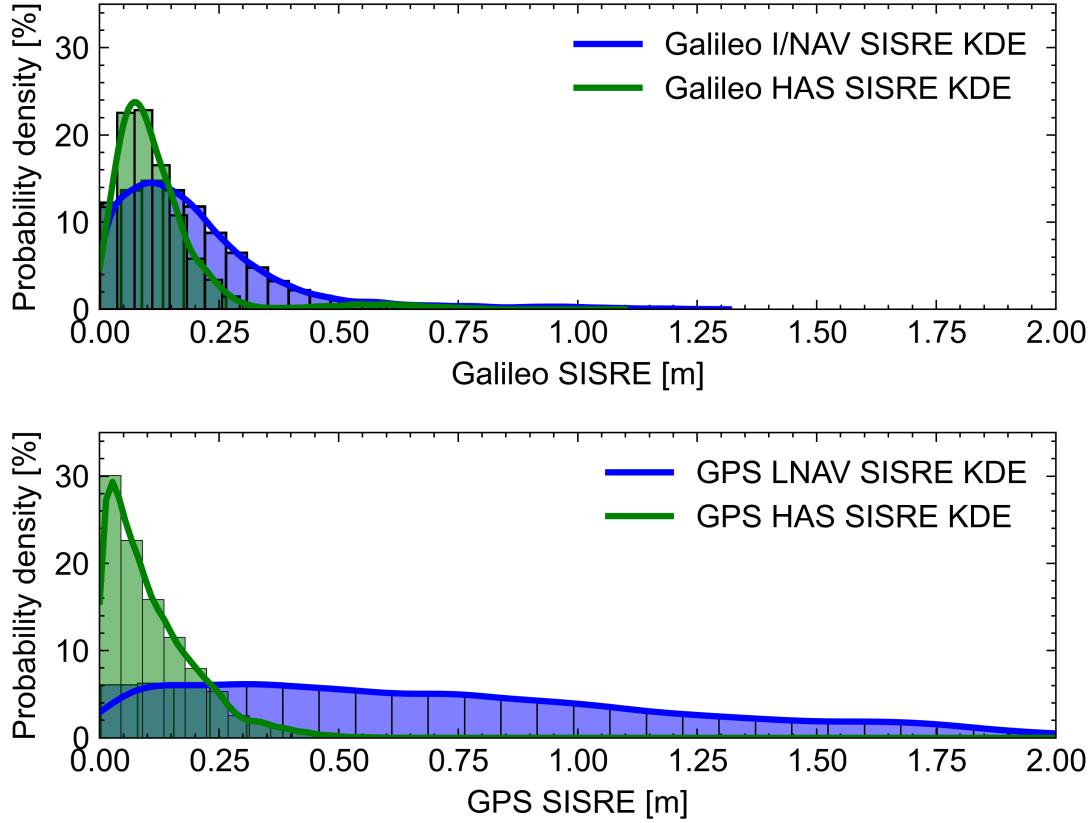


Figure 6.6: Galileo and GPS SISRE PDF and KDE from July 26 to August 1, 2023. The green and blue colours represent the SISRE with and without HAS corrections, respectively

To assess the accuracy of HAS OSBs, DCB products from the Chinese Academy of Sciences (CAS) are employed as a reference. Figure 6.7 indicates that the differences in DCB between HAS and CAS products are at the sub-nanosecond level, exhibiting rms values of 0.19 ns, 0.23 ns, 0.16 ns, and 0.19 ns for C1C-C5Q, C1C-C7Q, C1C-C6C, and C1C-C2L, respectively. This analysis aligns with the previous conclusion that HAS OSB accuracy remains consistent with the levels observed in 2022 (Naciri et al., 2023).

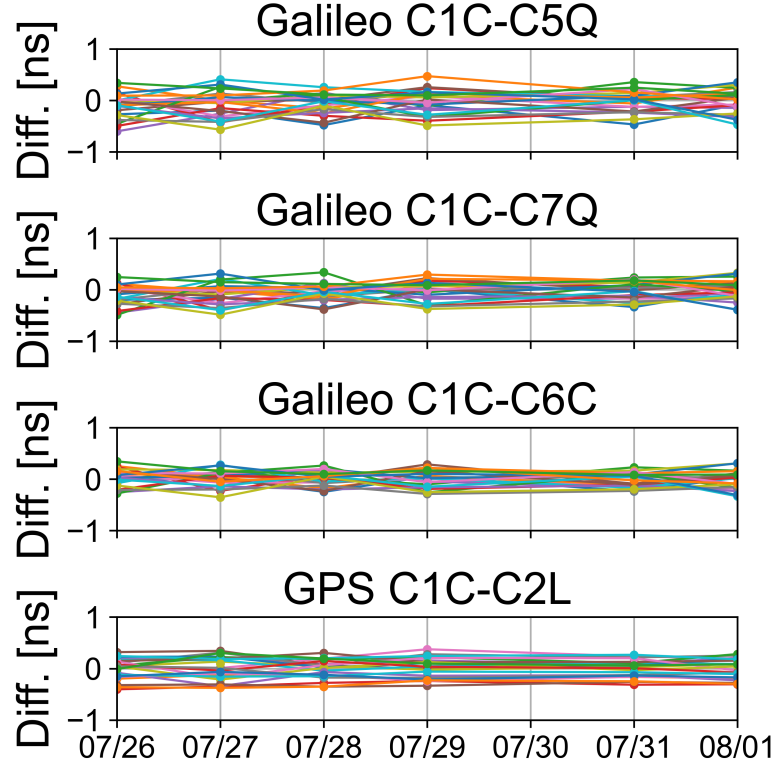


Figure 6.7: DCB differences of HAS and CAS products from July 26 to August 1, 2023

6.4.2 Smartphone positioning performance comparison using SPP, HAS PPP, and BRDC PPP with GPS and Galileo constellations

Figure 6.8 shows the time series of smartphone horizontal positioning performance for dataset 1. The conventional GNSS SPP solutions are represented by red lines, while the PPP with broadcast ephemeris (BRDC PPP) and HAS corrections (HAS PPP) are represented by blue and green lines, respectively. To ensure a fair comparison and to appreciate the benefits of using HAS corrections, the number of processed GPS and Galileo satellites is kept the same for both HAS PPP and BRDC PPP. Correspondingly, Figure 6.9 shows the time series of the number of processed satellites and observations for SPP and HAS PPP solutions. It is evident that although the SPP solution contains a larger number of processed satellites compared to HAS or BRDC PPP due to the limited availability of HAS corrections, the smartphone GNSS-PPP solutions demonstrate more accurate and stable

positioning performance against GNSS-SPP. This improvement is attributed to the incorporation of precise carrier-phase measurements.

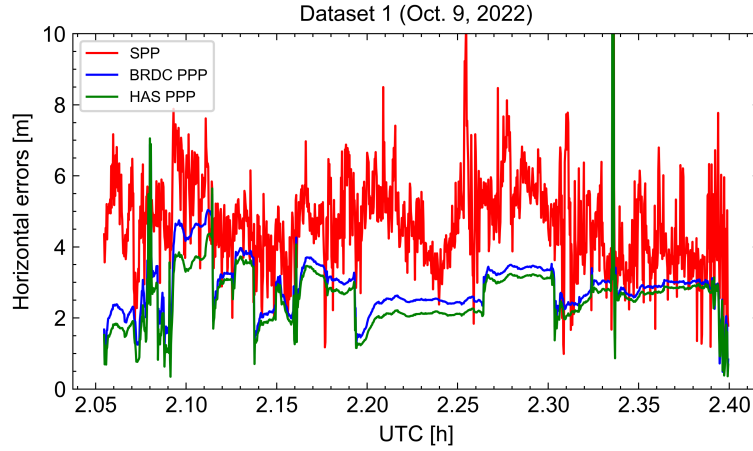


Figure 6.8: Time series of horizontal positioning errors for dataset 1

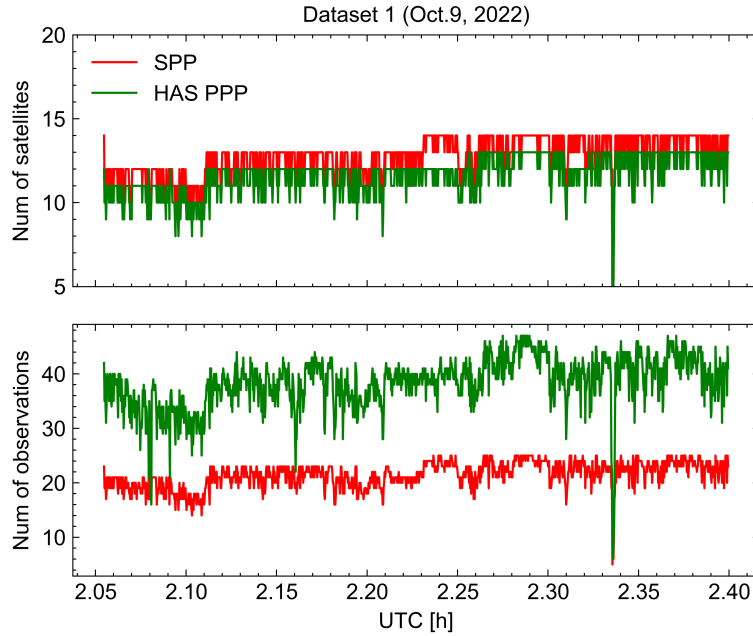


Figure 6.9: Time series of the number of processed satellites and observations for dataset 1

In Figure 6.10, the box plot illustrates the positioning errors processed by SPP, BRDC PPP, and HAS PPP methods, which are represented in red, blue, and green bars, respectively, for datasets 1 to 4. The coloured bins represent the interquartile range (IQR) from the first quartile (Q1) to the third quartile (Q3), while the distance between the whiskers and

the bin equals 1.5 times the IQR, indicating the range of error distribution without outliers. It is evident from the plot that the inclusion of HAS corrections enhances the accuracy of PPP solutions, effectively reducing positioning errors with smaller whiskers compared to SPP and BRDC PPP solutions. As highlighted in Table 6.6, there is an approximate horizontal positioning improvement of 0.1 m, 0.2 m, and 0.2 m in the horizontal domain for overall rms, 95th percentile errors, and 68th percentile errors, respectively, with the inclusion of HAS corrections.

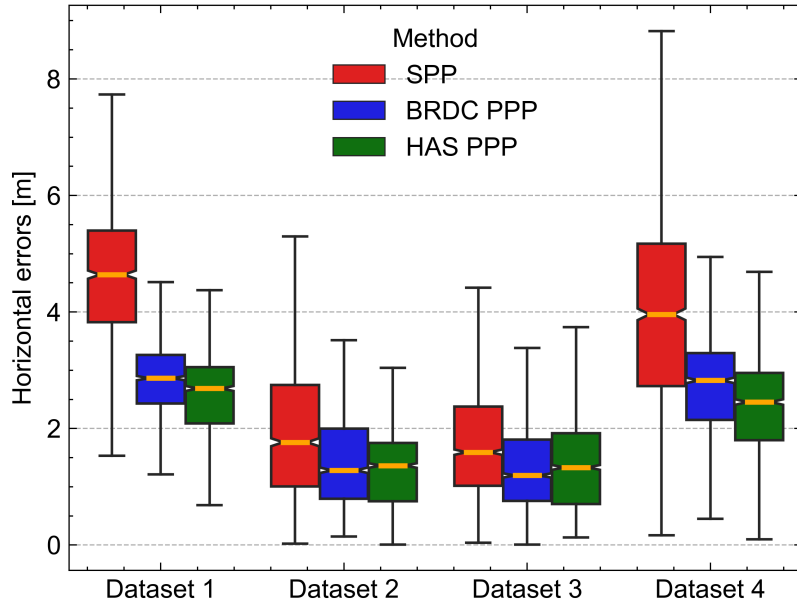


Figure 6.10: Box plot of positioning errors processed by SPP (red), BRDC PPP (blue), and HAS PPP (green) in the horizontal direction for overall datasets 1-4

Table 6.6: Overall horizontal statistics for all tested four datasets processed by SPP, BRDC PPP, and HAS PPP (unit: m)

Processing strategies	RMS	STD	95 th errors	68 th errors
SPP (GE)	3.3	1.9	6.1	3.4
BRDC PPP (GE)	2.1	1.1	3.5	2.3
HAS PPP (GE)	2.0	1.1	3.3	2.1

6.4.3 Smartphone PPP performance (GREC) with the combination of HAS and broadcast ephemeris

Figure 6.11 shows the time series of smartphone positioning performance for dataset 1 in the horizontal domain. The PPP solutions processed with HAS and CNES products for the GPS and Galileo constellations are represented in red and orange lines, respectively; the PPP solutions processing with the combination of HAS and broadcast ephemeris (HASandBRDC PPP) and CNES products (CNES PPP) with GREC four constellations are represented in green and blue lines, respectively. As expected, CNES PPP GE and GREC solutions show superior accuracy and stability in contrast to the other solutions. The performance discrepancy is attributed to the higher accuracy and availability of CNES ultra-rapid products compared to HAS and broadcast products. Additionally, the inclusion of GLONASS and BeiDou satellites makes a significant contribution to the overall positioning performance. As illustrated in Figure 6.12, the average number of processed satellites and observations for HASandBRDC PPP with GREC constellations is 20.3 and 55.5, respectively, significantly surpassing those of HAS PPP solutions with GE constellations, which are 11.6 and 38.4.

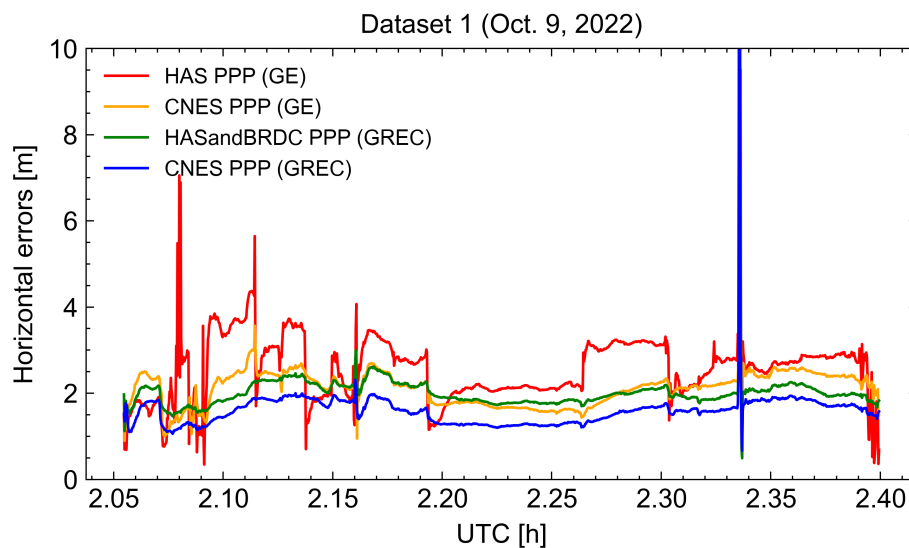


Figure 6.11: Time series of horizontal error for dataset 1, and the red, orange, green, and blue lines represent the positioning performance produced by HAS PPP (GE), CNES PPP (GE), HASandBRDC PPP (GREC) and CNES PPP (GREC) methods, respectively

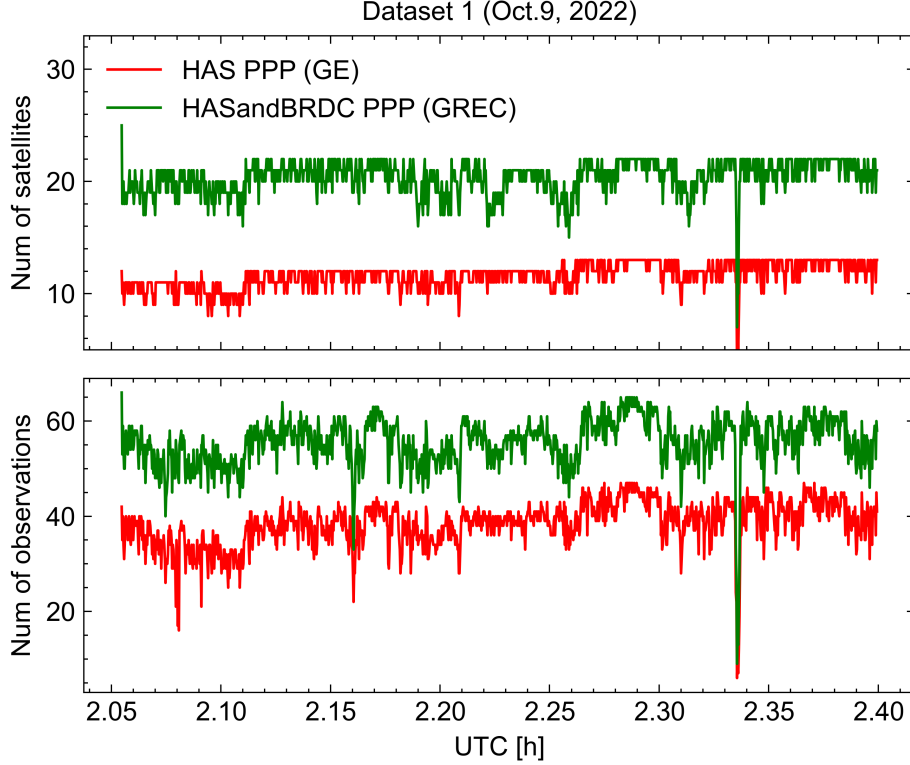


Figure 6.12: Time series of the number of processed satellites and observations for dataset 1, and the red and green lines represent the results processed by HAS PPP (GE) and HASandBRDC PPP (GREC) methods, respectively

In Figure 6.13, the overall experimental PDF and cumulative distribution function (CDF) in the horizontal direction for tested four datasets are presented. The solutions processed by HAS PPP (GE), CNES PPP (GE), HASandBRDC PPP (GREC), and CNES PPP (GREC) are represented in red, orange, green, and blue, respectively. Correspondingly, Table 6.7 provides the overall statistics for the four processing strategies. It is anticipated that PPP solutions with the CNES ultra-rapid products outperform solutions utilizing HAS corrections or broadcast products due to the lower SISRE involved in processing. However, it appears that the tradeoff of incorporating higher SISRE is relatively minor when considering the benefits gained from including additional constellations in the processing. Noticeably, the proposed HASandBRDC PPP algorithm improves the horizontal rms, standard deviation, 95th percentile error, and 68th percentile error from 2.0 m, 1.1 m, 3.3 m, and 2.1 m to 1.6 m, 0.9 m, 2.4 m, and 1.6 m, respectively, when compared to the HAS PPP GE solutions.

The proposed method also provides a comparable performance against CNES PPP GREC solutions, demonstrating significant potential in utilizing HAS corrections and broadcast ephemeris information, which users can directly access from satellites in real-time for the mass-market PNT applications.

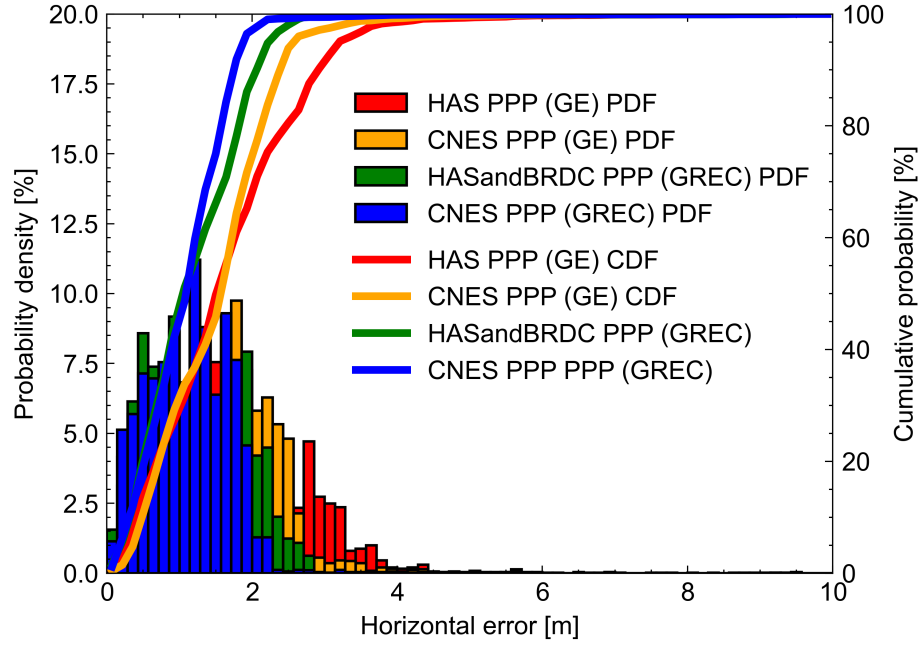


Figure 6.13: Overall horizontal errors PDF and CDF for all tested four datasets, and the red, orange, green, and blue lines represent the positioning performance produced by HAS PPP (GE), CNES PPP (GE), HASandBRDC PPP (GREC) and CNES PPP (GREC) methods, respectively

Table 6.7: Overall horizontal statistics for all tested four datasets processed by HAS PPP (GE), CNES PPP (GE), HASandBRDC PPP (GREC) and CNES PPP (GREC) (unit: m)

Processing strategies	RMS	STD	95 th errors	68 th errors
HAS PPP (GE)	2.0	1.1	3.3	2.1
CNES PPP (GE)	1.8	0.7	2.6	1.9
HASandBRDC PPP (GREC)	1.6	0.9	2.4	1.6
CNES PPP (GREC)	1.4	0.8	1.9	1.4

6.4.4 Smartphone PPP performance (GREC) with IMU integration

To mitigate the PPP gross errors caused by GNSS outages in realistic driving environments, this research further integrated the smartphone IMU measurements with GNSS

measurements in a tightly-coupled mode. Figure 6.14 presents the time series of horizontal positioning errors processed by HASandBRDC PPP (blue line) and PPP/IMU (green line) solutions for dataset 1 and dataset 4. It is worth noting that compared to dataset 1, dataset 4 poses more challenges for smartphone positioning due to frequent signal losses caused by harsh traffic conditions during traffic rush hours.

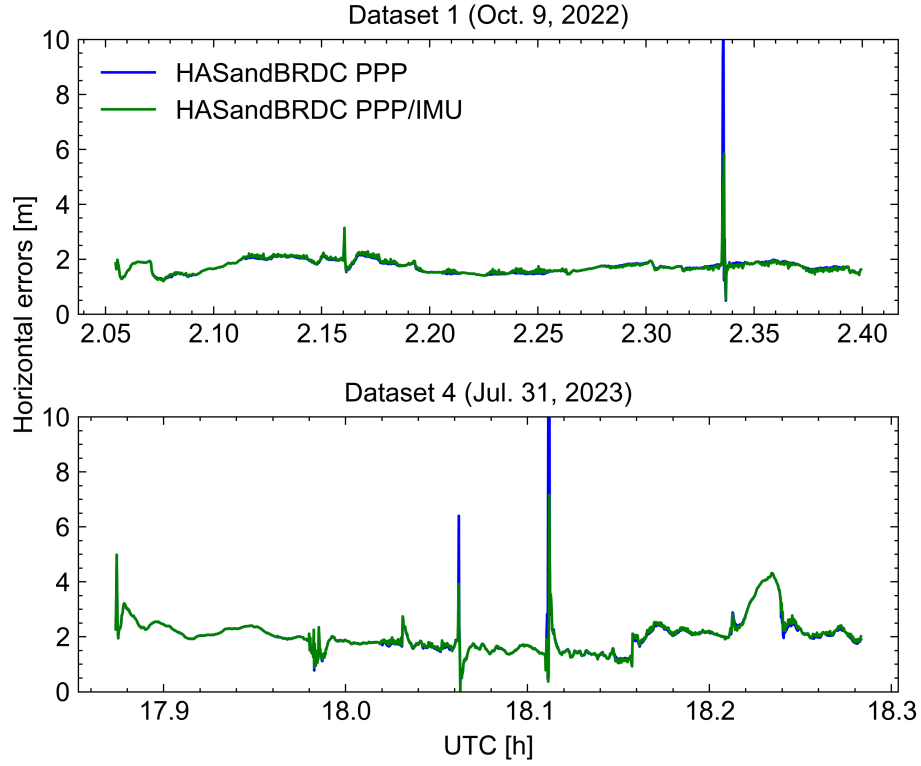


Figure 6.14: Time series of horizontal positioning error for datasets 1 and 4, and the blue and green lines represent the positioning performance produced by HASandBRDC PPP and HASandBRDC PPP/IMU with GREC constellations, respectively

Figure 6.15 summarizes the horizontal statistics for the tested four datasets, where the blue and green bars represent the horizontal maximum errors and rms processed by HASandBRDC PPP solutions, and the lighter colours represent those statistics obtained from HASandBRDC PPP/IMU solutions. As dataset 3 was conducted in open-sky and suburban roads with relatively good GNSS observation conditions, experienced minimal impacts from the IMU. In contrast, the integrated PPP/IMU solutions prove effective in reducing GNSS outliers for datasets 1, 2, and 4, resulting in a more accurate and robust

smartphone positioning performance. Specifically, the PPP/IMU solution notably reduces horizontal maximum errors by approximately 59%, leading to an overall rms reduction of 0.1 m. This highlights the effectiveness of the integrated PPP/IMU approach in enhancing positioning accuracy.

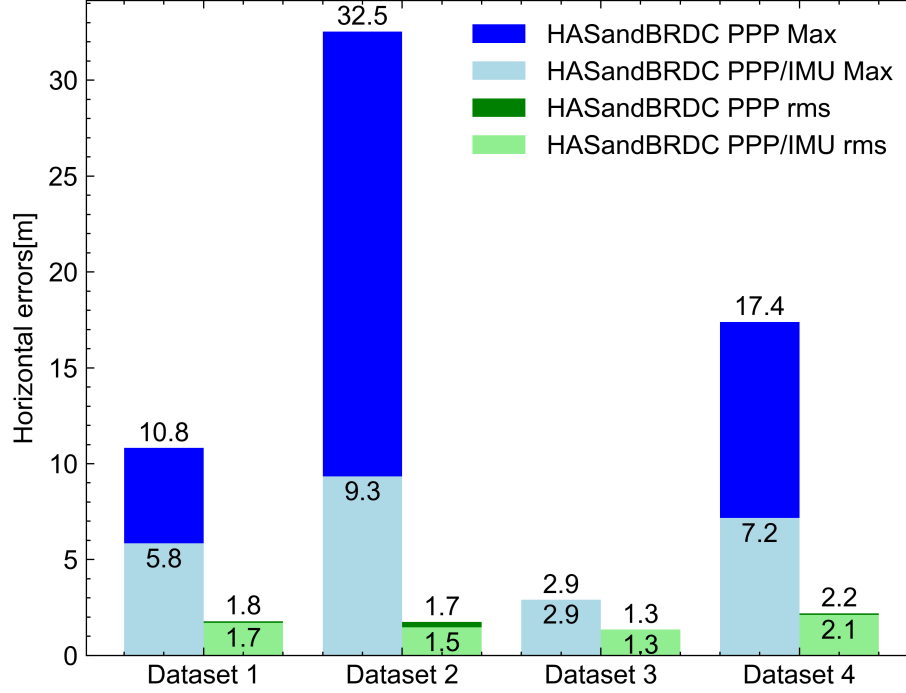


Figure 6.15: Overall horizontal errors statistics for tested four datasets. The green and blue bars represent the horizontal maximum error and rms obtained from HASandBRDC PPP solutions, respectively, and the light colours indicate their HASandBRDC PPP/IMU solutions

6.5 Summary

This chapter has presented a comprehensive assessment of the Galileo HAS and its potential impact on smartphone PPP in realistic driving conditions. By leveraging HAS corrections and broadcast ephemeris, the proposed algorithm demonstrated notable improvements in smartphone navigation, offering precise positioning capabilities with the information disseminated directly from satellites.

First, this research conducted a comprehensive comparison of the HAS corrections and corresponding GPS LNAV and Galileo I/NAV broadcast ephemerides for the years 2022 and 2023. The findings revealed a significant improvement in the availability of both GPS and Galileo HAS corrections in 2023 compared to 2022, with Galileo HAS corrections achieving a remarkable 99% availability, meeting the targeted performance standards (EUSPA, 2020). Moreover, the SISRE analysis demonstrated that HAS corrections can notably enhance the SISRE introduced by broadcast products, particularly for GPS satellites. For instance, the GPS clock rms and total SISRE were reduced from 91.7 cm to 15.2 cm and 93.5 cm to 15.8 cm, respectively.

Additionally, the analysis of the four vehicle experimental solutions revealed that the HAS PPP solutions exhibited superior performance compared to PPP solutions utilizing broadcast ephemerides, resulting in a reduction of 0.1 m and 0.2 m in horizontal rms and 95th percentile errors, respectively. Additionally, the combination of HAS corrections and broadcast ephemerides in smartphone PPP (HASandBRDC PPP) presented significant improvement when compared to HAS PPP solutions. This combination led to a decrease in the overall rms and 95th percentile error from 2.0 m and 3.3 m to 1.6 m and 2.4 m, respectively. The integration of smartphone IMU measurements can further improve the horizontal rms from 1.6 m to 1.5 m.

To sum up, this research introduced an augmented PPP algorithm that leverages HAS corrections and broadcast ephemerides, relying entirely on satellite-disseminated information. This enables real-time precise applications even in remote regions with limited Internet connections. Validated by vehicle experiments in urban environments, the results indicate that the HASandBRDC PPP solutions notably enhance positioning accuracy by incorporating GLONASS and BeiDou satellites with broadcast information into the processing. While there remains a slight performance discrepancy compared to PPP solutions utilizing CNES ultra-rapid products, the proposed HASandBRDC PPP/IMU algorithm achieves an overall rms of 1.5 m, demonstrating performance comparable to previous real-time smartphone PPP studies (Wang et al., 2021; Li et al., 2022b).

Chapter 7

Conclusions and recommendations for future work

The proliferation of GNSS-enabled smartphones has introduced considerable challenges to achieving precise positioning. Due to the inherent limitations of smartphone-grade, linearly polarized antennas and GNSS chipsets, traditional SPP methods typically only yield positioning at the accuracy of around 10 m, falling short of the increasing demands of the GNSS downstream market. Motivated by these challenges, this dissertation aimed to address these issues and comprehensively investigate the performance of low-cost smartphone positioning across various environments. Building upon the research outlined in the preceding chapters, significant advancements have been made in smartphone positioning performance by developing new positioning algorithms and integrating smartphone-native sensors. With the investigations and progress made thus far, the research questions posed can be addressed, and the objectives of achieving precise smartphone navigation with lane-level precision are attainable with the proposed algorithms in this dissertation. This chapter provides comprehensive conclusions based on the research questions and suggests recommendations for future studies aimed at further enhancing smartphone positioning.

7.1 Conclusions

The pursuit of real-time accurate smartphone positioning represents an ongoing research endeavour within the GNSS community. The primary challenges revolve around addressing

noisy measurements and constraints posed by limited tracking signals, as well as combating significant positioning degradation during GNSS outages. Additionally, traditional real-time PPP and RTK methods necessitate nearby infrastructure, such as base stations for RTK or Internet connections for receiving RTCM streams for PPP, thereby limiting precise smartphone applications in remote areas. To address these obstacles, this research has made significant contributions across four main areas, which are summarized as follows:

1. Applying GIM as an external ionospheric constraint with additional receiver DCB estimation to reduce the convergence time and enhance the positioning accuracy in smartphone PPP processing.
2. Enhancing smartphone PPP processing by optimizing the utilization of single- and dual-frequency observations, including pseudorange-only measurements, which are typically omitted in traditional PPP processing. Implementing an adaptive post-fit residual rejection quality control method to improve smartphone positioning performance in dynamic and challenging environments.
3. Evaluating smartphone positioning performance using both PPP and RTK configurations, and introducing a novel PPP and RTK hybridization algorithm to address the limitations of each technique. Additionally, investigating the performance of smartphone native IMU and integrating GNSS/IMU in tightly-coupled mode with initialization and re-sampling, pre-processing methods to improve smartphone positioning accuracy, particularly in GNSS blockage environments.
4. Assessing the PPP performance by comparing the utilization of the state-of-the-art Galileo HAS with the PPP performance of CNES ultra-rapid and broadcast orbit and clock products. Proposing a novel algorithm that leverages HAS and broadcast data to enable real-time PPP for smartphone positioning, incorporating IMU integration for enhanced positioning accuracy.

7.1.1 Effects of PPP-IC on smartphone navigation (Chapter 3)

Though not a novel concept, it is well known that PPP convergence and initialization can be significantly improved through accessing the slant ionospheric information (Odijk, 2002; Tu et al., 2013; Banville et al., 2014). Therefore, there is a growing trend to utilize the uncombined PPP and with ionospheric constraints (PPP-IC) as an alternate approach in the precise positioning community. However, the significance of PPP-IC for smartphone positioning remains unclear and deserves investigation. Given that, Chapter 3 applies the uncombined PPP-IC model with the GIM to reduce the initialization errors and convergence time for smartphone PPP.

Chapter 3 presents several noteworthy contributions by applying PPP-IC technology for three different grades of GNSS receivers, namely geodetic NovAtel OEM7, low-cost SwiftNav Piksi, and smartphone MI 8. With the aid of the GIM information and DCB estimation, the static and kinematic solutions verify that PPP-IC is a promising approach and has a greater impact on MI 8 and Piksi compared to OEM7. The performance criteria were assessed through convergence time, first epoch horizontal error, and horizontal rms over the first 20 minutes. Additionally, further corrections analysis for the smartphone provides insight into how ionospheric constraints affect PPP states. Based on these investigations, the questions posed for research topic 1 are addressed as follows:

1. *Whether the GIM ionospheric constraints help in terms of initialization for the dual-frequency smartphone, low-cost and geodetic hardware PPP?*

Yes, the answer to the first problem was validated through static and kinematic scenarios with and without ionospheric constraints, and results show that PPP-IC is capable of mitigating the initialization positioning errors significantly, e.g., the static experiment indicates that, with PPP-IC's help, the horizontal rms in first 20 minutes was improved about 57%, 26% and 23% for MI 8, Piksi and OEM7, respectively. Furthermore, smartphone PPP-IC takes 14 minutes to reach the convergence threshold of 1 metre, which is 11 minutes faster than conventional PPP.

2. *Whether the ionospheric constraints help under challenging environments for these three different grades of GNSS receivers?*

Certainly, this question is substantiated by the vehicle kinematic experiment conducted across various challenging environments, including areas with dense vegetation and suburban regions characterized by significant multipath effects. The outcomes reveal a noteworthy 72% enhancement in reducing horizontal rms error when applying ionospheric constraints to the MI 8 smartphone in suburban experiments. Specifically, the horizontal rms error decreased substantially from 15.9 m to 4.4 m, with the standard deviation reduced from 8.5 m to 1.8 m. These findings underscore the accuracy and consistency achievable with smartphone PPP-IC solutions, even in challenging suburban environments. Despite the significant performance discrepancy between smartphones and traditional GNSS receivers, PPP-IC methods effectively narrow this gap.

3. *How significant is the improvement of PPP-IC for the smartphone in comparison to low-cost and geodetic-grade receivers?*

Verified through both static and kinematic experiments, PPP-IC demonstrates a notably positive impact, particularly for improving low-quality measurements before convergence. In static reset experiments, PPP-IC yields significant enhancements for the MI 8 smartphone within the initial 20 minutes. This results in a reduction of horizontal errors at the first epoch for each reset time from an average of 6.9 m to 1.3 m, with the horizontal rms error decreasing from 1.7 m to 0.9 m. Similarly, in the kinematic test conducted in a parking lot scenario, a substantial improvement of 59% is observed in terms of smartphone horizontal rms error, surpassing that of Piksi (12%) and OEM7 (8%). Although OEM7 PPP-IC exhibits a 74% reduction in horizontal error at the first epoch, the corrected magnitude is not as significant as that of Piksi and MI 8 (see Table 3.6). This discrepancy can be attributed mainly to inherited errors from the GIM, ranging from 2 to 8 TECU, resulting in a difference of 32 to 130 cm on the L1 frequency.

4. *Do ionospheric constraints influence other PPP states, such as estimated receiver clock offset and ambiguities?*

Through the analysis of estimated corrections for the MI 8 smartphone, interestingly, it was found that not only the estimated slant ionospheric delays, but also the tropospheric delays, receiver clock offset, and float ambiguities estimation are affected by ionospheric constraints. As depicted in Figures 3.12a-3.12d, the utilization of ionospheric constraints contributed to stabilizing the estimated float ambiguity estimates, resulting in a reduction of the standard deviation from 3.1 m to 0.6 m. This improved performance can be attributed to the additional information provided by the GIM, which helps reduce errors in the estimated ionospheric delays. Consequently, these more accurate ionospheric delay estimates, along with scaled ionospheric variance, positively influence receiver clock offset and float ambiguities estimation, ultimately leading to enhancements in smartphone PPP solutions.

7.1.2 Enhanced smartphone PPP model with adaptive quality control method (Chapter 4)

Thanks to the emergence of multi-constellation, dual-frequency smartphone GNSS chipsets, it is now possible to utilize more observations and manage the ionospheric delays by means of, e.g., ionospheric-free combination, ionospheric error estimation, and ionospheric constraints (Chen et al., 2019; Wu et al., 2019; Yi et al., 2021). In this context, Aggrey et al. (2020) comprehensively compared the PPP performance with four released smartphones and an average horizontal error of 40 cm can be obtained for dual-frequency MI 8 solutions, which was superior to single-frequency solutions, but the performance degraded to 6 m in a kinematic test. Meanwhile, Wang et al. (2021) conducted a walking experiment and achieved 0.85 m and 1.09 m in horizontal and vertical components, respectively, with the aid of real-time ionospheric products. Zangenehnejad and Gao (2021) introduced the single- and dual-frequency processing mode in smartphone processing.

However, it is noteworthy that previous studies have often overlooked the utilization of all pseudorange and phase measurements for smartphone PPP processing, resulting in

the exclusion of valuable measurements. Moreover, smartphone observations frequently encounter carrier phase signal loss, leading to the observation record of pseudorange-only measurements for certain satellites. To optimize satellite geometry and leverage every available smartphone measurement, Chapter 4 proposes a novel PPP enhancement method. This method involves utilizing all smartphone code and phase measurements, coupled with an adaptive post-fit residual threshold rejection function designed to effectively filter out noisy measurements. The analysis involves ten vehicle datasets collected in suburban driving environments, encompassing open-sky highways, vegetation roads, and suburban roads. Based on these investigations, the research questions posed for research topic 2 can be addressed as follows:

1. *With regard to ultra-low-cost (smartphone-grade) hardware with an approximate cost of \$10, does pseudorange-enhanced PPP improve traditional PPP solutions and to what extent in harsh environments?*

This research employed the pseudorange-only measurements enhanced PPP algorithm to leverage all available smartphone observations. Through the validation of ten vehicle experiments, the results reveal comprehensive improvements. Compared to traditional single- and dual-frequency PPP solutions, the proposed pseudorange-enhanced PPP solutions demonstrate reductions in horizontal rms, standard deviation, mean error, and 95th percentile error from 2.8 m, 1.9 m, 2.1m, and 5.5 m to 1.7 m, 0.8 m, 1.3 m, and 2.1 m, respectively. A significant level of 46% improvement with 1.7 m overall horizontal rms can be achieved with the enhanced PPP, which notably surpasses conventional PPP solutions in smartphone processing. However, it is observed that the inclusion of pseudorange-only observations can be occasionally detrimental to PPP processing owing to the noisy measurements, which places higher requirements for GNSS quality control methods.

2. *Assessed through the range error correlation analysis, which indicators among the satellite elevation, C/N_0 value, pre-fit residuals, and post-fit residuals can infer smartphone measurement quality?*

The range error analysis depicted in Figure 4.10 illustrates scatter diagrams and correlation analyses between smartphone pseudorange range errors and several parameters: pre-fit residuals, post-fit residuals, signal strength, and satellite elevation. The results explicitly indicate that post-fit residuals exhibit the strongest positive correlation (0.67 on L1 frequency and 0.55 on L5 frequency) with range errors among other parameters. This suggests that utilizing post-fit residuals as a criterion for distinguishing measurement quality is more suitable and effective in smartphone positioning compared to other indices. However, it is also important to note that in rare cases, post-fit residuals may be deceptive compared to actual range errors. For instance, in Figure 4.10, there are instances where measurements display small post-fit residuals but large range errors.

3. *How to form the adaptive post-fit residual rejection function according to the learning statistic? And what is the ‘best’ GNSS PPP solution with smartphone-grade GNSS chipset and antenna that can be attained in real-world driving environments?*

This research introduced a novel adaptive post-fit residual rejection threshold aimed at identifying and excluding measurement outliers, while retaining valuable measurements. This adaptive threshold is determined through a three-segment function, as illustrated in Equation (4.7) and Equation (4.8). The learning statistics used to determine the adaptive threshold are derived from the ratio of visible satellites to pseudorange-only satellites, which can also be determined by other indicators such as PDOP that reflect the GNSS conditions. Based on experimental solutions, it is concluded that the enhanced PPP with adaptive post-fit residual rejection threshold can provide improved solutions among other traditional PPP strategies, and corresponding statistics indicate that the proposed algorithm can attain 1.8 m, 1.3 m, and 1.4 m horizontal positioning performance in 95th percentile error, 68th percentile error, and overall rms, respectively, under different driving environments.

7.1.3 Smartphone PPP and RTK hybridization with internal IMU (Chapter 5)

In 2016, Google announced the availability of code and phase raw measurements to smartphone users (Malkos, 2016), enabling more precise positioning techniques such as RTK and PPP, both of which are capable of delivering centimetre-level positioning in minutes with a standalone geodetic receiver (Zumberge et al., 1997; Eckl et al., 2001; Bisnath and Gao, 2009). In this context, Warnant et al. (2018) evaluated MI 8 positioning performance with ultra-short baseline, and their GPS+Galileo RTK solutions can achieve decimetre-level accuracy in static environments. Similarly, Dabove and Di Pietra (2019a) confirmed that the MI 8 has the capacity to reach centimetre-level precision with the single-baseline RTK approach. A subsequent contribution from Aggrey et al. (2020) compared the PPP performance with four released smartphones in static and kinematic environments, and the results indicated that an average of 60 cm and 40 cm horizontal positioning accuracy can be achieved for single-frequency and dual-frequency smart devices, respectively. Chen et al. (2019) and Wu et al. (2019) also proved that MI 8 is capable of providing decimetre-level PPP solutions in open-sky scenarios with final or real-time products.

However, both RTK and PPP have their own drawbacks. RTK requires a nearby GNSS infrastructure to serve as the base station, and its service accuracy is highly restricted by the baseline length. Conversely, PPP does not need an additional base station, but its convergence time is much longer than RTK due to the consideration of more errors, such as receiver clock and atmospheric errors. Furthermore, there is a growing need to provide robust and accurate solutions for smartphone applications in challenging GNSS environments with severe signal blockages. Addressing these issues, Chapter 5 proposes a novel PPP/RTK hybridization algorithm to leverage the advantages of both PPP and RTK in positioning. Additionally, this research integrates the smartphone's IMU with GNSS measurements in a tightly-coupled mode to further enhance the robustness and accuracy of the final solutions. Based on these investigations, the research questions raised by topic 3 can be addressed as follows:

1. *In realistic driving scenarios, how comparable is the ionospheric-constrained smartphone PPP solution to RTK performance?*

Without regard to the convergence period at the beginning, PPP is capable of providing the same grade of positioning accuracy as RTK for smartphone navigation. Validated by three vehicle experiments around York University, Toronto, Canada, RTK configuration can deliver better positioning solutions regarding 95th and 68th percentile errors. In contrast, PPP solutions are able to provide 2.1 m overall rms in realistic driving environments, which is more accurate than RTK solutions, due mainly to the negative impacts of GNSS outliers. This finding indicates that smartphone PPP has the capability to achieve RTK-like performance without limitations from the base station, which has not been seen in the existing literature.

2. *How does tightly-coupled IMU integration behave on RTK and PPP modes under different multipath profiles? Does the inclusion of smartphone native IMU harm the PPP or RTK solutions?*

Generally speaking, the inclusion of IMU does no harm to GNSS-only solutions since tightly-coupled solutions heavily rely on these GNSS-pure solutions in epochs with good GNSS availability. Correspondingly, TC solutions offer little position improvement for 95th and 68th percentile errors (see Figure 5.18). However, the integration of smartphone internal IMU can mitigate positioning gross errors significantly during GNSS outages, and significant levels of 34.8% and 19.0% improvement in horizontal overall rms are observed from RTK and PPP solutions, respectively. Thereby, the tightly-coupled smartphone GNSS/IMU solution has the capability to provide consistent horizontal positioning accuracy with overall rms <2 m through multiple multipath profiles.

3. *How does the proposed hybridization algorithm perform in comparison to RTK and PPP? And what is the optimal solution that smartphone precise positioning can achieve now with GNSS and IMU measurements in real-world driving environments?*

Based on experimental solutions and statistics, it is confirmed that the proposed hybridization algorithm captures the advantages of both RTK and PPP strategies. By adjusting the solution weights according to the estimated precision values, the PPP/RTK hybrid solutions can achieve 2.3 m of 95th percentile positioning accuracy and 2.0 m overall rms, which outperforms other GNSS-only solutions. Furthermore, by coupling with smartphone IMU, the hybrid solution accuracy can be further improved with 1.4 m overall rms in real-world driving scenarios, which is an optimal solution among other GNSS/IMU strategies and has not been seen in previous studies.

7.1.4 Smartphone navigation with Galileo HAS and broadcast products (Chapter 6)

Instead of broadcasting corrections through the Internet, a recent development in positioning communities has witnessed the emergence of the direct broadcast of SSR corrections via GNSS constellations, such as Japan’s QZSS CLAS and China’s BDS-3 B2b services (Saito et al., 2011; Tao et al., 2021). In contrast to regional services, the European Union is actively progressing in the development of the Galileo High Accuracy Service (Galileo HAS). The HAS system is expected to play a pioneering role by offering free Signal-in-Space corrections to users worldwide. Since January 24th, 2023 (EUSPA, 2023), Galileo HAS successfully declared its initial services operational, and these corrections have been accessible both through Galileo E6-B signals and HAS Internet Data Distribution service since then.

The operational announcement of Galileo HAS initial service has sparked significant interest in exploring the potential applications of HAS PPP in civil applications. Early trials conducted by several studies investigated the accuracy of HAS corrections through metrics such as SISRE and DCB differences (Hauschild et al., 2022; Fernandez et al., 2022). More recently, Naciri et al. (2023) provided insight into the capabilities of live HAS test signals in 2022, demonstrating that these signals could deliver Galileo and GPS orbit and clock corrections with SISRE values of 10.6 cm and 11.8 cm, respectively. Despite this progress, there has been no quantitative investigation into the use of low-cost smartphone sensors

with HAS solutions in realistic driving scenarios. Additionally, due to hardware disparities, smartphone processing with only two constellations cannot ensure sufficient observations in GNSS processing. To fully utilize all observed measurements and the broadcast information directly accessible from satellites, Chapter 6 introduces a new PPP/IMU algorithm utilizing HAS corrections and broadcast ephemeris (HASandBRDC PPP) for smartphone processing. Based on the results of vehicle experiments, the research questions raised by topic 4 can be addressed as follows:

1. *In comparison to the initial phase of Galileo HAS in 2022, how do the available HAS corrections and associated broadcast ephemeris perform after the official HAS announcement in 2023, in terms of orbit and clock availability, SISRE, and SISRE distribution*

The evaluation of HAS corrections for the years 2022 and 2023 demonstrates notable improvements in data availability. The average availability of GPS HAS corrections increased from 82% to 92%, and the average availability of Galileo HAS corrections rose from 91% to 99%. While there is no substantial enhancement in the accuracy of HAS, the corrections prove effective in reducing SISRE for both GPS and Galileo broadcast ephemerides. Additionally, the distribution of HAS SISRE closely resembles a Chi-squared distribution, indicating a more consistent and reliable performance against broadcast products.

2. *For smartphone-based navigation, how does the HAS PPP solution perform compared to traditional SPP and PPP with broadcast ephemeris (BRDC PPP) with GPS and Galileo constellations?*

As anticipated, both HAS PPP and BRDC PPP solutions exhibit superior performance compared to SPP solutions, primarily due to the inclusion of additional carrier-phase measurements. Notably, the utilization of HAS corrections contributes to further improvements in overall positioning performance. In comparison to BRDC PPP, it is

evident in the reduction of overall rms, 95th percentile errors, and 68th percentile errors from 2.1 m, 3.5 m, and 2.3 m to 2.0 m, 3.3 m, and 2.1 m, respectively.

3. *In realistic driving conditions, how does the proposed smartphone PPP with the combination of HAS corrections and broadcast ephemeris (HASandBRDC) behave compared to the smartphone PPP solution using traditional Internet-based ultra-rapid products with a full four constellations? Additionally, how does the inclusion of the smartphone IMU measurements impact the overall performance?*

Though smartphone PPP solutions with CNES products exhibit superior performance against other PPP solutions due to their higher product quality, the proposed smartphone PPP with the combination of HAS corrections and broadcast ephemeris demonstrates favorable behaviour compared to the HAS PPP solutions. The benefits of incorporating GLONASS and BeiDou satellites far outweigh the tradeoff of the higher SISRE inclusion associated with broadcast ephemerides. This leads to a reduction in the overall rms and 95th percentile error from 2.0 m and 3.3 m to 1.6 m and 2.4 m, respectively, which is highly comparable to the PPP solutions with GREC constellations using CNES ultra-rapid products. Furthermore, the PPP/IMU solution showcases its effectiveness by diminishing horizontal maximum errors from 10.8 m and 17.4 m to 5.8 m and 7.2 m for datasets 1 and 4, resulting in a 10 cm enhancement in the overall horizontal rms.

7.1.5 Smartphone positioning progress contributed by this dissertation

"Every mountain top is within reach if you just keep climbing", Barry Finlay, an author and mountaineer, said in his book "Kilimanjaro and Beyond: A Life-Changing Journey". This sentiment resonates with the challenges faced in smartphone positioning, akin to climbing to the top of a perilous peak. As shown in Figure 7.1, initially, relying on the internal smartphone positioning solution using SPP and single-constellation satellites can yield highly biased results, with positioning rms exceeding dozens of metres (Shinghal, 2020). Such accuracy falls far short of the precision required for, e.g., vehicle lane-level applications.

With the advent of smartphones supporting dual-frequency capabilities, dual-frequency smartphone PPP solutions, utilizing both GPS and Galileo constellations, have achieved an accuracy of 7 m. Expanding on this, this research employed GIM to constrain ionospheric delay errors in PPP processing, resulting in dual-frequency PPP-IC solutions with a horizontal rms of 3.1 m in kinematic parking lot environments (Yi et al., 2021). Additionally, this research further enhanced PPP solutions by incorporating both single- and dual-frequency observations, including pseudorange-only measurements, to optimize satellite geometry. An adaptive quality control method was proposed to effectively remove measurement outliers. With these advancements, the enhanced PPP solutions reached a horizontal rms level of 1.5 to 2 m (Yi et al., 2022, 2024). Continuing our research, a hybrid PPP and RTK algorithm is proposed, along with tightly-coupled integration of smartphone GNSS and IMU raw measurements. These solutions consistently provided horizontal rms better than 1.5 m, even in challenging GNSS environments (Yi and Yang, 2023). Lastly, this research explored Galileo High Accuracy Services (HAS) PPP and proposed a method that combines HAS and broadcast products for smartphone navigation, achieving a smartphone positioning rms of 1.5 m in real-time mode, solely relying on information from satellites and smartphones.

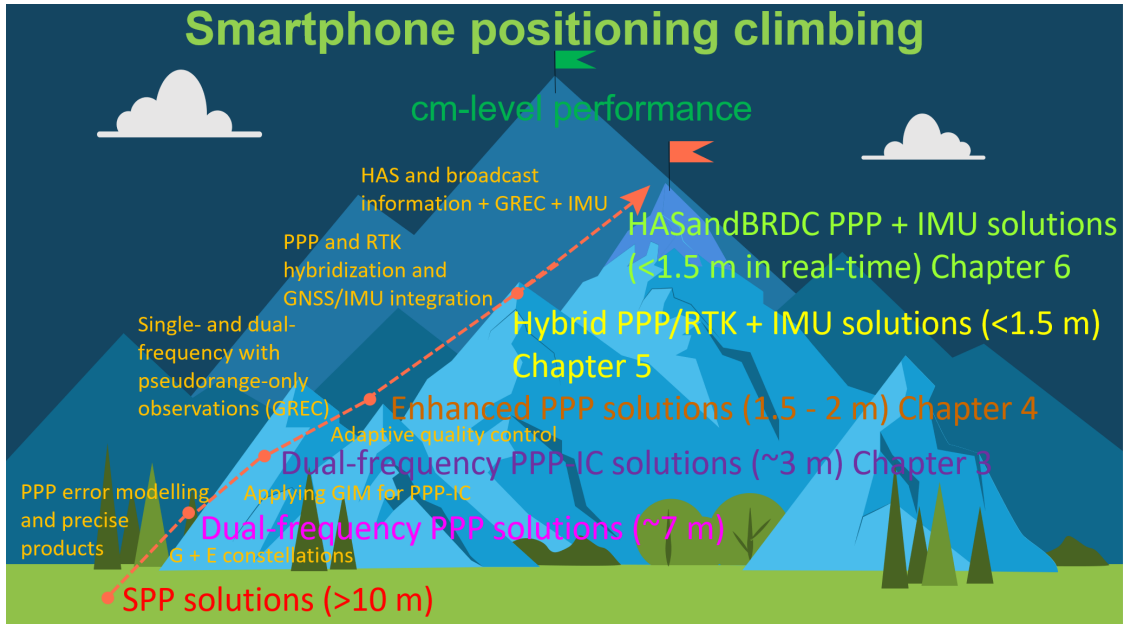


Figure 7.1: Smartphone positioning climbing map for this dissertation

7.2 Recommendations

While achieving centimetre-level smartphone PPP performance in realistic driving environments remains a challenge, this dissertation has made significant contributions in narrowing the performance gap between smartphones and other geodetic and low-cost GNSS receivers. However, there are still some limitations for improvement in future work. For instance, this dissertation did not leverage phase biases and the Decoupled Clock Model (DCM) model for PPP with Ambiguity Resolution (PPP-AR). The journey towards achieving high accuracy performance is ongoing, and there is no end in sight as long as further improvements are the goal. It is anticipated that centimetre-level smartphone positioning performance can be attained in the near future with continued research and efforts. In light of this, recommendations for future work are provided based on the current progress made in this dissertation to further enhance the robustness, accuracy, and reliability of smartphone positioning in urban driving environments.

7.2.1 PPP-AR for smartphone navigation

Ambiguity Resolution (AR) is a promising technique in reducing convergence time and initialization errors in PPP processing. It leverages the integer nature of carrier phase ambiguity to eliminate the waiting time typically required for float ambiguities to converge. Over the past few decades, numerous studies have explored AR in GNSS applications, resulting in the development of several AR models such as the DCM (Collins et al., 2008), Fractional Cycle Biases (FCB) model (Ge et al., 2008), and Integer Recovery Clock (IRC) model (Laurichesse et al., 2009). These models have demonstrated their equivalence in the mathematical domain and their ability to enhance user positioning performance (Teunissen and Khodabandeh, 2015). To efficiently and accurately fix ambiguities, widely used methods include the Least-squares AMBiguity Decorrelation Adjustment (LAMBDA) and the Best Integer Equivariant (BIE) approach (Temiissen, 1995; Teunissen, 2003).

AR is also a significant research area in smartphone navigation. Recent studies by Geng and Li (2019) and Wen et al. (2020) have demonstrated the feasibility of achiev-

ing centimetre-level AR solutions for RTK and PPP, respectively, with the aid of external geodetic antennas. However, achieving AR in PPP with internal smartphone-grade GNSS receivers and antennas remains challenging owing to noisy measurements. Therefore, smartphone PPP-AR is expected to be a focal point of research, with a potential approach to use the widelane (WL) combination with regional ionospheric and tropospheric corrections, which is expected to improve the success rate of smartphone AR and ultimately achieve centimetre-level accuracy in realistic driving environments.

7.2.2 HAS PPP-AR and PPP-RTK analysis

Galileo High Accuracy Services has emerged as a pioneer in providing GPS and Galileo precise orbit, clock, and bias corrections globally. Unlike other PPP services which rely on denser reference stations, current HAS corrections are generated by 15 Galileo Sensors Stations and related infrastructures, and they are broadcasted within the Galileo C/NAV navigation message through the E6B signal at 1278.75 MHz (de Blas et al., 2023). Since January 24th, 2023, Galileo HAS successfully declared its initial services operational, and these corrections have been accessible both through Galileo E6B signals and HAS Internet Data Distribution (IDD) service since then (EUSPA, 2023).

Currently, Galileo HAS operates at Service Level 1, and it is expected that HAS will soon disseminate satellite phase biases, paving the way for PPP-AR. Moreover, Galileo HAS is anticipated to expand its services to include atmospheric corrections for the European coverage area at Service Level 2, enabling the feasibility of PPP-RTK. With these advancements, the application of HAS PPP and HAS PPP-RTK for low-cost sensors would be an appealing topic for future research work.

7.2.3 HAS and other regional GNSS augmentation services combination

Instead of broadcasting corrections through the Internet, a recent development in positioning communities has witnessed the emergence of the direct broadcast of SSR corrections via GNSS constellations. Alongside Galileo HAS, which broadcasts satellite precise corrections globally, other GNSS augmentation services such as QZSS CLAS in Japan, MADOCA cov-

ering the entire Asia-Oceania region, and BDS B2b services in China have been actively providing accurate PPP-based services in their respective regional areas (Saito et al., 2011; Tao et al., 2021; Kawate et al., 2023). More recently, an open Python toolkit CSSRlib has been created to leverage SSR corrections from multiple free services, allowing for a straightforward utilization of different augmented PPP services (Hirokawa et al., 2023).

However, there has been minimal research focusing on comparing the SISRE errors and bias differences across different services, and the discrepancy in positioning performance with different products remains unclear for users. Furthermore, the inherent SISRE errors present in the broadcast products significantly impede the performance of the proposed HASandBRDC smartphone PPP solution. Therefore, future research will concentrate on integrating Galileo HAS with other regional GNSS augmentation services to improve correction accuracy and availability.

7.2.4 Adaptive robust EKF for smartphone PPP/IMU processing

There are numerous challenges exist and restrict the precision of smartphone positioning. Either smartphone GNSS or IMU measurements are vulnerable to challenged environments, indicating these raw observations are frequently contaminated with outliers. However, the empirical Kalman Filter’s system noise covariance Q and measurement noise covariance R are derived during the laboratory experiments or simulations, which cannot reflect the errors from realistic and unexpected scenarios sometimes (Groves, 2013). In other words, the key problem faced by the EKF is to balance the weight of the observations and prediction states. In this context, the Sage-Husa Adaptive Extended Kalman Filter (SHAKF) was initially proposed based on a time-varying noise statistical estimator for updating Q and R (Sage and Husa, 1969). In this context, Liu et al. (2019a) introduced an updated SHAKF method with improved computational efficiency, enabling real-time applications. Additionally, Narasimhappa et al. (2019) modified the SHAKF to incorporate a time-varying noise estimator and an Adaptive Robust Kalman Filter (ARKF), a three-segment approach is used to evaluate the adaptive scale factor based on learning statistics.

More recently, Vana (2023) applied the modified robust adaptive EKF in low-cost GNSS/IMU integration, achieving a significant 55% improvement in horizontal rms. Similarly, Raghuvanshi et al. (2023) validated the adaptive EKF in smartphone navigation, demonstrating improvements in overall rms error ranging from 6% to 28%. Despite these advancements, designing an optimal adaptive scale factor remains a challenging task, particularly in the context of smartphone processing. This challenge is primarily due to the constraints imposed by low-cost hardware and the dynamic nature of driving environments. Given these limitations, there is a need for further refinement of the SHAKF algorithm, with a focus on selecting the suitable adaptive factor. Such modifications are expected to make significant contributions to smartphone PPP/IMU integration, particularly in harsh and unpredictable environments.

7.2.5 Smartphone GNSS PPP, IMU, and camera integration

While not a novel concept, previous studies have highlighted the potential of visual sensors in optimizing GNSS/IMU performance, particularly in challenging GNSS scenarios with long signal outages (Jonghyuk and Sukkarieh, 2004; Wang et al., 2008; Chu et al., 2012). Qu et al. (2015) emphasized the importance of using an appropriate density of landmarks to correct accumulated errors in visual-inertial odometry (VIO). Integration of a monocular camera has been shown to significantly enhance GNSS/IMU performance in PPP and RTK processing (Li et al., 2019; Pang and Tiberius, 2020). Recently, Jiang et al. (2023) demonstrated successful tight integrating of smartphone GNSS RTK with VIO, resulting in a 25% improvement in horizontal accuracy and a 30% reduction in velocity estimation error in kinematic tests.

Camera sensors are integral components of commercial smartphones, and smartphones equipped with cameras have become the most popular devices for capturing photographs worldwide (Dabove et al., 2017). However, image quality issues stemming from low-cost hardware and small sensor sizes result in optical distortion, lens shading, and poor resolution, limiting the utility of smartphone camera positioning (Ashraf et al., 2020). In light of these challenges, tightly-coupled GNSS/IMU/camera integration is expected to leverage

the strengths of each sensor and provide enhanced solutions for smartphone navigation in urban environments.

7.2.6 Smartphone navigation with AI

With the growing popularity of ChatGPT and Sora, artificial intelligence (AI) techniques have significantly changed people's lives and become an essential part of our daily lives. As machine learning (ML) and neural network capabilities continue to advance, these emerging technologies have also catalyzed new research avenues and product innovations for GNSS technologies and applications. Countless publications have explored the application of ML to address diverse challenges in GNSS, such as signal classification, navigation in complex environments, and mitigation of multipath effects.

For instance, Goodall (2009) explored intelligent methods to determine Q and R in a low-cost GPS/IMU navigation system. Sun et al. (2020a) conducted a comparative study of decision tree (DT), k-nearest neighbour (KNN), and adaptive network-based fuzzy inference system (ANFIS) in signal detection and classification. Kanhere et al. (2022) developed deep neural networks (DNNs) to integrate information from available GNSS measurements, demonstrating superior performance compared to traditional weighted least squares GNSS solutions. Recently, Özarpacı et al. (2024) utilized four ML algorithms to enhance velocity estimation with geodetic GNSS receivers. It is anticipated that AI techniques will prove to be invaluable tools in enhancing smartphone positioning accuracy and reliability, particularly in challenging environments.

References

- Abd Rabbou, M. and El-Rabbany, A. (2014). Tightly coupled integration of GPS precise point positioning and MEMS-based inertial systems. *GPS Solutions*, 19:601–609.
- Abd Rabbou, M. and El-Rabbany, A. (2015). Tightly coupled integration of GPS precise point positioning and MEMS-based inertial systems. *GPS Solutions*, 19:601–609.
- Aggrey, J. (2014). Multi-GNSS precise point positioning software architecture and analysis of GLONASS pseudorange biases. Master’s thesis, York University, Canada.
- Aggrey, J. and Bisnath, S. (2019). Improving GNSS PPP convergence: The case of atmospheric-constrained, multi-GNSS PPP-AR. *Sensors*, 19(3):587.
- Aggrey, J., Bisnath, S., Naciri, N., Shinghal, G., and Yang, S. (2020). Multi-GNSS precise point positioning with next-generation smartphone measurements. *Journal of Spatial Science*, 65(1):79–98.
- Agnew, D. C. (1997). NLOADF: A program for computing ocean-tide loading. *Journal of Geophysical Research: Solid Earth*, 102(B3):5109–5110.
- Annex, A. (1995). Global positioning system standard positioning service signal specification. *United States Coast Guard Navigation Center*.
- Ashby, N. and Spilker, J. J. (1996). Introduction to relativistic effects on the Global Positioning System. *Global Positioning System: Theory and applications*., 1:623–697.
- Ashraf, I., Hur, S., and Park, Y. (2020). Smartphone Sensor Based Indoor Positioning: Current Status, Opportunities, and Future Challenges. *Electronics*, 9(6):891.
- Banville, S., Collins, P., Zhang, W., and Langley, R. B. (2014). Global and regional ionospheric corrections for faster PPP convergence. *Navigation: Journal of the Institute of Navigation*, 61(2):115–124.
- Banville, S., Lachapelle, G., Ghoddousi-Fard, R., and Gratton, P. (2019). Automated Processing of Low-Cost GNSS Receiver Data. In *Proceedings of Institute of Navigation GNSS+ 2019 conference*.
- Bisnath, S. and Aggrey, J. (2024). Current Limitations and Prospects for Smartphone GNSS Precise Positioning. In *Proceedings of the 2024 International Technical Meeting of The Institute of Navigation*, pages 1–24.

- Bisnath, S. and Gao, Y. (2009). Current state of precise point positioning and future prospects and limitations. In *Observing our changing earth*, pages 615–623. Springer.
- Blewitt, G. (1990). An automatic editing algorithm for GPS data. *Geophysical research letters*, 17(3):199–202.
- Caamano, M., Crespillo, O. G., Gerbeth, D., and Grosch, A. (2020). Detection of GNSS multipath with time-differenced code-minus-carrier for land-based applications. In *2020 European Navigation Conference (ENC)*, pages 1–12. IEEE.
- Cai, C., Gao, Y., et al. (2007). Precise point positioning using combined GPS and GLONASS observations. *Positioning*, 1(11).
- Carlin, L., Hauschild, A., and Montenbruck, O. (2021). Precise point positioning with GPS and Galileo broadcast ephemerides. *GPS Solutions*, 25(2):77.
- CDDIS (2024). GNSS Ultra-Rapid Combined Orbit Solution Product. https://cddis.nasa.gov/Data_and_Derived_Products/GNSS/gnss_igsuorb.html.
- Chen, B., Gao, C., Liu, Y., and Sun, P. (2019). Real-time precise point positioning with a Xiaomi MI 8 android smartphone. *Sensors*, 19(12):2835.
- Chen, K. and Gao, Y. (2005). Real-time precise point positioning using single frequency data. In *Proceedings of the 18th international technical meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2005)*, pages 1514–1523.
- China Satellite Navigation Office (2019). BeiDou Navigation Satellite System Signal In Space Interface Control Document). <http://en.beidou.gov.cn/SYSTEMS/ICD/>. Accessed 18 Feb 2024.
- Chu, T., Guo, N., Backén, S., and Akos, D. (2012). Monocular camera/IMU/GNSS integration for ground vehicle navigation in challenging GNSS environments. *Sensors*, 12(3):3162–3185.
- Collins, P., Lahaye, F., and Bisnath, S. (2012). External ionospheric constraints for improved PPP-AR initialisation and a generalised local augmentation concept. In *Proceedings of the 25th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2012)*, pages 3055–3065.
- Collins, P., Lahaye, F., Heroux, P., and Bisnath, S. (2008). Precise point positioning with ambiguity resolution using the decoupled clock model. In *Proceedings of the 21st international technical meeting of the satellite division of the Institute of Navigation (ION GNSS 2008)*, pages 1315–1322.
- Dabove, P. and Di Pietra, V. (2019a). Single-baseline RTK positioning using dual-frequency GNSS receivers inside smartphones. *Sensors*, 19(19):4302.
- Dabove, P. and Di Pietra, V. (2019b). Towards high accuracy GNSS real-time positioning with smartphones. *Advances in Space Research*, 63(1):94–102.

- Dabove, P., Pietra, V., and Lingua, A. M. (2017). Positioning techniques with smartphone technology: Performances and methodologies in outdoor and indoor scenarios. In *Smartphones from an Applied Research Perspective*, pages 163–183. InTech.
- Darugna, F., Wübbena, J. B., Wübbena, G., Schmitz, M., Schön, S., and Warneke, A. (2021). Impact of robot antenna calibration on dual-frequency smartphone-based high-accuracy positioning: A case study using the Huawei Mate20X. *GPS Solutions*, 25(1):15.
- de Blas, F., Vázquez, J., Hernández, C., Ostolaza, J., Lagrasta, S., Fernandez-Hernández, I., and Blonski, D. (2023). The Galileo High Accuracy Service (HAS): A Pioneer Free-of-Charge Global Precise Positioning Service. In *Proceedings of the 36th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2023)*, pages 449–468.
- Delchambre, L. (2015). Weighted principal component analysis: a weighted covariance eigendecomposition approach. *Monthly Notices of the Royal Astronomical Society*, 446(4):3545–3555.
- Eckl, M., Snay, R., Soler, T., Cline, M., and Mader, G. (2001). Accuracy of GPS-derived relative positions as a function of interstation distance and observing-session duration. *Journal of Geodesy*, 75(12):633–640.
- Elmezayen, A. and El-Rabbany, A. (2019). Precise point positioning using world’s first dual-frequency GPS/GALILEO smartphone. *Sensors*, 19(11):2593.
- ESA (2016). Galileo initial services. https://www.esa.int/ESA_Multimedia/Transmissions/2016/12/Galileo_initial_services. Accessed 18 Feb 2024.
- European Union (2022). Galileo High Accuracy Service Signal-In-Space Interface Control Document (HAS SIS ICD), Issue 1.0. https://www.gsc-europa.eu/sites/default/files/sites/all/files/Galileo_HAS_SIS_ICD_v1.0.pdf. Accessed 10 Sep 2022.
- EUSPA (2020). Galileo high accuracy service (HAS) info note. European GNSS Agency. https://www.gsc-europa.eu/sites/default/files/sites/all/files/Galileo_HAS_Info_Note.pdf. Accessed 10 Sep 2022.
- EUSPA (2023). Galileo High Accuracy Service goes Live! | EU Agency for the Space Programme. <https://www.euspa.europa.eu/newsroom/news/galileo-high-accuracy-service-now-operational>. Accessed 15 Apr 2023.
- EUSPA (2024). EUSPA EO and GNSS Market Report. https://www.euspa.europa.eu/sites/default/files/euspa_market_report_2024.pdf. Accessed 18 Feb 2024.
- Everett, T., Taylor, T., Lee, D.-K., and Akos, D. M. (2022). Optimizing the Use of RTKLIB for Smartphone-Based GNSS Measurements. *Sensors*, 22(10):3825.
- Farrell, J. and Barth, M. (1999). *GNSS/INS Integration*, volume 61, chapter 9. Wiley-Interscience.

- Fernandez-Hernandez, I., Chamorro-Moreno, A., Cancela-Diaz, S., Calle-Calle, J. D., Zoccarato, P., Blonski, D., Senni, T., de Blas, F. J., Hernández, C., Simón, J., et al. (2022). Galileo high accuracy service: initial definition and performance. *GPS Solutions*, 26(3):65.
- Fortunato, M., Critchley-Marrows, J., Siutkowska, M., Ivanovici, M. L., Benedetti, E., and Roberts, W. (2019a). Enabling high accuracy dynamic applications in urban environments using PPP and RTK on android multi-frequency and multi-GNSS smartphones. In *2019 European navigation conference (ENC)*, pages 1–9. IEEE.
- Fortunato, M., Critchley-Marrows, J., Siutkowska, M., Ivanovici, M. L., Benedetti, E., and Roberts, W. (2019b). Enabling high accuracy dynamic applications in urban environments using PPP and RTK on android multi-frequency and multi-GNSS smartphones. In *2019 European navigation conference (ENC)*, pages 1–9. IEEE.
- Fu, G. M., Khider, M., and van Diggelen, F. (2020). Android raw GNSS measurement datasets for precise positioning. In *Proceedings of the 33rd international technical meeting of the satellite division of the Institute of Navigation (ION GNSS+ 2020)*, pages 1925–1937.
- Gao, R., Xu, L., Zhang, B., and Liu, T. (2021). Raw GNSS observations from Android smartphones: Characteristics and short-baseline RTK positioning performance. *Measurement Science and Technology*.
- Gao, Y. and Chen, K. (2004). Performance analysis of precise point positioning using real-time orbit and clock products. *Journal of Global Positioning Systems*, 3(1-2):95–100.
- Ge, M., Gendt, G., Rothacher, M. a., Shi, C., and Liu, J. (2008). Resolution of GPS carrier-phase ambiguities in precise point positioning (PPP) with daily observations. *Journal of geodesy*, 82:389–399.
- Ge, M., Zhang, H., Jia, X., Song, S., and Wickert, J. (2012). What is achievable with the current compass constellation. *GPS World*, 1:29–34.
- Geng, J., Chen, X., Pan, Y., and Zhao, Q. (2019). A modified phase clock/bias model to improve PPP ambiguity resolution at Wuhan University. *Journal of Geodesy*, 93:2053–2067.
- Geng, J. and Li, G. (2019). On the feasibility of resolving Android GNSS carrier-phase ambiguities. *Journal of Geodesy*, 93(12):2621–2635.
- Geng, J., Li, G., Zeng, R., Wen, Q., and Jiang, E. (2018). A comprehensive assessment of raw multi-GNSS measurements from mainstream portable smart devices. In *Proceedings of the 31st International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2018)*, pages 392–412.
- Gikas, V. and Perakis, H. (2016). Rigorous performance evaluation of smartphone GNSS/IMU sensors for ITS applications. *Sensors*, 16(8):1240.
- Gill, M., Bisnath, S., Aggrey, J., and Seepersad, G. (2017). Precise point positioning (PPP) using low-cost and ultra-low-cost GNSS receivers. In *Proceedings of the ION GNSS*, pages 226–236.

- Glaner, M. F. and Weber, R. (2023). Breaking the One-Meter Accuracy Level with Smartphone GNSS Data. *Engineering Proceedings*, 54(1):16.
- Goodall, C. L. (2009). Improving usability of low-cost INS/GPS navigation systems using intelligent techniques. *Thesis*.
- GPS.gov (2024). Space Segment. In: Curr. Future Satell. Gener. <https://www.gps.gov/systems/gps/space/#generations>. Accessed 20 Feb 2024.
- Groves, P. (2013). *INS/GNSS Integration*, chapter 14. Artech House.
- Gunning, K., Blanch, J., and Walter, T. (2019). SBAS corrections for PPP integrity with solution separation. In *Proceedings of the 2019 international technical meeting of the Institute of Navigation*, pages 707–719.
- Gurtner, W. and Estey, L. (2007). Rinex-the receiver independent exchange format-version 3.00. *Astronomical Institute, University of Bern and UNAVCO, Boulder, Colorado*.
- Håkansson, M. (2019). Characterization of GNSS observations from a Nexus 9 Android tablet. *GPS Solutions*, 23(1):21.
- Hauschild, A., Montenbruck, O., Steigenberger, P., Martini, I., and Fernandez-Hernandez, I. (2022). Orbit determination of Sentinel-6A using the Galileo high accuracy service test signal. *GPS Solutions*, 26(4):120.
- Haustein, M. (2009). Effects of the Theory of Relativity in the GPS. *Chemnitz University of Technology*.
- Hernández-Pajares, M., Juan, J., Sanz, J., Orus, R., Garcia-Rigo, A., Feltens, J., Komjathy, A., Schaer, S., and Krankowski, A. (2009). The IGS VTEC maps: a reliable source of ionospheric information since 1998. *Journal of Geodesy*, 83(3-4):263–275.
- Herrera, A. M., Suhandri, H. F., Realini, E., Reguzzoni, M., and de Lacy, M. C. (2016). goGPS: open-source MATLAB software. *GPS Solutions*, 20:595–603.
- Hilla, S. (2016). The extended standard product 3 orbit format (sp3-d) 21 february 2016.
- Hirokawa, R., Hauschild, A., and Everett, T. (2023). Python Toolkit for Open PPP/PPP-RTK Services. In *Proceedings of the 36th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2023)*, pages 2514–2526.
- Hofmann-Wellenhof, B., Lichtenegger, H., and Collins, J. (2012). *Global positioning system: theory and practice*. Springer Science & Business Media.
- Hopfield, H. (1969). Two-quartic tropospheric refractivity profile for correcting satellite data. *Journal of Geophysical Research*, 74(18):4487–4499.
- Hu, J., Yi, D., and Bisnath, S. (2023). A Comprehensive Analysis of Smartphone GNSS Range Errors in Realistic Environments. *Sensors*, 23(3):1631.
- IGS (2024). Precise Orbit and Clock Products. https://igs.org/products/#orbits_clocks. Accessed 20 Feb 2024.

- Jee, G., Lee, H.-B., Kim, Y., Chung, J.-K., and Cho, J. (2010). Assessment of GPS global ionosphere maps (GIM) by comparison between CODE GIM and TOPEX/Jason TEC data: Ionospheric perspective. *Journal of Geophysical Research: Space Physics*, 115(A10).
- Jiang, Y., Zhang, Y., Lyu, Z., Guo, S., and Gao, Y. (2023). Tightly Integrated Smartphone GNSS and Visual odometry for Enhanced Urban Pedestrian Positioning. In *Proceedings of the 36th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2023)*, pages 2057–2072.
- Jonghyuk, K. and Sukkarieh, S. (2004). SLAM Aided GPS/INS Navigation in GPS Denied and Unknown Environment. In *the 2004 International Symposium on GNSS/GPS, Sydney, Australia*.
- Kaiser, S., Wei, Y., and Renaudin, V. (2021). Analysis of IMU and GNSS data provided by Xiaomi 8 Smartphone. In *2021 International Conference on Indoor Positioning and Indoor Navigation (IPIN)*, pages 1–8. IEEE.
- Kanhere, A. V., Gupta, S., Shetty, A., and Gao, G. (2022). Improving gnss positioning using neural-network-based corrections. *NAVIGATION: Journal of the Institute of Navigation*, 69(4).
- Kawate, K., Igarashi, Y., Yamada, H., Akiyama, K., Okeya, M., Takiguchi, H., Murata, M., Sasaki, T., Matsushita, S., Miyoshi, S., et al. (2023). MADOCA: Japanese precise orbit and clock determination tool for GNSS. *Advances in Space Research*, 71(10):3927–3950.
- Klobuchar, J. A. (1975). *A first-order, worldwide, ionospheric, time-delay algorithm*, volume 75. Air Force Cambridge Research Laboratories, Air Force Systems Command, United States Air Force.
- Kouba, J. and Héroux, P. (2001). Precise point positioning using IGS orbit and clock products. *GPS Solutions*, 5:12–28.
- Lachapelle, G. and Gratton, P. (2019). GNSS precise point positioning with android smartphones and comparison with high performance receivers. In *2019 IEEE International Conference on Signal, Information and Data Processing (ICSIDP)*, pages 1–9. IEEE.
- Laurichesse, D., Mercier, F., Berthias, J.-P., Broca, P., and Cerri, L. (2009). Integer ambiguity resolution on undifferenced GPS phase measurements and its application to PPP and satellite precise orbit determination. *Navigation*, 56(2):135–149.
- Leick, A., Rapoport, L., and Tatarnikov, D. (2015). *GPS satellite surveying*. John Wiley & Sons.
- Lewen, Z., Pavel, V., Shirong, Y., and Fengyu, X. (2018). Evaluation of long-term Beidou/GPS observation quality based on G-Nut/Anubis and initial results. *Acta Geodynamica et Geomaterialia*, 15(1).
- Li, G. and Geng, J. (2019). Characteristics of raw multi-GNSS measurement error from Google Android smart devices. *GPS Solutions*, 23(3):1–16.

- Li, T., Zhang, H., Gao, Z., Niu, X., and El-Sheimy, N. (2019). Tight fusion of a monocular camera, MEMS-IMU, and single-frequency multi-GNSS RTK for precise navigation in GNSS-challenged environments. *Remote Sensing*, 11(6):610.
- Li, X., Ge, M., Zhang, H., and Wickert, J. (2013). A method for improving uncalibrated phase delay estimation and ambiguity-fixing in real-time precise point positioning. *Journal of Geodesy*, 87(5):405–416.
- Li, X., Huang, J., Li, X., Shen, Z., Han, J., Li, L., and Wang, B. (2022a). Review of PPP-RTK: Achievements, challenges, and opportunities. *Satellite Navigation*, 3(1):28.
- Li, Z., Wang, L., Wang, N., Li, R., and Liu, A. (2022b). Real-time GNSS precise point positioning with smartphones for vehicle navigation. *Satellite Navigation*, 3(1):19.
- Liu, A., Wang, N., Li, Z., Zhou, K., and Yuan, H. (2018). Validation of CAS’s final global ionospheric maps during different geomagnetic activities from 2015 to 2017. *Results in Physics*, 10:481–486.
- Liu, K., Zhao, W., Sun, B., Wu, P., Zhu, D., and Zhang, P. (2019a). Application of updated Sage–Husa adaptive Kalman filter in the navigation of a translational sprinkler irrigation machine. *Water*, 11(6):1269.
- Liu, W., Shi, X., Zhu, F., Tao, X., and Wang, F. (2019b). Quality analysis of multi-GNSS raw observations and a velocity-aided positioning approach based on smartphones. *Advances in Space Research*, 63(8):2358–2377.
- Lou, Y., Zheng, F., Gu, S., Wang, C., Guo, H., and Feng, Y. (2016). Multi-GNSS precise point positioning with raw single-frequency and dual-frequency measurement models. *GPS Solutions*, 20(4):849–862.
- Malkos, S. (2016). User location takes center stage in new android OS: Google to provide raw GNSS measurements. *GPS World*, 27(7):36.
- Martini, I., Susi, M., Cucchi, L., and Fernandez-Hernandez, I. (2024). Galileo high accuracy service performance and anomaly mitigation capabilities. *GPS Solutions*, 28(1):25.
- Melbourne, W. G. (1985). The case for ranging in GPS-based geodetic systems. In *Proceedings of the first international symposium on precise positioning with the Global Positioning System*, pages 373–386. US Department of Commerce Rockville, Maryland.
- Miya, M., Fujita, S., Sato, Y., Ota, K., Hirokawa, R., and Takiguchi, J. (2016). Centimeter level augmentation service (CLAS) in Japanese quasi-zenith satellite system, its user interface, detailed design, and plan. In *Proceedings of the 29th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2016)*, pages 2864–2869.
- Montenbruck, O., Steigenberger, P., and Hauschild, A. (2015). Broadcast versus precise ephemerides: a multi-GNSS perspective. *GPS Solutions*, 19:321–333.

- Naciri, N., Yi, D., Bisnath, S., de Blas, F. J., and Capua, R. (2023). Assessment of Galileo High Accuracy Service (HAS) test signals and preliminary positioning performance. *GPS Solutions*, 27(2):73.
- Narasimhappa, M., Mahindrakar, A. D., Guizilini, V. C., Terra, M. H., and Sabat, S. L. (2019). MEMS-based IMU drift minimization: Sage Husa adaptive robust Kalman filtering. *IEEE Sensors Journal*, 20(1):250–260.
- NASA (2024). Navstar 2A-01. In: NASA Space Sci. Data Coord. Arch. <https://nssdc.gsfc.nasa.gov/nmc/spacecraft/display.action?id=1990-103A>. Accessed 20 Feb 2024.
- Nie, Z., Liu, F., and Gao, Y. (2020). Real-time precise point positioning with a low-cost dual-frequency GNSS device. *GPS Solutions*, 24(1):9.
- Noureldin, A., Karamat, T. B., and Georgy, J. (2012). *Fundamentals of inertial navigation, satellite-based positioning and their integration*. Springer Science & Business Media.
- Odiijk, D. (2002). Fast precise GPS positioning in the presence of ionospheric delays. *Publications on Geodesy*, 52.
- Odolinski, R. and Teunissen, P. J. (2019). An assessment of smartphone and low-cost multi-GNSS single-frequency RTK positioning for low, medium and high ionospheric disturbance periods. *Journal of Geodesy*, 93(5):701–722.
- Özarpacı, S., Kılıç, B., Bayrak, O. C., Taşkıran, M., Doğan, U., and Floyd, M. (2024). Machine learning approach for GNSS geodetic velocity estimation. *GPS Solutions*, 28(2):1–13.
- Pang, M. and Tiberius, C. (2020). Precision and reliability of tightly coupled PPP GNSS and landmark monocular vision positioning. *Sensors*, 20(5):1537.
- Parra, C., Schütz, A., Pany, T., Hugentobler, U., and Baumann, S. (2023). The Galileo High Accuracy Service: Assessment of the Quality of Corrections and Preliminary PPP Performance. In *Proceedings of the 36th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2023)*, pages 2581–2591.
- Pathak, V., Thornwall, S., Krier, M., Rowson, S., Poilasne, G., and Desclos, L. (2003). Mobile handset system performance comparison of a linearly polarized GPS internal antenna with a circularly polarized antenna. In *IEEE antennas and propagation society international symposium. Digest. Held in conjunction with: USNC/CNC/URSI North American Radio Sci. Meeting (Cat. No. 03CH37450)*, volume 3, pages 666–669. IEEE.
- Paziewski, J. (2020a). Recent advances and perspectives for positioning and applications with smartphone GNSS observations. *Meas. Sci. Technol*, 31(091001):091001.
- Paziewski, J. (2020b). Recent advances and perspectives for positioning and applications with smartphone GNSS observations. *Meas. Sci. Technol*, 31(091001):091001.
- Paziewski, J., Sieradzki, R., and Baryla, R. (2019). Signal characterization and assessment of code GNSS positioning with low-power consumption smartphones. *GPS Solutions*, 23(4):1–12.

- Pesyna, K. M., Heath, R. W., Humphreys, T. E., et al. (2014). Centimeter positioning with a smartphone-quality GNSS antenna. In *Proceedings of the 27th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2014)*, pages 1568–1577.
- Qu, X., Soheilian, B., and Paparoditis, N. (2015). Vehicle localization using mono-camera and geo-referenced traffic signs. In *2015 IEEE Intelligent Vehicles Symposium (IV)*, pages 605–610. IEEE.
- Raghuvanshi, A., Vana, S., and Bisnath, S. (2023). Precise positioning of smartphones using a robust adaptive kalman filter. In *Proceedings of the 36th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2023)*, pages 2749–2764.
- Realini, E., Caldera, S., Pertusini, L., and Sampietro, D. (2017). Precise GNSS positioning using smart devices. *Sensors*, 17(10):2434.
- Saastamoinen, J. (1972). Atmospheric correction for the troposphere and stratosphere in radio ranging satellites. *The use of artificial satellites for geodesy*, 15:247–251.
- Sage, A. P. and Husa, G. W. (1969). Algorithms for sequential adaptive estimation of prior statistics. In *1969 IEEE Symposium on Adaptive Processes (8th) Decision and Control*, pages 61–61. IEEE.
- Saito, M., Sato, Y., Miya, M., Shima, M., Omura, Y., Takiguchi, J., and Asari, K. (2011). Centimeter-class augmentation system utilizing quasi-zenith satellite. In *Proceedings of the 24th International Technical Meeting of the Satellite Division of the Institute of Navigation (ION GNSS 2011)*, pages 1243–1253.
- Schaer, S. (2016). SINEX BIAS—Solution (software/technique) INdependent EXchange format for GNSS BIASes version 1.00. In *IGS workshop on GNSS biases, Bern, Switzerland*.
- Schaer, S., Gurtner, W., and Feltens, J. (1999). *Mapping and predicting the Earth’s ionosphere using the Global Positioning System*, volume 59. Institut für Geodäsie und Photogrammetrie, Eidg. Technische Hochschule Zürich.
- Schmid, R., Dach, R., Collilieux, X., Jäggi, A., Schmitz, M., and Dilssner, F. (2016). Absolute IGS antenna phase center model igs08.atx: status and potential improvements. *Journal of Geodesy*, 90:343–364.
- Seepersad, G., Hu, J., Yang, S., Yi, D., and Bisnath, S. (2021). Changing Lanes with Smartphones Technology. In *Proceedings of the 34th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2021)*, pages 3021 – 3036.
- Seepersad, G., Hu, J., Yang, S., Yi, D., and Bisnath, S. (2022). Performance Assessment of Tightly Coupled Smartphone Sensors with Legacy and State Space Corrections. In *Proceedings of the 35th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2022)*, pages 2235 – 2255.

- Shi, C., Gu, S., Lou, Y., and Ge, M. (2012). An improved approach to model ionospheric delays for single-frequency precise point positioning. *Advances in Space Research*, 49(12):1698–1708.
- Shinghal, G. (2020). Analysis and Conditioning of Gns Measurements from Smartphones for Precise Point Positioning in Realistic Environments. Master’s thesis, York University, Canada.
- Shinghal, G. and Bisnath, S. (2021). Conditioning and PPP processing of smartphone GNSS measurements in realistic environments. *Satellite Navigation*, 2(1):1–17.
- Sun, R., Wang, G., Zhang, W., Hsu, L.-T., and Ochieng, W. Y. (2020a). A gradient boosting decision tree based GPS signal reception classification algorithm. *Applied Soft Computing*, 86:105942.
- Sun, W., Li, Y., and Duan, S. (2020b). Xiaomi Mi 8 smartphone GNSS data quality analysis and single-frequency RTK positioning performance evaluation. *IET radar, sonar & navigation*, 14(9):1410–1416.
- Takasu, T. et al. (2013). Rtklib. Available: <http://www.rtklib.com>.
- Tang, C., Hu, X., Zhou, S., Guo, R., He, F., Liu, L., Zhu, L., Li, X., Wu, S., Zhao, G., et al. (2016). Improvement of orbit determination accuracy for Beidou navigation satellite system with two-way satellite time frequency transfer. *Advances in Space Research*, 58(7):1390–1400.
- Tao, J., Liu, J., Hu, Z., Zhao, Q., Chen, G., and Ju, B. (2021). Initial Assessment of the BDS-3 PPP-B2b RTS compared with the CNES RTS. *GPS Solutions*, 25:1–16.
- Temiissen, J. (1995). The least-squares ambiguity decorrelation adjustment: A method for fast GPS integer ambiguity estimation. *Journal of Geodesy*, 70:65–82.
- Teunissen, P. (2003). Theory of integer equivariant estimation with application to GNSS. *Journal of Geodesy*, 77(7-8):402–410.
- Teunissen, P. and Khodabandeh, A. (2015). Review and principles of PPP-RTK methods. *Journal of Geodesy*, 89(3):217–240.
- Teunissen, P. J. and Montenbruck, O. (2017). *Springer Handbook of Global Navigation Satellite Systems*, volume 10. Springer.
- Tu, R., Ge, M., Zhang, H., and Huang, G. (2013). The realization and convergence analysis of combined PPP based on raw observation. *Advances in Space Research*, 52(1):211–221.
- Urlichich, Y., Subbotin, V., Stupak, G., Dvorkin, V., Povaliaev, A., and Karutin, S. (2010). GLONASS developing strategy. In *Proceedings of the 23rd International Technical Meeting of The Satellite Division of the Institute of Navigation (ION GNSS 2010)*, pages 1566–1571.
- Urlichich, Y., Subbotin, V., Stupak, G., Dvorkin, V., Povalyaev, A., and Karutin, S. (2011). GLONASS modernization. In *Proceedings of the 24th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2011)*, pages 3125–3128.

- Vana, S. (2023). *Continuous Urban Navigation with Next-Generation, Mass Market Navigation Sensors and Adaptive Filtering*. PhD thesis, York University, Canada.
- Vana, S. and Bisnath, S. (2020). Enhancing navigation in difficult environments with low-cost, dual-frequency GNSS PPP and MEMS IMU. In *International Association of Geodesy Symposia*. Springer.
- Vana, S., Naciri, N., and Bisnath, S. (2019). Low-cost, Dual-frequency PPP GNSS and MEMS-IMU Integration Performance in Obstructed Environments. In *Proceedings of the 32nd International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2019)*, pages 3005–3018.
- Wang, G., Bo, Y., Yu, Q., Li, M., Yin, Z., and Chen, Y. (2020). Ionosphere-Constrained Single-Frequency PPP with an Android Smartphone and Assessment of GNSS Observations. *Sensors*, 20(20):5917.
- Wang, J., Garratt, M., Lambert, A., Wang, J. J., Han, S., and Sinclair, D. (2008). Integration of GPS/INS/vision sensors to navigate unmanned aerial vehicles. *The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, 37(part B1):963–969.
- Wang, L., Li, Z., Wang, N., and Wang, Z. (2021). Real-time GNSS precise point positioning for low-cost smart devices. *GPS Solutions*, 25(2):1–13.
- Warnant, R., Vyvere, D., Van, L., and Warnant, Q. (2018). Positioning with single and dual frequency smartphones running Android 7 or later. In *Proceedings of the 31st international technical meeting of the satellite division of the Institute of Navigation (ION GNSS+ 2018)*, pages 284–303.
- Wen, Q., Geng, J., Li, G., and Guo, J. (2020). Precise point positioning with ambiguity resolution using an external survey-grade antenna enhanced dual-frequency android GNSS data. *Measurement*, 157:107634.
- Wu, J.-T., Wu, S. C., Hajj, G. A., Bertiger, W. I., and Lichten, S. M. (1992). Effects of antenna orientation on GPS carrier phase. *Astrodynamics 1991*, pages 1647–1660.
- Wu, Q., Sun, M., Zhou, C., and Zhang, P. (2019). Precise point positioning using dual-frequency GNSS observations on smartphone. *Sensors*, 19(9):2189.
- Xiang, Y. and Gao, Y. (2017). Improving DCB estimation using uncombined PPP. *Navigation: Journal of The Institute of Navigation*, 64(4):463–473.
- Xiang, Y., Gao, Y., and Li, Y. (2017). Ionospheric STEC and VTEC constraints for fast PPP. In *China Satellite Navigation Conference*, pages 257–269. Springer.
- Xu, Y., Yang, Y., and Li, J. (2021). Performance evaluation of BDS-3 PPP-B2b precise point positioning service. *GPS Solutions*, 25(4):142.
- Yan, W., Bastos, L., and Magalhães, A. (2019). Performance assessment of the android smartphone’s IMU in a GNSS/INS coupled navigation model. *IEEE Access*, 7:171073–171083.

- Yan, W., Zhang, Q., Wang, L., Mao, Y., Wang, A., and Zhao, C. (2020). A modified Kalman filter for integrating the different rate data of gyros and accelerometers retrieved from android smartphones in the GNSS/IMU coupled navigation. *Sensors*, 20(18):5208.
- Yang, S., Yi, D., Vana, S., and Bisnath, S. (2021). Resilient Smartphone Positioning using Native Sensors and PPP Augmentation. In *Proceedings of the 34th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2021)*, pages 4208–4222.
- Yang, S., Yi, D., Vana, S., and Bisnath, S. (2023). Resilient Smartphone Positioning using Native Sensors and PPP Augmentation. *NAVIGATION: Journal of the Institute of Navigation*, 70(2).
- Yang, Y., He, H., and Xu, G. (2001). Adaptively robust filtering for kinematic geodetic positioning. *Journal of Geodesy*, 75:109–116.
- Yi, D., Bisnath, S., Naciri, N., de Blas, F. J., and Capua, R. (2023). Performance Assessment of Galileo High Accuracy Service (HAS) with Low-Cost GNSS/IMU Sensors in Urban Driving Environments. In *Proceedings of the 36th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2023)*, pages 1403–1416.
- Yi, D., Bisnath, S., Naciri, N., and Vana, S. (2021). Effects of ionospheric constraints in Precise Point Positioning processing of geodetic, low-cost and smartphone GNSS measurements. *Measurement*, 183:109887.
- Yi, D., Hu, J., and Bisnath, S. (2024). Improving PPP smartphone processing with adaptive quality control method in obstructed environments when carrier-phase measurements are missing. *GPS Solutions*, 28(1):56.
- Yi, D. and Yang, S. (2023). Hybridization of Smartphone GNSS PPP/RTK with Native IMU in Realistic Driving Scenarios. In *Proceedings of the 36th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2023)*, pages 2227–2241.
- Yi, D., Yang, S., and Bisnath, S. (2022). Native smartphone single-and dual-frequency GNSS-PPP/IMU solution in real-world driving scenarios. *Remote Sensing*, 14(14):3286.
- Yong, C. Z., Odolinski, R., Zaminpardaz, S., Moore, M., Rubinov, E., Er, J., and Denham, M. (2021). Instantaneous, dual-frequency, multi-GNSS precise RTK positioning using google pixel 4 and Samsung Galaxy S20 smartphones for zero and short baselines. *Sensors*, 21(24):8318.
- Zangenehnejad, F. and Gao, Y. (2021). Application of UofC model based multi-GNSS PPP to smartphones GNSS positioning. In *Proceedings of the 34th International Technical Meeting of the Satellite Division of the Institute of Navigation (ION GNSS+ 2021)*, pages 2986–3003.
- Zhang, H., Gao, Z., Ge, M., Niu, X., Huang, L., Tu, R., and Li, X. (2013). On the convergence of ionospheric constrained precise point positioning (IC-PPP) based on undifferential uncombined raw GNSS observations. *Sensors*, 13(11):15708–15725.

- Zhang, K., Jiao, F., and Li, J. (2018a). The assessment of GNSS measurements from android smartphones. In *China Satellite Navigation Conference*, pages 147–157. Springer.
- Zhang, X., Tao, X., Zhu, F., Shi, X., and Wang, F. (2018b). Quality assessment of GNSS observations from an Android N smartphone and positioning performance analysis using time-differenced filtering approach. *GPS Solutions*, 22(3):70.
- Zhao, J., Hernández-Pajares, M., Li, Z., Wang, N., and Yuan, H. (2021). Integrity investigation of global ionospheric TEC maps for high-precision positioning. *Journal of Geodesy*, 95(3):1–15.
- Zheng, Y. (2007). *Generation of network-based differential corrections for regional GNSS services*. PhD thesis, Queensland University of Technology.
- Zhou, F., Dong, D., Li, W., Jiang, X., Wickert, J., and Schuh, H. (2018). GAMP: An open-source software of multi-GNSS precise point positioning using undifferenced and uncombined observations. *GPS Solutions*, 22:1–10.
- Zhou, R., Hu, Z., Zhao, Q., Cai, H., Liu, X., Liu, C., Wang, G., Kan, H., and Chen, L. (2022). Consistency Analysis of the GNSS Antenna Phase Center Correction Models. *Remote Sensing*, 14(3):540.
- Zumberge, J., Heflin, M., Jefferson, D., Watkins, M., and Webb, F. (1997). Precise point positioning for the efficient and robust analysis of GPS data from large networks. *Journal of Geophysical Research: Solid Earth*, 102(B3):5005–5017.