Teachers' Perspectives and Pedagogical Documentation in Early Years Mathematics: Tensions and Possibilities

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Abstract

In this dissertation, I endeavour to explore the transformative potential of pedagogical documentation in early years mathematics education with a focus on the implementation challenges and divergent perspectives within the context of the Ontario Kindergarten Program (2016). Pedagogical documentation offers teachers an opportunity to explore the possibilities of children's theories, while challenging discourses that underscore neoliberalist practices. Pedagogical documentation includes collecting materials (e.g., photos, videos, written transcriptions, etc.) to be revisited and considered by students and educators together, to create pedagogical trajectories that are emergent and responsive. This study examines the perspectives of 12 Ontario Kindergarten teachers regarding pedagogical documentation in their kindergarten math classrooms, revealing tensions and inconsistencies with the literature and practices as outlined in the Ontario Kindergarten Program (2016) document. The teachers convened for ten weeks where video and audio recordings of every meeting were kept. Teachers examined two specific examples of pedagogical documentation of learning events. 'The Price of Apples' documentation portrayed three children engaged in a learning event about selling apples (Appendix A). 'The Columns of the Municipal Theatre' documentation portrayed a class visiting the columns outside a municipal building in their town (Appendix C). Through phenomenography-inspired analysis, the data demonstrated how the participants thought about pedagogical documentation as a way to collect evidence of student ideas. Interestingly, an analysis of high-frequency words uncovered key terms such as 'centres', 'grades', and 'values' that stand out in contrast to existing literature on pedagogical documentation. The results of this study indicate that participants used pedagogical documentation to capture evidence of student ideas, however, they did not speak of pedagogical documentation as a "way of being" with children, as a practice that nourishes emergent curriculum making, or as a method to build upon students' mathematical ideas. This research not only exposes the tensions and inconsistencies

in the use of pedagogical documentation within the Ontario Kindergarten program, but also contributes to the discourse by highlighting nuanced perspectives of the participants. In turn this paves the way for a more comprehensive understanding of the transformative potential of pedagogical documentation in early years mathematics education.

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Chapter 1: The Background of the Researcher

I approach this work with passion and tenacity. I have been a teacher in Ontario for 18 years. During this period, I have held various positions in the education system. However, before my career journey began, I was a parent of young children in Ontario. Many parents would agree that your view of the world changes after having children. My view of the world certainly changed, as did my understanding of education and 'school' in Ontario.

My children did not like mathematics, nor were they 'good' at it. Well, that is understandable as I was not good at it as a child either. I guess we were just not 'math people'. My own mother confirmed that I spent hours with flashcards at the dinner table in Grade 3 trying to memorize my multiplication facts. Hours of repetition and tears, which resulted in the same thing week after week. "Heather you are good with the 2s, 5s, and 9s (if you can just remember the trick). But 7s and 8s need so much work."

As my children began their math learning, they too seemed to struggle, and the 'tricks' did not seem to stick. Although I accepted the notion that some people are math people and some people are not, based on my own personal experience and undying faith in my children's capabilities, I was not at peace. Using pasta and small toys, I explored operations with my children and not only did math become enjoyable—it also became memorable.

I had been teaching for several years when in 2011, I had the privilege of assuming a role as a Student Work Study Teacher (SWST) with my school board. The Student Work Study was a joint venture between school boards in Ontario and the Literacy and Numeracy Secretariat of the Ministry of Education. As a SWST, I had the opportunity to visit classrooms within the board to listen to students talk about their interpretation of their learning. I documented in a variety of ways, all that students communicated to me. If teachers were willing, I would collaborate with classroom teachers to try different strategies, change environments, or

vary prompts to improve student achievement. My role was to research the literature to find what could be brought to the classroom to improve student outcomes. It was in this role, while listening to students from kindergarten to grade six, it became clear what students were saying about their learning in mathematics. Math is hard. Math is boring. Math is useless. Students were not having positive experiences in math and had developed bad relationships with mathematics.

For three years I collaborated with various teachers and students from kindergarten to grade 6. The third year I spent entirely dedicated to kindergarten classrooms as a SWST. This meant every day for an entire year I visited kindergarten. I have always loved the ‘early years’ but this year of listening to small children was inspiring. At this time, Ontario was in the process of phasing in full day kindergarten across the province. Certain schools had been selected to begin the program, and slowly but surely, over a 5 year period, all schools in Ontario would have a full day kindergarten program.

Over the years as a SWST, I built relationships to varying degrees with teachers. One of these teachers was Hope. Hope was a kindergarten teacher. Hope was just beginning her career and came with the enthusiasm and passion that often, young teachers do. Hope volunteered to have her classroom be part of the Student Work Study. This began a working relationship that truly pushed my learning and understanding of young children and mathematics education to a different level.

It was in Hope’s classroom that I really began to think about the potential of children in general, and mathematically. This was the beginning of my research into the town of Reggio Emilia, Italy. I began to research how they schooled their young children and the nuances of viewing children with an asset-based lens, as citizens of a community. I was affirmed in what I

believed since I was a young parent. All children are competent, curious, and capable of complex thinking.

Motivation for the study:

As a SWST, I had listened to so many students in despair when attempting to ‘do math’. I had watched countless teachers stand at the board, showing their favourite way to do calculations, expecting students to then copy and remember what they had just been shown. I was motivated to begin a graduate program in Mathematics Education, learning and unlearning so much about how mathematical experiences could be so different from the way I experienced math as a student, or so different from what I was seeing in the classrooms I was visiting. It was at this time that I really committed myself to changing my own practice and investigating mathematics education further.

I began to understand that mathematics was not for a select group. I began to understand and accept that everyone was capable of doing math; it just might be at different times and in different ways. I began to form a different relationship with math where I was actually doing math and surprising myself with my own capabilities.

A few years later, I had the fortune of working at York University in the Faculty of Education as a Seconded Faculty Member. I was hired to teach mathematics methods courses and mathematics electives to pre-service teachers. It was astonishing to see how the previous experiences of pre-service teachers (also referred to as teacher candidates) really framed their present relationship with mathematics. Many teacher candidates working to be certified in Primary/Junior divisions were terrified. Many were convinced they were ‘not math people’. They thought they could not do math; therefore, they could not teach math. This sentiment was not restricted to just a few students, nor a specific class. Over a 5-year period, I observed more than

60% of teacher candidates in every class I taught, with hesitant or negative relationships with math.

While at York, I had the opportunity to collaborate with a team of math educators who pushed my thinking and made me want to learn more. I loved being part of discussions about research and all things ‘math ed’. It was here that I decided to continue my graduate work. Once accepted into the PhD program, there were so many areas of mathematics education that were of interest for further research.

Hope and I remained friends throughout the years, and we had conducted a research project in her kindergarten classroom. I found myself thinking a great deal about mathematics education in the early years, especially from the lens of Reggio Emilia. I wanted to conduct research with Hope so that we could develop our understanding together of mathematics using pedagogical documentation with children to help build understanding. The goal was to see how young students and educators could use pedagogical documentation to experience mathematics together in meaningful ways.

As a result of the global pandemic and the challenges I faced in attempting to do in person research, I decided that another excellent way to investigate how pedagogical documentation was used or could be used in the early learning mathematics classroom, was to study the perspectives of the educators. Pedagogical documentation has been part of the kindergarten program since 2016, so I thought many teachers must be using it and learning wonderful things from it. I then shifted my focus to consider how teachers viewed, enacted, and thought about pedagogical documentation in their kindergarten math classrooms.

The Research Questions

Research questions were developed to examine the perceptions of teachers and their engagement with pedagogical documentation in the kindergarten mathematics classroom.

I investigated the following research questions:

1. How do teachers conceptualize pedagogical documentation in the early learning mathematics classroom?
    a) What are the qualitatively different ways kindergarten teachers think about or experience pedagogical documentation that portray mathematical learning experienced inside and beyond the classroom?

Chapter 2: Literature Review

Chapter 2 is an extensive exploration of existing literature, offering a nuanced understanding of the content that provides the foundation for this study. The primary objective of this chapter is to conduct a thorough review of relevant scholarly works, synthesizing a variety of perspectives, theories, and empirical findings. By delving into the existing body of knowledge, the literature review aims to identify gaps, contradictions, and emerging trends that will inform and guide this study. This literature review serves as the intellectual scaffolding upon which the subsequent chapters of this work are built, offering a contextual framework that grounds the study in broader literature. Throughout this chapter, I invite the reader on a journey through the intellectual landscape, whereby examining the existing literature we pave the way to explore new ideas that will expand and add to what already is present.

Overview of Conceptualizations of Education

Currently, two popular views of education are neoliberalism and democratic education (Davies & Bansel, 2007; Fielding & Moss, 2012; Vodopivec, 2012). Before examining the related literature specifically pertaining to mathematics, early childhood education, or pedagogical documentation, I would like to explore an overview of neoliberalism and democratic education with the intention that this conversation will help situate my study and the literature that grounds it. This study is rooted in an ethos of relationships that occur between people, their ideas and theories, their beliefs and values, and the environment. The work of this study is contingent upon viewing education and the relationships within, in a manner that resists neoliberalism and embraces a spirit of democratic education.

Neoliberalism

The discourses of neoliberalism have existed in school systems of capitalist societies since at least the 1980's (Davies & Bansel, 2007). It is thought that the influences of

neoliberalism in education are necessary for individuals to be competitive and productive in a global economy (De Lissovoy & Cedillo, 2017). Education, examined through neoliberalism, positions “the child as knowledge reproducer, the parent as consumer, the teacher as technician, and the school as business” (Fielding & Moss, 2012, p. 1) competing in the free market. The system of education does this by ensuring that students are ready to compete in this market by having students achieve predetermined and standardized outcomes (Hickey, 2016; Clarke, 2012). The goal of neoliberalist education then becomes the production of autonomous subjects ready to take their place in the marketplace (Fielding & Moss, 2012).

Roberts-Holmes and Moss (2021) explain that neoliberalism is grounded in three principles: competition, choice, and calculation. Competition manifests itself through limited public funding, grants, and resources, therefore forcing institutions to seek out ways to be more efficient and cost-effective over considering quality of care and education. Choice becomes apparent, not as collective choice and consideration of the goodness for all, but as individual choice. Individual choice in education appears as individuals (e.g., parents or guardians) choosing “between competing offers, to find the product (broadly defined) that best suits their needs, preferences and pockets” (Robert-Holmes & Moss, 2021, p. 6). Finally, calculation must occur, as in business transactions, the seller (the one who has a service or product to offer), and the buyer (the one who is looking for the service or product), “must be able to calculate as accurately as possible what terms are in their own best interest, what will maximize their benefits and minimize their costs. To do this they need information, preferably reduced to the form of numbers…” (Robert-Holmes & Moss, 2021, p. 6).

Neoliberalism has a distaste for publicly provided services, which according to neoliberalism, are naturally less efficient and less innovative than its private counterparts (Robert-Holmes & Moss, 2021). When public service does exist or is necessary, neoliberalism suggests those services be run as competing businesses where private business is permitted

into the fold, to enhance further competition. In a market economy favouring neoliberalism, the citizen becomes a consumer and a capable market trader. They are self-interested, self-reliant, independent, and competitive. They are able to assess and calculate what is in their best interest (Robert-Holmes & Moss, 2021). They take care of themselves without the expectation of help from others. They find solutions independently without any form of social support (Robert-Holmes & Moss, 2021).

As a result, some believe that society is best served through a free-market economy which in turn values an education system entrenched in competition and individual accountability (Fielding & Moss, 2012). By having students ready to be part of the global competitive market by demonstrating their **individual** achievement through world class standards, measured by standardized tests, competition and accountability are often demonstrated (Clarke, 2012; Hickey, 2016). Keddie (2016) describes students as “children of the market” (p. 115). As ‘children of the market’, students describe themselves and their value in the context of external benchmarks, often engaging in comparisons and assessments of their worth relative to one another. This results in “relentless performative pressure for academic excellence and improvement” (Robert-Holmes & Moss, 2021, p. 59) which comes with a cost that is dear. All too often these ‘children of the market’ suffer endless self-doubt, disappointment, and anxiety, as they struggle to achieve success and maintain it at the highest levels (Robert-Holmes & Moss, 2021).

Neoliberalism sees economic growth and prosperity not coming from the investment of governmental funds in education, but from transforming that education into a product that can be exchanged in the free market like any other product that can be bought or sold (Davies & Bansel, 2007). Clarke (2012) says that neoliberal education pervasively appears “in the form of marketization, privatization, standardization and accountability measures” (p. 298). Standardized testing suggests that all students and schools will adhere to the same outcome

driven policies and will be evaluated against the same criteria, expectations, and norms (Robert-Holmes & Moss, 2021). It would seem then, according to neoliberalist ideals, that training children to be ready for competition in the free market and ready to pursue **personal** gain and prosperity, is the goal of public education.

This neoliberal image of the child references a child as one who is lacking. In neoliberalist society there are those who are able to compete and choose for themselves, and there are those who cannot. Tronto (2017) reminds us that "… any 'free market' excludes some people, those who are deemed incapable of 'choosing' and are reduced to the status of 'learning'. Children are among those who are, historically, not ready to choose" (p. 34). In effect, this leaves the child "incapable of functioning as homo economicus and lacking that essential quality of human capital, so not ready yet to join the fray of life as a market actor and as an entrepreneur of the self" (Robert-Holmes & Moss, 2021, p. 97). This is the view of the child as an empty vessel, incapable of contributing, ready to be filled with another's infinite wisdom. This child must set out on a journey ready to be filled with knowledge, attitudes, and skills that will help them travel through this stage of 'becoming'. Robert-Holmes and Moss (2021) explain, in this view, childhood is in fact a stage that must be experienced in order to reach the desired destination of adulthood where the adult will now have the required skills and knowledge to be an independent, responsible, skilled and compliant consumer-citizen, ready and capable of positively impacting the growth of a nation's wealth.

However, a neoliberalist approach in education only helps to erode the idea that education is an issue of public interest and the responsibility of all citizens (Fielding & Moss, 2012). If education is approached through the lens of neoliberalism, education becomes a mechanical exercise, for which decisions are made solely by those who have been tasked to "define, assess and improve standards of performance" (Fielding & Moss, 2012, p. 2). This leaves little room for educators to capitalize on opportunities to work with important and original

ideas that children might have (Fielding & Moss, 2012). According to these authors, through this lens the diverse, varied, and complex ways of thinking that students may have, cannot come to fruition. Students are expected to repeat and regurgitate knowledge rather than create it, because of the predetermined standards and expected outcomes (Keddie, 2016). Cristina Vintimilla (2014) calls us to consider neoliberalism "not only as a state's economic orientation, but as particular modes of subjectivity, and how one becomes (or may resist becoming) a neoliberal subject, especially through the project of education" (p. 80). The impacts of neoliberal approaches on education become like ripples in water, which increasingly have influence on the way in which a society views social welfare and a commitment to community values (Fielding & Moss, 2012; Keddie, 2016).

Democratic Education

In my work, I would like to consider "democracy as a way of thinking, being and acting, of relating and living together, as a quality of personal life and relationships" (Fielding & Moss, 2012, p.2). I frame these considerations particularly focusing on ideas documentation that are articulated by scholars in early childhood education research. It is important to consider the themes of democratic education to understand the philosophy behind this work, which will be investigated and discussed in this study.

Loris Malaguzzi, founder of Reggio Emilia schools in Reggio Emilia, Italy, based his beliefs and pedagogy on democratic education (Cagliari et al., 2016; Edwards et al., 2012; Gandini, 2012). The schools of Reggio Emilia emerged after the second World War as the community of Reggio Emilia was feeling the effects of a fascist regime in their public schools (Moss, 2016). Parents of Reggio Emilia looked to Loris Malaguzzi, an educator and philosopher of the area, to lead them to an education that would offer their children meaningful learning opportunities in an environment that was grounded in community and not politically driven. Malaguzzi's philosophy is grounded in the way in which the child is viewed (Malaguzzi, 1993). In

a democratic education, the child is seen as rich in resources, powerful, competent, and capable of finding out about their world through interactions with others (Edwards, 1995; Malaguzzi, 1993). This child is not one who needs to be filled with knowledge but is one who is capable of having thoughts and ideas of their own. This child has desire, confidence and takes immense pleasure in being the authors of their learning (Malaguzzi, 1993). The image of this child is one who is beautiful, strong, intelligent, and has “very ambitious desires and requests” (Malaguzzi, 1993, p. 56). Meaning, an adult sharing learning with this child is not the transmitter or manager of knowledge. This adult is the cultivator of relationships between people, things, thoughts, and environments (Malaguzzi, 1993). This adult is a researcher, a learner, a prompter and a listener. This adult shares the space, time, and learning WITH the child.

Themes of democratic education are evident in the literature of mathematics education as well. Ball, Goffney and Bass (2005) offer that mathematics education is a key resource for building democratic thought, hinging largely on the way in which it is taught. Mathematics is grounded in problem solving and deliberation, where students share, compare, and analyze one another’s mathematical ideas. When approached as such, it can offer alternative ways to interpret or represent, or even a different path to a solution (Ball, Goffney & Bass, 2005; Stemhagen, 2011). Allen (2011) supports this notion stating that the mathematics classroom is a rich opportunity for students to engage in “carefully reasoned, thoughtfully articulated, fact-based classroom discourse” (p. 3). Allen (2011) continues by suggesting that when the community of learners include the teacher in the role of learning, this disrupts traditional roles of power and authority. As a result, the community engages and fosters democratic practices. Students learn to listen carefully to others, to consider the viewpoints of others, to value the ideas of others, and to respectfully dialogue to resolve misunderstandings and differing opinions.

Views on Teaching and Learning

The concepts of teaching and learning are explored throughout this dissertation. Therefore, I believe it is important to begin with an examination of these ideas and terms, in relation to the literature in both mathematics education and early childhood education. In doing so, I hope to provide a context through which this work can contextualize the terms 'teaching and learning'. These are the ways that I frame my conceptualizations of teaching and learning.

Brent Davis (1997) has noted that school math has traditionally upheld a neoliberalist notion as it has "been cast in the same terms as material commodities, and we have tended to be pre-occupied with such matters as the efficiency of production (i.e., acquisition of knowledge)" (Davis, 1997, p. 3). However, that is not the perspective upheld in this dissertation. The view of mathematics in this dissertation draws attention to ideas of learning being a complex process (Davis, 1997). Learning is not a simple transmission of knowledge from teacher to student, but a dynamic process that involves interactions of multiple factors, including learners, the social context, the materials used in the learning process, and the teacher. Also, Davis (1997) describes this learning as an activity that involves the whole person. Knowledge arises through the interactions between people and their environment, making it a deeply personal and contextual process. Additionally in this dissertation, I embrace the idea that mathematics is a communal activity facilitated by conversation and interaction. Therefore, it is important that we share ideas, challenges, and insights between educators and teachers. Davis and Sumara (2007) also acknowledge the ethical dimensions of teaching and learning mathematics. They suggest that educators must consider the implications of their teaching practices and actions on students' development as ethical beings. Therefore, students and teachers must experience math through curiosity, its beauty, and its intrinsic value. Additionally, knowledge is emergent; something that arises through the interactions within the learning

environment, including entities that are human and non-human, rather than being a static set of facts to be transferred (Davis, 1997).

Another view I would like to consider regarding teaching and learning comes from Gert Biesta. Biesta (2005) builds upon the conversation of neoliberalism infiltrating education suggesting that the most recent language of learning helps promote education as an economic transaction. Biesta also identifies the learner as the consumer, the teacher or institution as the provider. The learner, teacher, or institution acquire the commodity of the education itself (Biesta, 2005). Biesta (2014) proposes that when the language shifts from 'teaching' to 'learning', this radically changes the notion of what teaching, and the teacher are. Biesta has labelled this shift, the 'learnification' of education (Biesta, 2014). Biesta (2014) suggests that when education uses the language to 'learn from', this suggests using the teacher as a resource, much like a book or the internet. Biesta (2014) states the teacher is someone, in the very general sense, who brings something new to the educational encounter. This means the teacher presents "students something that is neither derivable from nor validated by what [the students] already know but that truly transcends what they already know" (Biesta, 2014, p. 50). Therefore, in this way we can understand teaching as something that comes from the outside and brings something radically new to the educational encounter.

The critique that Biesta (2009) brings to the discussion of learning is that what is valued in education comes from an implicit reliance on the beliefs of the goals of education. Typically, these goals favour some groups over others. The mainstream notion regarding what is valued in education lies in the idea that what matters most is academic achievement in a small number of curricular areas like language, science and mathematics (Biesta, 2009). Biesta explains that "whether academic knowledge is indeed of more value than, for example, vocational skills, all depends on the access such knowledge gives to particular positions in society" (p. 37), which is precisely how education continues to reproduce social inequality. Naturally then, it is in the

interest of those who benefit from such goals to keep the status quo and not open a discussion about what education might or could be (Biesta, 2009). Herein, the value of educational content reduces education to mere skills acquisition. Biesta (2009) believes by prioritizing measurable learning over critical thinking and collective well-being, the broader purposes of education, like democracy and social responsibility, are undermined. Therefore Biesta (2014) indicates that it is important to consider education processes and practices including aspects of content, purpose, and relationship. Biesta (2014) references the work of Freire saying that education involves learning that is not reproductive, but constructive and generative. Teachers have a vital role in teaching, as learning is not a natural process that simply 'occurs' (Biesta, 2014).

It is also important to look at learning in terms of how the Reggio Emilia approach to education references it. Lenz Taguchi (2010) describes this as "a pedagogy that works with and makes use of—rather than working against—differences, diversity and increased complexities of learning and knowing" (p. 9). Lenz Taguchi (2010) identifies this pedagogy as an intra-active pedagogy. This involves educators engaging in "collaborative knowledge-production" with children beyond what they or the children believe is possible. This intra-active pedagogy draws specific attention to "all living organisms and the material environment" (Lenz Taguchi, 2010, p. 10). This pedagogy resists normative views of knowledge production that suggest a single, most important theory of learning and knowing. Where educators then apply this theory to evaluate specific learning outcomes against a standardized framework.

Additionally, Malaguzzi explains in a conversation with Lella Gandini (2012), that it is necessary to discuss the role "children assume in the construction of self and knowledge, and the help they get in these matters from adults" (Gandini, 2012, p. 57). Malaguzzi suggests that it is important for adults to stand aside for a time, listening very carefully to the children. Then, he says, if the educator has truly understood, the teaching they offer may very well be different than anything they have offered before (Gandini, 2012). Malaguzzi also recognizes that the

goals and conditions of the teacher are different from the learner. Malaguzzi explains that teaching and learning “should not stand on opposite banks and just watch the river flow by; instead, they should embark together on a journey down the water” (Gandini, 2012, p. 58). It is through meaningful, reciprocal encounters that teaching can support learning and how to learn (Gandini, 2012).

In my conceptual framework drawing upon the work of Clark and Peterson (1984), I propose that we should be researching teacher conceptions and perceptions in order to understand the human enterprise of teaching (Clark & Peterson, 1984). Clark and Peterson (1984) suggest that in order to understand teaching, researchers should endeavour to understand teacher thinking by listening to teachers. Teachers are required to assimilate a great deal of information (i.e., research literature, board directives, curriculum, etc.) with their own expectations, attitudes, beliefs, and goals, to make judgements and choices on a daily basis (Clark & Peterson, 1984). This means that teachers are steadily bombarded with information that they must reconcile with the thoughts and beliefs they already hold. In turn, these thoughts and beliefs are brought together to create considerations which guide behaviour in ‘teaching’ (Sigel, 1985). Therefore, the concept of ‘teaching’ arises from the actions and choices that teachers make in their classrooms on a routine basis.

I will discuss in more detail throughout the dissertation how teacher thoughts and beliefs impact their actions. However, at this moment, I would like to draw attention to the work of John Mason and Brent Davis (2013). Mason and Davis (2013) have pointed out that when working with teachers in mathematics education the challenges the teachers faced in teaching were not with their content knowledge of mathematics, nor with their understanding of pedagogy. Mason and Davis (2013) suggest that the teachers “seemed to lack the vital connective tissue between mathematical awareness and in-the-moment pedagogy” (p. 183). The authors note that it is one

thing for a teacher to notice something that a student lacks or is missing, but it is altogether a different thing to be able to think of an appropriate pedagogical action in those moments.

Leatham et al. (2015) are in line with Mason and Davis (2013) as they suggest that the reason that it is difficult for teachers to use student mathematical conceptions, is that teachers find recognizing and interpreting student ideas complex. Leatham et al. (2015) explain that much research on student thinking has focused on how teachers respond to that thinking, rather than "identifying attributes of student thinking that teachers need to understand to make effective decision about which thinking is most productive to pursue" (p. 89). Hence, the connective tissue Mason and Davis (2013) speak of.

Mason and Davis (2013) are in line with Biesta's ideas around the 'learnification' of education. Mason and Davis (2013) state that teachers are not simply distributors of knowledge, nor are they the 'guide on the side'. Mason and Davis (2013) suggest that what teachers do in teaching must be based on intentional and deliberate action. Mason and Davis (2013) state that teachers must not simply react. Internalized reactions are automatic and habitual. The authors suggest that teachers must respond. Responses are considered. These are the in-the-moment choices that teachers make as they attend to both the choice itself and the conditions around the choice (Mason & Davis, 2013). The authors further develop this idea by adding that teachers must develop this kind of knowing by way of "being mathematical with and in front of their students" (Mason & Davis, 2013, p. 188). Teaching is then a way of being where 'knowing the answer' can be used to inform listening to what students say and to the ways in which students display their ideas, rather than simply listening for a desired or correct answer. Teachers then support students to use their own mathematical powers to construct understanding in ways that are meaningful for the student.

Mason and Davis (2013) do not suggest that teachers just need to ‘know more’. In fact, what they suggest is important in teaching, is ‘noticing more’. The authors explain that in ‘noticing more’ teachers will know “more deeply and richly in the sense of having possible actions” (Mason & Davis, 2013, p. 192) which come to mind when they are needed. Therefore, what teachers know about teaching is reflected in,

> a ‘way of being’ *WITH* mathematics knowledge that enables a teacher to structure learning situations, interpret students actions mindfully, and respond flexibly, in way that enables learning to extend understanding and expand the range of their interpretive possibilities through access to powerful connections and appropriate practice.
>
> (Mason & Davis, 2013, p. 194)

Mason and Davis (2013) conclude in saying that teachers can teach by responding to students through awareness, noticing, and sensitivity rather than depending on their habitual actions. Yet, this requires being present ‘in the moment’. The authors suggest that teachers must work to expand and enrich what they notice, meaning paying attention to what they are noticing and in what manner they notice. The authors point out that this enhances teachers in developing their way of being mathematically.

In summary, throughout the study I will challenge neoliberalist views of education (teaching and learning) in efforts to explore practices that resist neoliberalism and embrace democratic ways to engage with education. The literature examined will support and echo this goal. The following literature review will include examining pedagogical documentation, kindergarten, and math reform.

Pedagogical documentation

Pedagogical documentation sometimes is thought of simply as a tool (Turner & Wilson, 2009), however that would be an oversimplification of pedagogical documentation and a significant missed opportunity. A large body of literature looks at pedagogical documentation as part of a way of being with children (Dahlberg, 2012; Forman & Fyfe, 2012; Rinaldi, 2012). Some important ideas that emerge from this literature are the way in which the child is viewed, the act of teacher listening, the notion of children being caring, competent and capable beings, and the importance of relationships in learning. Dahlberg (2012) states "pedagogical documentation …is a specific attitude about life" (p. 226). These ideas come together to position pedagogical documentation as part of a specific approach to education. This approach includes viewing school as a place of "democratic political practice" (Dahlberg, 2012, p. 226). This approach includes active listening (I will discuss listening in more detail below) and construction of an ethical relationship, also known as an ethics of encounter (Dahlberg & Moss, 2005). This approach to education prioritizes the child as a capable and competent protagonist and as someone that is not predefined by developmentally appropriate practices. It values collaborative learning (including learning with the educators), emergent curriculum (rather than predefined) and an appreciation for aesthetics.

The term 'pedagogical documentation' refers a discursive-material praxis and points out to both a process (always discursively embedded) as well as the important materials (photos, transcripts of student conversations, student work, and videos) that create and result from that process (Dahlberg et al., 1999). The material of pedagogical documentation could be hand-written notes of what children say and do, audio recordings, video recordings, still photos, and children's work that are collected during the learning event. These materials or artefacts are recordings of what the children say and do and how the educator interprets and relates to the children and their work (Dahlberg et al., 1999). The materials are the interpretation of the

documenter (the educators) and become the materials that make the thinking visible (Rinaldi, 2021). These materials become the ‘memory’ of the ideas of the children and educators, which seem to be objective (Rinaldi, 2021). However, the person documenting infuses each piece of documentation with their own established discourses and practices. Educators then offer the documentation up to others knowing their interpretation will also be subjective (Rinaldi, 2021). Yet that is part of the process. The documentation and the revisiting of the documentation by people involved in each specific project, becomes part of a process of creating and recreating, knowing and revisiting; and figuring out what might be pedagogically significant. Essentially the materials become part of a “collective knowledge building event” (Rinaldi, 2021, p. 42).

The process of pedagogical documentation involves the creation and assembly of those materials as a means to reflect on children’s ideas not after the fact, but as they are happening. In defining what pedagogical documentation is, it is imperative to discuss what it is not. Pedagogical documentation is not child observation (Dahlberg et al., 1999). Child observation is assessing children according to predetermined categories of development or academic outcomes, which define what a ‘normal’ child should be doing at a particular age or stage. The focus of this type of observation is mainly to determine if the child is conforming and is not necessarily interested in the child’s learning processes (Dahlberg et al., 1999). The process of pedagogical documentation and the content, which is referred to as ‘pedagogical documentation’ or ‘documentation’, is not the aforementioned type of observation. Pedagogical documentation has a central role in meaning making that is not measured against a predetermined measure of quality (Rinaldi, 2001). Pedagogical documentation plays a part in making children’s ideas and theories visible and emerges from the context of the individuals participating in the meaning making at that moment in time.

Lenz Taguchi (2010) calls pedagogical documentation “an active agent in generating discursive knowledge” (p. 63). The author explains pedagogical documentation as part of a

process that generates materials in helping to construct meaning of children's ideas. Lenz Taguchi (2010) notes that "it [the material] is not a fixed matter with a fixed essence but a substance in a process of intra-active performances and becoming" (p. 63). Pedagogical documentation is not about documenting the learning event, but documenting a representation of the learning event as it was when it happened (Lenz Taguchi, 2010).

Considering documentation, Lenz Taguchi (2010) elaborates on the concept of an intra-active pedagogy. In this way, it is important to consider the "agency of the material in the production of knowledge" (Lenz Taguchi, 2010, p. 65). An intra-active pedagogy draws attention to the phenomena that occur "in the inter-relations, inter-connections, interferences, and waves of diffractions that emerge *in-between* the material, the discursive and the human beings" (Lenz Taguchi, 2010, p. 65). This means that it is not simply the interactions between the humans that are considered as what is said and done, but the interactions with materials as interactive agents, is also considered. Lenz Taguchi (2010) states that "things, matter, artefacts, materials, furnished environments, and architecture" (p. 65) are as important to the encounter, as the relations between the human beings. Materials, objects, and artifacts take on an active, almost lifelike role. However, this perspective emerges primarily through the discussions and analyses conducted by the children, themselves, as revealed through detailed examination of the documentation (Lenz Taguchi, 2010). This further underscores the importance of relations and relationships when engaging with pedagogical documentation.

Giamminuti et al. (2022) present pedagogical documentation as an opportunity to examine early childhood dogmas. Giamminuti et al. (2022) draw upon the work of Derrida and take his notion of 'dogmatic slumber' and apply it to the idea of pedagogical documentation as practiced in the educational project of Reggio Emilia. They propose that documentation risks to be taken up as a normative tool of data collection. In these ways, pedagogical documentation exists as a vehicle to perpetuate dogmas of "compliance, surveillance, and standardization of

knowledge" (Giamminuti et al., 2022, p.213) often practiced in educational systems affected by neoliberalism.

These authors describe some 'dogmatic' practices of pedagogical documentation that are taken up in contexts outside of Reggio Emilia, continuing to uphold neoliberal principles. The authors situate these ideas in the dogma of children's interests, the dogma of implementation, and the dogma of method. A further examination of these dogmas will allow an in depth understanding of how pedagogical documentation can be used as a practice to challenge dogmas rooted in neoliberalism and open up spaces to uncertainty and endless possibility.

For instance, the dogma of children's interest lies in the fallacy that being 'child-centred' is of utmost importance and that everything revolves around those interests (Giamminuti et al., 2022). Focusing solely on the child's interests overlooks the complex web that makes up those interests in the first place (this echoes Biesta's ideas previously elaborated). Children's interests are not isolated but emerge from an interconnectedness of elements, both human and non-human. Giamminuti et al. (2022) explain that pedagogical documentation is not a tool for tracking student interest but should be used to challenge comfort and serenity often sought after in classrooms, pursuing spaces where discomfort and questioning flourish. Giamminuti et al. (2022) suggest that a traditional child-centred approach is oversimplistic and exclusive of the complex relationships (including non-human entities) that influence children's interests. Pedagogical documentation can act to investigate a more nuanced, relational understanding of education, challenging status quo and conventional approaches.

The dogma of implementation takes up pedagogical documentation as a vehicle to uphold "the power and unquestionability of government-mandated, and often content-driven, curriculum policy" (Giamminuti et al., 2022, p. 217). This dogma espouses neoliberalist ideals that enforce the compliance of teachers, the resistance of critique or reform, and the negation of

any thoughtful experimentation (Giamminuti et al., 2022). When pedagogical documentation is used within this dogma it acts as a tool that reduces teachers to technicians whose sole objective is to deliver pre-determined curricular programs. Instead, if the dogma of implementation is challenged, teachers should be considered as intellectual professionals who think and respond to the daily encounters in their classrooms.

The dogma of method reduces pedagogical decision-making to regulated step-by-step approaches, which suggests that teaching is simply administered by technique (Giamminuti et al., 2022). This dogma, as that of implementation, offers a deficit model of the teacher and suggests that teaching is merely a series of methods, within which pedagogical documentation exists. Pedagogical documentation is then restricted to a prescription of method, templates, and compliant practice and uniformity. When taken up in this way there is no space for pedagogical documentation to act as a vehicle for independent thought or resistance (Giamminuti et al., 2022).

When considered outside of these dogmas, educators see children as capable individuals by engaging with pedagogical documentation (Dahlberg, 2012; Forman & Fyfe, 2012) and then create curriculum that is responsive, situated, and meaningful. Educators may then take the learning of the children as the base line for that individual child, rather than comparing the child's competence against some external measure. The creation of a specific project and the situated learning of the children involved is the priority, not some other standardized measurement (Macdonald, 2007).

Pedagogical documentation has the potential to disrupt the status quo and generate "thinking that is situated, affective, and relational" (Giamminuti et al., 2022, p. 214). Giamminuti et al. (2022) endorse that "pedagogical documentation opens possibilities for engaging with a relational-responsive approach that is driven by affect" (p. 222). Using pedagogical

documentation in this way illuminates the possibilities of “the relational entanglements” (Giamminuti et al., 2022, p. 222), and becomes a practice of listening with others and resisting the ‘dogmatic slumber’ of neoliberalism. In this way educators can engage with “cultures of thinking and doing, … and imagining the not yet possible” (Giamminuti et al., 2022, p. 222).

Pedagogical documentation is not a plan. Educators and students engage with the documentation of learning events to make meaning. Pedagogical documentation is emergent. Pedagogical documentation enables educators and students to take responsibility for making meanings and coming to decisions about what they see is transpiring. Pedagogical documentation produces documents that make visible the ideas and learning of the individuals and the group so that the individuals and the group can return to see themselves from an external point of view, both during and after the learning process (Rinaldi, 2001). What results from this is beautiful knowledge that is created by the contributions of many people. In these captured moments there is the past, in terms of what has already occurred, but there is also opportunity for the future by asking ‘What else could happen if…’ (Rinaldi, 2021).

Pedagogical documentation engages and brings life to curricular work, in ways that may help alter the institutionalized constructions in more democratic ways (Giamminuti et al., 2022). Pedagogical documentation can trouble the dogmatic slumber of education, in that it can continuously work to change boundaries of practice in education (Giamminuti et al., 2022). In doing so, pedagogical documentation “may be conceptualized and enacted as ‘the new’” (Giamminuti et al., 2022, p. 224). By virtue of its distinctiveness and the promotion of questioning and wondering, pedagogical documentation fundamentally challenges the very essence of neoliberal education, where rigid curricula, frequent testing, and performance metrics that give precedence to uniformity and quantifiable success, are prioritized.

Pedagogical documentation as part of a specific approach to education

I have chosen to adopt the perspective of pedagogical documentation from the literature which views pedagogical documentation as part of a specific approach to education and curricular processes (e.g. Dalhberg, 2012; Rinaldi, 2012; Wien et al., 2011). Pedagogical documentation invites the educator to remain open to that which is not known but is emerging (Giamminuti et al., 2022). Pedagogical documentation invites educators to resist the temptation to fall into dogmatic slumber (Giamminuti et al., 2022). Malaguzzi and his colleagues used pedagogical documentation in the preschools of Reggio Emilia to give voice to the child (Lenz Taguchi, 2010). Rather than using pedagogical documentation to observe the normative outcomes of the child, they used it to reveal "the multiplicity of differences of children's strategies and conceptualizations, and without any desire to categorize what it was they heard or seen" (Lenz Taguchi, 2010, p. 72).

This approach to pedagogical documentation encompasses the view of the child as a caring, competent, and capable individual in their own learning. Children have agency and are able to contribute to their own learning as much as the educator might (Dalhberg, 2012; Malaguzzi, 1993; Rinaldi, 2012). This approach involves an entire way of being. Pedagogical documentation allows the educator to be with children in ways that may not seem to be in line with traditional pedagogy.

> It's necessary that we believe that the child is very intelligent, that the child is strong and beautiful and has very ambitious desires and requests. This is the image of the child that we need to hold.
>
> (Malaguzzi, 1993, p. 56)

The view of the child as described above is a fundamental tenet of Reggio Emilia schools (Malaguzzi, 1993). Children ask questions, wonder, collect information, make

hypotheses, try out their hypotheses, develop and revise their theories (Cagliari et al., 2016). Malaguzzi suggests that when he refers to the competent child, he is not suggesting a self-sufficient child. He refers to this child as an “agent-child” (Cagliari et al., 2016, p. 369). Meaning that this child interacts in a context of learning in which they are co-authors. This child is ready to situate themselves in a context of communication and discussion. Malaguzzi (1993) promotes that children are rich with ideas, with a natural inclination to interact and construct the world around them. Educators should offer children the opportunity to create their own ideas, to feel pleasure in discovery, and become the authors of their own learning (Malaguzzi, 1993). Herein lies the potential of pedagogical documentation.

Pedagogical documentation acts as a living record of student ideas and offers a way of revisiting and reviewing students’ theories and learning events. The act of documenting preserves the nuances of student ideas, allowing for deeper analysis and reflection in the future, whether educators capture the documentation through photos, videos, or transcriptions of conversations. Pedagogical documentation facilitates interactive reflection between students, educators, and others. When students and educators review documentation together, they discuss different interpretations and collectively construct new understandings. The roles of the educators and the students shift from traditionally accepted roles, when using pedagogical documentation. As suggested previously, pedagogical documentation challenges traditional pedagogy because educators adopt a more inquisitive and open stance, positioning themselves alongside their students. Educators and students, along with the environment and other materials, shape the learning journey together. Using pedagogical documentation this way in mathematics allows educators to “capitalize on the diversity of student ideas, strategically selecting, sequencing, and connecting solution strategies in ways that require students to attend to approaches that are different from their own” (Singer-gabella & Shahan, 2016, p. 415).

Pedagogy of Listening

Listening is a critical part of teaching and learning. However, the pedagogy of listening within the context of the Reggio Emilia approach to education, is a complex and multi-dimensional concept (Dahlberg & Moss, 2005). The pedagogy of listening involves a variety of forms of communication, relational, dialogic, and subject to interpretation. This type of listening is not simple. “It requires a deep awareness, and at the same time a suspension of our judgements, and above all our prejudices; it requires openness to change” (Dahlberg & Moss, 2005, p. 100). It commands that we are willing to embrace the unknown and be comfortable in the uncomfortable.

To listen in this way means to be open to the Other. It means considering the Other from their own position and experience and not corralling the Other into sameness. When working with children the pedagogy of listening recognizes the dialogue is rooted in relationship (Dahlberg & Moss, 2005). This means that educators and students discuss and negotiate understanding. Here the educator listens openly to the ideas, questions, and answers of the children, with whom they share time and space. The pedagogy of listening “emphasizes relationships, ‘social bonds’, and the importance of being in a community for creating and re-creating theories (many, diverse and provisional) as part of a continuous process of learning that involves theorizing, dialogue, reflection and negotiation” (Dahlberg & Moss, 2005, p. 101).

When engaging with the pedagogy of listening, the educator must refrain from giving predetermined meanings to ideas children share. A pedagogy of listening challenges ideas of developmental psychology and child development (developmentally appropriate practices) as these classifications can oversimplify, generalize, dismiss, or trivialize children’s experiences and development (Dahlberg & Moss, 2005). A pedagogy of listening invites the educator to engage with children with a more open, attentive, and receptive approach; one that acknowledges and values children’s ideas as significant and insightful. For instance, educators

must pause existing frameworks of expectations or norms to hear and see what the student is sharing in their learning, with the meaning that the student intends (Wien et al., 2011). The pedagogy of listening is grounded in “radical dialogue” (Dahlberg & Moss, 2005, p. 101) where the educator participates with the child, sharing a space where both people are sharing and listening and trying to make meaning together.

Listening requires the educator to put themselves in a seemingly vulnerable position; they must learn to “open themselves to others… competent listening creates a deep opening and predisposition toward change” (Rinaldi, 2006, p. 114). When educators engage with this type of listening, they focus on the ideas and words of the individual student; educators should not be constrained by that which is already known (Merewether, 2018). This listening requires educators to see uncertainty as an opportunity for possibilities, not as a danger or a threat. The educator must be open to surprises and curiosity, while accepting doubts and mistakes, which are all part of the process of deep knowledge and creation (Dahlberg & Moss, 2005, p. 105). The pedagogy of listening is about being open to students’ ideas, as comprehension manifests itself in the moment.

The pedagogy of listening is different from what has historically been the paradigm of teaching in North America. Traditionally our North American education system has largely prescribed to a pedagogy of transmission, where a hierarchy of control existed, and perhaps continues to exist (Freire, 2002). Authors like Cannella and Viruru (2004) have noted how children have “been categorized in opposition to those who are older, as children versus adults” (p. 64). The authors explain that even the children have accepted the limits bestowed upon them through the distinction of their difference. Labels abound that classify the children further, like slow, gifted, hyperactive, innocent, and incompetent (just to name a few); some of the institutionalized manifestations that mentally and physically maintain control (Cannella & Viruru, 2004). This dichotomy creates a position of privilege from which adults operate. With this

privilege comes power. Cannella and Viruru (2004) explain that society views the innocent child as ignorant compared to the knowledgeable adult. Therefore 'the child' becomes a group "for which surveillance, limitation, and regulation is necessary for their own good" (Cannella & Viruru, 2004, p. 67).

In education, teachers were (and perhaps are) often thought of as holding the knowledge. Teachers taught the students a concept and expected the students to repeat that which the teacher shared, in the way that the teacher explained it (Freire, 2002). Then students' work would be examined after the student 'demonstrated their learning'. The feedback would then come from the teacher only, and often not be such that would move learning forward (Wien et al., 2011). With traditional pedagogy, it is thought that once something has been taught, learning has occurred and is ready to be tested (Wien et al., 2011). Yet simply because the educator has stated something doesn't guarantee it has been learned by the student (Wien et al., 2011). In contrast, teachers aligned with Rinaldi (2012) are open to students' ideas and seize the opportunity to build knowledge in the moment with the children, when engaged with a pedagogy of listening. Not surprisingly, mathematics reform implicates educators to engage in the same type of listening.

In mathematics education literature, Davis (1997) identifies the importance of listening. Davis discusses three types of listening that may occur in a classroom: evaluative, interpretive and hermeneutic listening. Often educators engage in evaluative or interpretive listening. For example, educators who engage in evaluative listening often already have a predetermined 'answer' or statement in mind and are listening for those responses in the statements made by students, as correct or incorrect. This educator is "listening but only at a trivial level" (Davis, 1997, p. 360). This means that ideas that do not fit the predetermined 'answer' or response are generally dismissed or ignored. Interpretive listening occurs when the educator is listening for the purpose of making sense of what the student is saying. This type of listening means that the

listener is deliberately attending to what the speaker is sharing and is “aware of the fallibility of his or her sense making” (Davis, 1997, p. 364). Although the educator does not assume authority for the correctness or incorrectness for what the student says, the listener does direct the conversation by means of the questions and prompts they ask (Aizikovitsh-Udi & Star, 2011). However, hermeneutic listening is a different type of listening altogether.

We can align the pedagogy of listening with hermeneutic listening. Davis (1997) describes hermeneutic listening in mathematics classrooms as listening without evaluation or judgement. This type of listening attempts to understand in far more nuanced ways than simply what the educators hears. Hermeneutic listening involves listening to the actions and words that are occurring in the moment, but also listening to all that has made that moment what it is (Merleau-Ponty,1962). As a listener engages in hermeneutic listening, the listener is drawn into the very being of the other. Merleau-Ponty (1962) describes hermeneutic listening,

> …there is constituted between the other person and myself a common ground; my thought and his are inter-woven into a single fabric, my words and those of my interlocutor are called forth by the state of the discussion, and they are inserted into a shared operation of which neither of us is the creator. (p. 413)

Hermeneutic listening is not merely an act. In line with pedagogical documentation, Davis (1994) describes listening as an orientation, a way to be in the world (para.35). Davis (1994) explains that listening is not a procedure or a series of guidelines a person follows. Davis (1994) describes the orientation of listening as a state into which people enter. A person listens with their entire body, not just their ears and minds. People listen by feeling and touching, smelling, and observing. A person is then in a state of listening. Therefore, listening is not something people do, but a space people occupy. The author explains hermeneutic listening in saying,

> … so listening is not a technique that can be reduced to a set of prescriptions or guidelines. It is something that we enter into, something that we are, emerging from our occupation with others and with their meanings. In this way, listening has much in common with ‘attention.’ (para. 37)

This type of listening brings “the collected weight of our experience” (Davis, para.37) to affect and influence emerging ideas, thoughts, and new understanding.

Bronwyn Davies (2014a) corroborates Brent Davis’s (1994) view on listening. Davies (2014a) calls this type of listening ‘emergent listening’. Emergent listening opens up the possibilities to new ways of being and knowing, for both the educators and the children. The author states that emergent listening “opens up the possibility of valuing difference, not as categorical difference, but as an emergent, differentiating or becoming” (p. 42). Davies (2014a) also refers to this type of listening as “a focused attention” (p. 42). The author explains that this attention is focused on the minute details; it is more than just decoding sound for meaning. This attention turns its entire body’s orientation to the Other. This attention allows the individual to “come to know the other through an intimate, social synaesthesia, where the words, the sonority, the affect of one are heard in the ears of the other, but also in their mouths, their eyes, their hearts, their gut” (Davies, 2014a, p. 42).

Pedagogical Documentation and the Role of the Educator

The role of the teacher becomes an important theme when considering pedagogical documentation as part of a different way of being with children. Here the educator shifts from the holder of the knowledge to a learner and researcher, interacting with the child. The educator assumes a critical role by being attentive and adopting a reflective stance to the needs and curiosities of the children.

As Carla Rinaldi (2021) describes, “The task of the teacher is to create a context in which children’s curiosity, theories, and research are legitimated and listened to…” (p. 91). In this context the teacher works to ensure children feel safe and at ease, while at the same time widening and extending “children’s horizons by creating complexity in the child’s environment by introducing new theories, concepts, languages, and materials, as tools for children’s theorizing and meaning making” (Dahlberg & Moss, 2005, p. 103).

Biesta (2014) highlights that the presence of the teacher is essential in education. The teacher is not just “a fellow learner or a facilitator of learning, but as someone who, in the most general terms, has to bring something to the educational situation that was not there already” (Biesta, 2014, p. 6). As was previously mentioned, Biesta (2014) draws attention to the idea of the disappearance of teaching and the teacher. Biesta (2014) argues that teaching must be more than just the facilitation of learning. According to Biesta (2014), the idea of teaching must carry with it the notion of transcendence. This could be taken in two meanings of the word transcendence: a) to rise or go beyond the limits of, and b) to outstrip or outdo some attribute, authority or power (Merriam-Webster, 2024). Therefore, the role of the teacher is not of a disposable or dispensable resource, but an individual that has something to give. This teacher does not “shy away from difficult questions and inconvenient truths, … they work actively and consistently on the distinction between what is desired and what is desirable, so as to explore what it is that should have authority in our lives” (Biesta, 2014, p. 57).

The teacher’s role changes from a person who provides guidance to a person who supports the competent child (Edwards, 2012). The teacher is not an observer, but a person who documents and researches. This teacher is a listener and a negotiator of meaning. This teacher looks for occasions to provoke “discovery through a kind of alert, inspired listening and stimulation of children’s dialogue, co-action, and co-construction of knowledge” (Edwards, 2012, p. 152). Specifically, the teacher may provoke the learning of a group of children by using the

ideas of one child to frame the action of the group. This may be a whole group meeting where the teacher writes down what children say, reads their comments back to them, and searches for the ‘spark’ that will motivate more questions and more group action (Edwards, 2012). As the teacher documents, they act as the ‘memory’ of group or individual action. They share the documentation with the children, which allows children to revisit their own ideas and actions and reinterpret them in deeper ways (Edwards, 2012).

Lenz Taguchi (2010) explains that teachers are collaborative designers and creators of learning encounters with children and other colleagues. Teachers become whole-heartedly involved in “a collaborative process of constructing new knowledge with and about specific phenomenon and children and ourselves as teachers” (Lenz Taguchi, 2010, p. 94) all the while exploring their own limits and possibilities. Wien et al. (2011) explain that pedagogical documentation requires the educator to be open to what a child is considering, rather than assuming learning has occurred because teaching has occurred. Pedagogical documentation therefore challenges the position of the teacher “as all-knowing judge of learning” (Wien et al., 2011, p. 2). When engaged with pedagogical documentation students and teachers collaborate and reflect collectively to share feedback and co-learning.

Edwards (2012) explains some of the nuances of change that occur when we consider a teacher’s role in a traditional sense, to a teacher’s role when engaging with pedagogical documentation. For example, if traditionally a teacher was a curriculum planner, their role changes to a co-constructor of knowledge. The author explains, if the teacher assumed the role of a program planner, their role changes to the creator of the environment as a third teacher. The role of a teacher facilitating play changes to the teacher as an exchanger of understandings (Edwards, 2012). The teacher who provides guidance changes to the teacher who supports the competent child (Edwards, 2012). The teacher changes from a parent educator to a partner with

parents. Finally, the teacher who is a communicator with various stakeholders, becomes a "listener, provocateur, and negotiator of meaning" (Edwards, 2012, p. 149).

The role of the teacher is to develop relationships with the children, the environment and what is to be learned, when engaging with pedagogical documentation. The relationship between the teacher and the children is crucial because it develops trust and security and therefore fosters collaboration between the teacher and the children. As Malaguzzi explains to Lella Gandini, the "relationship is the primary connecting dimension of our system" (Gandini, 2012, p. 45), but not merely as a warm and comfortable environment. Malaguzzi clarifies that the kind of relationship is "a dynamic conjunction of forces and elements interacting toward a common purpose" (Gandini, 2012, p. 12).

Malaguzzi used the metaphor of 'Ariadne's thread' to explain the role of the teacher in pedagogical documentation (Rinaldi, 2021). (In the classical myth Adriadne, the daughter of King Minos of Crete, fell in love with Theseus. Theseus had come to slay the Minotaur who lived in a labyrinth. Ariadne gave Theseus thread that he unwound as he entered the labyrinth. Theseus slayed the Minotaur and successfully followed the thread out of the labyrinth to reunite with Ariadne.) Malaguzzi stated that the teacher's task was to give orientation to children in their learning (Rinaldi, 2021). In other words, teachers are those who hold the thread that provides the path out of the labyrinth, for the students. In holding the thread, teachers "construct and constitute the interweavings and connections, the web of relationships, to transform them into significant experiences of interaction and communication" (Rinaldi, 2021, p. 29).

Rinaldi (2004) states the educator must view the learner as a protagonist in constructing their own knowledge. Even noticeably young children are collaborators and communicators (Rinaldi, 2004) and in classrooms where pedagogical documentation is enacted, teachers acknowledge and embrace this. The children and teachers use the environment around them to

help build onto their ideas and theories. The educator acts as co-researcher and co-learner to help make the learning visible to the community at large, for example the student, the other students and the teacher (Buldu, 2010). We can draw parallels of this approach to connections to reforms in mathematics education. Many researchers (e.g., Fennema, 1996) have argued that a way mathematics pedagogy can improve is for teachers to learn how to understand and listen to the thought processes of their students. The research in this area states that children can have complex ideas about mathematics and that teachers should use this knowledge to change instruction (Fennema, 1996). Reforms in mathematics occur as relationships are built and learning occurs between students, educators, and the environment through the use of pedagogical documentation.

Educators must also think about their position in the interpretation and documentation of children's learning, to embrace the essence of pedagogical documentation. This means that educators must re-examine their relationship with learners and their position of authority in the classroom (Giudici et al., 2001). Documentation that gathers evidence of many perspectives and offers up many different ways to solve problems, helps guard against an unnecessary authority shift in favour of the educator. Ryerson (2017) suggests that to really understand the processes with which students engage in learning, it is crucial that teachers have students participate in the study of their own learning. Teachers who practice this type of co-learning pedagogy have described how when enabling students to direct their own learning, engagement, and ownership of learning increases (Ryerson, 2017). Ryerson (2017) remarks that when teachers implicate students in decision making and managing learning, students persevere more, are more engaged in assessment and they take more responsibility for their own learning. When educators encourage students to function as active contributors to their own learning and take charge of that learning, rich meaning making occurs. This is precisely the involvement reform mathematics generates.

Mathematical reform practices also require the educator to form relationships and build a learning community for practices to flourish. Educators, students, and the learning community engage in making meaning (mathematical and otherwise) and solving problems in ways that make sense to the learners (Smith & Stein, 2011). The educator acts as a member of the community, encouraging members of the community to communicate and justify their reasoning, using the documentation for reflection (Wien et al., 2011). Singer-gabella and Shahan (2016) recognize how mathematics education uses generative problems/situations to elicit student views which feed various forms of discussion that capitalize on the diversity of students' ideas and theories. This in turn leads to more elaborate understandings (Singer-gabella & Shahan, 2016). A parallel between documenting students' mathematical ideas and engaging with pedagogical documentation suggests the two in tandem could provide a very rich platform for teachers to listen to student voice and use student learning as a basis for instruction, in early learning mathematics.

The Ministry of Education of Ontario expects teachers in Ontario to engage with pedagogical documentation (Ministry of Education, 2016) in the kindergarten classroom to make children's ideas visible. The kindergarten document suggests that educators need to use pedagogical documentation to capture a child's perspectives and opinions so that the educator and the child can discuss and interpret the child's ideas (Ministry of Education, 2016). The educator engages with the student in making sense of the world. Educators must consider not only how students are making sense of the world, but also how their own actions as an educator can affect that knowledge construction (Dahlberg et al., 1999). This parallel can also be seen in mathematics education. A mathematics teacher must consider student ideas and carefully consider the questions or prompts that they will use, consider responses of other students, and consider the work that came before the current interaction, to fully be present with the student (Singer-gabella & Shahan, 2016; Smith & Stein, 2011). Through pedagogical documentation,

this teacher will build on learning and take part in constructing new meaning. This teacher will co-evolve with the students and the environment. These teachers are participants in the new knowledge production that is emerging (Dahlberg, Moss, & Pence, 1999). Although much literature about pedagogical documentation refers to many areas of learning in education, a specific link to the recommendations of mathematics reform can be made regarding teachers as participants in the learning process.

Research suggests that using student mathematical ideas, opinions and learning as the basis of practice offers students a rich approach for constructing meaning (e.g., Fennema et al., 1996; Stein & Lane, 1996). Yet, these practices can be complex and difficult not only to understand, but also to enact (Leatham et al., 2015). Teachers can use pedagogical documentation to foster this practice. Pedagogical documentation can be a tangible means through which mathematical reform can begin to take root in the early learning classroom.

The Affordances of Pedagogical Documentation

The kindergarten curriculum document (Ministry of Education, 2016) refers to pedagogical documentation as “the process of gathering and analyzing a wide range of evidence of a child’s thinking and learning over time and using the insights gained to make the child’s thinking and learning visible to the child and the child’s family” (p. 36). Educators gather information about children’s learning from observations, notes, photos, videos, voice recordings, work samples, and interactions with children. Educators then analyze and interpret this information, or evidence of learning, in collaboration with the children to gain insights into the children’s learning paths and processes (Rinaldi, 2012). The insights gained from these interactions may become the impetus for next steps in the child’s learning, while thinking and reflecting with the child in the process.

In using pedagogical documentation in this way, an asset model replaces a deficit model of the child. Interpretations of a child's learning now occur within the context of what the educator and learning community already know and recognize about that specific child (Bath, 2012). An ethos of trust develops while educators acknowledge the importance of a variety of different perspectives and theories. The result becomes a continuous knowledge-building opportunity by all involved (Bath, 2012).

Children can not only learn from their own documentation but their peers' as well (Buldu, 2010; Rinaldi, 2004). Pedagogical documentation holds exciting potential to support students to propose their own mathematical ideas and conjectures, learn to evaluate their own ideas, and understand the ideas of others, which fosters mathematical reasoning skills. Rinaldi (2004) recognizes that learners enrich their knowledge and understanding by listening to others and being open to their ideas. The author states that as children work and learn together, they are developing their own processes by learning from the processes of others (Rinaldi, 2004). In other words, pedagogical documentation is not simply about documenting the results of the learning, but it functions as part of the learning process. By using pedagogical documentation, children develop their self-confidence and self-awareness during the learning, as educators use the documentation throughout the learning experience and not simply at the end.

Pedagogical documentation is a negotiation between a child's imagined theories and what they understand to be reality (Wien et al., 2011). The documentation allows the educators to further the learning by way of a collaborative process of listening, discussion, and interpretation. Initially, the educator may be uncomfortable and frustrated as meaning is sought; however, educators must assume a different disposition to allow students to own their learning (Wien et al., 2011). As previously mentioned, Lenz Taguchi (2010) describes pedagogical documentation as an "active agent in generating discursive knowledge" (p. 63). It is part of the process of creating meaning of children's thoughts by offering material to consider. However,

Lenz Taguchi (2010) states that the material is not simply a 'thing' but is something that comes alive in the intra-active process. The author explains that the material – as a performative agent – "… will also put things in motion by means of its own agentic force and materiality. Thus, new possibilities for intra-action with other matter and organisms will emerge" (Lenz Taguchi, 2010, p. 64). The author clarifies that this intra-active pedagogy explicitly focuses on the events produced through the "inter-relations, inter-connections, interferences and waves of diffractions that emerge *in-between* the material, the discursive and human beings" (Lenz Taguchi, 2010, p. 65). In other words, the author is suggesting that the process involves human and non-human entities (for instance, materials, spaces, objects, etc.) coming together to produce something that together is more complex and more abundant together, than the sum of their parts. Pedagogical documentation plays an important part in the dialogue of meaning making (Dahlberg et al., 1999). This process allows students and teachers to take on the responsibility for co-constructing meaning collectively; documentation is a central piece to negotiated learning. Fyfe (2012) suggests that the educator, through documentation, can have an in-depth examination into the learning paths that children take and the approaches to learning they use. The co-constructed documentation then contributes to subsequent emerging ideas and theories through the shared, iterative experience of discussion and interpretation.

Pedagogical documentation can be implicated in this learning process because children examine a concept, ask questions, and make hypotheses about it, experiment with solutions, while documenting or being documented in numerous ways. Pedagogical documentation offers transparency to learning experiences and creates visible records of the 'how and what' of learning (Wien et. al., 2011). The process of documentation makes it feasible for the learner to look inwardly at their own learning to construct and reconstruct their new and prior thoughts (Fyfe, 2012). Researchers (e.g., Rinaldi, 2004) assert that when educators record students' ideas and use the recordings as a learning tool, children at noticeably young ages have

potential to engage with sophisticated ideas which include mathematics. Much literature pertaining to pedagogical documentation, where evidence of children learning through these recordings (visual or audio or written representations) exists (Bath, 2012; Rinaldi, 2004). Therefore, my work is an opportunity to examine what teachers know and wonder about using pedagogical documentation, specifically in mathematics.

Pedagogical documentation is an important part of reform structures that are intended to foster learning and to change the learning-teaching relationship (Giudici et al., 2012). Educators recognize the child as an active participant in their own learning, with whom educators explore and attempt to find meaning (Rinaldi, 2012). Some scholars describe pedagogical documentation as enhancing learning by offering the child a concrete and visual memory (Giudici et al., 2012). In this approach, young children create documentation in numerous ways and use that documentation to make their ideas visible to their peers, educators, and the community at large (Dahlberg, 2012). This record of what the child said and did acts as a point of departure for the next steps in learning (Buldu, 2010). Documentation presents an opportunity to track students' learning and to have students talk about how they have learned.

In conclusion, pedagogical documentation allows the recording of student learning and students' descriptions of how they learn. Much literature, specifically from the region of Reggio Emilia exists, discussing the effectiveness of pedagogical documentation in the early years. Nevertheless, the field of education needs more research examining how educators could use pedagogical documentation in the Ontario context, specifically in mathematics. As the Ontario Kindergarten curriculum (Ministry of Education, 2016) implicates educators to use pedagogical documentation to document children's learning, there is little that describes how that looks specifically in Ontario Kindergarten programs. This work will examine teachers' conceptions and wonderings of using pedagogical documentation to record mathematical experiences in the Ontario Kindergarten classroom. My work will extend the existing literature in pedagogical

documentation and mathematics reform and aims to influence classroom practice as it evidences the authentic experiences of teachers in Ontario Kindergarten classrooms.

Values and Choice

Value and choice are critical aspects when considering democratic education, influences of Reggio Emilia, and pedagogical documentation. Although I have already discussed the concept of democratic education, I believe I must offer a more careful examination of how decisions and choices based on values implicate what happens in kindergarten classrooms.

Carla Rinaldi (2021) explains that the ethos from the schools of Reggio Emilia offers something different. This approach offers "different values, different relationships, and a different way of living" (p. xxx). The difference to which Rinaldi (2021) is referring occurs when children are protagonists in their learning and important participants in the democratic process of education. Reggio inspired schools and programs make a conscious decision to challenge the rationality of neo-liberalism as it stands in contrast to values of – "dialogue and relationships of cooperation – not competition" (Rinaldi, 2021, p. xxii). As a result, educators, education, and society must recognize values and choices as both "political and ethical, but made in relationship with others, not as decisions made by individual consumers" (Rinaldi, 2021, p. xxii).

Putnam (1993) speaks to the idea of pedagogy and choices being neutral in saying:

> Pedagogy like school is not neutral. It takes sides, it participates in deep and vital ways in the definition of this project whose central theme is not mankind, but his relations with the world, his being in the world, his feeling of interdependence with what is other than himself. So, pedagogy implies choices, and choosing…[it] means having the courage of our doubts, of our uncertainties, it means participating in something for which we take responsibility. (p. 127)

Rinaldi (2021) builds on this idea, clarifying how the process of pedagogical documentation is intrinsically linked to a way of being, therefore cannot be neutral either. The author explains how documentation is an integral part of the process of fostering learning and for enhancing the learning-teaching relationship. Rinaldi (2021) states that “assumptions should be stated that … will aid in understanding that our choice and practice are neither random nor indifferent” (p. 37).

Rinaldi (2021) indicates that the notion of choice begins with how we choose to view the child. In viewing the child as a protagonist in their own learning, and the adults as co-learners, we may consider a different value of ‘being together’ that may not be experienced in many Kindergarten classrooms in North America. Educators and education view the ‘child’ as competent and capable. This image of the child sees the child as “rich in potential, strong, powerful, competent, and most of all connected to adults and other children” (Malaguzzi & Gandini, 1993, p. 10). The adult in this scenario is not one who transmits knowledge but becomes “a curator of relationships” (Malaguzzi & Gandini, 1993, p. 4). This adult concerns themselves with curating relationships between children and adults, as well as between things, thoughts, and the environment (Malaguzzi & Gandini, 1993). Teachers must see themselves as researchers and co-learners. Teachers take time to create an authentic learning experience produced with the children (Malaguzzi & Gandini, 1993). The authors state that often adults impose adult time and adult thought on children’s time and ideas. Unfortunately, this then “negates children being able to work with their own resources” (Malaguzzi & Gandini, 1993, p. 4). When educators become curators of relationships, this then becomes a unique way of being with people (teachers, students, parents, etc.). This way of being “offers a sense of belonging to people longing for other values, relationships and way of living” (Rinaldi, 2021, p. xxx). Rinaldi (2021) points out that this in turn helps disrupt the dominant discourse of neoliberalism and competition in the learning environment.

The values espoused in this way of being include “friendship, solidarity, respect for differences, dialogue, feelings and affection” (Rinaldi, 2021, p. 63). Rinaldi explains that people can only share these values by living them. This brings us to the values of differences, participation, democracy, and learning.

In valuing differences, educators embrace individuals in terms of their own background and personal story. We must employ the pedagogy of listening, as hermeneutic listening, to truly be open to knowing individuals. This type of listening “means letting go of any truths that we consider to be absolute, being open to doubt and giving value to negotiation as a strategy of the possible” (Rinaldi, 2021, p. 103).

In valuing participation, this means that feeling a part of, or having a sense of belonging applies to all members of the community. This means offering spaces, languages (literal and metaphorical), organizational methods, and strategies open enough that everyone can find themselves included. Rinaldi (2021) suggests that this type of participation “requires a certain sense of indefiniteness and ample spaces of possibility” (p. 103).

Valuing democracy is also a critical piece. Rinaldi (2021) states that educators and society must not forget “how closely the school is connected to the society in which it is situated” (p. 103). The author explains how school is not a place to simply transmit the culture of the society in which it lives. School can be a place where people can live and create culture through democracy. This means that the children are members of such society and acting contributors to the creation therein.

Rinaldi (2021) also describes learning as one of the fundamental values held in this particular view of the child. Rinaldi explains that educators must understand learning as a relational endeavour. This means that as humans we must be open to the “complex, conflictual, and unpredictable nature of human learning, wherever it takes place, both inside and outside

the institutional contexts" (Rinaldi, 2021, p. 104). Rinaldi (2021) clarifies that "learning is the emergence of that which was not there before" (p. 104). There should be a "constant relational reciprocity between those who educate and those who are educated, between those who teach and those who learn" (Rinaldi, 2021, p. 104). Educators and students must value this reciprocity. Rinaldi (2021) is clear that within this view of the child, the child does the learning and the teaching, as the educator participates in relationship to learn and grow with the child.

The view of the child and pedagogical documentation include many specific values, but also comprise a wider definition of 'value'. Value in this case refers to principles and standards people hold in general. Values are the principles and standards that inform peoples' judgements and opinions (Dahlberg, 2012). These very principles and personal values inform the choices teachers make when engaging with pedagogical documentation. Educators must understand that pedagogical documentation is not an innocent activity (Dahlberg, 2012). The author points out that "it always has social and political implications and consequences" (p. 229). Dahlberg (2012) describes the work of Michael Foucault (1970), in saying, although an educational system may intend to present a state of "formal neutrality and equality, despite its best intentions, may still be viewed as a potential act of power and control" (p. 229). Dahlberg (2012) continues to emphasize that educators must be alert and observant, otherwise pedagogical documentation can become a tool to "'predict and control' children more effectively through processes of normalization and surveillance" (p. 229). In this way, children can easily become "mere 'objects' for our understanding" (Dahlberg, 2012, p. 229). With this in mind, awareness must heighten to the choices being made within documentation. We must question if the documentation process is rooted in democracy and values as listed above, and "realizes its potential for active listening and for taking in multiple perspectives" (Dahlberg, 2012, p. 229). Or is documentation merely used as a tool to uphold the "power/knowledge nexus" (Dahlberg, 2012, p. 229) in classrooms?

Educators must accept that values exist within the process of engaging with pedagogical documentation as a way of being with children. Educators choose what to observe and then what to interpret. That is subjective (Rinaldi, 2012). In making a choice this means that educators are giving value or evaluating this experience as meaningful for the children's learning (Rinaldi, 2012). As educators share these processes with children, the children then see what is valued as meaningful from their work. Therefore, educators record, transcribe, and reflect upon the children's words and thoughts for meaning. In doing so, children feel that their voice and thoughts are valued. (Rinaldi, 2012). Children may then begin to understand that the recordings and transcriptions are artefacts of research and offer opportunity for dialogue as the basis of important learning for everyone.

Kindergarten in the literature

A literature search was restricted to publications since 2012 to determine the connection between ideas such as Reggio Emilia, inquiry, play-based learning and kindergarten. Searches were conducted using the search terms "inquiry in kindergarten", "Reggio in kindergarten" and "play-based learning in kindergarten". I chose these search terms as search terms that were most relevant to research on pedagogical documentation in kindergarten. The following review describes a portion of the literature pertaining to the view of the whole child in kindergarten. I also offer a review of an example of the literature applying to the Reggio Emilia approach in kindergarten. I then explore a sampling of the literature regarding play-based and inquiry-based learning in kindergarten and examples of various inquiries in kindergarten classes.

The Whole Child

To begin, I offer an examination of the literature on the development of the whole child. For example, Saracho and Evans (2021) conducted a literature review of research describing the "pioneers" of Early Childhood Education. The authors found that the development of the

whole child in kindergarten was a common theme in literature. Their list of “pioneers” included Friedrich Froebel, Maria Montessori, Susan Isaacs, Reggio Emilia, Magda Gerber, and some contemporary pioneers. Saracho and Evans (2021) highlighted that “pioneers” advocated for hands-on education with a focus to assist children to function successfully in society. Generally, their approaches involved paying attention to the whole child rather than simply being concerned with a child’s academic gains. Hustedt et al. (2018) found a similar focus on the whole child, as they investigated teachers’ beliefs about what children entering Kindergarten should be able to do. Hustedt et al. (2018) noted that policies often promoted an academic focus when discussing children’s competencies. The teachers involved in the study took a more nuanced view of children and the teachers focused on nonacademic skills (e.g., participation, cooperation, etc.) as a key component of determining kindergarten readiness (Hustedt et al., 2018). Keung et al. (2019) studied teachers’ conceptions of “effective play-based learning development and how their perceptions of effectiveness relate to whole-child development” (p. 627). These authors determined that when parents are involved and when the teacher’s enactment of play pedagogy occurs, these factors help enhance a child’s whole personal development. Bautista et al. (2016) discovered that teachers ranked social and emotional development as the top learning priority in kindergarten, for the development of the whole child. This was followed by language, whereas numeracy was ranked in sixth place.

Reggio Emilia approach in kindergarten

The Reggio Emilia approach of early childhood education is gaining much attention as an “excellent model to facilitate optimum learning in young children” (Harris, 2021, p. 123). The goal of Reggio Emilia inspired pedagogy is “to educate children into critical thinkers and guardians of democracy” (Vodopivec, 2012, p. 1). Reggio Emilia approaches are based on the rights of adults and children, democratic values and the rule of law (Vodopivec, 2012). Reggio Emilia approaches to education view childhood as a socially constructed concept; hence the

children and the educators are co-founders and co-creators of knowledge and society (Vodopivec, 2012). This is significant as this connects to the foundational principles of democratic education where respect, equity, and concern for each other, occur in learning encounters. It is important to highlight that this is in opposition to a more neoliberal, top-down educational model where the curriculum is adult designed and directed, and children are viewed as passive recipients of knowledge rather than active contributors.

Harris' (2021) research revealed that parents, despite pressures to choose an 'academically' focused program, found Reggio Emilia inspired programs better value for their children. Parents shared ten perceived benefits of choosing a Reggio inspired program over a more 'academic' program. Some of those benefits included much more interest, inquiry and project-based learning, individualized care and learning, classroom environment and choice of materials, viewing children as capable, and constructivist-based learning [learning through doing] (Harris, 2021).

The research suggests that transplanting Reggio Emilia approaches into other contexts and locations other than Italy can have challenges. Many factors influence the application of Reggio inspired approaches (Emerson & Linder, 2021). Researchers continue to engage with research to examine how they may apply Reggio approaches in different contexts. Some of the research trends into Reggio approaches include teacher and administrative perspectives and practices, teacher voice, focused inquiries, child and parent participation, the efficacy of Reggio inspired approaches, and reframing Reggio inspired research (Emerson & Linder, 2021). Literature also reveals that researchers are investigating assessment in kindergarten within the context of Reggio Emilia approaches. Some key priorities of assessment for kindergarten inspired by Reggio Emilia, were identified (Deluca et al., 2019) in the literature. First, there must be a commitment to child centred and developmentally appropriate pedagogies. Second, educators must value formative assessment and be continuously collecting data to that end.

Third, a variety of methods must be available for students to demonstrate their learning (Deluca et al., 2019). This diverts from Reggio approaches to assessment. Within Reggio approaches to assessment, educators would not consider traditional assessment tools like paper and pencil tests as one of the methods for students to demonstrate their learning.

Play-based learning in Kindergarten

There is also much literature exploring play-based learning in kindergarten. Research is beginning to define play, looking at diverse types of play, examining the social and academic benefits of play, and considering the ways educators can facilitate play-based learning in kindergarten (Taylor et al., 2019). Although researchers continue to have challenges to specifically define what constitutes play in a classroom (Taylor et al., 2019), a study conducted by Pyle and Danniels (2017) has identified a continuum of play ranging from child-directed play to teacher-led play. Pyle and Danniels (2017) identify on the side of completely child-directed play, what we might consider as free play. Here the child has unlimited choice and freedom to direct the play in whichever way the child wishes. This is often pretend or imaginative play (Pyle & Danniels, 2017). In the middle of the continuum occurs a collaboratively designed play where teacher and child structure and control the play together. At the other end of the continuum, the teacher completely orchestrates the 'play'. This play is often structured games and activities created by the educator, which the educator deems playful (Pyle & Danniels, 2017). Many educators, researchers, and sociologists recognize play as a valuable vehicle for academic and social learning (Pang & Simoncelli-Bulak, 2017). Research has proved the benefits of play scientifically as "the neural pathways in children's brains develop through exploration, thinking skills, problem solving, and language expression that naturally occurs during play (Pang & Simoncelli-Bulak, 2017, p. 24). Pang and Simoncelli-Bulak (2017) note that play-based learning can lead to more developed skills socially, emotionally, and academically. Play allows children

to engage with problem solving, inquiry, and helps them understand the world around them (Trehearne, 2003).

Researchers have acknowledged play as having a key role in early learning (Pyle et al., 2017). Research also examines the role the teacher has in providing free play, and guided play opportunities. The teacher's role must be flexible and responsive to meaningfully support play. A teacher's involvement may be very little as students engage in free play. A teacher may assume a more active role in co-constructed play with children (Pyle & Bigelow, 2014; Pyle et al., 2017) and when supporting social interactions (Danniels & Pyle, 2022). Regardless, research continues to emphasize the importance of adult interaction in play (Colgan, 2016).

Some research has examined factors that influence kindergarten teachers to implement play-based learning. Researchers identified strong administrative leadership as a factor that positively influenced teachers' intentions to implement play-based learning (Yin et al., 2021). Teachers, who believed in the efficacy of themselves and of play-based learning, were also more likely to implement play-based learning in their programs (Yin et al., 2021). Despite school programs all over the world mandating play-based learning, researchers continue to see a discrepancy between policies that governments and school boards make and practices that educators enact (Pyle et al., 2020). Additionally, even with the recommendations for teacher involvement, Pyle et al. (2020) found that teacher involvement in play happened in less than 50% of classrooms studied.

Inquiry based learning in Kindergarten

Inquiry based learning is a pedagogical approach that is often employed in kindergarten and research continues to reveal affordances and challenges with inquiry based learning. Inquiry based learning follows the methods and processes, much like professional scientists (Pedaste et al., 2015). The learner formulates hypotheses and tests them by conducting

experiments and making observations. Inquiry based learning is an approach to solving problems. Active participation and responsibility for one's own discoveries are important aspects of inquiry-based learning (Pedaste et al., 2015). Dewey (1938) explained that students must be actively engaged in their own learning process and have a degree of control over their learning. In turn, the teacher takes on the role more like a facilitator or a guide (Dewey, 1938). When considering the role of the teacher within the context of pedagogical documentation, the literature suggests the teacher assumes the role of a co-researcher and co-learner, more than a facilitator or a guide (Rinaldi, 2001).

Examples of Kindergarten Inquiries

Much literature gives concrete examples of various inquiries enacted in kindergarten. MacDonald and Breunig (2018) studied how kindergarteners experienced an outdoor schoolyard inquiry. The school yard in this case consisted of a large green space with open fields and built play equipment near the school yard entrance. The space outside included horseshoe-shaped stone seats for children with a small platform for the teacher. There was a built playground and row of trees acting as a divider, a permanent weather station and a bin storing other teaching and learning resources (MacDonald & Breunig, 2018). Students gave the researchers a student-guided tour of the outdoor space and highlighted aspects of the space that described their experiences. The authors posed open-ended questions about what was meaningful for the students and what types of activities they engaged in (MacDonald & Breunig, 2018). These authors reported that the inquiry was an excellent example of democratization of education. Students learned from their own decisions and grew from interactions with peers. The authors reported that participating in the outdoor schoolyard inquiry offered a platform for students to engage with their learning in ways that were meaningful to them (MacDonald & Breunig, 2018).

Pecaski McLennan and Bombardier (2015) described an inquiry in music and movement they conducted in their kindergarten class. These educators explained that the inquiry was an opportunity for students to construct their own meanings and interpretations of their learning using a wide variety of materials and having plenty of time for students to explore them. These authors were educators in a kindergarten class where an inquiry emerged from a few grass skirts being left in a recycled materials bin. Three girls put on the grass skirts and began to dance. From there more interest in the inquiry grew. Some students made their own grass skirts, some students set up chairs for an audience, and some students made tickets for purchase to see the show. All the while, the educators supported the play by introducing new materials like books about dance and engaging with the students in their play. The inquiry grew into an inquiry over several weeks as the educators continued to observe students' play and prompted and offered materials to extend that play (Pecaski McLennan & Bombardier, 2015).

Johnson et al. (2019) described a study of kindergarteners engaged in a specific exploration involving project-based inquiry. The teachers, Ms. Jones and Ms. Monroe, wanted to do an inquiry project with their kindergarten students using the Project Based Inquiry Global process (Johnson et al., 2019). The Project Based Inquiry Global process included five stages of inquiry: a) ask a compelling question, b) gather and analyze information, c) creatively synthesize claims and evidence, d) critically evaluate and revise, e) publish and share (Johnson et al., 2019). The teachers wanted to launch "a project that embraced cultural awareness while also giving their students a chance to share their backgrounds through the use of technology" (Johnson et al., 2019, p. 609). The teachers asked the students, "How can we explore life in other countries through the 5 senses?" This launched their compelling question along with a couple of read alouds about the senses and diversity. Educators sent letters home asking parents for artifacts of the children's cultures and traditions. Students began to consider what they might share with their peers regarding themselves and their artefacts. Students were able

to use Flippgrid to record themselves and re-watch their presentation. Students received peer feedback and teacher feedback on their products at this point in the inquiry. Students made revisions based on that feedback and were able to re-record. To share the final piece, educators invited parents to come to the school to see what the students had discovered. Johnson et al. (2019) claimed that because of this inquiry, students developed mindsets for global awareness. Students were successfully able to discuss how life in other countries around the world was similar or different to their own (Johnson et al., 2019).

Ghafouri (2014) described one Ontario classroom's experience with a nature inquiry. Ghafouri (2014) explained the importance of students playing an active role in constructing their own learning and students connecting emotionally to their learning. This author studied a kindergarten class in Ontario, where Reggio Emilia approaches inspired the classroom context. This classroom teacher promoted a multidisciplinary approach to teaching and learning. Play and naturalistic inquiry were the foundation of the activities that happened in the classroom. Students engaged in self-directed inquiries, activities, or play, as the teacher supported their exploration responsively with materials, space, time, and her engagement. There were many natural materials throughout the classroom such as rocks, shells, twigs, leaves, and pieces of bark. The researcher gathered data through observation of the emergent and self-directed experiences of children and the natural world, as well as the teacher's pedagogical responses to the children (Ghafouri, 2014). Teacher interviews, a parent questionnaire, and students' artefacts also provided sources for data. The author stated that it was observed that the students' engagement interest and learning was deeper, richer, and more sustained as a result of engaging with natural materials in the classroom (Ghafouri, 2014).

A bee inquiry used dramatic inquiry to investigate the lives and behaviours of bees. In this inquiry, teachers used the dramatic approach called "Mantle of the Expert" where the educators presented students with a problem in the form of a letter or video. The letter (or video)

was from an important client (in this case a local farmer) who asked them as experts (e.g., entomologists) to help the client solve their problem (in this case the bees disappearing from the orchards) (Farrand et al., 2019). Through drama, the students engaged with various aspects of being an entomologist, like examining insects to understand their anatomy and keeping a scientific log of their observations and discoveries. Farrand et al. (2019) described that students became competent problem solvers as the students exercised their strengths through dramatic inquiry activities.

"Waiting for the Monarch" was an inquiry in another kindergarten class where students were learning to study biodiversity of their own community. As the students researched Monarch Butterflies, they examined the life cycles of plants and butterflies in their ecosystem (Forest & Hechter, 2017). The authors of this study noted by "using outdoor spaces as authentic learning environments, students can become inspired to be stewards of their own futures" (Forest & Hechter, 2017, p. 38).

Research conducted by Harris et al. (2017) demonstrated how kindergarten students can generate scientific explanations in various contexts, when the teacher employed various kinds of discourse strategies. In this inquiry, the students also investigated the life cycle of the Monarch Butterfly. The educators presented the monarch larvae to the students in the classroom and placed them on milkweed the educators kept in the classroom. Students made predictions and observed over time the growth and changes of the larvae. The teacher facilitated and scaffolded the students' dialogue of scientific explanations, by employing various discourse strategies (Harris et al., 2017). Some of the discourse strategies used by the teacher included asking explanatory questions, asking students for more detail, or asking students to offer an example with their ideas. The author coded the discourse strategies, and in their results, described the frequency and duration of the various strategies used by the teacher.

Murcia et al. (2020) described an integrated science and mathematics inquiry from one kindergarten class. This inquiry followed the children's interest in the sun. The inquiry involved following shadows, examining the need for sun protection by making hats. (Murcia et al., 2020). The researchers described how intentionally planning the learning environment ensuring it was rich with material resources, and using inquiry questioning practices, supported young children's engagement, and learning in the inquiry. The teacher in this classroom planned specific inquiry, problem-based questions, which she proposed to small groups of children. Then through inquiry questioning practices, she guided the students through the inquiry. For example, the teacher began with the question "Help, I forgot my hat! How can we make a hat to keep the sun off my face?" Educators then provided time for the students to investigate methods and materials to solve the problem. The teacher engaged in open-ended, inquiry-based questions such as "What might you need to know to begin solving this problem?" or "What tools might you need or use?" The classroom teacher reported students were more personally aware of science and math (for example by examining the measurement of shadows and in hat construction) in their everyday activities after the inquiry. The teacher also stated that as she engaged in more inquiry questioning, students were more willing to spend time on tasks problem solving and engage in higher order thinking (Murcia et al., 2020).

The Ontario Kindergarten Program Document

In 2010 kindergarten began a transformation in Ontario. At the time, the Ministry of Education in Ontario decided that over a 5 year period all half day kindergarten programs in Ontario would transition to full day, full-time programs. The Ministry of Education developed a draft document of a new curriculum that would focus more on inquiry-based, play-based learning for kindergarten students. By 2016, every kindergarten class in Ontario was full day programming and a new version of the curriculum was published. This curriculum promoted inquiry-based, play-based programming and demanded that educators transition "from a

traditional pedagogy to one centred on the child and informed by evidence from research and practice about how young children learn" (Ministry of Education, 2016, p. 4). This new document is a 327 page resource for educators including research based descriptions of what the new program should look like. Some areas discussed in the document are: i) play-based learning in a culture of inquiry, ii) the learning environment, and iii) assessment and learning in kindergarten: Making children's ideas and learning visible. The expectations are situated in four frames: belonging and contributing, self-regulation and well-being, demonstrating literacy and mathematics behaviours, and problem solving and innovating. There are detailed descriptions of the frames before the document describes the actual expectations for each frame.

The kindergarten document contains 318 pages with 31 overall expectations and 126 specific expectations. The document numbers the overall expectations which describe an overarching concept or skill. The document states the overall expectation first then a list of specific expectations follows the overall. For example, the curriculum document lists the overall expectation fifteen as "OE 15 As children progress through the kindergarten program, they: demonstrate an understanding of numbers, using concrete materials to explore and investigate counting, quantity, and number relationships." Under this overall expectation are ten specific expectations (which are also numbered) that describe in more detail what those skills and concepts look like. For example, specific expectation # 15.1 states: "(As children progress through the kindergarten program they…) investigate (e.g., using a number line, a hundreds carpet, a board game with numbered squares) the idea that a number's position in the counting sequence determines its magnitude (e.g., the quantity is greater when counting forward and less when counting backward)" (Ministry of Education, 2016, p. 217). Continuing in this way # 15.2 states: "investigate some concepts of quantity and equality through identifying and comparing sets with more, fewer, or the same number of objects (e.g., find out which of two cups contains more or fewer beans [i.e., the concept of one-to-one correspondence]; investigate

the ideas of more, less, or the same, using concrete materials such as counters or five and ten frames; recognize that the last number counted represents the number of objects in the set [i.e., the concept of cardinality])" (Ministry of Education, 2016, p. 218). The specific expectations continue to list, one after the other, the minutia of the skills and concepts that educators are to cover over the two year program.

For each specific expectation, the document offers a chart which suggests ways children might demonstrate their learning. This is subdivided into 'saying', 'doing', and 'representing'. Directly adjacent to the ways children may demonstrate is another column suggesting the educator's intentional interactions, for example, 'responding', 'challenging', and 'extending'. Under each of the subheadings are potential responses and prompts educators could use to interact with students.

The kindergarten program document discusses the way in which educators must engage with pedagogical documentation. The discussion of pedagogical documentation in the kindergarten document comes under the heading "Assessment and Learning in Kindergarten: Making Children's Thinking and Learning Visible" (Ministry of Education, 2016, p. 36). The document refers to pedagogical documentation as a process "of gathering and analyzing a wide range of evidence of a child's thinking and learning over time" (Ministry of Education, 2016, p. 36) for the purpose of making children's conceptions visible. The description of pedagogical documentation that continues in the kindergarten document reveals neoliberal undertones in the choice of words used like: "eliciting and interpreting evidence", "rigorous evidence of student learning", and "educators are becoming 'knowledge leaders'. However, some pieces of the document's description do reflect an approach more reminiscent of a Reggio Emilia approach. The document refers to the 'pedagogy of listening' and acknowledges that pedagogical documentation is not summative assessment. The description states that pedagogical documentation is an ongoing process that involves revisiting student ideas. The document also

acknowledges that the educator must be willing to ‘slow down’ and take time to listen and observe using an inquiry stance. It appears the description of pedagogical documentation in the kindergarten program document attempts to reconcile the use of pedagogical documentation in a system that is heavily influenced by neoliberalist practices, which in turn creates a tension for pedagogical documentation to be used as a conduit for democratic practices in education.

Centres

Centres (also known as learning centres or play centres) have long been a staple of pre-school and elementary classrooms (King-Sears, 2005) in Canada and United States. ‘Centres’ is a structure that educators use in kindergarten to define an area where an activity may be set up for students. To understand the context of centres in the current kindergarten classroom, it is important to explore the literature on centres. But first we must investigate the connections between learning centres, neoliberal ideas, and developmentally appropriate practices.

A dominant discourse that orients the education of young children is based on the notions of developmentally appropriate practice (DAP). Based on some of these theories, certain beliefs of how children should progress through various stages with various kinds of skills and knowledge, have been accepted. Many of these theories form the basis of how teachers are educated to be prepared to teach in education. DAP describes how children must be at certain ages or display certain benchmarks to be able to engage with certain academic activities. However, many scholars critique theories of child development and developmentally appropriate practices based on their development, largely by male philosophers in Western Europe, during the Enlightenment era.

Land et al. (2022) critique discourses of child development suggesting that these discourses have limited ideas about what it means to be a child. These authors explain that the notion of the ‘universal child’ has been created in accordance with “strategies and techniques of

power in accordance with political, social, judicial, and economic conditions of society" (Land et al., 2022, p. 111). The authors suggest that developmental psychology is one the factors that has produced a particular understanding of the child in Canada. This understanding shapes the ways in which we socialize and enculturate the child in education, so that they gain mastery of the world and become contributing members of society (Land et al., 2022). Children become knowledgeable through development, and development is the way in which educators can know children. Child development is linear and requires a path to specific outcomes. With developmental trajectories, time is framed in a way that "located children in the future as productive citizens and in the present as humans-in-progress" (Land et al., 2022, p. 117). The authors suggest this is a colonial concept of time where distinctive divisions are created between the past, present and future to protect the authority of the people "making decisions toward control and mastery in a present that prepares for a better future" (Land et al., 2022, p. 117). This progress is determined by the skills a child develops along the journey of becoming, one skill building on the previous, eventually arriving at adulthood, well trained to contribute to the free market.

In early childhood education, many educators believe that learning centres or centres used in the classroom, support the implementation of developmentally appropriate practices because they are intended to 'meet learners where they are' in terms of their developmental stages. It is thought that centres support DAP because they are child centred, allow for hands-on learning, offer social interactions, and integrate multiple developmental domains (like cognitive, social, emotional, physical, and language) simultaneously (Cobb & Meter, 2000).

Learning centres have long been adopted to promote developmentally appropriate practices as the activities at the centres can seemingly be created for specific students at specific stages of development (Ricard, 2002). However, some scholars have critiqued developmentally appropriate practices as being culturally biased. Some scholars have pointed

to the lack of inclusivity in these practices for children with diverse backgrounds and values (Lopez & Mac Kenzie, 1996; Tobin et al., 1989). Graue (1993) critiqued the rigid implementation and application of some of these approaches in education. This author pointed out in her research how DAP has great potential to limit children's learning opportunities. Erica Burman (2017) reminds us that developmental psychology, from which theories of child development come, is tied to the culture and time that produced it. This author states that a model that represents development as a unified, predictable phenomenon "irrespective of culture, class, gender and history has meant that different can be recognized only in terms of aberrations, deviations – that is, in terms of relative progress on a linear scale" (Burman, 2017, p. 278). Rogoff (2003) highlighted the overemphasis on individual learning of developmentally appropriate practices, which in turn neglect the social and community contexts that influence children's development. The concept of individual learning and accountability raises other concerns of how this may lead to an overreliance on standardized assessment and tools that track achievement (Meisels, 1992). Tools that track achievement and standardized assessments do not fully capture child's learning experiences and quickly digress to fulfilling the neoliberalist agenda of standardization, competition, and individual choice – preparing the child to be a productive consumer in the free market.

The idea of learning centres is not new. Early in education, educators realized that large-group instruction was not the most effective way to demonstrate new learning to children, nor the best way to differentiate (King-Sears, 2005). It was evident teachers needed to conduct small group instruction for best learning, and to do so, the rest of the children needed to be engaged in activities that could be done independently so that the teacher's attention could remain with the small group (Keiff, 2005; King-Sears, 2005; Pattillo & Vaughan, 1992).

Some research on learning centres has been conducted, resulting in mixed findings. Bautista et al. (2019) conducted a study specifically in a preschool classroom regarding

thematic play centres. They found that typical thematic centres in early childhood classrooms included dramatic play, arts, blocks, language and literacy, and science centres. Children would engage with materials and resources that related to the theme, which were engaging, fun and open-ended (Bautista et al., 2019). Bautista et al. (2019) discovered that of 108 classrooms studied, only 36 of those employed learning centre time. They also discovered that merely having learning centres in the classroom was not indicative of teachers spending time engaging with students at those centres. The level of involvement of the teachers in their study varied from those who never engaged with children to those who engaged every time students were at learning centres (Bautista et al., 2019). Another discovery the authors made was the fact that ¼ of the time students spent at learning centres was to complete class assignments independently. This meant that while students were at a centre completing class assignments, other students were at thematic centres playing. Teachers spent more time supervising students completing class assignments than engaging with those playing in thematic centres (Bautista et al., 2019). The authors also found that educators limited the children's freedom of choice in the centres, to the extent that the authors questioned whether the children's behaviours really qualified as play (Bautista et al., 2019) The learning centres were rigid and constrained, where children completed pre-determined activities with specific materials and resources provided. When teachers did engage, the interaction tended to be superficial (Bautista et al., 2019).

However, in another study, Bhattacharya et al. (2018) described a classroom where educators used learning centres with effectiveness and positive results. In a grade 2 classroom an educator invited students to explore living and non-living things at learning centres. In this case the learning centres facilitated the students' learning by breaking down the science into understandable sections (Bhattacharya et al., 2018). Students were able to move freely around the classroom visiting and revisiting centres as they saw fit. Students could revise their illustrations and hypotheses and talk with their friends. As a result, students were able to identify

many important characteristics that make an object living or non-living (Bhattacharya et al., 2018).

Educators often define learning centres or play centres as areas in the classroom, dedicated to specific small group play or tasks (Badger, 2016; Bell, 1983). Learning centres provide a means to help children construct “knowledge from interactions with materials and other people” and become “autonomous, self-directing, responsible individuals” (Pattillo & Vaughan, 1992). Teachers often determine who uses which centres and what will occur at the centres (Bell, 1983). Educators store materials that apply to a given learning centre in cupboards or shelves near the area, where students can have access to materials easily (Erturk Kara, 2017).

Historically, teachers wanted to monitor students’ engagement in the learning centres, and therefore implemented various organizational strategies to do so. Educators felt that “[m]onitoring how well students accomplish tasks at learning centres is important for students’ accountability and for teachers’ awareness of when tasks need to change” (King-Sears, 2005). Teachers “wanted all kinds of proof that the students were doing what they were supposed to do during centre time” (Keiff, 2005, p. 292-L). Timers, sign-in sheets, checklists, learning contracts, and bulletin boards are diverse ways that teachers established for accountability (Keiff, 2005). Bulletin boards helped guide students to specific learning centres and kept track of which centres they have yet to visit. Teachers even began to control the traffic to and from centres and they also controlled the number of children permitted in each centre (Keiff, 2005). Educators and education generally believed that the success of the children and the learning centres depended on strict routines that children understood and followed.

As time has passed, the intentions and goals of learning centres have shifted. More recently, it has been thought that the use of learning centres shifts the focus of control from the

teacher to the student, putting the student in charge of developing their own problem-solving skills (Badger, 2016). Learning centres should provide opportunities for individual and collaborative learning, while developing self-regulation, independence, and responsibility (Badger, 2016). Badger (2016) states that classrooms that employ effective instruction value elevated levels of communication and emphasize student collaboration and minimize "individualized work tasks such as worksheets" (p. 628). These classrooms also employ learning centres as a way for students to become active learners by regulating their learning through reflection and self-evaluation and asking questions to clarify understanding rather than simply accepting information without discernment (Badger, 2016).

Some educators understand that learning centres offer a natural form of differentiated learning and instruction in the classroom (Andreasen & Hunt, 2012). Andreasen and Hunt (2012) state that by using learning centres, teachers can "enhance learning by matching student characteristics to practice and assessment" (p. 240). The authors note that specifically in the math classroom, students can use learning centres to work on different activities that relate to the same concept, through differentiation in content, process, or product. In the model Andreasen and Hunt (2012) studied, educators used four math learning centres. The four stations (centres) included a teacher's station where students could receive additional help or enrichment. The "Shop" station where students would examine a fictitious piece of work to conduct an error analysis. The "Practice Plaza" where various practice opportunities were available to students to give them more exposure to a given concept. Finally, the "Proof Place" would have students using tools and models to solve, explain, and justify their mathematical strategies and ideas to a given problem (Andreasen & Hunt, 2012). Within these four centres, students had a choice in determining which centre they felt was best to visit for their own learning.

In conclusion, learning centres can offer some rich learning opportunities in kindergarten classrooms, depending on numerous factors. Some factors include how open-ended the activities are at each centre, what kinds of tasks are available at the centre, if students have flexibility in visiting different centres as they choose, and how much and what kind of teacher interaction occurs at the centre. It appears the more choice and responsibility students have for their own learning at centres, the more students may develop skills like self-regulation, independence, responsibility, and critical thinking.

Mathematics Reform

Beginning in the 1980's, the poor mathematical achievement of American students was noted (Ellis & Berry, 2005). Researchers began to realize that, in the movement toward a more technological world, achievement in mathematics needed to be approached in a different way. In learning mathematics, priority was given to mathematical ideas, rather than rote procedural memorization and repetition (Ellis & Berry, 2005). Researchers and educators began to engage with new goals in mathematics education. These new goals included making conceptual connections, articulating original ideas, explaining, and justifying arguments, and applying knowledge in new contexts (Ellis & Berry, 2005). Additionally, a growing interest and awareness came in considering the significance of student culture and the classroom culture on the creation of meaningful mathematical understanding (Cobb, Wood, & Yackel, 1990; Ladson-Billings, 1997).

Emphasis changed from mathematics being distant from the human experience to mathematics being part of the human experience (Ellis & Berry, 2005). Researchers and educators came to understand that for students to really understand mathematics they needed to dialogue about experiences with mathematics, form connections between concepts, engage in critical contemplation, and reflect upon the ways that mathematics could help them make sense of their world (Ellis & Berry, 2005). This reform required flexibility with how teaching in

mathematics was approached and how learning in mathematics is evaluated. Hence, this required a movement away from the “static and deterministic models of the PFP [procedural-formalist paradigm]” (Ellis & Berry, 2005, p. 12).

Reform based mathematics is deeply focused on listening to students’ ideas and problem solving (e.g. NCTM, 2000; Drake, 2006; Chapman, 2016). The Ontario Mathematics Grade 1-8 curriculum (2005) aligns with this as it states that educators must give students opportunities to use problem solving to investigate concepts. The understanding then is, teachers are present and involved in curating and evolving with student ideas to carefully accompany students “into an understanding of the mathematical principles involved” (Ministry of Education, 2005, p. 4). The National Council of Teachers of Mathematics (NCTM) offers recommendations built on the assumption that learning is an active process and not something that students derive understanding through memorization and procedural mechanics (Schoenfeld, 2004, p. 266). Schoenfeld (2004) acknowledges that mathematics education has followed a highly “content oriented” (p. 269) approach to pedagogy in the last century. A general notion that mathematics instruction is focused on procedures and is not necessarily something that students can “figure out”, has taken root because of this content-oriented approach to pedagogy (Lindsey & Stigler, 2012, p.191). Jardine, Friesen, and Clifford (2006) concur and suggest that ‘understanding’ has largely been equated to memorization and recall on demand. As a result, ‘understanding’ has become taken up as a possession or an object which has been disseminated from other sources (e.g., knowledge passed down from people, books, other resources, etc.); again, deeming such ‘knowledge’ or ‘understanding’ as a commodity to be sought. Further to that, Jardine, Friesen, and Clifford (2006) critique that for some time, it has been understood that curriculum topics “must then be broken down and doled out in carefully monitored ‘zero-sum’ exchanges” (p. 4). For example, a teacher offers knowledge by way of transmission, the student must ‘take up’ this information, memorize it and then regurgitate it on another occasion

determined by the teacher. It is the student who is solely responsible for the 'holding' of this information until a time when a figure of authority demands that the student release the information. These knowledge/information fragments are those which then become perceived as the necessary 'basics' of education.

Key Aspects of Reform Mathematics

Mathematics education reform focuses on transforming traditional teaching methods to foster deeper understanding, critical thinking, and active student engagement. There are specific aspects involved in mathematics pedagogy that are associated with mathematics reform. I have chosen to reference NCTM's (2014) Principles to Action, Executive Summary to outline some of the aspects of mathematics reform that relate to this work. First, we must consider the role of the teacher when discussing reform mathematics. This discussion will build upon ideas that have already been presented in the section "Pedagogical documentation and the role of the educator". Educators must reconceptualize their roles as teachers, when engaging in reform mathematics. The teacher is no longer the sole authority and source of knowledge in the mathematics classroom, whose purpose is to transmit knowledge or information to the students (Wood et al., 1990). The teacher is now expected to actively engage with students' learning by negotiating mathematical meaning with the students (Wood et al., 1990). This means that teachers do not 'pre-teach' the mathematical concepts, but rather present a problem for the students to collaboratively solve. With this in mind, let's consider some significant aspects of mathematics reform:

1. Students are active participants in their learning, they construct understanding in ways that make sense for them,

2. Tasks and materials should promote critical thinking, mathematical reasoning, and problem solving,

3. Tasks and materials should offer multiple entry points and varied solution strategies (Teachers may also anticipate student ideas in a variety of ways so they can consider pedagogical moves in advance),

4. Discourse is valued and encouraged among students and educators to build shared understanding by analyzing and comparing students approaches and arguments,

5. Listening to students' ideas and use student ideas in ways that support and advance learning,

6. Conceptual understanding is promoted to act as a foundation for procedural fluency, so that students can develop flexibility with their ideas.

(NCTM, 2014)

I will explain these points in further detail below.

1. Students are active participants in their learning, the construct understanding in ways that make sense for them:

The role of students in reform mathematics changes from students being a passive recipient of knowledge to one who is an active participant in constructing "knowledge by reorganizing their current ways of knowing" (Wood et al., 1990, p. 496). It is the belief of reform mathematics that children learn by interacting with their environments in combination with their past experiences. Children learn when current ideas intersect with problematic situations "and students construct new concepts to restore coherence and meaning to experience" (Wood et al., 1990, p. 496). In reform mathematics, it is expected that students will engage with mathematics by selecting appropriate strategies and justify solutions when solving new problems, making predictions about the solutions they are seeking, and constructing new

understanding (Richland et al., 2012). Students are to ask each other questions and build on other students' suggestions. Students in a reform mathematics learning environment are encouraged to take more responsibility for their own learning (Brown et al., 1996).

2. Tasks and materials should promote critical thinking, mathematical reasoning, and problem solving:

In mathematics reform teachers create tasks that offer students the opportunity to engage in mathematical reasoning and problem solving (Maas, 2011; NCTM, 2014). This is a more constructivist approach where students actively engage in mathematical problems and tasks to explore various solutions and various ways to think about solutions and reasoning (Leijen et al., 2023). Students are encouraged to explore, conjecture, and solve complex problems, which helps them develop critical thinking and reasoning skills. This approach values students' active engagement and creativity in the learning process (Lesh & Zawojewski, 2007).

3. Tasks and materials should offer multiple entry points and varied solution strategies (this may also include the teacher anticipating student solutions to consider pedagogical moves in advance):

In mathematics reform tasks must offer multiple entry points and the opportunity for varied solution strategies. Scott (2015) explains that "people learn in a variety of ways and may take multiple pathways to skills acquisition" (p. 4). This requires that teachers recognize that personalized learning will occur as students consider their ideas in ways that are meaningful to them. Scott (2015) states that personalized learning means that "individuals approach problems in their own way, grasp ideas at their own pace, and respond differently" (p. 4) to feedback. As a result, teachers must be prepared for a variety of student responses and misconceptions, which allows them to plan instructional strategies that address these diverse ideas. Anticipating various approaches a student might use during the solving of the problem, "makes it possible for

the teachers to recognize when his or her students produce it [specific solutions or misconceptions] and carefully consider beforehand what actions" (Smith et al., 2009, p. 551) the teacher could take. This proactive approach helps in creating more responsive and adaptive teaching practices (Stein, Engle, Smith, & Hughes, 2008).

4. Discourse is valued and encouraged among students and educators to build shared understanding by analyzing and comparing students approaches and arguments:

Promoting student discourse is essential for developing mathematical understanding. Encouraging students to explain their reasoning, ask questions, and engage in discussions with peers helps deepen their conceptual knowledge and problem-solving skills (NCTM, 2014; Smith et al., 2009). Discourse-rich environments support collaborative learning and the co-construction of knowledge (Hiebert & Wearne, 1993; NCTM, 2014). This collaborative learning environment encourages students to "discuss their ideas with peers, exchange different points of view, question others, seek clarification and participate in higher-order thinking such as managing, organizing, critical analysis, problem resolution, and the creation of new learning and deeper understanding" (Scott, 2015, p. 6).

Walshaw and Anthony (2008) provide strong evidence of how important discourse is in effective mathematical instructional practice. The authors explain that these instructional practices must include "developing a community of learners where participation carries rights and obligations; careful listening, synthesizing, and questioning of student responses to move mathematical thinking forward; sharing and modeling conventional mathematical language; and providing regular opportunities for students to engage in mathematical argumentation" (p. 522). Bruce and Flynn (2011) found that when student dialogue in mathematics classrooms was supported with math-talk guidelines (sentence stems that are provided to support or initiate mathematical conversations), students were able to co-construct mathematical concepts in ways that clarified ideas and empowered students. Research has found that norms (like math-

talk guidelines) are important to produce mathematical debate and effective mathematical explanations (Bruce & Flynn, 2011; Yackel & Cobb, 1996).

5. Listening to students' ideas and use student ideas in ways that support and advance learning:

Listening to students is fundamental in a reformed mathematics classroom. It involves paying close attention to students' ideas, reasoning, and misconceptions to guide instruction effectively. This approach values students' voices and recognizes the importance of understanding their thought processes (Kazemi & Stipek, 2001). This aspect links back to previous discussions about hermeneutic listening and the pedagogy of listening as described by Davis (1997) and Rinaldi (2006).

6. Conceptual understanding is promoted to act as a foundation for procedural fluency, so that students can develop flexibility with their ideas:

The development of conceptual understanding is also an important aspect of reform mathematics. I refer to the work of Richland et al. (2012) to give context to the term 'conceptual understanding'. Richland et al. (2012) describe conceptual understanding as

> an expert-like fluency with the conceptual structure of a domain. This level of understanding allows learners to think generatively within that content area, enabling them to select appropriate procedures for each step when solving new problems, make predictions about the structure of solutions, and construct new understandings and problem-solving strategies. (p. 190)

Yurekli et al. (2020) describe conceptual understanding as the mathematical connections and comprehension of and between concepts, operations, and relations. Mathematics reform places a strong emphasis on developing students' conceptual understanding rather than mere procedural fluency (Leijen et al., 2023). Students develop understanding in ways that make

sense to them and are built on their own prior knowledge. Understanding the underlying concepts allows students to apply their knowledge flexibly and to solve novel problems (Leijen et al., 2023). This approach contrasts with traditional methods that often prioritize rote memorization and the application of algorithms (Hiebert & Grouws, 2007).

According to Lestari and Surya (2017) individuals are displaying conceptual understanding of mathematics when they can do the following:

- Give a clear explanation of the concept found, both verbally and in writing,
- Identify problems and provide examples of non-examples,
- Use diagrams or symbols to present a concept,
- Change one form of representation to present a concept,
- Know various meaningful concepts and be able to interpret them,
- Identify given concepts and understand them,
- Compare and differentiate concepts.

Ultimately conceptual understanding refers to the understanding of why and how something works mathematically. For example, a student may know 3 + 2 = 5 without understanding the concept. A student can memorize this fact but can have no idea of its real meaning. However, if a student understands that numbers can be composed and decomposed, they are developing conceptual understanding of numbers. The student would then understand that 5 can be split into various combinations, such as 1 and 4, or 2 and 3, and we would identify that as building conceptual understanding. The student would also be able to represent 5 in a variety of ways.

Individuals choosing education as a profession have inadvertently developed their understanding of pedagogy (as students) long before they even enter pre-service education programs (Gainsberg, 2012). As a result, many teachers have very little experience with reform-based mathematics and often revert to rote learning and endless textbook drills (often referred to as 'traditional' North American pedagogy of mathematics). This pedagogy does extraordinarily little to promote deep learning and build conceptual understanding which is required as a foundation to be able to move into abstract thought (Lindsey & Stigler, 2012; Melville & Kajander, 2013). Often perceived to be more easily implemented than reform-based mathematics (Handal & Herrington, 2003), it is not a wonder that this 'traditional' pedagogy has a stronghold in school systems.

Mathematics is often perceived as a universal and culture-free subject (Bishop, Seah, & Chin, 2003). This perception leads to the notion that mathematics and values are separate, with math seen as a technique-oriented curriculum devoid of social aspects (Bishop, 2005, 2012). However, values and beliefs, while closely related, are distinct; values are the actions of beliefs (Bishop, 2005). Values come into play whenever teachers make pedagogical decisions, as these choices are based on certain values.

Teacher beliefs about mathematics and how it should be taught significantly influence daily teaching decisions. Maas (2011) explains that these beliefs shape instructional practices, impacting student engagement and learning outcomes. Ernest (1989) categorizes the nature of mathematics into three views:

1. **Instrumentalist View**: Mathematics is seen as an accumulation of facts, skills, and rules used for external purposes (Ernest, 1989).
2. **Platonist View**: Mathematics is a static body of pre-existing knowledge awaiting discovery (Beswick, 2011).

3. **Problem Solving View**: Mathematics is a dynamic and creative human invention, seen as a process rather than a product (Beswick, 2011). This view reflects the contemporary position of mathematicians (Beswick, 2011).

Maas (2011) builds on Ernest's constructs, identifying two types of teachers based on their beliefs about effective mathematics teaching:

- **Transmission Teacher**: Prioritizes logical ideas and knowledge transmission, where teaching is a one-way process focused on reproduction and repetition. Teachers present solutions to students and students are expected to mimic those solutions. (Leijen et al., 2023; Liljedahl, 2020; Maas, 2011).
- **Constructivist Teacher**: Believes in engaging students in thinking processes and problem-solving tasks, fostering deeper conceptual understanding and meaningful learning (Leijen et al., 2023; Maas, 2011).

The relationship between teacher beliefs and instructional practices in mathematics education is well-documented. Ball (1990) explores how teachers' mathematical knowledge and beliefs influence their instructional strategies. Teachers who embrace mathematics reform and understand the value of a deep understanding of mathematical concepts and the importance of conceptual understanding tend to use instructional practices that promote deep thinking and problem-solving (Ball, 1990). These teachers also view mathematics education as a means to develop logical reasoning and problem-solving skills, and they prioritize tasks which support such processes (Boaler, 2002). They are more likely to use manipulatives, real-world problems, and open-ended questions to engage students in meaningful mathematical discourse (Ball & Bass, 2000).

What Math Do Teachers Need?

There are many ways in which the literature considers what mathematical knowledge teachers need for teaching. Let us consider the way in which Brent Davis and Moshe Renert (2013) discuss the knowledge teachers need for teaching mathematics. The authors explain that the knowledge teachers need to teach mathematics is not a simple, clear cut set of discrete skills, but a sophisticated mix of elements "with various realizations of mathematical concepts and awareness of the complex processes through which mathematics is produced" (Davis & Renert, 2013, p. 247). Davis and Renert (2013) take the term 'realizations' from the work of Sfard (2008) where Sfard referred to the connections and associations that a learner might use to make sense of mathematics. Davis and Renert (2013) explain that the term 'emergent' refers to the "adaptive and evolutionary dynamics described by complexity researchers" (p. 247) and they use this term to describe the "coherent-but-never-fixed character" (p. 247) of the multifaceted form of mathematics that teachers need for teaching.

Davis and Renert (2013) suggest that school mathematics tends to be quite limited, specific, and static and therefore the teacher's knowledge seems to be the same. Yet the authors suggest that rather than looking at that body of knowledge as being a discrete body of basic knowledge, they suggest that this body of knowledge could be viewed as a flexible and vibrant type of knowledge that exists across a body of professionals. In this light, they position "mathematics knowledge for teaching in terms of a learnable participatory disposition within an evolving knowledge domain" (Davis & Renert, 2013, p. 247). This notion is gleaned from the authors' extensive research which suggests that when teachers participate in shared mathematical activities, these activities have an immediate and substantial influence on their knowledge of mathematics and in turn, their teaching practice.

When we consider mathematics as a participatory body of knowledge, we can identify this as a knowledge that is emergent. Davis and Renert (2013) share that the idea of

emergence describes what is happening in the experiences teachers are having. They explain that their understanding of emergent knowledge suggests that the math teachers need for teaching is “a responsive and autopoietic system of realizations that is distributed across a body of educators” (Davis & Renert, 2013, p.250). The diversity of realizations among educators contributes to powerful, conceptual, and adaptable understandings, that allow the educator to move flexibly among the interpretations, combining them and then choosing the one that applies to the circumstances at hand.

The mathematical knowledge for teaching is complex in its development and existence. Davis and Renert (2013) explain that the process of developing and identifying realizations in a collective setting is neither simple, apparent, nor linear. The realizations are all the different ways a learner (in this case teacher) might associate or connect to a given mathematical concept. Every individual has and applies a personal set of realizations. Where some of those realizations may be relatable by many, some other realizations may be idiosyncratic or shared by only a few people. Additionally, realizations are not static. Realizations change and evolve throughout the process. Sometimes more realizations are added, and sometimes other realizations are discarded.

Therefore, when considering Davis and Renert’s (2013) vision of mathematics teachers need for teaching, we can see that essentially, they are suggesting that this mathematics “is a way of being” (p. 263) with knowledge that allows educators to structure learning contexts, thoughtfully consider student actions, and be responsive in a flexible way. This also connects with the discussion mentioned previously where Mason and Davis (2013) explain ‘in-the-moment’ actions of teachers. Seemingly, the above authors all suggest that mathematics is something that is understood through and in interactions with others. Teachers should engage with mathematics in this way developing a ‘way of being’ with mathematics and their students. Teachers need to be aware of the interactions and listen to students carefully, hermeneutically,

as has already been discussed. Through this listening teachers can then make purposeful and responsive choices as their 'in-the-moment' actions. This teacher allows students to expand their learning and understandings and interpret possibilities by accessing powerful connections and representations.

The Gap Between Theory and Practice

There continues to exist a gap between theory and practice in education (Allen, 2011; Kaufman, 2003). Specifically in mathematics education, much literature (e.g., Laletas et al., 2022; Shaharabani & Yarden, 2019) discusses the gap and potential reasons for it continuing. Scott (2015) acknowledges that there is worldwide agreement about what learners need. Learners in the 21st century need to be able to think critically, communicate effectively, innovate, and solve problems through negotiation and collaboration (Scott, 2015). However, the author states that "pedagogy has not adapted to address these new challenges" (Scott, 2015, p. 2). Scott (2015) states that the 'transmission' model reigns as the dominant model of instruction in education throughout most of the world.

Shaharabani and Yarden (2019) discovered in their work that teachers seem to not be able to relate to research or theories, as the teachers feel theories are irrelevant and far from the work they do every day. Laletas et al. (2022) concur that the theory-practice divide continues to occur as teachers struggle to integrate the academic knowledge, they have learned with the real-world practice they enact every day. Ribaeus et al. (2022) add that often individuals see research-based knowledge and experience-based knowledge as two different knowledge domains and therefore practitioners have difficulty assimilating the two. Kaufman (2003) builds on this idea in saying that the gap created between academics and practice leads to "a perception of theory as belonging to an 'ivory tower' and not relevant to practice" (p. 213).

Teachers' thoughts about reform versus the practice they currently hold also have an impact on the gap between theory and practice. Teachers have formed their thoughts regarding what math is and what is not, by their experiences as math teachers and math learners (Allen, 2011). There has been a "longstanding tradition of a particular lesson structure commonly found in American classrooms" (Allen, 2011, p. 2), which also affects reform. Teachers often desire certainty in their practice and educators are going to draw certainty from practices that they know well, for example rote memorization (Melville et al., 2013). Therefore, Melville et al. (2003) explain that teacher uncertainty is also a factor that contributes to the divide between theory and practice.

There is considerable literature that documents the difficulties teachers are having with changing practices in the mathematics classroom. Chapman (2015) states that meaningful change in practice continues to be a challenge for many teachers. Drake (2006) recognizes that there are some examples of teachers successfully adapting their learning and teaching practices in ways which align with reform practices, but they maintain that research also demonstrates how many teachers' practices have remained quite disparate from practices proposed by current mathematics reform. Allen (2011) acknowledges that change is slow, despite years of research on how people think and learn. Many teachers are not accustomed to teaching using student ideas and problem solving and lack the understanding of how to cultivate student conceptions to deeper, more meaningful learning (Allen, 2011). There is evidence that suggests teachers find using students' mathematical ideas in their lessons challenging because teachers struggle with the complexity of recognizing and interpreting student ideas (Leatham et al., 2015). Despite teachers wanting to do what is best for their students, teachers remain confused and anxious about reform practices (Sun Lee et al., 2009). Research suggests that traditional math teaching is easier than trying to adopt approaches that are more progressive; innovation puts additional demands on teachers (Handal & Herrington, 2003). Therefore,

teachers may rely on their own ideas, knowledge, and experiences, rather than the most current research in pedagogy (Handal & Herrington, 2003). Albeit NCTM (2000, 2017) is explicit in their recommendations for teacher practice and methods of mathematics teaching including problem solving and reasoning. There is convincing evidence (Allen, 2011; Chapman, 2015; Gainsberg, 2012; Stemhagen, 2011) that the suggestions from NCTM (2000) are not being implemented in an expansive way in classrooms across North America.

Research explains what practices educators should consider in reform-based mathematics and those that educators should abandon. Traditional approaches like rote memorization and isolated skill practice will not advance learners' critical thinking abilities (Scott, 2015). The research also states that students should be engaged in dialogue. Students should be in control of the dialogue and have many opportunities to dialogue about their ideas and the ideas of others (Allen, 2011; Schoenfeld, 2020). However, Allen (2011) states that this rarely happens in classrooms. In fact, Allen (2011) explains that teacher talk typically far outweighs student talk in most classrooms. In one example, researchers recorded students in math classrooms and student-initiated talk occurred for only 7 to 8 minutes out of 150 minutes, with the remainder taken up by teacher talk (Allen, 2011).

Educators must offer students opportunities to explore mathematics within the context of inquiry (Melville et al., 2015; Schoenfeld, 2020; Scott, 2015). Scott (2015) states that "learners are more successful at acquiring new competencies when they build strong metacognitive abilities, reflect objectively on new concepts learned, and integrate that information with their existing knowledge and skills" (p. 3). Scott (2015) explains that when students are not required to "memorize and repeat disconnected facts and knowledge" (p. 3) but are encouraged to contemplate difficult concepts, evaluate innovative ideas, and make conclusions based on their own reactions and insights, deeper learning occurs. Much research promotes this type of problem solving (Hiebert et al., 1996; Melville et al., 2013; NCTM, 2014). NCTM (2014) advises

that the learning of mathematics must include tasks that promote reasoning and problem solving, opportunities to make connections and use representations, pose intentional questions, build understanding conceptually, encourage productive struggle, and use student ideas to promote mathematical conversations.

Literature does suggest ways to narrow the gap between theory and practice. Scott (2015) suggests that in 21st century learning contexts, teachers will need to change their position from “content conveyors to content curators” (p. 9). This lines up with the ethos of a pedagogy that embraces pedagogical documentation. Teachers change their position from a traditional ‘giver of knowledge’ to one who co-learns and co-researches, participating in meaning making with the child. Chapman (2016) states that teachers need to realize the need for change by way of their own conceptions and teaching. Teachers need to understand the difference between their current practice and what research suggests, and then they need the confidence to carry it through successfully (Chapman, 2016). Melville et al. (2013) explain that time spent in professional development on mathematical reform should focus on supporting teachers with their efforts to learn and relearn practices of how to teach mathematics.

Although not specifically math reform, some professional development opportunities have come through action research and work with case studies, in efforts to bridge theory and practice. Shaharabani et al. (2019) examined teacher’s questions of their own practice in order to support teachers to bridge the divide between theory and practice. Through questioning practices, the participating teachers formed two distinct types of connections. First, teachers were able to identify theory’s contribution to their practice. Second, teachers began to recognize their practices confirming the theories. Shaharabani et al. (2019) suggest that in their work teachers demonstrated a desire to develop professionally through their questions and in effect bring the two worlds of theory and practice closer together.

Literature discusses approaches to mathematics reform. Much research supports teachers being able to reflect on their own practice and consider other ways to enact teaching of mathematics. For example, Chapman (2016) describes work with teachers where change occurred based on the teacher's own awareness of their practice. The development of reflective awareness had a positive and meaningful impact on their practice. "Self-based questioning, meaning-based questioning, and creating pedagogical models" (Chapman, 2016, p. 322) were all central features that helped teachers develop their own reflective awareness.

Leatham et al. (2015) also describe work that helped support teachers develop their practice. In this study teachers developed their awareness about student ideas and what attributes of these ideas were productive to pursue. Through noticing student ideas, teachers began to recognize and understand aspects of student conceptions that could produce rich mathematical learning in the classroom. Leatham et al. (2015) discuss three interrelated skills teachers developed. First teachers developed recognizing what is important in mathematical situations, then they needed to be able to make connections between the specifics of the mathematics in context, and finally teachers needed to be able to reason about the mathematics of the given situation. The specifics of student conceptions that came to be recognized as having immense potential to provoke mathematical ideas became known as **Mathematically Significant Pedagogical Opportunity to Build on Student Thinking (MOSTs)** (Leatham et al., 2015).

Laletas et al. (2022) describe a professional development experience whereby leaders encouraged teachers "to critically reflect and learn together through 'dialogic pedagogy'" (Laletas, 2022, p. 488). Through co-teaching, teachers engaged in a process of posing critical questions about practice and co-constructing knowledge. Through this mutual learning process, teachers were able to construct their knowledge together by "questioning, interrogation and negotiation of ideas and opinions" (Laletas et al., 2022, p. 488). This dialogic process served as

a means through which the teachers could raise their own awareness and address complex issues in their practice (Laletas et al., 2022).

Chapman (2016) discusses how the body of research literature in mathematics education has grown in the last two decades. Studies have offered insights to support mathematics teacher educators in helping pre-service and in-service teachers change their interpretations and practice to move in the direction of the current reform perspectives of mathematics education (e.g., NCTM, 2000). It is curious that many university pre-teacher education programs promote and teach reform-oriented methods, but mathematics teaching remains largely characterized by direct instruction and students repeating procedures for memorization (Allen, 2011, Gainsburg, 2012). UNESCO (2015) also recognizes the disparity between 21st century skills and pedagogies that still exist in our classrooms. UNESCO (2015) acknowledges that although there is agreement worldwide that people in this society need to be able to think critically, be innovative, solve problems collaboratively, and have skills to negotiate professionally, the pedagogy in classrooms has not changed significantly to address these changing needs. The 'lecture' model of instruction remains the dominant instructional approach. As a result, these trends make it difficult to proclaim that education has achieved mathematics reform on a large scale.

Pre-service teacher (PST) education or teacher education in general, has perhaps expected too much from teachers in having them take general ideas they have learned and apply those to specific classroom activities (Gainsberg, 2012). Research suggests that the more natural order of learning would be from specific to general (Gainsberg, p. 363). Gainsberg suggests that teacher education programs should give teachers practical tools and strategies to use to inform their learning. Most teachers, especially pre-service teachers, must 'do' before they can conceptualize the theory. Some research suggests that reform practices in mathematics may only be aligned in practice once a teacher's beliefs have been changed. Yet,

Gainsberg proposes that teachers are more apt to change their beliefs once they try a practice or strategy and the teacher (or PST) sees and experiences the positive outcome of that work.

Chapter 3: Conceptual Framework

The landscape of educational research has evolved significantly over the past six decades, with a notable shift from scrutinizing teachers' observable behaviors to delving into the realm of their thoughts and beliefs. In this dissertation I will explore the concept that in order to understand teaching, exploring what teachers think is crucial. The trajectory of research in the 1980s witnessed a pivotal turn as cognitive psychology gained prominence, prompting a transition from analyzing teachers' actions to actively listening to their ideas about their practice. The conceptual framework I have chosen helps focus the discussion on two pivotal aspects in research—teacher thinking and teacher beliefs—both realms that shape instructional decisions and, consequently, student outcomes. By listening to teachers' stories and delving into the depths of their reflections, this research seeks not only to comprehend the complexity of educators' thoughts but also to acknowledge the profound impact their beliefs have on shaping the decisions in the classroom. The following chapter explains the dimensions of teacher thinking and teacher beliefs, delving into the nuances of these constructs and how they apply to this study, as well as contextualizing their significance in the broader body of educational research.

Listening to what teachers think

Research that includes interviewing teachers and focuses on listening to teachers talk about what they do has been ongoing since the 1980's, but research into teacher practice has been ongoing for more than 60 years (Fives & Buehl, 2012). In the beginning, research about teaching focused on teachers' actions and what occurred because of those actions (Fang, 1996). There was an emphasis on teachers' behaviours, as researchers thought and accepted that their behaviours influenced students' behaviours which then impacted student achievement (Fang, 1996). Therefore, data analysis became decontextualized to search for positive and

generalizable information that could help develop theories about teaching and learning (Fang, 1996).

In the ‘80’s research became influenced by the rising interest in cognitive psychology (Fang, 1996). This was impactful as interest in cognitive psychology meant that more researchers started to interview teachers and really listen to how they describe what they do. Teacher thinking began to appear more in the literature. Researchers began to understand and represent teacher’s thoughts in a more cyclical or iterative way (Fang, 1996). Research showed that teachers must consider many factors and make many decisions in their practice (Kennedy, 2016). This research demonstrated that teachers sometimes must deal with conflicting ideas and goals in their work and those factors all impact the decisions teachers make (Kennedy, 2016). It also became apparent, from this research that teachers move iteratively between their own decisions and the feedback that students provide back to their own decisions again. The research suggested that teacher behaviour does influence student behaviour, but student behaviour also influences teacher behaviour (Fang, 1996). The author suggests that as a result the iteration between the two impacts student achievement. So began the shift of research from how teachers behave to how teachers think. A couple of underlying assumptions resulted from the work that involves how teachers think (Fang, 1996). Fang (1996) explains first, teachers are professionals who must make informed decisions daily, amidst variable factors like classroom environments, students, and other factors that play upon the daily actions of teachers. Second, teacher thoughts, actions, and decisions do impact teacher behaviour (Fang, 1996).

An examination of the literature on teacher thinking reveals that there are a variety of ways to describe what informs teacher thinking and action. Researchers, educators, and education often describe teacher thinking using terms like teacher perspective, teacher dispositions, teacher voice, teacher thinking and teacher beliefs. In the following sections, I will explore teacher thinking and teacher beliefs as the two areas of focus, considering why it is

important to listen to teachers. The following section will help me define and discuss teacher thinking and teacher beliefs, to provide context for the conceptual perspective being used in this study.

Teacher Thinking

Researchers suggest that it is obvious that what teachers do is directly related to what teachers think (Blanton & Moorman, 1987; Clark & Peterson, 1984; Nisbet & Warren, 2000; Sato & Takahashi, 2004; Tsui, 2003). In my work, we can consider 'thoughts' as an intellectual product or organized view an individual holds (Connors, 1979). 'Thinking' is the action of using the mind to produce thoughts (Merriam-Webster, 2023). 'Thought processes' are how teachers gather, organize, interpret, and evaluate the information they think about (Connors, 1979). Clark and Peterson (1984) suggest we should research teacher thinking so that we can understand the human endeavour of teaching. Teachers think and make a host of instructional judgements (Fang, 1996), which are unobservable, but greatly influence what teachers say and do.

Academics claim that studies produced around teacher thinking help inform organizational structures for teacher educators and researchers (Clark and Lampert, 1986). Clark and Lampert (1986) assert that these structures can help teachers become more reflective professionals, and research on teacher thinking could become a resource for work in self-directed professional development. Moreover, researchers recommended that research on teacher thinking should occur to help teachers understand their practice rather than having their practice dictated to them (Clark & Lampert, 1986). In other words, researchers should not be looking to research for prescriptions of how teachers ought to think or be trained. Clark and Peterson (1984) suggest that the knowledge gleaned about teacher thinking that is most important, is "about the mental lives of teachers directly from the descriptions of the way teaching is" (Clark & Lampert, 1986, p. 30). This means the best source from which to discern what is important in teacher thinking, is from the teachers themselves.

Teachers often talk about their work and what they are thinking in very spontaneous ways and in narrative form (Kelchtermans, 2009). Teachers share their thinking using "anecdotes, metaphors, images and other types of storytelling to recall, share, exchange or account for their experiences in classrooms and schools" (Kelchtermans, 2009, p. 260). Kelchtermans (2009) tells us storytelling is a natural way that people make sense of the way they experience the world. The author has made a case that teachers' thinking passes through a lens which helps inform the decisions teachers make. The teachers guide their actions and interpretations through this lens in any given context. However, actions and experiences within various contexts can also change the lens (Kelchtermans, 2009). Therefore, the lens is both "a condition for and a result of the interaction(s)" (Kelchtermans, 2009, p. 261) of teaching. The author uses the analogy of a person wearing glasses. The person wearing glasses becomes so accustomed to wearing the glasses that they are not consciously aware they are doing so. The glasses only become noticed when the wearer's vision becomes blurry, or someone points out the glasses are out of date. It is then that the wearer becomes conscious of the glasses and begins to question how the glasses influence what and how they are seeing. Kelchtermans (2009) explains that this triggers an examination of the glasses and the potential to get new lenses. A teacher's thoughts pass through the lens without the teacher ever realizing. The lens informs the thoughts and guides the teacher to make decisions about their actions. The teacher may see the need to change their thinking when factors challenge their practice and blur their thinking.

Deborah Britzman (2003) draws our attention to the significance of teachers' experience and educational autobiographies in their approach to teaching. Britzman (2003) explains that teaching and the role of teachers are heavily influenced by cultural myths. The author suggests cultural myths are broadly held beliefs and assumptions about what teaching is and should be. The author reminds us that the knowledge pre-service teachers bring to the profession is based

on “approximately thirteen thousand hours observing teachers” (p. 27). Britzman (2003) explains that this observation is not passive, but charged with relations of power as students learn to interact with the formal curriculum and learn to successfully read the sub-text of the teacher in terms of their moods, behaviours, judgements, values, communication, and classroom strategies. Therefore, pre-service teachers come to the pre-service education with cultural myths as part of their thoughts. The author explains the conceptions of teacher thinking are often contradictory within given contexts and are always changing. Hence teacher thinking is overly complex and influenced by many factors. Pre-service teachers come to their education with cultural myths (Britzman, 2003) they hold, only to add a great deal of theoretical and practical experience. Britzman (2003) suggests that the myths provide the teacher and the system “a semblance of order, control, and certainty in the face of the uncertainty and vulnerability of the teacher’s world” (p. 222). The deep needs of the institution of education construct these systems of order and limitations because of the significant emphasis on social control in the school context and system (Britzman, 2003).

Britzman (2003) itemizes three cultural myths that are pervasive in education: a) everything depends on the teacher, b) the teacher is the expert, and c) teachers are self-made. The myth of everything depends on the teacher exacts an impression that unless the teacher has full control, there will not be learning. This view is problematic in that it constructs the idea that the teacher must be perfect. This means that if the teacher is at all uncertain, administrators, parents, other educators, and even students could view this as a character flaw and a case of bad management (Britzman, 2003). This leads to equating learning with social control and viewing pedagogy as a process of instilling knowledge, rather than considering “the practices that construct both knowledge and our relationships to it” (Britzman, 2003, p. 225). The reality is that not everything is contingent upon the teacher. When society and the institution of education construe the position of teaching as if everything did depend on the teacher, these

notions reduce the teacher's work to controlling the classroom and environment and exerting institutional authority in efforts to uphold the fallacy (Britzman, 2003).

Following behind the myth of everything depends on the teacher, is the myth of the teacher as an expert. Britzman (2003) explains there is a large cultural expectation that teachers are subject experts. This myth reduces knowledge to specific, discrete, and isolated units of content specific information. Whereas, if the teacher were to present any level of uncertainty, this would be considered a threat to their authority (Britzman, 2003).

The third myth, as explained by Britzman (2003) is that teachers are self-made. This belief devalues the relevance of "teacher education, educational theory, and the social process of acknowledging the values and interests one brings to and constructs because of the education encounter" (Britzman, 2003, p. 230). This gives credence to the idea that there are natural teachers; teachers who are 'born' into the profession. This myth helps deny the fact teaching is a complex web of relationships and contradictions, where struggle occurs, and teachers come to negotiate their 'teaching style.' In effect, when teachers enter the field, they must address many institutional expectations and cultural myths. Britzman (2003) sympathetically recognizes and respects the type of pressures that influence teacher thinking, because there are many.

To add to these ideas, we can consider Britzman's (2003) discussion of superficial knowledge. The author shares that superficial knowledge is deeply entwined with 'visceral knowledge' or the intuitive (gut-instinct) understandings that people have. Tacit knowledge builds superficial knowledge (like cultural myths as mentioned above) and ways of talking about and understanding the world; societal norms often influence these ways of knowing (Britzman, 2003). In this case the norms may be those that teachers have assimilated from observing as a student and engaging as a teacher. Cultural myths offer a set of images, representations,

definitions, and reasons which help measure a teacher's thoughts, emotions, and practice. Education at large governs the image of the teacher by the "taken-for-granted views of power, authority, knowledge, and identity" (Britzman, 2003, p. 30). It is the over-familiarity of the teacher's role, the teacher's classroom appearance, the school structure, and the understanding of the institution, which offer the predictability of the teacher's role and rob the teacher of their own subjectivity. In turn, the teacher's superficial knowledge also impacts teacher thinking.

We must also consider external factors that play upon teacher thinking. Pitt and Phelan (2008) explain that being autonomous, as teachers are often believed to be, means that teachers must think "for oneself in uncertain and complex situations in which judgment is more important than routine" (Pitt & Phelan, 2008, p. 189). This requires teachers to place their autonomy at the service of children. Simultaneously, often teachers are faced with "prescriptive, outcome—based curricula and systems of accountability" (Pitt & Phelan, 2008, p. 190) that also constrain their autonomy and therefore their thinking. Pitt and Phelan (2008) draw our attention to the fact that even though teachers are educated and highly skilled professionals, with specialized knowledge, who technically have the freedom to choose, they are controlled by governing bodies and systemic dogmas that regulate, or even strip them of their autonomy and in turn influence their thinking.

Goodson (1991) recognizes that to understand teacher thinking, researchers need to know much more about teachers' priorities. Goodson (1991) also believes that researchers can know more about teachers' priorities by knowing about teachers' lives and listening to their stories. In the past, researchers would often dismiss the data on teachers talking about their thinking, as researchers would edit out the data viewing it as being "too `personal', `idiosyncratic' or `soft'" (Goodson, 1991, p. 37). This led to a selective application of teacher communication in research (Goodson, 1991). Goodson (1991) explains that there are some

very valid reasons to study what teachers say about their thinking. The following points demonstrate some of Goodman's (1991) reasons to study what teachers say:

1. Social sciences are interested in a range of personal accounts of events which help clarify a more collective and generalizable understanding of teacher thinking.

2. Life experiences and background are important pieces of who a person is and what they do. The teacher invests themselves in what they do, therefore life experience and background influence their thinking and therefore their practice.

3. A teacher's lifestyle, including identities and cultures, influences thinking which influences practice. Goodson (1991) gives an example of the connection between youth culture and the curriculum reform movement of the sixties. A teacher in the sixties may have identified with youth culture (rock music, fashion, etc.) but could not bring that part of them into their teaching as they were still tied to what the curriculum dictated. Goodson (1991) points out that this is valuable information to consider for curriculum development. Leaders should design curriculum to fit teachers, rather than teachers fit to curriculum.

4. A teacher's life cycle also impacts teacher thinking and action. Research has shown that a person's perspectives at various stages of their lives affect their professional work in diverse ways.

5. Critical incidents throughout a teacher's career can also impact teacher thinking, styles, and practices. Information about teacher burnout and stress is also vital information to access as burnout and stress are also factors that affect teacher action.

Therefore, it is important to acknowledge and act upon teacher thinking in research. Goodson (1991) suggests starting with teachers' stories. Furthermore, researchers recommend it is important to start by listening to teachers' stories because teachers engaging in research with researchers may feel very vulnerable (Goodson, 1991). It is not productive for researchers to begin with the most vulnerable parts of a teachers' practice with the intention to reform the practice, as this is where the teacher will feel most scrutinized (Goodson, 1991). From the stories, researchers can begin to collect data that will help indicate areas for growth and improvement (Goodson, 1991).

Teacher Beliefs

Beliefs is another prominent theme in research that involves interviewing and listening to teachers describe teaching (or what they do). Beliefs distinguish themselves from thoughts and thinking as individuals hold beliefs within thoughts and thinking, much like opinions and judgements are held. Therefore, beliefs are a specific kind of thought. The term "teacher belief" does not have a unified definition, in fact many definitions coexist (Pajares, 1992). For example, Ableson (1979) describes beliefs as the mind arranging and organizing knowledge for a specific purpose under specific circumstances. Others have said that beliefs are dispositions that are time and context specific and determinants of actions and behaviour (Brown & Conney, 1982). Additionally, Sigel (1985) has pointed out that beliefs are "mental constructions of experience-often condensed and integrated into schemata or concepts" (p. 351) that guide behaviour.

Nonetheless, there has been no universal definition of 'teacher beliefs' to which scholars have agreed (Cross, 2009). Cross (2009) defines beliefs as "embodied conscious and unconscious ideas and thoughts about oneself, the world, and one's position in it, developed through membership in various social groups; these ideas are considered by the individual to be true" (p. 326). Cross (2009) suggests that beliefs influence thinking in the way people solve problems and structure tasks. Even more than knowledge and facts, beliefs guide and influence

how people act, think, and emote (Bandura, 1987; Cross, 2009). This means that thinking is the action of using the mind to produce thoughts, and beliefs are a specific kind of thought produced. My work aligns with Cross' (2009) definition of beliefs as well as that provided by Russo et al. (2020). Russo et al. (2020) discuss beliefs as referring "to a teacher's personal view on a specific issue or practice that they consider to be true" (p. 2). Therefore, the beliefs of the participants in this study are situated in the understanding that beliefs include conscious and unconscious thoughts on specific issues or practices which an individual considers to be true, which participants have developed through membership in various social groups.

It is important to consider teacher beliefs, as beliefs about teaching are major determinants of what choices teachers make in the classroom (Nisbit & Warren, 2000; Sato &Takahashi, 2004; Tan, 2011; Tsui, 2003) and the environments that people create for themselves and people around them (Conners, 1978). Teachers make decisions in the moment as their instruction and response to students unfolds, and in doing so they draw upon their beliefs to help decide what to say or how to act (Schultz & Cross, 2020). Fives and Buehl (2012) note that beliefs can therefore support or impede classroom practice. Beliefs function as filters for interpretation, structures for defining problems, and/or guides for action (Fives & Buehl, 2012).

Errington (2004) suggests that linked to a teacher's belief system are ideas about teaching and learning that a teacher holds. These ideas could include beliefs about what they should be teaching, what students should be learning, and the respective responsibilities each have (Errington, 2004). These beliefs all play a part in the decisions teachers make daily. Many factors influence teacher beliefs (Fang, 1996). Such influences include, just to name a few, the quality of the teacher's pre-service program, the kind and quantity of reflection done in the pre-service program, and the subculture of the discipline (Fang, 1996). Teacher beliefs can manifest themselves in everything a teacher does. For example, beliefs could exist in the teacher's

expectations of their students or in the teacher's understanding and interpretation of a specific subject area (Fang, 1996). In any case, a teacher's beliefs can have a significant impact on student learning (Fang, 1996).

Kagan (1992) explains that much of a teacher's belief system is tacit. Most teachers do not even recognize that their belief systems exist or have the language to describe and label their beliefs (Kagan, 1992). This suggests that beliefs function as any other type of knowledge; they are implicitly held assumptions and understandings that individuals bring to their work (Kagan, 1992). This is critical to contemplate when considering the power beliefs have on behaviour and action. Nespor (1987) explains that belief systems do not need general or group consensus to be valid or appropriate. Belief systems are "disputable, more inflexible, and less dynamic than knowledge systems" (Pajares, 1992, p. 311).

Posner et al. (1982) have used Piaget's concepts of assimilation and accommodation to describe change in beliefs. Assimilation occurs when an individual incorporates a new belief into existing beliefs. Accommodation occurs when an individual cannot incorporate a new belief into existing beliefs so the mind must reorganize existing beliefs to allow space for new beliefs (Pajares, 1992). When beliefs are strong it is more likely that a person will accommodate rather than assimilate their new beliefs (Pajares, 1992). People do not necessarily hold beliefs because of how logical they are or how indisputable they are. People often hold beliefs because they align with other beliefs in the system (Schutz & Cross Francis, 2020). Beliefs are unlikely to change unless something challenges a belief, and a person cannot assimilate them into existing systems. Resistance occurs and the belief change happens only as a last resort (Pajares, 1992). This illustrates how powerful belief systems can be. Teachers may have beliefs they do not even realize are there but are brought to bear in the daily work the teacher does. These beliefs do not need consensus with other people, or even with other beliefs, and are valid merely by being held by the individual.

Researchers have put forward that how knowledge is viewed in a specific area of study, along with the content that is taught and how it is taught, has a significant impact on a person's beliefs about knowledge in that discipline (Cross, 2009). Beliefs emerge over years of education and experience in various capacities. Beliefs are difficult to change and remain fixed despite the acquisition of more education or experience (Cross, 2009). Cross (2009) describes relationships between beliefs in terms of 'dimensions' to help understand the resistance beliefs must change. His 'dimensions' are as follows:

1. The first dimension suggests that beliefs can be organized like premises and conclusions. This means that a belief could be a proposition to an argument where another belief could be a proposition to a different argument. The conclusion is a result of the relationship between the first two propositions. For example, mammals are warm blooded. Whales are mammals. As a result of proposition one and two the conclusion is, whales are warm blooded (Cross, 2009).

2. The second dimension deals with the psychological strength of a belief. Those beliefs with strong psychological strength are called core beliefs. Those beliefs with less psychological strength are called peripheral beliefs. As these beliefs can be separate and independent of each other, a person can hold conflicting beliefs at the same time. For example, a teacher may believe that students should try multiple strategies in mathematics. The same teacher may only accept the specific strategies they taught the class. If a student offers something altogether different, the teacher would not accept the strategy. These two beliefs contradict each other. People can hold these beliefs simultaneously if the person does not have to consider the beliefs concurrently (Cross, 2009). So as long as the teacher is not considering the true meaning of their first belief about students learning multiple strategies while they are only accepting the strategies

they taught, there will not be any cognitive conflict and the teacher can continue to hold both beliefs.

3. The third dimension discusses the way that beliefs are clustered. Clustering of beliefs offers protection from competing beliefs. Sometimes the clustering of beliefs is upheld by another belief. For example, a core belief could be that a person believes all students should have a chance to be successful at school. The same person could believe that students who are not in the gifted program should not be able to take advanced math classes. Although these two beliefs are conflicting and inconsistent, a person holds these beliefs together by the person's belief that abilities are fixed (Cross, 2009). The belief that abilities are fixed acts like a shield protecting the two other conflicting beliefs from butting up against one another.

Teachers may hold implicit beliefs based on hypothetical facts or principles about their role, their students, and their responsibilities. These implicit beliefs based on hypothetical facts are called theories. These theories influence teachers' decisions around their practice (Fang, 1996). Theories are an important part of the general knowledge teachers hold and become part of the way teachers assimilate information and act upon this information in their classrooms (Fang, 1996). A set of theories becomes such a reality for teachers that they then rely upon those theories as a guide for their personal thoughts and actions (Fang, 1996). In other words, it is important to consider teachers' theories and beliefs as they are a part of the teacher's knowledge and thinking. Teachers depend on their theories and beliefs, knowingly or unknowingly, to inform their thoughts and actions in the classroom.

Additionally, research has extensively explored teacher beliefs in mathematics education, highlighting their impact on instructional practices and student outcomes. Peter Liljedahl (2014) notes the significant interest in how beliefs, attitudes, and self-efficacy shape

classroom practice. Philipp (2007) describes beliefs as the lens through which teachers make sense of the world, influencing their approach to teaching mathematics. For instance, beliefs about the nature of mathematics—whether viewed as static facts (Instrumentalist view), pre-existing knowledge (Platonist view), or a dynamic, creative process (Problem-solving view)—affect teaching methods (Beswick, 2011; Ernest, 1989). Research also has demonstrated how views on beliefs in mathematics education are still divided (Beswick, 2011; Ernest, 1989; Hannula, 2016; Liljedahl, 2014). However, I am not exploring mathematical beliefs or the research that describes it in this dissertation in any detail. Instead, I reference beliefs in mathematics to highlight that much research in this area does exist.

In conclusion, some authors have recognized the challenges teachers face daily in their work, which play upon and draw upon a teacher's thinking and beliefs. Clark and Lampert (1986) describe how teachers encounter many related and competing decisions in the work of planning and teaching which may be influenced by a teacher's beliefs. For example, that which a teacher chooses for one student might be a missed opportunity for another student. Therefore, teachers face many practical dilemmas daily in the work they do (Clark & Lampert, 1986). Clark and Peterson (1984) explain that a teacher must aggregate and make sense of a great deal of information from various sources about each individual student and their class collectively. Teachers must study, enact, and integrate a growing body of research literature on education (Clark & Peterson, 1984). Teachers must take this knowledge and integrate it with their own expectations, attitudes, beliefs, goals, and purposes (Clark & Peterson, 1984). Finally, the teacher must respond, make judgements and decisions, reflect, and then regroup their thinking to begin the process all over again (Clark & Peterson, 1984). This means that teachers are incessantly encountering information on "how to" teach, which they must continually integrate with their currently held thoughts and beliefs. Then they must use this amassed knowledge to respond to student need in planning and in the moment. Considering all that

influences what a teacher does and thinks, it is evident that researching teacher thinking is necessary and important.

Applying Clark and Peterson's Work

Clark and Peterson (1984) have played a major role in establishing how important it is to listen to teachers talk about teaching. Their work provides a framework to understand how teacher knowledge and beliefs influence teacher actions. I have chosen to use Clark and Peterson's (1984) framework to qualify how important it is to listen to teachers. I have made this choice primarily because researchers have used this framework extensively in mathematics education (e.g., Russo et al., 2020; Sullivan, 2018). Clark and Peterson's (1984) work offers a way to conceptualize how teachers talk about their thoughts and beliefs and what thoughts they think about, as well as how teachers' thinking informs their actions. Clark and Peterson (1984) suggest that there are three interrelated thought processes that influence teachers' intentions in the classroom. The three thought processes are as follows: a) teacher's planning, b) teachers' interactive thoughts and decisions, and c) teachers' theories and beliefs. (Clark & Peterson, 1984). Clark and Peterson (1984) illustrate the importance for research to collect data on the thought processes of teachers.

Allow me to explain these three thought processes further:

a) Teacher planning includes the thinking the teacher has prior to instruction. This would also include thoughts that the teacher has after instruction that informs what the teacher will plan for the next lesson.

b) Teachers' thoughts and decisions include several facets of knowledge which will be explained in further detail below.

c) Finally, teacher's theories and beliefs encompass the general knowledge and beliefs of objects, events, people, places, and their intertwining relationships (Fang, 1996).

There had previously been a fourth thought process of judgement examined in research (see Clark & Yinger, 1977). However, in Clark and Peterson's (1984) work they included the area of judgement under both a) teacher planning and b) teachers' interactive thoughts and decisions and chose not to separate judgement into a thought process of its own.

The following section will explore how other studies have employed and referenced Clark and Peterson's (1984) framework to examine a) thinking, and b) beliefs. Several works have highlighted the importance of Clark and Peterson's (1984) work. According to Kelchtermans (2009), Clark and Peterson's (1984) work has helped identify that the thinking processes and representations of teachers must be understood to conceptualize and influence teachers' decision making and actions.

Ball, Knoblock, and Hoop (2007) used Clark and Peterson's (1984) framework to inform their study to explore and understand the planning of intern and novice teachers in agricultural education. Ball, Knoblock, and Hoop (2007) stated they used the framework, as it represents a "bidirectional relationship between teachers' thought processes and their actions and their observable effects" (p. 56). Ball et al. (2007) adapted Clark and Peterson's (1984) framework to specifically look at the constraints and opportunities teachers have related to planning.

In 2018, Sullivan engaged with a project that was informed by Clark and Peterson's (1984) framework and how it proposes that teachers' actions and are directed by their beliefs and intentions. Sullivan (2018) elaborated that one of the benefits of this framework is that it provides a vehicle for anticipating reasons that teachers might be reticent in doing certain things. For example, he found that teachers may be reluctant to incorporate challenging tasks

with pedagogies that fit for those challenging tasks, because their own knowledge of the math or the accompanying pedagogies do not align with the challenging tasks (Sullivan, 2018). In effect, teachers may not have the understanding of the math itself, to support students' learning in those challenging tasks.

Russo et al. (2020) used Clark and Peterson's (1984) framework, as it emphasized the importance of teacher beliefs. Russo et al. (2020) recognized that previous frameworks had been used to examine changes in mathematics content knowledge of teachers, but they were looking for a framework that would specifically allow them to examine beliefs. Clark and Peterson's (1984) framework allowed them to examine how beliefs inform both the teachers' intentions to act and their actual teaching actions.

Other authors have referenced Clark and Peterson's (1984) work to understand teacher beliefs. For instance, Kagan (1992) explained that in the late '80's and early '90's, a variety of indirect methods emerged, for eliciting teacher beliefs. This research included asking teachers to think aloud as they considered classroom vignettes or as they watched videos of their own teaching. Kagan (1992) explained that Clark and Peterson (1984) used semi-structured interviews where teachers described specific classroom events or decisions. The researchers invited teachers to represent their understanding of certain pedagogical terms using concept maps. The researchers carefully examined the language teachers to understand teacher thought and action, informed by their beliefs (Kagan, 1992). Also, Cochran-Smith and Lytle (1999) recognized Clark and Peterson (1984) as contributing to research about the "new" image of teacher learning that was developing at the time, which was informed by research on how teachers think about their work.

In conclusion, Clark and Peterson (1984) emphasize how teacher thinking and beliefs should be the focus of education research. Clark and Peterson (1984) illustrate how research could inform education practice in ways that previous research agendas had not. For example, Clark and Peterson (1984) demonstrate how research can position teachers as professionals like doctors, lawyers, or architects, rather than simply technicians who carry out skills or procedures that were determined by others (as had been thought of teachers' roles previously). Meaning Clark and Peterson (1984) highlight the teacher as a reflective professional. Clark and Peterson (1984) state that teachers often must make quick decisions in their work and that their beliefs do influence what they say and do.

Therefore, it is vital that researchers conduct research that gathers data specifically about how teachers talk about their planning, thoughts, decisions, theories, and beliefs. In doing so this type of research will give insight into teachers' theories and beliefs. Clark and Peterson (1984) have made significant contributions to mathematics education research in developing a framework for looking at teachers' thoughts and actions. Many other authors have used and referenced Clark and Peterson's (1984) framework in many other studies. In this work, I use Clark and Peterson's (1984) work to lay significance to how important it is to listen to teachers. It is evident that beliefs are important and inform teachers' decisions, therefore listening to teachers' ideas about their decisions is important.

Clark and Peterson's (1984) foundational framework, emphasizing the intricate relationship between teachers' thoughts, beliefs, and actions, significantly informs the methodological choices for this study. I have selected phenomenography as it allows for a nuanced exploration of the numerous ways teachers perceive and experience phenomena related to their conceptions and decision-making processes. This aligns with Clark and Peterson's (1984) emphasis on understanding the diversity and complexity of teachers'

perspectives. Content analysis complements phenomenography by offering a systematic approach to examining the teachers' ideas, enabling the identification of recurring themes and patterns, across a broader range of the teachers' ideas and conceptions. By combining phenomenography and content analysis, I aim to capture the rich spectrum of teachers' thoughts and beliefs, as advocated by Clark and Peterson's (1984) framework, providing a comprehensive understanding of how educators conceptualize and navigate their professional practices.

Chapter 4: Study Design

In chapter four, I offer a comprehensive examination of the research design employed in this study, delving into the philosophical underpinnings, methodology, and specific methods chosen to investigate the research question: “How do teachers conceptualize pedagogical documentation in the early learning mathematics classroom?” I have chosen to situate my work in qualitative methods, using a combination of phenomenography and content analysis as the primary analytical methods. Phenomenography is employed to explore the diverse ways in which participants experience and think about specific instances of pedagogical documentation. Whereas I have applied content analysis to scrutinize transcripts from ten weeks of Zoom meetings and participants’ journals, offering a broader perspective on the participants’ conceptualizations of pedagogical documentation. This chapter provides detailed insights into the data collection methods, including the use of virtual meetings, audio-visual recordings, and participant-driven discussions and explains the rationale behind the selection of qualitative methods. I made my choices for the methods for my study, with the intention to offer an understanding of how teachers engage with and think about pedagogical documentation in the context of their encounters with children in the early learning mathematics classroom.

Participants

I was fortunate to have 11 Kindergarten teachers to participate in the study, which I held over the summer. We met once a week through online ‘Zoom’ meetings. Through my various networks during my career as a classroom teacher, like conferences, teaching various additional qualification courses, my work at York University, and other relationships that I had built over the years, I had connected with many teachers. Through casual conversations, teachers began showing interest in the work I was considering. As teachers reached out to

other kindergarten teachers they knew, I found myself with a group of kindergarten teachers with various experiences and years of experience, interested in participating in the study. The participants, who either were full time Kindergarten teachers, teachers who had kindergarten as a part of their teaching assignment, or pre-service teachers who had placements in kindergarten, joined the study voluntarily. I sent an email to known kindergarten teachers or kindergarten teachers who had shown interest in participating in the study. The participants volunteered to participate without any affiliation to their school or school board.

To this day, Hope remains one of my most cherished colleagues and an inspiration for my work in early learning mathematics. Hope had been teaching about 12 years at that time, spending about ten of those years in kindergarten. Hope had been through the professional development and implementation of the full day kindergarten program right from the beginning. Many of those sessions we were able to attend together. I knew that Hope had been experimenting and using pedagogical documentation in her classroom for years. Some of that work we did together when I was a Student Work Study Teacher.

I did not know Lee before the study. A colleague of mine at York University knew Lee and told her about my study. Lee was also a full-time kindergarten teacher. Lee had been teaching 22 years with ten of those in kindergarten. Lee did not consider herself good at math in the beginning of her career and did not like math. In 2011, Lee decided she would get reacquainted with math so that she could instill the love of math in her kindergarten students. Lee indicated she was extremely comfortable with pedagogical documentation and had been using it in her kindergarten classroom since 2009.

Karen was a Special Education Resource Teacher who supported in kindergarten. I met Karen through a math additional qualification course I taught. I told Karen about my study, and she was interested in participating. Karen had been teaching for over 25 years. Two years prior,

Karen had taught kindergarten full time for several years. Karen's school had received funding to transition the commercially bought toys in their classroom to natural loose parts. Karen said she had used pedagogical documentation in her kindergarten classroom as well and was extremely comfortable doing so.

Jean Claude had been a teacher candidate in my classes during his Bachelor of Education degree at York University. Jean Claude was extremely interested in mathematics education. Jean Claude and I stayed connected throughout his master's degree and into the beginning of his career. Jean Claude was a second year teacher who had just finished providing prep periods to kindergarten classes in drama, physical education, and health. Jean Claude said he was new to pedagogical documentation.

Jacques was a teacher candidate who had just finished his first year of his Bachelor of Education. I did not know Jacques before my research, but Jacques was a friend of Jean Claude. Jacques had just finished a placement in kindergarten and shared with Jean Claude that he was interested in participating in the study. Jacques had seen pedagogical documentation but said he had not done much work with pedagogical documentation during his placement. Jacques knew he would be returning to the same kindergarten placement in the fall and wanted to participate in the study for his own learning.

Reagan was a teacher candidate who had just finished her first year of her Bachelor of Education. I had taught Reagan while at the Faculty of Education at York University. Reagan had just finished a placement in kindergarten. Reagan also wanted to participate in the study for her own learning. Reagan stated her experience with pedagogical documentation was limited.

Donna was a very experienced teacher. Donna had taught 32 years in primary division. I met Donna during my time at York University as she had offered up her classroom as a space for me to take teacher candidates to explore math pedagogy. Donna had done prep coverage

for kindergarten and a reading program with kindergarten. Donna stated that her experience with pedagogical documentation was limited to photos taken during ‘reading buddy’ times.

I did not know Jane before the study. Jane was a colleague of Donna and Jean Claude. They worked in the same school board. Jane was a teacher librarian who supported kindergarten in the library. Jane had been teaching for 30 years. Jane taught kindergarten but before the implementation of the full day kindergarten program. Since the full day kindergarten implementation, Jane provided prep coverage for kindergarten teachers. Jane explained she used pedagogical documentation in the library, generally by way of photos, to provide records of the learning in the library, for the kindergarten teacher.

Hannah was a first year teacher who had just finished a long-term occasional position in kindergarten. Hannah had also been one of my students while I was at York University. Hannah said she had been using pedagogical documentation in her kindergarten classroom. Hannah’s class was online, therefore parents participated in collecting documentation for Hannah.

Stacey was a first year teacher who had just finished her first year in kindergarten. I did not know Stacey before the study. Stacey knew Jean Claude and he told her about the study. Stacey’s class had been in person that year. Stacey stated she used pedagogical documentation in her classroom.

Shira had also been a student of mine at York University. Shira and I also stayed connected as she began her career as a new teacher. Shira was a second year teacher who had just finished a long-term occasional position in kindergarten. Shira explained that she had used pedagogical documentation in her classroom. Shira followed the original teacher’s program that included parts of play-based, inquiry-based learning.

Data Collection

I chose qualitative research methods for this study, as I felt quantitative measures and/or statistical analysis did not fit the research interest of my work nor the goal of understanding participants' perspectives. I wanted to explore the research question, "How do teacher conceptualize pedagogical documentation in the early learning mathematics classroom?" by examining the variation of perspectives and experiences the participants had. The responses to this research question were contingent upon the variety of ideas offered by the participants. I did not investigate data from a statistical or numerical point of view. Qualitative research is used to explore and understand one single phenomenon and doing so involves qualitatively understanding the forces that play upon the phenomenon (Creswell, 2015). Therefore, I felt that qualitative approaches were the best choice for the type of research question being asked and the type of data I was interested in collecting.

The goal of qualitative research is to examine a complex, detailed, and nuanced understanding of a phenomenon, (Creswell, 2007; 2015). Qualitative researchers often use an emergent qualitative approach to their work. They collect data in natural settings being thoughtful and respectful of the people and places that are being studied (Creswell, 2007). The focus of anlaysis is often to establish patterns or themes (Creswell, 2007). The detail and complexity is achieved by speaking directly to people and understanding their perspectives without applying predetermined theories or expectations that have been established in the literature (Creswell, 2007). Qualitative research can also be employed when researchers want to empower people to tell their stoires and diminish the power dynamic that sometimes exists between researcher and participants (Creswell, 2007).

Creswell (2015) reminds us that the intial plan for qualitative research is not tightly prescribed. The qualitative researcher knows that phases of the process may change, including the research question, forms of data or even the site of data collection (Creswell, 2015). These

changes are all dependent upon the data that emerges during the collection process. Qualitative researchers are not bound by cause and effect relationships, but are motivated to identify the complex interactions in any given situation (Creswell, 2015). The goal is to interpret and describe the rich, detailed picture that emerges from the data (Creswell, 2015).

Creswell (2015) states that researchers make a personal assessment about the description that emerges from the data. The researcher chooses the description that fits the "themes that capture the major categories" (Creswell, 2015, p. 237) of data. This is how this qualitative research is interpretative (Creswell, 2007). Researchers offer an interpretation of what emerges from the data. Researchers are implicated by their own background, context, and understandings which cannot be separated from the interpretations they make (Creswell, 2007).

I am drawn to the ideas of Bronwyn Davies (2014a, 2014b) to explain further how I, the researcher, am implicated in my own work. Davies (2014a, 2014b) draws on the metaphor of diffraction from physics which is the way light waves combine when they overlap and create patterns of difference. Davies (2014a, 2014b) explains that qualitative research creates a diffraction in the way the interconnected nature of the phenomena, knowledge production, and the research process itself, bends and overlaps. This means that the research encounters (between myself, participants, conversations, and topics of discussion) will create spaces where being and knowing are intertwined and inseparable (Davies, 2014a, 2014b).

As Davies (2014a, 2014b) describes when using the diffraction metaphor, I am not setting out to discover objective truths; my work focuses on producing subjective truths that emerge in the specific moments of research encounters. As mentioned above, these truths are contingent upon the interactions between myself, the participants, the subject of research and the broader environment and context. The work is not linear. At the time, I did not presume to

know what will emerge from these interactions. This work values the unexpected and what emerges in the process. This work values honouring the perspectives of my participants.

I also think about the idea that this type of analysis resists standardization (Davies, 2014a, 2014b). I am attuned to how different elements come to matter in the context of the discussions and ideas of the participants, as well as the research practice itself. I acknowledge how this approach is not fixed, it recongizes and makes visible the entanglements between myself, the participants, and the world, including how these factors come into relationship and how they influence one another (Davies, 2014a, 2014b). Davies (2014a, 2014b) suggests that the notion of diffraction sheds light on the reality of 'entanglements'—not just as a theoretical construct but as an active element in the research process. This means that I, as a researcher, acknowledge that the phenomena being studied cannot be understood in isolation but is part of a web of relationships and influences that include me, the researcher, the researched, the contexts, the methods, and methodologies employed. In short, this method of qualitative research proposes a more dynamic, relational, and reflective way of engaging in my work, where my focus is on the complex interplay of factors that shape the production of knowledge. I choose to apply this approach as I aim to explore the nuanced and varied, often invisible, and interconnected forces of human experiences. My study occurred over a period of ten weeks. Every week, participants and I met through 'Zoom' virtual meetings for one and a half hours. Every session was video, and audio recorded. Some participants attended all sessions, whereas other participants missed some sessions.

The discussion topics were suggested sometimes by the participants and sometimes I suggested ideas according to the conversations we had together.

The dates and general conversation topics were as follows:

July 4th	Introductions and setting meeting times
July 8th	More introductions. Discussing what participants are interested in learning in the study. Defining 'pedagogical documentation'.
July 14th	During the week prior, I sent three articles to participants. Participants could choose one of three to read. PI attempted to capture thoughts shared about the articles on a PowerPoint slide. Definition of 'pedagogical documentation' continued.
July 28th	The week prior to this session, I provided the participants with a slide set of key learning from a graduate course I took in pedagogical documentation. Discussion of the information on the slides. Discussion of "philosophy of education" being confused with pedagogical documentation.
August 4th	Discussion of being vulnerable with each other.
August 11th	Discussion of choices being informed by values. 'The Price of Apples' documentation examined and discussed as an exemplar of mathematics and pedagogical documentation in early learning.
August 18th	Shared a video from the Ministry of Education showing a kindergarten teacher engaging with students while students play. Teacher shows how he could derail students' ideas to try to steer their 'learning' in the direction he deemed important, then he showed how he could be present and learn with students, allowing the students to direct the learning. Participants in the study brought examples of math that they had done in their own classrooms. One participant had suggested the week before they have a 'show and tell' of math.
August 25th	It was thought that sharing the participants' ways of documenting would be useful. Teachers shared math activities and ways they documented the activities.
September 1st	'The Columns of the Municipal Theatre' documentation examined and discussed as an exemplar of mathematics and pedagogical documentation in early learning.

September 5th	Participants shared their concluding thoughts about the study and how it has impacted their ideas, understandings, and practice.

Data analysis:

I have chosen to use two different categorical approaches to data analysis to examine the data: phenomenography and content analysis. Initially I chose to focus on two conversations occuring in weeks six and nine. I chose a phenomenographic approach to analyze this data. I chose to examine the variety of ways participants thought about pedagogical documentation in the context of a two specific pieces of documentation: 'The Price of Apples' and 'The Columns of the Municipal Theatre'. Content analysis was used to analyze the transciptions of all 10 weekly meetings. Content analysis was used because a significant amount of data existed and content analysis allowed specific high frequency words that occured over the 10 weeks of the study, to be analyzed. Phenomenography analysis was used to examine how the participants thought of the two specific examples of pedagogical documentation. Subsequently it made sense to examine a broader context of the data, which was accomplished through content analysis.

Phenomenographic analysis and content analysis share emergent and categorical characteristics, however, there are differences between the approaches. With these methods, the process for data analysis is iterative; data is analyzed for initial or preliminary categories and then is further refined to more specific and descriptive categories or themes (Kinnunen & Simon, 2012; Krippendorff, 2019; Weber, 1999). Phenomenographical approaches require categories to stand in relation to each other (Kinnunen & Simon, 2012). Similarily, content analysis is the subjective coding of data into categories to identify themes and patterns (Hsieh and Shannon, 2005). Finally, the analysis of data using phenomenography is usually conducted after data collection. The data analysis consistent with a content analysis approach usually also occurs

after data collection, but there are occasions when the analysis may occur during the data collection.

Allow me to indicate the differences of the two methods of analysis. First, phenomenographic analysis takes a second order approach, in that the researcher is examining the participants' interpretation of the experience or phenomena. This means that the research "aims at description, analysis, and understanding of experiences; that is, research which is directed towards experiential description" (Marton, 1981, p. 180). On the other hand, content analysis can offer the opportunity to provide new insights to a researcher's understanding of a given phenomenon (Krippendorff, 2019). The difference between these approaches is noteable as the researcher uses a method of analysis like phenomenography, to understand the participants' experiences of the phenomenon. This means that in this study, I am interested in the variations in the participants' perspectives of the experience. The other method, content analysis, is used to better understand the participants' ideas about the wider context of the data, for example all conversations about pedagogical documentation in the early learning mathematics classroom (Hsieh & Shannon, 2005). This means that by using content analysis to analyze high frequency words, I understand the data in a wider context and I gain a broader understanding of the conversations we had, as a whole. Second, in phenomenography, data unfolds as "ways in which people experience, interpret, understand, apprehend, perceive or conceptualize various aspects of reality" (Marton, 1981, p. 178) through conversations or interviews, but in content analysis themes arise from the frequency of words or concepts that appear repeatedly in a text (Hsieh & Shannon, 2005).

Analysis Inspired by Phenomenography:

History of phenomenography. A phenomenographical approach was used to analyze the way in which participants experienced the examination of the two examples of pedagogical

documentation ‘The Price of Apples’ and ‘The Columns of the Municipal Theatre’. Ference Marton (1986) wrote:

> Phenomenography investigates the qualitatively different ways in which people experience or think about various phenomena. This implies that phenomenography is not concerned solely with the phenomena that are experienced and thought about, or with the human beings who are experiencing or thinking about the phenomena. (p. 31)

The purpose of this method of analysis is to attempt to uncover the varying understandings people may have about a phenomenon and then sort these understandings into conceptual categories (Marton, 1986). Descriptions of the participants’ perceptions that result from a phenomenographic analysis are “descriptive, experiential, content-oriented, and qualitative” (Marton, 1986). Essentially, the phenomenographic analysis presents the variety of ways in which different individuals understand the same content or phenomena, as an aspect of their world (Marton & Neumann, 1989).

The process of phenomenographic analysis expresses variations of an experience in hierarchical statements or ideas moving from lower order to higher order. The lowest level presents the most simplistic way of experiencing or thinking about the phenomenon, whereas the highest level presents the “most sophisticated and developed way of experiencing the phenomenon” (Han & Ellis, 2019, p. 6). Marton (1986) stated that looking carefully at different ways people think about an experience could help transition from one way to view something to a qualitatively different way.

Data that is analyzed through phenomenography, are recorded as isolated statements that address the research question. These statements are continually sorted and re-sorted to determine distinctive characteristics in the ‘pool of meaning,’ as described by Marton (1986). The analysis is termed ‘dialectical’ as the meaning of the data is determined by bringing

statements together and comparing them in qualitatively different ways (Marton, 1986). The goal is to examine the range of meanings within a sample group, as a group and not as individuals within the group (Akerlind, 2005). Therefore, no one transcription of data can be understood individually, separated from the others (Akerlind, 2005).

Application of phenomenography. I have chosen to use a phenomenographical approach as phenomenography is well established in mathematics education (Erdogan, 2012; Gooya et al., 2013; Inan et al., 2010; Neuman, 1997). In this work, I wanted to understand the variations of the perceptions and experiences the kindergarten teachers had in working with pedagogical documentation. I feel that Clark and Peterson's (1984) work aligns with phenomenography due to its emphasis on understanding the nature of teachers' thoughts and beliefs, which resonates with the core principles of phenomenography. As mentioned, phenomenography, as a qualitative research approach, seeks to explore and describe the variation in individuals' experiences and perceptions of a phenomenon. I feel that Clark and Peterson's (1984) framework, emphasizing the interconnected thought processes of teachers, provides a conceptual foundation that complements phenomenography's focus on capturing the diversity of perspectives.

Clark and Peterson's (1984) framework, highlighting the complexity of teachers' thinking, aligns with phenomenography's goal of revealing the various ways teachers conceptualize their practices, beliefs, and decision-making processes. The framework's recognition of the bidirectional relationship between teachers' thoughts and actions resonates with phenomenography's interest in exploring the connections between individual experiences and their subsequent actions.

Undoubtedly, there are other approaches that would have been appropriate to use to categorize and analyse the data, however, I chose phenomenography as it seemed to be the method that would offer the variations of experience that interested me. Seeing as the two areas

of interest in this dissertation are mathematics education and early childhood education, I recognize that other options, like discourse analysis, could have also been in line with the intentions of my work.

Therefore, I chose Clark and Peterson's (1984) work as their work is established in mathematics education and their work fits well with phenomenography as it shares a common goal of delving into the rich, varied dimensions of teachers' experiences and perceptions, offering a complementary perspective for understanding the complexities inherent in educators' thinking processes.

Phenomenographical analysis

The first two analyses were phenomenographical and came from conversations of two examples of pedagogical documentation named 'The Price of Apples' and 'The Columns of the Municipal Theatre'. Both of these examples of pedagogical documentation revealed a learning event that occurred with a small group of children in their classroom and in an outdoor context. The two conversations were not combined, rather they were analyzed separately. The first was the "Price of Apples" followed by the "Municipal Theatre".

Phenomenography is a well-established qualitative research method and has been adopted for much research in education (Han & Ellis, 2019), and specifically mathematics education. Phenomenography was an appropriate choice for this study, as the study did not set out to confirm or affirm a predetermined idea (Hansson Halleröd et al., 2015). The intention of the study rather, was to explore and discover new insights into a deeper understanding of the participants' varied conceptions of pedagogical documentation. Eleven different teachers with varying backgrounds and experience participated in my study. The teachers were from different school boards, lived in different communities, and even had different experiences in teaching kindergarten. Therefore, to capture the rich variation of what these teachers might bring to our

conversations, I decided that phenomenography would be a method that would allow me to do just that. I have used phenomenography before and felt comfortable with the process. Additionally, as has been previously mentioned, phenomenography is a well-established method in mathematics education. Contrasting categories of meaning were developed and compared for similarities and differences, ultimately reaching a description of the unique characteristics of each category and a higher level of abstraction (Ljunggren, Carlson, & Isma, 2021), in the process.

Statements made that related to the research question in the conversation about 'The Price of Apples' were transcribed onto cards. These statements were those that responded to the question "What are the qualitatively different ways kindergarten teachers think about or experience pedagogical documentation that portray mathematical learning experienced inside and beyond the classroom?" Data came from the transcriptions of the meetings, as well as the participants' journals. The researcher engaged in a phenomenographical approach to make a pool of meaning of the statements that occurred in that conversation. Cards were sorted into categories. Cards were sorted and categorized repeatedly through an iterative process. Initially two broad categories of 'pedagogical documentation' and 'big math ideas' emerged. Under the broad category of 'pedagogical documentation' came themes like 'what is pedagogical documentation', 'provocations', 'factors', 'considerations', 'challenges', and 'questions'. Under the broad category of 'big math ideas' came themes like 'student action', 'teacher action', 'math happening' and 'number'. Over 13 iterations of data analysis occurred. In the end two broad categories of 'pedagogical documentation' and 'big ideas in math' were created with several themes within each category.

As with the data of 'The Price of Apples', the data of 'The Columns of the Municipal Theatre' was also analyzed using a phenomenographical approach. All statements that occurred in the conversation about 'The Columns of the Municipal Theatre' pedagogical

documentation, were transcribed onto cards to respond to the research question “What are the qualitatively different ways kindergarten teachers think about or experience pedagogical documentation that portray mathematical learning experienced inside and beyond the classroom?”. The cards were sorted and categorized in the same iterative process as ‘The Price of Apples’. Several general themes were developed in the first iteration of categorizing the data. The following illustrate the initial themes: ‘has the word curriculum’, ‘math’, ‘teacher led or student led’, ‘Reggio’, ‘environment’, ‘connections’, ‘what kids can do’, and ‘constructivism’. After 9 iterations of ‘The Columns of the Municipal Theatre’ data, it was evident that teachers thought about the pedagogical documentation of ‘The Columns of the Municipal Theatre’ as: 1) concepts relating to a Reggio approach, 2) relationships and connections between educators, students, and curriculum, 3) various actions of students and teachers, and 4) relating to the curriculum documents and curriculum expectations.

Analysis Inspired by Content Analysis

History of Content Analysis. Content analysis is a method that allows the analysis of texts addressing the themes and core ideas found in qualitative data texts (Drisko & Maschi, 2015). Content analysis is a group of analytical approaches that range from “impressionistic, intuitive, interpretive analysis to systematic, strict textual analyses” (Hsieh & Shannon, 2005, p. 1277). Historically, content analysis was used as a quantitative method which coded the text data into explicit categories then described that data statistically (Hsieh & Shannon, 2005). Over time, as content analysis began to be used qualitatively, the goal then became to provide a qualitative understanding of the phenonemenon being studied (Hsieh & Shannon, 2005). Krippendorff (2019) explains that replicable and valid inferences from texts and the way they are used in context, can be discerned by using content analysis as a method.

There are many definitions of qualitative content analysis, however, the definition that most closely aligns with the analysis used in this study is provided by Krippendorff (2019).

Krippendorff (2019) states that this type of content analysis allows information to emerge in the process of researchers analyzing texts within a given context. Krippendorff (2019) explains features of texts which apply to the definition used for this analysis. Krippendorff (2019) states that the text itself does not have a goal, but it is the researcher who gives meaning to the text. When examining text through context analysis, Krippendorff (2019) notes that texts may have multiple and varied meanings which are relative to their context.

Researchers (e.g., Drisko & Maschi, 2015) state that qualitative content analysis shares some techniques with other forms of qualitative analysis. For example, Berg (2001) states that 'open coding' is the first step in Glaser and Straus's grounded theory method and is also used in content analysis. Therefore there are parallels with this method and many other qualitative and categorical methods. Weber (2011) suggests there are many advantages of using content analysis. By using open coding I am able to identify and categorize themes in a manner that remains open to the data's nuances, enhancing the analysis' reliability and validity.

Weber (2011) states that communication is a basic aspect of human interaction. Content analysis works with the original texts or transcripts of that interaction (Weber, 2011). Content analysis works with all texts. Content analysis works especially well with texts that occur in natural settings (unlike a structured interview for example, that is obviously intended for the purpose of collecting data). In this way participants are more likely to communicate in natural ways not censoring their messages because they are conscious that those messages are being recorded as 'data' (Weber, 2011). Content analysis supports capturing genuine perspectives, beliefs, or practices authentically and in context. Therefore, by using content analysis I am able to analyze data that was captured in natural discussion, meaning in the back and forth during dialogue. The naturalistic communication of the teachers in my study allowed me to understand the conceptualizations, values, affordances, and potential challenges they associate with pedagogical documentation in their math classrooms more organically than if I had collected the

data through a structured interview. By using content analysis, I uncovered the word ‘centre’ which revealed a tension between the traditional use of centres in the early learning math classroom and the educational encounter using pedagogical documentation. The word ‘values’ also was revealed through content analysis.

Content analysis, a widely used research method (Drisko & Maschi, 2015), finds application in education and various disciplines (De Wever et al., 2006; Graneheim & Lundman, 2004). De Wever et al. (2006) explore its usage in analyzing asynchronous discussion group transcripts, presenting an overview of fifteen analysis instruments with critical considerations. Graneheim and Lundman (2004) outline essential content analysis concepts in nursing research. Wang et al. (1990) conduct a meta-review revealing over 3700 ratings on variables influencing learning, emphasizing the significance of students, classrooms, homes, communities, metacognition, classroom interactions, and peer groups.

Application of Content Analysis. As I noticed the emergence of certain terms repeated, I considered what type of method I could use to analyze the data that was presenting itself. Content analysis seemed to be an appropriate choice as the words were of interest because of their unexpected, repeated use. Content analysis allowed me to focus on language use (e.g., the repeated use of the word “centre”) and permitted underlying assumptions, priorities, or other influences that shaped the participants’ conceptions, to be exposed. Content analysis offered me a way to quantify the occurrances of the words, then systematically explore their context and potential implications in response to my research question: ‘How do teachers conceptualize pedagogical documentation in the early learning mathematics classroom?’

Clark and Peterson’s (1984) work also aligns well with content analysis to answer the research question “How do teachers conceptualize pedagogical documentation in the early learning mathematics classroom?” due to its focus on understanding the complex interplay

between teachers' thoughts, beliefs, and actions. I chose content analysis as it is a method that systematically analyzes the content of textual, visual or audio data to identify patterns, themes, and meanings. In the content of pedagogical documentation, the framework provided by Clark and Peterson (1984) offer valuable insights into the thoughts processes and beliefs that may influence how teachers conceptualize and engage with documentation practices.

Content analysis allows for the systematic examination of textual data, such as the teachers' ideas and reflections related to pedagogical documentation. By applying content analysis to teachers' written or verbal expressions, research can uncover recurring themes, patterns, and categories that reflect how teachers think about and approach pedagogical documentation.

Again, Clark and Peterson's (1984) emphasis on the bidirectional relationship between teachers' thoughts and actions provides a conceptual foundation for content analysis to explore the nuances in how teachers conceptualize pedagogical documentation. The method enables a detailed examination of the language, concepts, and underlying beliefs present in the teachers' discussions.

Therefore, content analysis, when applied to teachers' narratives and reflections, is well-suited to capture the intricacies of how teachers conceptualize pedagogical documentation, complementing Clark and Peterson's (1984) framework by providing a systematic and rigorous approach to analyzing the rich content of teachers' perspectives.

Analysis using Content Analysis. The third analysis of the data examined high frequency words. Essentially, I used a categorical coding approach to analyze the high frequency words. This approach is consistent with and inspired by content analysis in that content analysis is the systematic analysis of texts addressing the themes and main ideas found in texts (Drisko & Maschi, 2015). This approach was used to examine all of the data from the

study, not simply the data from the conversations about the two examples of pedagogical documentation. After the phenomenographical analysis of the two data sets ‘The Price of Apples’ and ‘The Columns of the Municipal Theatre’, much data remained. By examining high frequency words the data could be examined, not for the variety of ways participants thought about the pedagogical documentation in the context of a specific learning event like ‘The Price of Apples’ or ‘The Columns of the Municipal Theatre’, but for the significant ideas the participants held about pedagogical documentation over the period of the study.

The key aspect of this method of analysis is that high frequency word analysis allowed major ideas to emerge. This was accomplished by extracting high frequency words from the data set. The words were identified as high frequency in that the amount of times they occurred were of highest frequency compared to other words. The process was inductive as the categories came from the data and the high frequency words were not analysed with a preconceived theory or idea in mind. As the high frequency words were identified, themes were created based on what was said in the statements that contained those high frequency words. This process contextualized the high frequency words to help create the themes.

There were over 15 hours of transcribed meetings plus the journal entries of the participants that made up the entire data for the study. The data acquired from the documentation ‘The Price of Apples’ and ‘The Columns of the Municipal Theatre’ provided context specifc themes. Therefore, it was necessary to examine the data for broader ideas of pedagogical documentation, in the other conversations during the study. This approach was used to respond to the same initial research question: “How do teachers coneptualize pedagogical documentation in the early learning mathematics classroom?” Instead of looking at a variety of ways the participants experienced the phenomenon, this analysis examined the significant ways the participants thought about the phenomenon.

It was decided that high frequency words would be examined. The word ‘centres’ appeared frequently in the meeting where the documentation “The Price of Apples” was discussed. The documentation itself did not reference the idea of a centre, however the participants quickly related their ideas to the notion of centres. All meetings were then analyzed for the word “centre” or “centres”. The website https://databasic.io/en/wordcounter/ was used for the word count analysis. Once all statements containing the word “centres” were extracted from the transcriptions and journals, each statement was placed on a card. Therefore the unit of analysis was a sentence. Cards were then sorted into initial categories. There were no predetermined categories and the categories emerged through the coding process. The cards were then sorted into themes and labelled accordingly. This was an iterative process and took several iterations to categorize and assign themes. After the themes were established conclusions were documented. All the meetings were then analzyed for other high frequency words. Expected words like “teaching”, “teacher”, “class”, “students”, etc., were not counted. Two other words of interest emerged as frequent, unanticipated words. “Grades” and “values” appeared as words of interest. For example, the term grade first appeared as a word of interest in the second meeting of the study. The term ‘value’ appeared largely in a session that occurred in week five of the study. Categories and themes were established for each of the words separately. For example, all phrases where the word ‘centre’ occurred, were analyzed first. Categories and themes were established for the word ‘centre’ only. Then the word ‘value’ was analyzed in th same way with the word ‘grades’ following. The data was analyzed to describe the emergence of the frequently used words and understand the relationship of the words to the participants’ thoughts, beliefs, and connections to pedagogical documentation in the math classroom.

Chapter 5: Findings

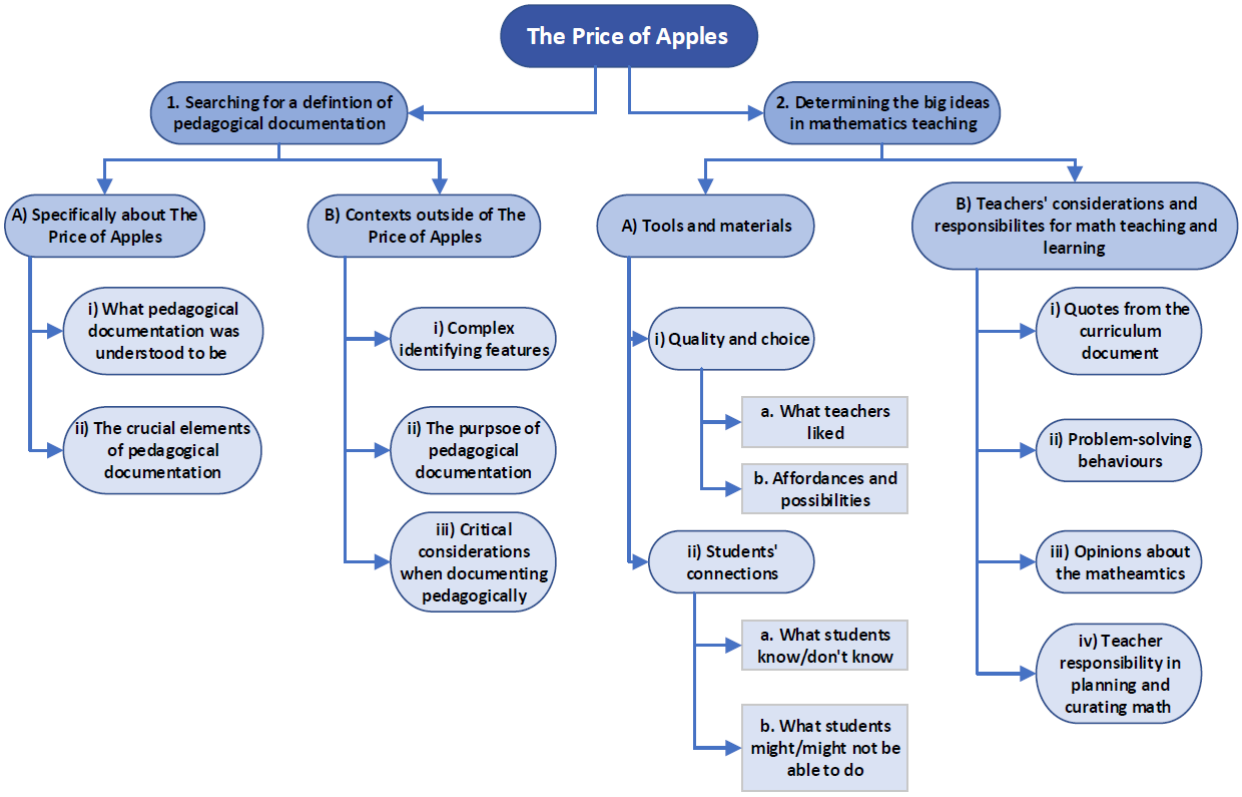
The Price of Apples

The documentation described in Chapter 5 entitled 'The Price of Apples', offers a glimpse into the dynamic world of early childhood mathematics education. In this narrative (Appendix A), three young protagonists, namely Daniela, Marco, and Tommaso, find themselves in a classroom setting transformed into a store. Here, the children not only craft their own currency for transactions but also actively participate in organizing the items available for purchase. This chapter examines the participants' experiences and conceptions surrounding 'The Price of Apples,' unraveling the layers of their engagement with and understanding of pedagogical documentation and significant mathematical ideas. The diagram in Figure 1 explains the categories that capture the participants' experiences and conceptions.

The children created coins that they used for purchasing items and they helped organize the items for sale. The documentation features Daniela with a change purse totaling 60, ready to purchase apples. The children determine the price of the apples by weight. A second collection of documentation entitled ‘Thoughts about Calculating Cost’ (Appendix B) continues the story of ‘The Price of Apples’ and shows Daniela trying to find 133 with the coins she has in her purse. Daniela attempts to construct the total 133, digit by digit. She uses a 1 coin for the hundred, a 1 coin plus a 2 coin for the first 3 (the tens) of the 33, and she uses a 1 coin plus a 2 coin for the second 3 (the ones) of the 33. Tommaso points out that the value is actually 7 and the children begin to negotiate how Daniela can put together enough coins to equal 133.

The participants in this study experienced the pedagogical documentation ‘The Price of Apples’ as a) searching for a definition of pedagogical documentation and b) determining the big ideas of mathematics teaching. I explain the sub-categories of responses under these two overarching categories in the diagram below (Figure 1).

Figure 1: Categories of Participants' Experiences and Conceptions of "The Price of Apples" Documentation



Category 1: Searching for a definition of Pedagogical Documentation

All statements in this category provide insight into how the teachers/participants conceptualized the overarching idea of pedagogical documentation from examining 'The Price of Apples'. Some of the statements were made specifically about 'The Price of Apples' while others were more involved and referred to contexts outside of 'The Price of Apples'. The assertions of participants not only indicate how they were conceptualizing pedagogical documentation but also go beyond, to identify what they saw as some critical considerations when implementing pedagogical documentation.

A) Specifically about "The Price of Apples":

The statements that teachers made specifically within and about the context of 'The Price of Apples' provide much insight into what they understood pedagogical documentation to be, and the crucial elements of pedagogical documentation. For instance, Shira simply stated she really liked 'The Price of Apples' example and indicated 'the story' is a crucial element in pedagogical documentation when she explained: "This is a story of the children in this centre, and this is the learning that happened organically from this place." Hope indicated that 'connection in tasks' is an essential element of pedagogical documentation by saying: "I really love how interconnected the tasks are [in this pedagogical documentation]. I want to be there because there is so much math." These statements demonstrate how Shira understood pedagogical documentation as a story and Hope identified how interconnected the tasks were. Other statements about 'The Price of Apples' came from questions and statements about student involvement as a crucial element of pedagogical documentation. These statements provide insight into how the research participants view students in relation to pedagogical documentation. Stacey implied that student voice is important to pedagogical documentation when she asked: "Where is the student voice [in these photos]?" Hope continued with another suggestion, "I would have the students take the pictures!" The role of story, connections and student voice are deeply rooted in the ways that these teachers thought about the crucial elements of pedagogical documentation.

B) Contexts outside of "The Price of Apples":

Statements that occurred further from the specificity of the 'Price of Apples' provide insight into more complex identifying features participants hold regarding pedagogical documentation, the purpose of pedagogical documentation, and critical considerations when documenting pedagogically.

Participants shared their conceptions of more complex identify features of pedagogical documentation when Hope explained: “Those 10 pictures are the pedagogical documentation; the rest of the stuff that they had to cull through was just documentation”. Hope was clearly indicating that pedagogical documentation is a special form of documentation. Hope generalized that intentional and purposeful choices are part of the process of pedagogical documentation. Lee mirrored this idea as she stated, “You might have taken a handful of kids with rich samples and photos. But not all kids have rich samples and photos every time.” Meaning that pedagogical documentation requires the characteristic of “rich”. Lee further elaborated and used the word ‘path’ to suggest that when the ‘path’ is evident, that is evidence of pedagogical documentation: “When we look at it and see the path, we have pedagogical documentation.” Another identifying feature of pedagogical documentation is ‘provocations’. Hope’s ideas explained ‘provocation’ with more complexity and expressed that ‘open’ is essential. Hope asserted that: “You are going to hope that whatever provocation you put out isn’t some closed activity that’s just specific to make a repeating pattern.” Hope realized that the provocation needs to be open ended enough to allow the students to investigate and inquire more deeply, and hence demonstrate learning that is ‘worthy of documentation.’

Participants also spoke of the importance of the purpose of pedagogical documentation. For example, Hope suggested that in terms of deciding how to do pedagogical documentation: “It depends on the purpose”. Hope continued to elaborate, “If it [pedagogical documentation] is evaluative, it makes more sense to come from the teacher.” In this statement Hope is suggesting that when teachers use pedagogical documentation as evaluation, the teacher should make the choices of what to document and how to document, and not the students. Hope stated that there may be other times (not evaluation) when students choose what to document and do the documentation themselves. Lee also indicated that she agreed that there are gains to be had with students doing the documentation when she said: “They (the students)

started coming out with things that I have no idea… sometimes I'm shocked… like naturally that's where the pedagogical documentation comes in, because I will think one of us understands it. Then twelve of us understands it." Lee explained how pedagogical documentation can allow the teachers and the students to see, understand, share, and learn together from ideas students hold.

Finally, participants shared statements that illustrate critical considerations when teachers engage with pedagogical documentation. Shira stated: "To do documentation justice, you need to be intentional with it, but to do your students justice you need to be present and active and engaged with them." This statement reveals the dilemma Shira felt when engaging with pedagogical documentation. While being present with students as student ideas emerged, Shira also realized she needed to be intentional with her choices of events or interactions that she was documenting pedagogically. Shira identified the challenge of doing both at the same time. Shira went further to state, "I want to be authentic to the learning journey, but I have to realize me being present, is authentic to the learning journey." Shira worried that her presence would influence student learning and that the interactions of the children would not be authentically student led as a result. However, she also knew that her presence was also an authentic part of the learning experience and was trying to reconcile that in her mind.

The above statements and examples illustrate the participants' ideas about pedagogical documentation relating to the specific example of "The Price of Apples" as well as ideas that were more generalized. Participants spoke of identifying features of pedagogical documentation, the purpose of pedagogical documentation, and critical considerations when working with pedagogical documentation.

Category 2: Determining the Big Ideas in Mathematics Teaching

The statements that fell under “Big Ideas in Math” either related to A) tools and materials or B) they related to teachers’ responsibilities and considerations for math teaching and learning. These statements refer to what teachers need to prepare and consider for math teaching and learning in the classroom and in relation to the curriculum. This category of comments does not directly speak to mathematics in relation to pedagogical documentation, as those statements appear in the category ‘Pedagogical Documentation’.

A) Tools and materials

In the category of ‘Big Ideas of Math’, the subcategory ‘tools and materials’ emerged. The participants’ statements in this subcategory emphasize the use of tools and materials as they relate to mathematics. Some statements refer to ‘The Price of Apples’ and pointed to the quality and the choice of the tools and materials in the learning space. Other statements move beyond to discuss students’ connections to tools and materials, not specifically in ‘The Price of Apples’ example, but more generally.

Some teachers simply made comments about what they liked about the tools and materials and what they might include if they were to enact “Price of Apples” in their own classrooms. For instance, Stacey stated: “The simplicity of these materials is beautiful.” When looking at the documentation of the Price of Apples example, Shira offered examples of tools and materials she would consider for her classroom, “I would think about a cash register, play money, labels, and markers for signs, price tags and flyers”. These simple statements demonstrate what came to mind for participants when looking at the pedagogical documentation of ‘The Price of Apples’.

Some statements were more complex, as participants suggested the affordances and possibilities of tools and materials more generally. In addition, these comments were different

from the previous statements, as they reach beyond the documentation of ‘The Price of Apples’. They included generalizations about math understanding, inquiry-based teaching, and the importance of having materials available. For instance, Stacey referred to the paper circles she saw in ‘The Price of Apples’ documentation and generalized how she felt that deeper understanding is uncovered through the use of that particular material to represent money. She was very clear that using circles of the same size with different number leads to understanding when she followed up with:

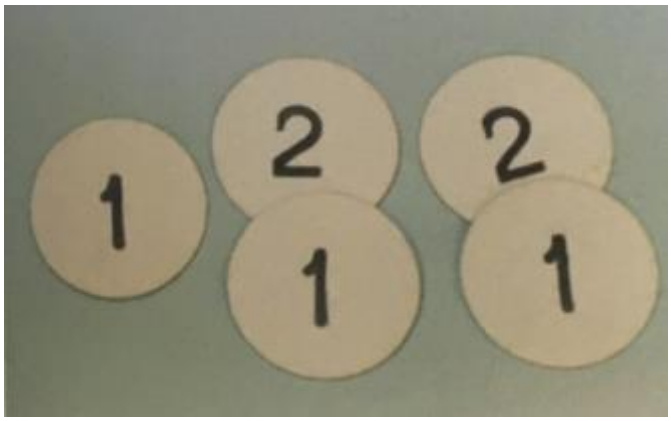
> I just really enjoyed how it wasn’t play money, and how the manipulatives were the same, they just had different numbers on them. And I feel like that actually brings in a deeper value and then a concept of understanding. I like, would love to throw away my play money and bring this out.

Hope generalized about how one can identify an inquiry-based classroom when she said: “If you have manipulatives in many areas in the classroom, it is a sign that you run an open inquiry-based program”. This comment demonstrates that Hope considered the use of manipulatives in a variety of spaces in the classroom, to be an indicator for an inquiry-based program. Lee wondered about possibilities when she said: “Do I have a number line in every centre?” Lee suggested having a number line in every centre (whether a math centre or not), could disperse math learning throughout the classroom. Lee also recognized the value of open-ended manipulatives in the learning space and how they add value to an inquiry-based program. Additionally, Lee suggested: “Maybe with all those circles, maybe you have tons that are blank.” Lee spoke of the paper circles with numbers for money being available in various areas in the classroom. She felt that could lead students to writing down different numbers, making the tool more open-ended. These comments show how teachers considered what they saw in the pedagogical documentation of ‘The Price of Apples’ and how they began to apply those examples to other contexts. These statements show how teachers in the study reflected

upon what the educators in the documentation offered students by way of tools and materials and how the teachers in the study would consider using them in their classrooms.

At the deepest level, comments emphasized students' connections to 'tools and materials'. These comments are different from the previous groups of comments because they refer to the way teachers imagined or anticipated 1) what students know/do not know and 2) what students might/or might not be able to do. All of the comments here share a transparent characteristic of focusing on students. Some participants described what students knew and did not know regarding their mathematical interactions with tools. Referring to the coin manipulatives used in 'The Price of Apples' story, Shira shared her idea that "kids don't have that at that age … that concept of a looney is one, and then the same size basically, a coin in a different color is somehow worth two". However, Shira continued to say that students "can still have [a] very rich and meaningful level of engagement with cost and purchasing". Teachers also discussed what students are capable of doing with tools and materials. For example, as the conversation shifted from the "Price of Apples" Lee described how a "kindergarten student made a tool to show 2 metres for how far they wanted everyone to be away from them." The student wanted everyone to be 2 metres away from them because of COVID precautions. Lee continued to explain: "As kids made tools and tried to throw an airplane 2 metres – it started to become an understood measurement." This statement shows, as students continued to explore and play with two meters in various contexts, 2 meters became familiar to them. Some statements in this sub-category referred to what teachers saw students do mathematically, in 'The Price of Apples' example. Hope also shared an example: "It [the play in 'The Price of Apples'] was really exploring math and problem solving, from a real-world context and I think that that's what was so cool…" Shira remembered the example from the documentation where Daniela was trying to make 133 with the coins in her change purse (as described above).

Figure 2: Daniela's thought process



(Rinaldi et al., 2016, p. 55)

Tommaso pointed out to Daniela that the value she actually had was 7, not 133. Considering this example Shira said: "She [Daniela] was making… was it one, one, three? and she put … one, three, three? She put the two together and then a two and a one together because she did not have a three (Figure 2). So, that is how she wrapped her head around value in terms of 'this is how much I pay you'." To this, Hope added: "She didn't have unitization." Hope described how the girl saw a "1" coin and put that as the value of 100. Then she took a "2" coin and a "1" coin together to make 3. The student placed that three as a "30". Then the final "2" coin and "1" coin represented a "3" as in the units of 33. Therefore, Daniela thought the total value she was representing was 100 + 30 + 3= 133. Shira and Hope noticed how Daniela used the coins but not in the way of adding up the coins' value to equal 7, but rather in a way to represent each digit in the number 133. Stacey remarked that her students knew to ask for certain tools even when they could not see them. Stacey stated:

> Even when you rotate things or move … this year, a lot of time and they were rotated out because of cleaning, then kids will come and say, 'Can I have the magnifying glass?' or 'Can I have the "this"?' because they needed it in their play to do something … then they knew that those tools were available and they were tools for learning. It was very powerful.

Stacey concluded: "And that's when those connections and those deeper learnings come through". Lee and Stacey's comments illustrate how they perceive tools and materials as a

catalyst for learning in complex ways. Clearly Lee and Stacey feel that tools and materials as part of the environment, have a distinctive and important role.

The comments in the category of “Tools and Materials” refer to the way in which the participants consider which tools and materials to place in learning spaces in the classroom. The comments also refer to the affordances and possibilities for the application of tools and materials, as well as comments that discuss students’ connections to tools and materials. The comments are talking about the way that students engage with mathematics, but specifically in terms of tools and materials, therefore the comments do not belong in any previous category about mathematics.

B) Teachers’ considerations and responsibilities for math teaching and learning

The participants shared statements where they were making direct reference to the curriculum document. For example, Lee said: “They're counting by twos, by fives and 10s, which are the like the best way to encourage counting, right.” Lee quoted right from the curriculum document when saying, “… counting by twos, by fives, and 10s”. Hope added to this thought with: “Those are the typical numbers that we would anchor to, and you know, when you're considering currency?” Hope knew that anchors were also an expectation in the curriculum document. Lee quoted the curriculum document again when she stated: “Construct and deconstruct numbers to ten.” Hope added: “It is interesting to see how interconnected all the math was.” Hope noticed that the example demonstrated interconnected math and they were examples of math right out of the curriculum expectations. These statements demonstrate the teachers were targeting specific curriculum expectations as they were identifying the mathematics occurring in ‘The Price of Apples’.

Some statements referred to students’ problem-solving behaviours in mathematics, as they related to the kindergarten curriculum document. In Ontario, educators are to provide

kindergarten students opportunities to apply problem-solving strategies and learn to solve challenging problems. Hope stated that when engaged in open-ended play she noticed “they [students] were more open to risk taking.” In kindergarten, educators are to support students to develop their understanding of mathematics through their individual approaches to learning and their own prior knowledge. Lee explained how this happened in her classroom. She stated that surprisingly “four [students] might have ideas that the teacher didn’t even consider.” She continued going deeper by adding: “Even when students have miscommunication together, they can solve their problem together.” Reagan also stated: “It’s the students that were asking the [math] questions [in play]. We should be assessing the students by the questions they are asking.” Reagan knew, from reading the curriculum document, that in the kindergarten program students are expected to pose questions, generate ideas and develop theories. Stacey agreed with these ideas as she explained when kids ask the questions and use the tools to solve problems: “that’s when those connections and deeper learning come through”. These comments describe what teachers know and have observed in students’ mathematical and problem-solving behaviours.

More complex statements occurred as the teachers offered their opinions about the mathematics. For example, Hope said: “If kids don’t have the ‘muchness’ of a number, it is going to be more challenging to unitize.” Hope continued by saying: “Unitization is a tricky concept to grasp.” These comments reveal how teachers were conceptualizing student knowledge and understanding of mathematics, in this learning experience.

The statements in this paragraph are categorically different from those in the paragraphs above as these refer specifically to the responsibilities teachers have in planning and curating learning opportunities in mathematics. Shira noted: “The educators in the room would have used so much intentional thought when planning this [The Price of Apples] play centre.” Lee suggested: “The teacher can be the conduit [of the learning] because I know what the

curriculum says." She explained: "The curriculum actually says, 'I can construct numbers to ten.' So that's what I'm going to make sure we get." Here, Lee confirms the idea that it is a teacher's responsibility to design learning experiences so that students explore the mathematics as suggested by the curriculum. Lee explained, "Someone said, 'if you can't see math in every centre in the room, then how can you bring it in?' That really forced me to think about what was missing." These comments demonstrate the level of reflection the teachers considered when considering the mathematics in 'The Price of Apples' example. Shira recognized the thoughtfulness required to choose materials and the setup of the materials to provoke students' ideas. Lee recognized the responsibility of the teacher in knowing the curriculum to ensure they designed learning experiences to provoke exploration as the curriculum states.

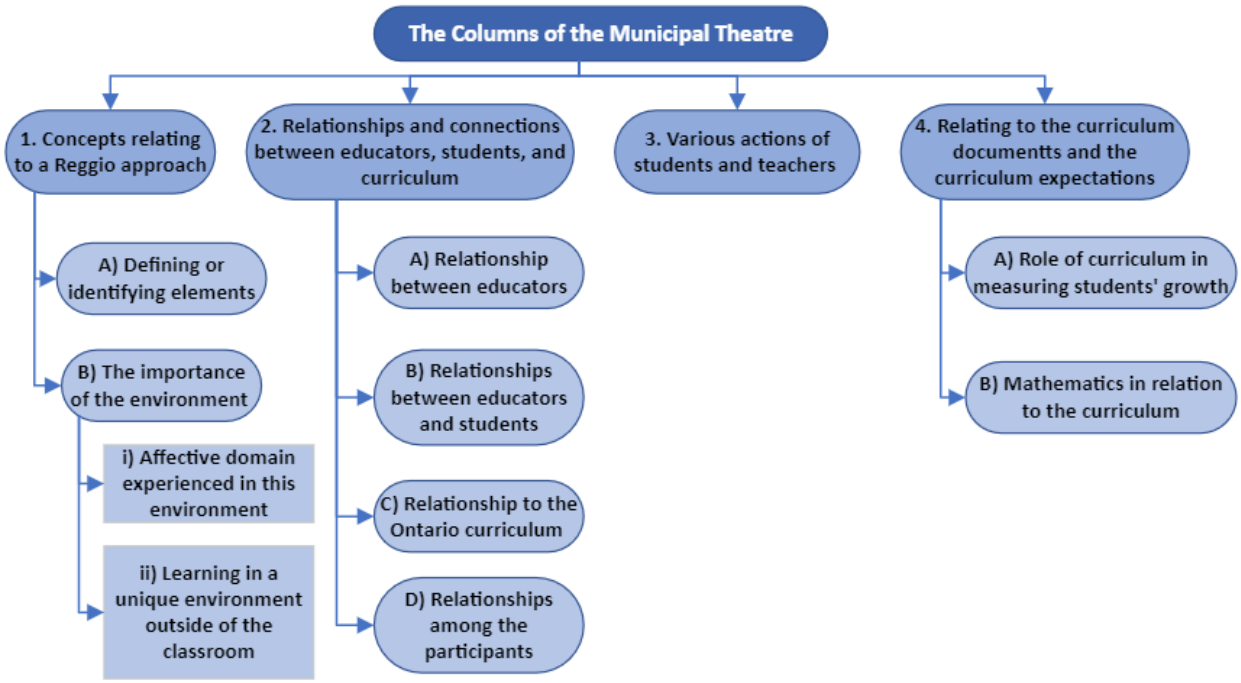
Chapter 6: Findings

The Columns of the Municipal Theatre

Chapter 6 highlights the findings from the conversations with participants about the pedagogical documentation example ‘The Columns of the Municipal Theatre’. The pedagogical documentation of ‘The Columns of the Municipal Theatre’ featured a class of children visiting the colonnade of a Municipal Theatre. (Appendix C). The children were very interested in the columns at the front of the building. The children began to explore the columns from a variety of perspectives. The following analysis draws attention to the variety of ways the participants conceptualized this pedagogical documentation. The diagram in Figure 3 explains the categories that capture the participants' experiences and conceptions.

The teacher participants conceptualized pedagogical documentation of ‘The Columns of the Municipal Theatre’ as 1) concepts that related to a Reggio approach, 2) relationships between educators, students, and curriculum, 3) actions of students and teachers, and 4) relating to the curriculum document to measure student growth and mathematics in relation to the curriculum. The following table demonstrates the categories and sub-categories that emerged from this example of pedagogical documentation:

Figure 3: Categories of Participants’ Experiences and Conceptions of “The Columns of the Municipal Theatre” Documentation



Category 1: Concepts related to a Reggio Approach

Participants in the study conceptualized pedagogical documentation in this category as concepts related to a Reggio Emilia approach. A characteristic feature of some of the statements in this category was the term and concept of Reggio Emilia. Statements either included the word ‘Reggio’ or referred to aspects that are important in the early childhood education approach of Reggio Emilia. Participants spoke to A) defining or identifying elements of a Reggio approach, and B) the importance of the environment in a Reggio approach. Within the subcategory of the B) the importance of the environment in a Reggio approach, emerged as i) the affect experienced in this environment, and ii) learning in a unique environment outside of the classroom.

A) Defining or identifying elements of a Reggio approach:

The statements about pedagogical documentation that used the word ‘Reggio’ were essentially trying to determine defining aspects of the Reggio Emilia early childhood education approach. Jean Claude thought that teachers having a very open perspective to students’

learning experiences would be part of a Reggio approach. Jean Claude exemplified this perspective by stating: “My current understanding, at this point in time, is if the teacher says, ‘Whoa! There’s a lot of cool learning there, that’s Reggio.’” He continued trying to make sense of his understanding: “But if the teacher says, ‘Well there’s a lot of cool learning about patterns there, that’s not Reggio anymore. Maybe I’m right, maybe I’m wrong.” Hope tried to reconcile that, regardless of what we think Reggio is, we must remember we are not Reggio: “We keep having to keep checking in—we are not Reggio. Our system runs differently. That’s just the reality check we always need to remember.”

When participants talked about pedagogical documentation, they also recognized social constructivism as an important identifying element of the Reggio approach. For example, as Jean Claude spoke of the dance that occurs between teacher, early childhood educator, the students, and the curriculum, he described these interactions as a square. Hope responded: “That’s the social constructivism.” Hope described teacher reflection as an important part of ensuring that her program was rooted in social constructivism: “That’s what I envision is the constant reflection of, am I really embodying this constructivist belief that I hold within my classroom?” Hope felt that the Reggio approach embraced social constructivism.

Participants also spoke of pedagogical documentation as an important part and an identifying element of the Reggio Emilia approach. Lee considered how she might embrace the experience of going to the Municipal Theatre through documentation, in a way that would align with a Reggio approach: “What if we brought clipboards, some string, some chalk, and I sat down and I just documented? I sat and said please stay within the columns; … but you just let it go and see where they go. Is that like Reggio? Or is that me going ‘I know what they’re going to do?’” Reagan spoke to the importance of giving students freedom to explore, so that teachers have the opportunity to document: “By giving students this freedom the teacher is able to document what the students already know and see how they can apply it.” Reagan identified

‘freedom’ as a defining element of a Reggio approach. Jean Claude suggested that the difference between documentation and pedagogical documentation was a very fine line. Jean Claude pointed out: “That’s not two camps on the opposite sides of the education world, that’s like one word.”

B) Environment:

Participants conceptualized the role of the environment in the example ‘The Columns of the Municipal Theatre’, as being an important part of a Reggio Approach. Statements in this sub-category were further divided into statements referring to the i) affective domain experienced in the environment, and ii) learning in a unique environment outside of the classroom.

i) Affective domain experienced in this environment:

Participants experienced pedagogical documentation through the role of the environment using terms like boring, cool and fun. In her written journal, Hannah explained at first glance she did not think much of this environment: “Even for me, at the beginning I thought ‘What a boring field trip’.” In the study meeting, Lee questioned ‘boring’ and thought that the children would be engaged and she as a teacher could support the children’s interests:

> If you started doing it [playing and running] and seeing if you could make the noise and see if you could hear it, your interest… they’d [the students] be like ‘What are you doing?’ I [as the teacher] was listening to the noise. Someone else might make another noise and we could do a parallel… there’s so many interesting things when we [as teachers] get engaged.

Lee suggested that the students seemed to have fun outside, in this environment. "…because they [the students] seem to have a lot of fun when they were out." Lee recognized that learning could happen in various environments, which is a feature of the Reggio Emilia approach.

Hannah speculated that students would have fun having a race and trying to push over the column. Hannah predicted that students might say: "Let's race to see who's first. Let's push it all together to see if we can move it.'" Lee agreed that running was fun and thought a race would be likely: "Because you started asking what do you think they might do? ... They are going to start a race, because that's great, and they are going to do inside, outside, inside." Lee continued: "What if they all just ran up and down and went 'Weeeeeeee', is that okay? Because I literally thought if one of them starts running, we're just going to run, like it would look so cool."

ii) Learning in a unique environment outside of the classroom:

Teachers' conceptualizations also spoke to the possibilities for learning while in this environment outside of the classroom. Jacques stated: "Math is found in every aspect of what the children explore [at the Municipal Theatre]." Jacques added: "The teacher can now refer back to this experience when creating word problems or teaching other aspects of math to remind students of the real-world application." Jacques emphasized the importance of this real experience: "This experience is priceless. Students have the opportunity to learn and explore a real-world example of mathematics without even realizing it." Jean Claude agreed with Jacques. Jean Claude explained the depth of understanding that could result in children's mathematical experiences because of this visit to the Municipal Theatre. He described how the children could use this real-world experience to describe proportion and size more accurately: "When Hope was talking about spatial sense, I was struck by this idea of size, and these students are going out and seeing this really tall tower. Such a wonderful experience because it gives them a sense of richness of proportions."

In her journal, Donna wrote how important going to this environment was: “This experience allows students to see math in their environment.” Donna continued to explain how important being physically present at the theatre was for the students’ learning: “This real-life experience is much better than if the teacher was to show a picture of this theatre. The students would not be able to understand how big, wide and strong this structure is.” When we gathered as a study group, Donna described in more detail the benefits of being present in the environment, rather than just showing a picture of the environment. Donna felt this lived experience would help students better connect to their learning and the creation of their own structures:

> What a difference if the teacher was to just show a picture on the SMART board. They [the students] would just say it looks tall. The teacher would say: ‘Okay, create it.’ But it would not be as rich and as great an understanding. They actually could feel how wide it is. It may have taken five kids, their arms around the columns, and when they create, they'll have to know that the base has to be wider like there was rectangular prisms on the bottom of the cylinders to hold it up, and when they are to create their own structures, they'll learn from that too.

Jean Claude summarized the importance of being present, as it allows students to bring learning ‘back into the classroom’: “After having this experience [seeing it in real life] when they [the students] go back into the classroom they build a block tower, the way they will talk about it will be so much more informed and proportional and grounded in data.”

Considering the way in which students and educators could explore distance and measurement in this environment, participants made simple, direct statements. Lee stated how she felt she and her students might explore distance: “I also thought we would all jump off the side and start jumping and maybe do distance.” Lee added to her own idea in terms of

measurement: “And if you had chalk, we could spend all week there. You could measure, you could count if you’d like.” Jean Claude explained in more detail, how educators and students could investigate distance and measurement:

> Another example of a provocation would be Reagan stepping in and going ‘All right let me put this backpack really far.’ Like equal distance with the other poles and add another pole and then try to make it really far away, and then say, ‘What does that tell us?’ Anything that comes out of their mouth is [an] amazing demonstration of understanding.

Jean Claude felt that a teacher could provoke a learning opportunity about distance and measurement in this environment very easily by using the environment in specific ways, for example, placing the backpack in strategic places. Using the environment in specific ways is an important part of the Reggio Emilia approach.

Lee connected the idea of there being learning potential for everyone in this environment. Lee felt it was significant that teachers participated in the same activities as the students in this environment. Lee thought that in this way teachers would also engage with the space and the learning: “

> We go to the place, and they start hitting the wall. Then if Reagan [as the teacher] goes and says ‘You know what? You’re inspiring me to go and do that too, …’ we’re taking the documentation and the learning and we’re showing that we can inspire others, like the learning doesn’t end when we’re an adult.

Lee also suggested that this environment offers opportunities for teachers to consider what students might do in their learning there: “But that’s awesome if you can predict if you have some ideas where they’re going to go if you take them to that space.” Jean Claude added onto

Lee's ideas: "The level to which the teacher predicted what was going to happen in this environment might give us a window into where they [the students] were [in their thinking]."

Many statements in this category demonstrated the ways in which the participants were trying to make sense of aspects of the Reggio Emilia approach to early childhood education. The comments in this category highlighted the significance of the affective domain experienced in the new environment as well as the learning experienced in the environment of The Municipal Theatre. Participants shared various perspectives about changing the learning environment from the classroom and allowing learning to happen in a new space. The statements in this category demonstrated a growth and change in the teachers' conceptions of 'environment', the effect environment has on learning, and the importance 'environment' has in a Reggio approach.

Category 2: Relationships and connections between educators, students, and curriculum

In the second category, participants conceptualized pedagogical documentation of 'The Columns of the Municipal Theatre' as A) relationships and connections between the educators, B) relationships and connections between educators and students, C) relationship to the Ontario curriculum, and D) relationships among the participants.

A) Relationships and connections between the educators:

The comments that referred to connections in terms of relationships between educators all highlighted an idea of synergy. For example, Hope explained that when she was working with the Early Childhood educator in her room, they worked so seamlessly/synergized together, a person would not know which official role either of the adults occupy in the classroom (i.e., either teacher or ECE). This is very evident when Hope stated that she recalled years ago when "somebody walked in… they didn't know which role we were." As described in the kindergarten

program document, the two educators (the teacher and the designated early childhood educator (DECE) work within a special partnership of collaboration and reflection. In Ontario, the kindergarten curriculum document (Ministry of Education, 2016), explains that the two educators have complimentary skills and training, which should create a relationship, built on trust, mutual respect, and open communication. Jean Claude affirmed his idea about collaboratively/collectively working when he stated: “He [Charles Pascal, Author of ‘With our Best Future in Mind: Implementing Early Learning in Ontario’] said that about the teacher/ECE relationship that you shouldn’t be able to tell who’s leading, that it should be this dance.” Jean Claude expanded on the meaning of this relationship by saying: “I love this idea. I’m hearing from Hope is this idea that the whole is greater than the sum of its parts.” Jean Claude used this expression referring to the relationship between the teacher and early childhood educator as one that has a certain synergy rather than that which they could do working individually.

B) Relationships and connections between educators and students:

The comments that spoke to the relationship between educators and students during discussion about pedagogical documentation emphasized the learning of the teachers in community, teacher listening, and teachers in collaboration. Lee suggested: “It’s how you build community and show you’re learning together.” Lee explained how educators and students create relationships by ‘being’ with each other, building community, and learning together. Lee added: “You [the teacher] can be really internal and just let things come to you…or you can be reciprocal.” Lee suggested that a teacher can wait for students to come to them, or a teacher can make a concerted effort to build relationships and make connections, by being reciprocal with students. Jean Claude also identified that there is a similar synergy when educators and students build relationships: “When both are actually listening to each other and doing one step at a time rather than 17, you get this moment in the process that’s the whole is greater than the sum of its parts.” As Jean Claude previously suggested the relationship between the two

educators in the classroom involved a certain synergy, he also suggested the relationship between the student and the educators together involved a special combined effect that without the relationship would not be as effective.

C) Relationship to the Ontario curriculum:

Participants also made statements in the context of pedagogical documentation that reflected the relationship of the educators to the Ontario curriculum, for example, the way the teacher knows the curriculum, and the way the teacher and designated early childhood work together within the expectations of the curriculum. Hannah explained: “I like the way Donna worded it: ‘It should be a balance of both [relationships between students and curriculum], so we need to know the curriculum well in order to make the connections.” Hannah also signaled the importance of knowing the curriculum so educators can help students make connections. Hope expanded on that idea: “I love that idea of the teacher connected with the DECE, with the kids, with the curriculum.” Hope recognized that for a rich classroom environment and learning to occur, there must be relationships between the people and the curriculum. Jean Claude also showed the connection to the curriculum when he said: “… that idea of reciprocity, which by the way is connecting and belonging, contributing and belonging.” ‘Contributing and Belonging’ is a frame in the kindergarten curriculum that Jean Claude noted promoted relationships between educators, students, and curriculum. Jean Claude drew attention to the fact that reciprocity within relationships is a key factor in ‘Contributing and Belonging’ in the curriculum.

D) Relationships among the participants:

Interestingly, the discussion about relationships in the classroom extended to the relationships between the participants of the study. Statements reflected how participants felt the study group was also making connections. Lee stated simply: “We did it! We became a group!” This statement came in the midst of the conversation around becoming a cohesive whole in the classroom where relationships between educators, students, and curriculum

flourished because of the interrelationship of the whole and not the operation of the distinct parts. Lee was comparing that we too, as a study group, had arrived at that same type of relationship. Jean Claude added to Lee's idea: "We became a group which is the goal of the kindergarten too!" Jean Claude continued to support Lee's idea by saying our relationship as a group became more like what would be the goal in a kindergarten classroom. Jean Claude continued to expand on his ideas: "… where the whole is greater than the sum of its parts. Where you can't always tell who's the lead and people are doing that dance." Again, Jean Claude referred to the synergy of the factors that come together to be greater together than that which individuals could be or do individually. Jean Claude took his statement further by offering a metaphor to explain: "Like if Jacques and I go for a walk in the neighbourhood. The route we take is going to be different than either of our routes if we both went on walks separately, because the walk we take together is the product of our interaction with each other." Jean Claude explained how important the interaction of the relationship is in producing a result that could not be achieved by each individual doing the 'work' on their own.

The above statements and examples illustrate the categorical differences in the participants' ideas about connections and relationships in the 'Municipal Theatre' example of pedagogical documentation. These statements present the participants' ideas about relationships and connection between educators, students, the Ontario curriculum, and each other.

Category 3: Various actions of students and teachers

In this category, participants conceptualized pedagogical documentation as various actions of students and teachers. The discerning features of statements in this category pertain to the student actions, educator actions, and shared actions by students and educators, which could occur at the Municipal Theatre.

Some statements in this category spoke to types of student actions or learning that could occur at the Municipal Theatre. Jacques suggested: “The kids could ask further questions either to the teacher or to one another to bounce ideas and concepts off one another.” Donna stated she felt that students could learn other things besides math: “Students will also learn the science behind this.” Hannah explained that students have great capacities that may not be obvious to adults: “Students are capable of being interested in things many adults wouldn’t think they’re interested in.” Other statements in this category emphasized students’ actions of controlling or exploring their own experiences. One participant suggested that learning would not be as rich if educators left students to their own devices. Jean Claude suggested that a lesson that was entirely led by students could look the same as students playing alone at home. Jean Claude suggested that the students playing alone could limit their learning potential. Jean Claude stated: “A 100% student driven lesson would look the same as something they did Thursday evening when they’re waiting for dinner. They are sitting talking to the wall, going ‘I’m playing school today’, talking about whatever.” Jean Claude and Jacques did not have the same perspective in terms of what might happen if students lead or explored their own learning before teacher instruction. In contrast Jacques thought: “In allowing students to go and explore the columns at their own pace, looking at different elements, it opens a door that would otherwise have been very difficult to open.” Jacques offered a different perspective suggesting that if students were permitted to examine the environment of the Municipal Theatre before teachers ‘instructed’ them, students may discover ideas and theories that they would not have discovered had the teacher intervened prematurely. Jacques added to his thought suggesting that if teachers give students the opportunity to explore first, then teachers could follow up student ideas in meaningful ways. Jacques asserted: “The teacher can now use the ideas they [students] explored by looking at the columns and implement them [the ideas] in other aspects of student learning in order to ground the learning in a real-world context.”

Some statements in this category emphasized the actions of educators, as they relate to pedagogical documentation, in terms of educators directing and leading learning. Jean Claude felt if educators were entirely directing the lesson, there would not be space for students to insert their ideas. Jean Claude felt the teacher had a necessary role in the classroom in moving learning forward. However, Jean Claude used the metaphor of acting a scene to clarify how a teacher could bulldoze a lesson or learning experience in a classroom. Jean Claude cautioned the actions of a teacher over directing by explained his perspective further:

> It makes me think of acting and having a scene partner in improv, and how you can bulldoze a scene, and by the way it also applies to conversations and classrooms and discussions. You can bulldoze and sort of only giving out energy and directing everything.

Other participants spoke to the actions of teachers in terms of directing learning specifically when considering mathematics teaching. Jacques stated: “Moving forward teachers should ask: ‘Why do you think that is?’ or ‘How do you think that happened?’ because it would force the students to begin to think critically about the things they are exploring.” Jacques explained that these types of questions were good to use to be able to draw out student ideas in mathematics. Jacques continued to explain that teachers could use this example as a good exemplar of teaching math: “I would interpret this as an example of how to properly teach math to kindergarten students.” Lee suggested that the role of the teacher was not necessarily to provide answers: “You [the teacher] don’t always have to have the answer to it. We don’t even have to say, ‘Here’s the mathematical idea I have.’” Hope agreed with Jacques. Hope felt that the teacher had an explicit role in helping students express their learning: “Kindergarten is experiential learning. That’s how the child is demonstrating their understanding. You as the educator are a wiser, smarter, more experienced person would make that connection whereas they can’t express that, right?”

Several statements in this category spoke to the actions of teachers in terms of the teacher letting go of their control over students and students' learning. Lee questioned the idea of letting students go: "We go back and forth. I love the idea of a scene. How do we get them engaged and then how do we let them go?" Lee wondered about the teacher's actions of letting the students go in their own direction. Reagan stated how the educators gave students open opportunities to respond when playing at the Municipal Theatre: "I really like how the teacher wasn't asking point blank questions but allowing the students to elaborate on what they are noticing and explain their thinking." Hope wondered if she was giving up control and allowing students to take the lead in their own learning. She wondered if ultimately, they were co-constructing the learning. Hope also questioned her own approach: "Am I listening to the kids? Am I giving them control? Are we constructing this learning together?"

Other comments in this category highlighted the way actions could be shared between the educators and students. To this idea of not knowing who started or who was leading learning, Donna simply stated: "… chicken or egg." Jean Claude clarified that the connections that occurred between the educators and the students would make it unclear as to who was leading the learning. He explained this idea further: "Maybe it's not just this dance and marriage between teacher and ECE but also this triangle with the students, as well. And so, you can't tell who's leading at all." Jean Claude had previously referenced the idea of having great moments in the classroom where a person could not tell who was leading the learning, where the educators and students 'passed' the leading around. He continued: "[It is those great moments] We want to see in a kindergarten class." Jean Claude tried to question whether or not it was important to be able to define who led 'it'. Jean Claude stated: "It becomes semantics as to who led it because you're right… who led it in that moment? Did the students lead by bringing it in? The teacher led it by bringing the space?" Jean Claude continued his questioning by asking complex questions about who had ownership over the learning. Jean Claude asked: "Who's

learning, is it? Who's got that ownership? Is ownership subjective? Could multiple people disagree about who has ownership in a given moment? Can you feel like you have ownership and not? Or can you not have ownership and feel like you do?"

The statements in this category describe the way in which the participants of the study perceived ideas of student action, teacher action, and the way that actions could be shared between educators and students. Several statements highlighted actions like shared learning or not knowing who was leading the learning. Statements that referred to connections between educators and students appeared in this category as the statement was focusing on the fact that it was not obvious who was leading the learning; the focus was not about the connections through connections, and therefore do not appear in the category about relationships and connections.

Category 4: Relating to the curriculum documents and curriculum expectations

The participants of the study conceptualized pedagogical documentation in this category as relating to the curriculum documents and the curriculum expectations. The statements in this category spoke to A) the role of the curriculum in measuring student growth, and B) mathematics in relation to the curriculum. I have further divided the statements that occurred in the subcategory of mathematics into groups of i) numeracy and balance, ii) spatial sense and structures, iii) measurement, and iv) patterning.

A) The role of curriculum in measuring student growth:

Discussion about pedagogical documentation included participants explaining the role of the curriculum document in terms of how the curriculum document must measure student growth. Hannah stated directly that this is what she was trained to do: "But we understand growth through the documents we were taught to use to measure student thinking." Hannah

explained further, how the curriculum documents are part of a bigger system making decisions on how to judge students:

> Administration and people of the 'higher ups' are picking and choosing what we must teach and what we can 'do for fun', and this is what I understand, as what educators are instructed to determine a child's growth, when the 'for fun' stuff should be measured as growth, as well.

Hannah explained how she felt that educators should assess the whole child in kindergarten. She felt that educators should evaluate a child based on all areas of growth and not simply against pre-determined expectations listed in the curriculum documents. Hannah stated: "I would absolutely start with the students and measure their 'growth'. Rather than understanding/knowing/memorizing the curriculum document, I'd rather measure it as growth."

B) Mathematics in relation to the curriculum document:

Participants conceptualized the pedagogical documentation of "The Columns of the Municipal Theatre" through evidence of number sense in relation to the curriculum document. As Donna was identifying the math in "The Columns of the Municipal Theatre', she stated: "I listed the strands of math and in each strand there's an example. Number sense we can count the number of columns, the number of steps between columns." Hannah added to Donna's ideas, contributing what she identified as number sense: "I see estimation when the children are attracted to the columns and state 'There are so many of these things!'" Donna continued to expand on the examples of number sense: "In picture eight, I liked how the child put his arms around the column. The teacher can ask – 'How many students wrap around the column?' You could introduce fractions… 'If 3 students wrap their arms around the cylinder…each student is 1/3 of a cylinder.'" The statements in this group simply identified examples of what participants named as number sense in 'The Columns of the Municipal Theatre'.

Some statements in this category spoke specifically about equality and inequality as a mathematical concept in ‘The Columns of the Municipal Theatre’ example. Jean Claude simply stated: “But watching this now, [picture #9] I thought of a lot of math, in that it made me think of equalities and inequalities.” Some statements referred to the concept of equality as it related to the students running between the columns rhythmically. The notion of ‘not equal’ addressed by participants was that of the columns bearing the weight of the building equally. Jean Claude noticed how students were referring to the columns and what might happen if a column fell down. Jean Claude suggested:

> Not that students would express it this way… but the understanding they are demonstrating is this idea that when you take one away, the others are not going to be able to balance, so there is this tension, this is inequality. When you take more away, it’s going to be even more unstable.

Jean Claude felt that the students were demonstrating an understanding of inequality because the students knew that if one column fell the other columns would have a hard time holding the building up. Hope explained how she had explored the idea of equality in her classroom and how important this type of hands on experience was in building understanding: “Something we tried… where we kind of looked at the balance and the equality first and understanding that in order to then look at it from number sense and what the equal sign actually means, what it means to be equal, because they’ve had that experience.”

Participants also made simple statements about what they observed as spatial sense and measurement in pedagogical documentation of ‘The Columns of the Municipal Theatre’, in relation to the curriculum document. Donna listed what she felt was spatial sense: “Cylinder, round edges, the base is a rectangle square prism, shapes of doors and windows.” Hope went deeper with her observation. She described how students gained understanding of spatial

sense: “I love the experience they are having with spatial sense in terms of size, perspective, because they took them to the one side: ‘If I stand this way I disappear’. Like all of a sudden, they are developing a core understanding of spatial sense.”

Some comments involved conceptions of structures and buildings occurring in the Municipal Theatre experience. Lee directly stated: “Anything architectural is math.” Donna added to what math was evident in relation to the curriculum documents: “Strong stable bases will hold up the columns and structure above.” Donna knew this language was directly from the curriculum documents. Lee elaborated on the idea of building as math: “Someone’s building structures and you’re like, ‘All they do is build block things!’ – so much math!” Hannah recognized the importance of building and structures in children’s play: “I would interpret it [the Municipal Theatre experience] as the children becoming interested in architecture and structures.” Donna identified additional math apparent in the Municipal Theatre experience: “Looking at the whole place student can see the structure is very symmetrical. In picture one each window is between 2 columns.”

A few statements used the word structure and/or referred to what students might do with structures (as identified in curriculum documents), based on their experience at the Municipal Theatre. Donna stated that when students return to the classroom after visiting the Municipal Theatre, they would learn when engaging with building their own structures: “When they [the students] are to create their own structures, they’ll learn from that too.” Reagan added a suggestion to build onto what students might do: “Maybe the students could draw out how they remember the structure and then build it together in class.” Shira explained what she felt students understood when playing at the Municipal Theatre: “They also demonstrate that there needs to be regular equal intervals of stable structure, so the roof doesn’t collapse. They know that because if certain ones are gone, then there’s added weight and the more you take away the harder it’s going to be for support.”

Participants spoke to ideas about measurement in the conversation about pedagogical documentation. Donna listed what she saw as measurement: “Measurement is how tall and wide. The distance is also using non-standard measurement… like their steps.” Hannah indicated what she saw as measurement: “I see measurement when students observe and hug the columns, when they state they are very, very tall and they are so fat.”

Some of the statements in this group involved conceptions of what the participants thought children might say or do in relation to measurement and size, as concepts from the curriculum document. Lee thought the students would play and compare the size to what they know: “Maybe they’re going to do the length of the condo at the door, play hide and seek, count and compare the CN Tower height … compare to their condo heights.” Jacques agreed with Lee, suggesting that students would compare to objects they already know: “They might say things like: “‘This is heavier than my whole bed!’ showing that they are exploring. They are also exploring the idea of size and weight.” Jacques also thought the children might ask questions about the size of the columns: “Children might ask why they’re so heavy and what are they made out of.” Donna explained what students would need to consider in size when building in the classroom: “… and when they create, they will have to know that the base has to be wider … like there was rectangular prisms on the bottom of the cylinder to hold it up.” Jean Claude explained further the depth of student ideas when it came to them understanding size. Jean Claude stated that the students would be able to relate the size of objects in the classroom after having the visit to the Municipal Theatre: “They are not going to say, ‘Oh it’s so tall it can touch the sun’ [referring to their own block tower in the classroom] because they have seen something way taller.”

Donna commented on what she thought the students might do upon returning to the classroom after their experience: “Children could try to build strong, tall columns, and structures with blocks, Lego, and recycled materials.” Reagan wondered about what a student was doing:

“In picture #8, a student is wrapping his arms around a column. I wonder if he is trying to measure the circumference of the column with his arms.” Donna felt that students could use their arms to measure the size of the columns: “Children can touch and wrap their arms around the columns which helps further their understanding of the largeness and size of the columns. Jean Claude confidently explained that students’ understanding of height would be solidified by having visited the columns: “[After this experience] they are going to be able to articulate it’s pretty tall. Which is this mathematical sense of ‘How far up to that column that I saw last week, would it come?’” Teachers connected their comments in this group to mathematics of measurement and size as they knew from the curriculum documents.

Other statements in this category demonstrated how the participants identified patterning in ‘The Columns of the Municipal Theatre’, as concepts from the curriculum documents. Some participants identified patterns in rhythm as math that was experienced at the Municipal Theatre. Lee explained directly: “Someone will say they only want you to do dance. But dance is rhythm, but it’s also math. It’s also many other things.” Hannah agreed with Lee, highlighting the connection of rhythm to patterning: “I see distance and music rhythm. Alessandro runs to the columns and touches each one as if to perceive the rhythmic sequence of each column.” Hannah described as Alessandro ran, he pointed out the pattern of going inside and outside of the columns. Both Lee and Hannah felt that rhythm had a strong connection to patterning. Donna continued to expand on this idea of patterning. Donna stated: “Patterning – students running inside, outside.” Then Donna speculated: “What if students ran in a different pattern? For example – in, in, out, out…” Hope commented on how she observed students in the example exploring patterns: “So it’s interesting that they’re interacting with patterns in different ways.”

Hope referred to a conversation that might occur when she and the students arrived at the Municipal Theatre. Hope stated very directly: “I know a kid … if I said to them, ‘What do you

see?' they would say, 'I see a pattern.'" Hope confirmed: "The kids recognize that a pattern repeats, and they can identify that pattern in their world." Jean Claude described the full potential of what he thought students could do: "Predicting a pattern is a really sophisticated mathematical idea that we can tell is there, just by the fact that they're playing and laughing and having fun and singing." This comment appears here rather than in the group of comments referring to interest, as Jean Claude is emphasizing the sophistication of the concept of predicting patterns, albeit evidenced by the children having fun.

The statements in this category featured the participants' conceptions about the role of curriculum in measuring students' growth and mathematics in relation to the curriculum document, as was experienced in the documentation of "The Columns of the Municipal Theatre".

Chapter 7: Findings

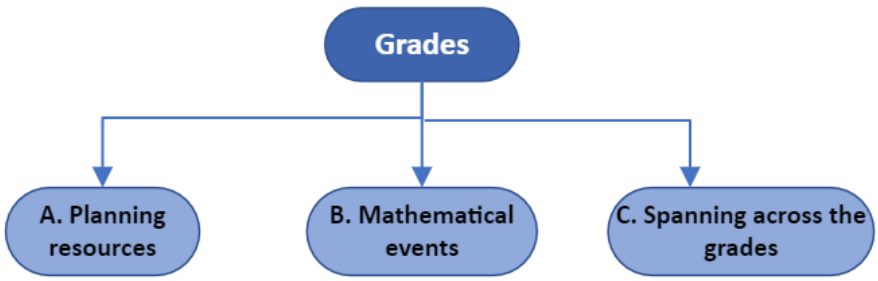
Grades, Centres, and Values

Chapter 7 highlights the findings described under the categories of grades, centres, and values. These categories all reflect data analyzed through content analysis of high frequency words. As described in the study design, high frequency words were identified and analyzed to examine the data from a different perspective, specifically to gain perspective in a broader context of pedagogical documentation and mathematics.

Category: Grades

The statements in this category contained the word “grade” or “grades". The statements were grouped into three sub-categories. The first sub-category, A) ‘planning resources’ described the statements participants made referring to what resources they used or could be used when planning in various grades. The sub-category B) ‘mathematical events’ evidenced the participants’ descriptions and anecdotes of the ‘mathematical events’ that occurred when they were students or that occurred in their classrooms when they were teachers. In the sub-category C) ‘spanning across the grades’, participants spoke about issues that span across grades in terms of documentation, tensions between kindergarten and other grades, and mathematics. The following diagram illustrates the sub-categories and subsequent groupings:

Figure 4: Categories Emerging from High-frequency Word Analysis of “grade” or “grades”



A) Planning resources:

In discussions about pedagogical documentation and/or mathematics, conversations about planning and what resources teachers could use emerged. Jane stated: “I use Marian Small. I haven’t used her books in kindergarten, but I’ve used it for grade one.” Lee supported Jane’s choice, as she too used books authored by Canadian mathematics educator, Marian Small. Lee explained: “I like this one a lot because there’s lot of questions… it goes all the way to grade 2.” Donna was also interested in Marian Small’s books. She asked: “Lee, does that book have a grade 3-4?” Donna was moving to a new grade in the next school year, so she spoke of her goals regarding the use of textbooks as a resource in planning. Donna explained:

> A lot of the teachers at my school use the textbook especially in the junior grades. I want to take away the textbook. I want them to have fun learning. I want them to talk more rather than just independent work and busy work. I want something fresh. I want to reenergize myself. Especially now that I’m teaching grade 4, which is a new grade for me.

B) Mathematical events

Some conversations about grades led participants to recall specific mathematical events that had occurred in specific grades in their lives. Lee wanted to share an experience she had as a math learner. She explained: “I have to talk about how I was taught math… because I got really stuck in grade one and two. And they just told you to subtract by putting the top number on, then the bottom number.” Jean Claude also shared an experience he had as a student: “In grade 10, I was sitting in a computer programming class with variables and functions or whatever, and a light bulb clicked, and I understood grade 10 math for the first time, halfway through the year because of what I was doing in computer programming.”

Some statements that used the word “grades” also used the word “mathematics”. Some statements referred to a specific mathematical context (e.g., when Lee’s students built nets in her kindergarten class). Other statements described what students do mathematically in specific grades. Donna felt that the progression from kindergarten to grade 1 was a large step. Donna stated: “There’s such a jump in math and language from kindergarten to grade 1.”

Some participants described what they knew students could accomplish in mathematics in specific grades. Lee shared how one student was inspired to create nets in kindergarten. Lee explained: “We found an old textbook in the staff room, and they had this pattern for grade 5 or 6, and I brought it down [to the classroom] because it looked like net shapes. And one of the students is like, ‘I can make that!’” Lee continued to explain some mathematics her kindergarten class tried in mathematics. Lee explained how the students were inspired to make dodecahedrons: “They got really excited, it was supposed to be for grade 6, we got really excited and tried to make one, ‘cause we thought we could make one out of paper.” As participants described what was happening mathematically in their classrooms, Karen questioned if those stories exemplified all learners. Karen was doubtful that what participants were saying students could do, represented the majority of students in a kindergarten classroom. Karen wondered: “I really felt I had a strong math program, but the language and the words that you’re using as examples, I’m like, ‘Really?!’ Those are your midrange SKs? Or is that maybe grade one and grade two?”

C) Spanning across the grades

In this sub-category the participants referenced the words ‘grade’ and ‘grades’ when discussing ideas and concerns that spanned across the grades. Some statements in this category spoke to the idea of documentation across the grades. Jean Claude felt that documentation had more application than just kindergarten. Jean Claude explained: “Everything we were discussing about pedagogical documentation talks about how it’s not just kinder, it sets

students and teachers up for success across the grade span. I can use pedagogical documentation and my learnings about it in a grade seven classroom." Jean Claude explained that whether people viewed pedagogical documentation simply as a tool, or whether people viewed it as a way of being with children and people, applied throughout the grades. Jean Claude expanded on his thoughts: "One or both of those depending on what pedagogical documentation means… like the way of being with children and the relationships applies across the grades." Jean Claude also felt that the Reggio inspired approach, including pedagogical documentation, could have a greater impact across the grades if educators began to embrace the view of the child in a more Reggio inspired way. He felt that our current view of the child where education happens to them rather than with them is very different from that which a Reggio inspired approach would entail. Therefore, it could have a systemic impact if this view of the child were to 'catch on'. Jean Claude explained: "You could talk about marks and grades and every single aspect of the way school is structured is endangered if pedagogical documentation catches on."

As conversations occurred regarding pedagogical documentation and mathematics, the subject of the curriculum of other grades besides kindergarten, emerged. Participants discussed kindergarten in relation to other grades and the tensions that exist between the two contexts. Jean Claude tried to make sense of the tension between mathematics in kindergarten and mathematics in older grades. Jean Claude described his concern:

> In play based kinder we embrace the idea of being present and connecting to the curriculum after. But then in grades 1-8 math we make a big deal about being intentional in our teaching with teacher moves to guide students along a series of predetermined learning goals. They feel kind of opposed.

Jean Claude also expressed his concern about the tension between the kindergarten program being more of an emergent curriculum compared to older grades that have very detailed expectations and learning goals need to be presented up front. Jean Claude explained:

> So, the problem for me is not just that kinder is hazy in terms of whether we should plan before or after, but rather every single grade is suddenly thrown into question, because of course it can and should be and is done together, but we need to acknowledge that these two things don't mesh automatically. That there's actually a conflict to wrestle with here that lasts through the grade span.

Hope supported Jean Claude's idea of the tension between teaching and learning in kindergarten compared to that of older grades. Hope explained: "And the fact that the program [kindergarten] values pieces like documentation and it values communication, there's so many other values, that maybe aren't as portrayed in the older grades." Hope continued to support Jean Claude's claim of the tensions between kindergarten and older grades. Hope stated: "I don't know if learning goals necessarily have as strong a connection in kindergarten as it may in some of the older grades. I have to agree with Jean Claude in the sense that you don't always know your goal in kindergarten." Lee asked: "Do we have to follow the curriculum exactly and say we only teach in grade 1 when they're ready. While still examining how educators could enact pedagogical documentation in other grades and considering the tensions with the curriculum documents, Karen explained her perspective about the curriculum. Karen stated: "I'm a very strong believer in that curriculum we need, it's our cornerstone. And there's nothing that irks me more than teachers going beyond the curriculum of that grade."

As participants were considering how pedagogical documentation may be enacted in other grades, ideas of how the design of the mathematics curriculum spanning the grades, emerged. Participants engaged in a conversation about mathematics in terms of the 'scope and

sequence' across the curriculum and grade levels. For example, Jean Claude spoke of a Ministry of Education publication called the "Paying Attention to…" Series. Jean Claude explained how reading that series helped him understand the scope and sequence of some mathematical ideas from kindergarten to grade 12. Jean Claude explained: "I found that those were so cool and so helpful because it really connects the dots of between what you're doing in kindergarten and what you're doing up to grade 12." Jean Claude also expressed his concern about who may be making decisions in terms of the scope and sequence of learning in various grades, at political levels. Jean Claude stated:

> I'm really interested in the idea of the math curriculum, the Ontario program for mathematics, [someone] is maybe making these arbitrary or maybe they are research-based decisions about what's a kindergarten idea, what's a grade 3 idea, what's a grade 8 idea, with the intention that they build on each other as you go from grade to grade.

Participants also expressed interest in the scope and sequence across the grades in terms of student readiness for the next grade. For example, Donna stated: "I remember Karen saying she worries kids aren't going to be ready for grade 1." Donna continued: "I think in kindergarten there's a pressure to be ready for grade 1." Lee added to Donna's idea: "I'm going to build on that, and just say that grade 1 teachers always say that, and then they don't understand the curriculum." Stacey explained that the scope and sequence does not just impact the progression from kindergarten to grade 1. Stacey expressed a further implication: "Because if kids I'm working with in JK cannot figure out the difference between a triangle and a circle, they are not going to have that foundation, and that really solid foundation to move forward, to take it to grade 3 or grade 4." Karen agreed: "They need to know that in and out [addition of numbers 1 – 10], backwards, upside down, everything. It needs to be solid so that when they get to grade 1, they are ready for those teens."

Comments in this category reflected statements that included the word “grade” or “grades”. Interestingly, statements reflected “grades” in terms of grade levels and at no time reflected “grades” as a mark or evaluation.

Category: Centres

Statements in this category all use the word “centre” or “centres”. One statement described ‘The Price of Apples’ as a centre and other statements described what materials could or would be placed in a centre. Other statements spoke about the planning of centres and some participants explained their struggles in how to engage with children in centres.

When examining ‘the Price of Apples’ documentation, Shira stated how she recognized this example as a play centre. Shira simply remarked: “This is a story of children approaching this play centre in their classroom.”

Participants also described and questioned what materials educators could offer in various centres in the classroom. Lee explained how she and her DECE would prepare materials for the centre: “We used to make bank cards for all the kids, so they could have one at the drama centre. We had a bank there.” Stacey explained what she did to prepare the centre: “I bring in grocery store flyers every week because I still get them at my house, and I put them into the dramatic play centre and my kids absolutely loved it.” Shira noticed in ‘the Price of Apples’ example the unique addition of a balance to the centre. Shira remarked: “You think about your stereotypical store or kitchen centre area, I don’t necessarily think ‘Oh, in my store area I should have scales.’” As Lee began to think more broadly about materials in her centres, she questioned the materials she has offered at centres in the past: “Did I have a number line in every single centre? Did I put one across the floor so you could measure across the floor?” Lee thought if she were to offer a number line in many centres in the classroom, the children would have more opportunities to think mathematically at every centre. Lee stated: “You could just

count across the floor, and anybody could count, right? … like in EVERY centre." Lee continued to expand on her thoughts. Lee realized the importance of materials in the centres to support mathematical perspectives and processes. Lee explained that the materials would be critical in offering mathematical experiences to students. Lee felt that teachers should consider what they are offering as math at every centre. Lee stated: "In every centre if you can't see most of the strands of math somewhere, how can you bring it in?"

Some participants commented on the planning aspect of centres. Stacey had visited a private school where she saw centres established. At the private school, the teachers had placed photos at each centre of students engaging in the work of that centre. She felt it was especially important for students to know what was expected of them at each centre and if planning for the centres would include photos, this would be very helpful for students. Stacey explained: "I know that at the private school, their junior kindergarten class, they have a lot of that at each centre … pictures of the children in the learning centre demonstrating play and they narrate it, which is really unique." Shira was impressed as she considered the planning that must have occurred when establishing the environment in 'the Price of Apples' example. Shira explained: "One thing that resonated with me is how the educators in that room would have used so much intentional thought when planning this play centre, to have all these different … like to have the scales." Hope described how she would engage students in the planning of the centres. Hope explained: "They [the students] started to use data management techniques in order to gather their data and then interpret that data in order to determine where the dramatic centres should go." Hope explained how she endeavours to include the students in planning. Hope explained it was very easy to involve the children: "For example, determine what we should do or turn our dramatic centre into, or determining what snacks we should offer at the movie theatre, those are easy pieces that I can turn around and offer to the kids."

Some statements addressed the way in which teachers could engage with students in the centres. When discussing how to use pedagogical documentation while students are at centres, much discussion focused on the possibility of bringing students back to the learning that occurred in previous centres. Some participants suggested that students could reflect on the documentation from previous centres. Karen felt uneasy with the idea of taking students away from their present play to reflect on something they had done previously: “And how do I take them over to the block centre … okay now remember what we were doing?” Karen explained the challenges with trying to get students to reflect on previous centre play. Karen questioned whether or not that was truly honouring student learning: “I found it difficult to take what they were doing that day and then do those next steps naturally, and still honour what they were doing the next day, because I didn’t want to take them away from what they were now in … like the water centre.” Lee described how she liked to engage with the students as they were working at centres: “I really like the concept of just being able to let them play and record like just that is their normal conversation, like this is what they’re talking about at a centre.”

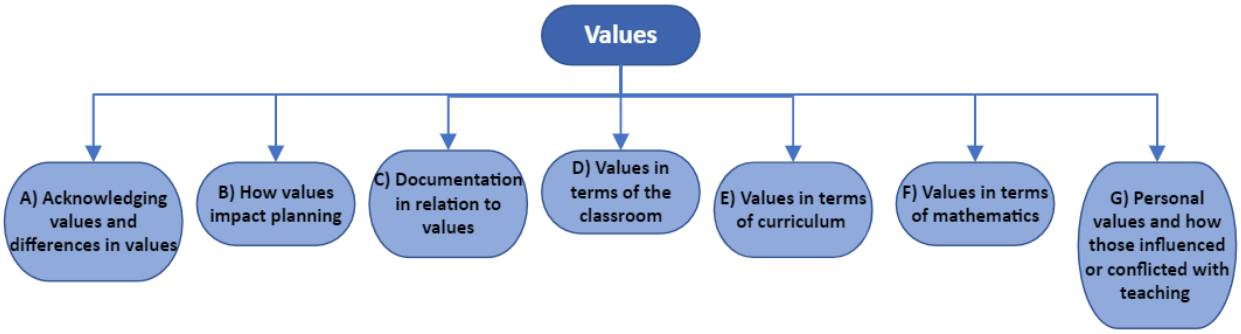
Statements in this category focused on the word centre and how centres exist in the kindergarten classroom. Participants expressed how it is possible to engage students in the planning of the centres, while others explained sometimes engaging with students in centres means simply observing the students and documenting their play.

Category: Values

As the discussion of pedagogical documentation progressed, the concept of values and how values pertain to pedagogical documentation entered the dialogue. The idea of ‘values’ and how values relate to pedagogical documentation are important because these statements demonstrate one aspect of how the teachers in the study conceptualize pedagogical documentation. Statements in this category include every statement where participants used the

word “value” (or “values”) throughout the 10-week sessions. Several sub-categories emerged in this category and the following diagram describes the sub-categories:

Figure 5: Categories Emerging from High-frequency Word Analysis of “value” or “values”



A) Acknowledging values and differences in values:

Participants made statements that pertained to acknowledging values and acknowledging differences in values, in this category. During a conversation about values in relation to pedagogical documentation, Reagan spoke to the idea of values that teachers hold. Reagan felt it was necessary that teachers acknowledge they bring values with them to their work: “It doesn’t’ necessarily have to be that all values are totally equal, but you just need to acknowledge what your values are, to make sure that it’s the right proportions.” Reagan continued to highlight the importance of acknowledging values and checking the position or influence of those values: “… but just to acknowledge what your values are and then kind of work backwards from there to say, ‘Is this something that could be skewed or not’. Just being reflective of them, I guess.”

B) How values impact planning:

During conversations about mathematics and organizing their programs, some participants spoke about value in terms of planning. These statements spoke to examples of

planning that were valued, and how values influence planning. For example, Stacey was very interested in seeing a long range plan that came from an exemplary class that was valued: “I’d love to see in your ideal world or someone you look up to, or a kindergarten class that you value, a year at a glance in math.” Stacey continued to explain why this was so important to her: “Long range plans are so valued and still written, but I’d love to see that and maybe even deconstruct a long-range plan.” Other participants, like Shira, explained how she saw value was considered in the planning of the learning in the example the “Price of Apples”. Shira stated: “Just the simplicity, without compromising the depth of the learning. The simplicity of the organization, but without sacrificing that depth and that real value.” Stacey also reminded the group how important it was to consider the value of each individual child when planning. Stacey explained: “I think it’s really important to value each individual learner, and how we plan for that.”

C) Documentation in relation to values

Statements in this category also reflected the participants’ conceptions of how values relate to pedagogical documentation. Hope explained how her values played out in her documentation. Hope stated: “So again, you are going to see that in my documentation that I really value following the kids’ leads and you’ll see that.” Hope explained in more detail how her values influence her choices: “I think the way that you value the elements of your program, and your pedagogy should be apparent within, or would be apparent within your documentation.” Hope continued to explain:

> It’s interesting that I like the way you sort of ordered it, value this so you can see it in my documentation. For example, I really value open play. I really value having provocations that are open ended and giving kids the time to explore that.

Hope expressed what makes kindergarten different than older grades and how documentation is truly valued. Hope stated: “The fact that the program values pieces like documentation and it values communication and there’s so many other values that are maybe not as portrayed in the older grades.”

D) Values in terms of the classroom,

Alongside the specific discussion about values and pedagogical documentation were statements that spoke to how values relate to the classroom. In the sense of the group of people who come together in that space. Jacques explained that he was looking forward to getting a full time job to establish his own values, rather than those that he felt were espoused by his faculty of education. Jacques explained: “I want to be in a classroom so I’m not just monotonously repeating what my school has taught me the values of the classroom should be.” Lee explained how value manifests itself in the classroom in terms of how she values students. Lee explained that students have opportunities to play, but also, she works with small groups as she sees fit, depending on what students need. Lee stated: “There’s lots of play, there’s lot of opportunity to work in small groups and do something that I’m really concerned about. There’s lot of things that can be done and there’s lot of play space for them because everybody is valued.”

E) Values in terms of curriculum,

When discussing pedagogical documentation, the conversation pointed towards values in relation to the curriculum documents. Statements that addressed the idea of value within the curriculum, referred to value in terms of valuing the curriculum, and the values that are inherent in or are given to curriculum. For example, some statements spoke to the values that the participants had about curriculum. Karen explained, “After we finished off last time, I realized that the one thing that I really value is curriculum.” Karen shared that what appears in the curriculum document is very important to her. She explained further: “If I could see the

curriculum happening, whether it's social, emotional, or whatever, that's what I valued." Other statements in this group spoke to values or value systems that participants held, which might present themselves through the choices the teachers made in enacting the curriculum document. Reagan explained this idea:

> We might not you know explicitly say 'These are the value we should have' or 'These are my values' or anything like that, but by the way you implement the curriculum, by focusing your time on something versus another thing … that's kind of teaching your values.

Reagan continued to explain that it is inevitable that teachers will bring their own choices and values into their work in their classrooms. She added to her ideas: "If you were to say I'm going to teach everything equally in math and teach it just as the curriculum states, without values, then you're going to have your internal biases and your values skewed towards certain things you're used to, and you prefer [anyway]."

F) Values in terms of mathematics,

An important part of the discussions the study group had related to mathematics in relation to pedagogical documentation. Therefore, the idea of value in and of math emerged. A key feature of this group of statements is how value is connected to math. Value took on different meanings in this group. For example, some statements addressed value as in the actual value of math manipulatives, for example play money. These comments addressing the value of play money appear here, as they did not speak to curriculum specifically. Another meaning value took was connected to how participants valued mathematics. When discussing the way that children understood the value of money through manipulatives, Shira explained: "I think it's more rather than having the child associate… this coin has this value… it's more that this coin represents this number, and this piece of paper represent this number." Stacey stated

that she liked students to be able to play with manipulatives rather than real money. Stacey explained, “I feel that actually brings in a deeper value and then a concept of understanding.” Shira shared one concern she had about play or real money. Shira stated: “I think we talked in another week about how coins that look basically the same [toonies and loonies] having different value is a really abstract concept for kids to grasp.” Some statements referred to other mathematical ideas of value. Lee stated: “They [students in kindergarten] don’t always know value.” Jean Claude built on the idea of value. Jean Claude said: “… in grade one they have this conceptual understanding that money can purchase things and money is worth a certain amount, and then maybe you could start talking about a specific cents value for dollar value.” Other statements in this group addressed the values of and in mathematics. Shira explained the challenges she faced: “I know that math connects through everything, but how do I translate that into enriching experiences that value this interconnectedness of math while also realize that kindergarten is a program such that it is general, and things happen when they happen.” Jane explained that whether teachers value math or not, teachers must teach it. Jane explained: “Let’s say math, because math isn’t my thing, but I still have to teach math, right? Whether I like it or not or value it or whatever, I still have to do it.” Lee spoke to the value of revisiting concepts in math. Lee stated: “One of the articles really kind of fed back to what you were saying Karen, about repeating stuff. There is real comfort and value in doing certain things over and over again.”

G) Personal values and how those influenced or conflicted with teaching.

The final group of statements in this category referenced how personal values influenced or conflicted with teaching. Lee wanted to be sure she understood what was meant by “values”. Lee stated: “So that when we start talking about values… I would love to know what you mean by values.” Lee continued to clarify her perspective about the word “value”: “I think that we’re using that word that doesn’t necessarily need to be in there, what do we need to teach the

curriculum, is not about my value system." Hope agreed that the group was stuck on the word 'value', but explained why the word is important: "I think we're getting really stuck in this values word, and yet I really think it's your values that really determine the program." Lee still felt uncomfortable with the use of the word "value":

> I don't want my teaching "values" as I don't care about this or I do care about that … it feels moralistic whereas if I'm teaching curriculum, I like exactly what you are saying but I think it's the word… I think it's the 'value' word that really upsets me.

Hope tried to help the group come to a collective understanding of the meaning of 'values'. Hope tried to explain how the values being discussed were specific to those that pertained to the classroom. Hope stated: "I think the more you say values I don't think you necessarily mean like moralistic values. The way I interpret that is more your values in terms of… not who you are as a person… but what do you value with your classroom."

Some participants named what they valued. Karen explained: "I don't know if you guys ever talked about your key values. For me, it's my family, my health, and then my teaching. Teaching is very, very high up." Jacques explained what he valued: "What I value most is that the students feel comfortable and feel safe sharing their answers and speaking in class and speaking around the other students in the class and being okay getting the wrong answer because that's all part of learning." Shira stated what she valued: "So I guess when I think of what I value, I value really learning from them [the students] by watching them explore." Sam also stated her values: "I value going outside every day." Other statements were general statements that spoke to the implications of values. Jane stated: "What we do value has to be on the sideline because it's about the kids and what they need to do and our curriculum, right?" Jane expressed her discomfort in discussing values. She explained: "I just kind of feel like when

you say you value, it makes it personal, about you." Hope offered her ideas to explain how values come into their work. Hope stated:

> I think the word value, like I wrote here in my notes, like yes you need to cover the curriculum. However, I think it's your values that help you map out how you're going to cover it, your values are what are going to, I think, help you to create the program that you believe, as the educator, will help those kids be successful.

Reagan tried to explain how the concept of having values does not necessarily need to be viewed negatively. Reagan thought there might be merit to naming and acknowledging personal values. Reagan explained: "I could also go on a positive lens where like, you could value respect and you might not realize it but you're implicitly teaching children to respect each other and that's a huge value that they're learning." Other statements spoke to the way values may or may not influence choices. Jane explained how she did not really feel that our personal values should influence our choices. Jane stated:

> I guess to me 'value' is sort of a personal word and you might have your kindergarten program and there may be things you value in it more than others. But when it comes right down to it, it doesn't really matter what you value because you still have to cover it all.

Hope tried to explain her reasoning about personal values and choices. Hope expressed:

> So, when I hear the talk about the value, like what do I value as a teacher? I think that word value connects really well with the whole idea of pedagogical because it connects to the word pedagogy. Like what are my values in the classroom, what do I value in terms of my program, in terms of how I am with kids, that's at least my interpretation.

Statements in this category revealed the participants' views about values in relation to many aspects, as well as how values influenced their choices. The statements demonstrate that

participants had varied opinions on the importance of values and the types of values that may or may not pertain to their teaching.

Chapter 8: Discussion

In Chapter 8, I offer up five distinct yet interconnected sections that form the heart of the discussion. In the first section I focus on the values that educators place on the curriculum and their work, providing insights into the driving forces (e.g., thinking and beliefs) that shape their instructional approaches. The second section discusses pedagogical documentation, recognizing its potential to capture student action but underscoring a gap in the discourse as a transformative force to 'be with students' in ways that are not traditionally understood in Ontario pedagogy. Moving forward, the third section contributes a critical analysis of the Ontario Curriculum, shedding light on its rigor and its impact on teaching practices. I then shift the focus to the kindergarten classroom (section four), to the exploration of learning centers reveals a discomfort with the use of these spaces and the intended goal for their use. Finally, within the realm of mathematics education (section five), I trouble the relationship the participants have for teaching mathematics. We will discuss what seemed to be the nature of their current knowledge and how we might create opportunities for future realizations in math. Through these interwoven threads, I intend to contribute insights to the ongoing examination of engaging with children and pedagogical documentation in the kindergarten math classroom.

Values

I suggest that the findings of this study reveal that a critical gap exists between the educators' awareness of their values and values in general, and the choices that values play in pedagogical decisions in mathematics learning and otherwise. Although a couple of participants acknowledged values in terms of their own practice, many teachers in the study did not connect values they had about their practice or students, to the work they do in their mathematics classroom. Three participants were adamant that values did not have anything to do with their work. In fact, Lee pointed out that the discussion of 'values' made her very uncomfortable as it

felt ‘moralistic’ to talk about values. Surprisingly, many teachers in the study had a prevailing belief that the curriculum document, specifically the expectations, was the deciding factor as to what should be considered and enacted in the classroom.

Lee explained that teachers must teach the curriculum and that we did not even need to discuss the word “value”. Jane agreed with Lee by saying it does not matter what educators value because they still must cover the curriculum expectations. Karen also reiterated the importance of curriculum as what she valued when she stated that seeing the curriculum happening in the classroom, was what she truly valued. Karen spoke about the curriculum, as others had, as the determining factor of her choices. These teachers conceptualized values as being irrelevant and not applicable to the work they do in the mathematics classroom and felt the focus should be on covering the curriculum expectations.

As I consider Karen’s views, I am again drawn to the neoliberal undertones of the teachers’ ideas. The learning events occurring in classrooms where teachers believe the curriculum is all that has value, are reduced to ‘zero-sum exchanges’ as coined by Jardine, Friesen, and Clifford (2006). I also wonder about Karen’s autonomy. Pitt and Phelan (2008) draw our attention to the fact that even though teachers are educated and highly skilled professionals, with specialized knowledge, who technically have autonomy, they are controlled by governing bodies and systemic dogmas that regulate, or even strip them of that autonomy. Karen truly felt that her sole role was to deliver curriculum. I question how much of Karen’s belief that only curriculum could dictate what she did in the classroom was determined by the constraints of her autonomy by the system.

In math, this may mean that Karen might look for very specific evidence of the curriculum expectations and may miss some of the important aspects of reform mathematics. For example, an expectation like “15.5 subitize quantities to 5 without having to count, using a

variety of materials" could easily be satisfied by seeing a student identify a pattern like 5 on a dice. However, that evidence would not demonstrate any other connections, representations, or conceptualizations the student has of 5, which is built through conceptual understanding. Karen stated her only value was ensuring the curriculum expectations were delivered, however, with this being her only true value, there is a possibility that Karen could miss an opportunity to understand and help build her student's conceptual understanding.

The sentiment of Lee, Jane, and Karen invites one to consider if these teachers may be swept up in the dogmas of education, as Giamminuti et al. (2022) explained. Specifically, I consider the dogma of implementation where the educator fulfills the government mandate without resistance or question. The experience with the teachers became a point of possibility to further consider the idea of the teacher as a technician whose sole purpose is to deliver pre-determined, government controlled curricular programs. Additionally, this relates to Britzman's (2003) discussion of teacher subjectivity. These teachers were caught up in the cultural myths that accompany the image of the teacher. Because of institutional pressures, these teachers viewed 'values' as a subjective force that did not have a place in their work. Finally, we could consider the work of Pitt and Phelan (2008) and how they have troubled the idea of teacher autonomy. Pitt and Phelan (2008) explain that teacher autonomy refers to the teacher thinking for themselves in unpredictable and complex situations where judgement is more important than routine. However, this autonomy occurs in the service of children. To add another layer to teacher autonomy, the teacher is supposed to be autonomous, but within the confines of a governing body of licensure and certification and a "prescriptive, outcome-based curricula of systems of accountability" (Pitt & Phelan, 2008, p. 190). Therefore, I can understand Lee, Jane, and Karen's hesitation to identify values as being something that they could identify and acknowledge in their practice. They are unknowingly reacting to the dogma of implementation,

their own teacher subjectivity, and their lack of autonomy due to the paradox of teacher autonomy.

This revelation carries substantial implications, as Dahlberg (2012) cautions that if educators are not alert and observant about what determines the choices they make in their practice, pedagogical documentation may easily get "swept up into strategies to 'predict and control' children more effectively through processes of normalization and surveillance" (p. 229). Meaning that without heightened awareness about values, educators may make choices about what to document based on perceived standardized measures of normalization from the curriculum expectations. Notably, this may include watching children and waiting for them to demonstrate specific desired outcomes, rather than accepting the range of student ideas that may organically present themselves for documentation. As teachers watch for the normative outcomes of mathematics in the kindergarten classroom, their actions may in fact resist aspects of reform mathematics. For example, teachers may watch carefully for predetermined outcomes (either by their own accord or by curriculum) which could potentially discourage or not allow for student dialogue about the mathematics. Because there isn't any dialogue occurring, this would lead to teachers not needing to or having the opportunity to listen to student ideas. Both dialogue and teachers listening to students are both important aspects of reform mathematics.

However, when we think back to the affordances and possibilities of documentation, we can consider how pedagogical documentation allows us to disrupt the normative demands of education, especially in the Ontario context. Educators use their values to make judgements as to what to document. In recognizing this subjectivity, educators engage with pedagogical documentation openly and unapologetically pointing to what is important to them.

Renaldi (2001, 2004, 2021) tells us that values are, in fact, what inform educators' decisions when they engage with pedagogical documentation. An educator will demonstrate to

children what they value by selecting and documenting specific interactions they have with children. Although the importance and the impact of values on teacher practice and pedagogical documentation is well documented in the research (e.g., Giudici et al., 2001; Rinaldi, 2001, 2004, 2021), it was surprising and concerning that a subset of participants expressed a dismissive attitude toward the significance of values and the role of values in their work.

Reagan, however, was one participant who conceptualized how her work might reflect her ‘values’. Reagan expressed that it was necessary, as teachers, to acknowledge that teachers do have values that influence their choices. Reagan suggested that it is important for teachers to realize that some values could ‘skew’ decisions or influence choices that teachers make. With her example of a teacher implicitly teaching respect, Reagan suggested that some values may be taught inadvertently, perhaps by modelling respect for the students and holding respect as an important trait among the students. Reagan did draw attention to the fact that teachers have values and should acknowledge them. Reagan did suggest that values could influence what a teacher might do in the classroom. However, Reagan did not reference how values might implicate choices in the process of pedagogical documentation.

Reagan was at the beginning of her career, and I found it interesting that she did recognize the implications values may have for her as a teacher. Reagan’s ideas connected to Putman’s (1993) suggestion that pedagogy and school are not neutral. Reagan recognized that values do implicate the relationships of the teacher with other teachers, with curriculum, and with students. In one conversation Reagan explained how she liked how the educators at ‘The Municipal Theatre’ were asking open-ended questions. This demonstrates that Reagan values giving students open-ended questions, in other words, questions with multiple responses or entry points. This is one important aspect of reform mathematics. Reagan said she liked how the teacher wasn’t asking directed and limiting questions but allowed the students to elaborate on what they noticed and explain their ideas. Reagan was acknowledging how important

student dialogue is which highlights another important aspect of reform mathematics. Reagan knew that we, as teachers, must take responsibility for our values as they influence our choices.

Hope also referred to values as having an impact in her work. Hope suggested that what she values in her classroom and how she values the children, may help shape the program she designs. Hope's perception of her experience partially aligns with what the literature describes in terms of values in teaching. The Ontario Kindergarten curriculum document (Ministry of Education, 2016) states that the values educators hold deeply influence the way an educator documents a child's learning. The curriculum document emphasizes "[t]he choices educators make about what to document reveal their values and what they deem important to notice about children" (Ministry of Education, 2016, p. 38). Rinaldi (2012) supports this idea as she explains, "Children feel that we value what they say, and therefore what they think, because we record it, transcribe it, and reflect on what it might mean" (p. 242). Rinaldi (2012) states that in making choices as to what to document, educators are giving value to what students do. In this process, children see and feel what teachers value about their voice and thoughts (Rinaldi, 2012). Although Hope did not refer to her values in terms of what she documented, she did acknowledge that her values were going to influence her choices in general.

Implications:

The implications stemming from the discussion of this study are not only noteworthy but hold paramount importance in reshaping the discourse and practices surrounding pedagogical documentation and mathematics education in Ontario. The glaring misalignment between the theoretical perspectives of values within pedagogical documentation literature and the actual conceptions of how teachers used the word "values" in this study, suggests that there may be missed opportunities for teachers to use pedagogical documentation as a vehicle for democratic education, co-construction of learning with students, and the development of mathematical ideas and theories in early learning.

It is important to consider the conceptions of the participants of the study, in terms of how values intersect with the work they do, as it is evident that what educators think will affect what they do in their classrooms (Clark & Peterson, 1984). The Ontario Kindergarten curriculum document requires that pedagogical documentation be the vehicle to make students' ideas visible. Therefore, it is critical that educators pay attention to their own values and how values at large implicate what happens in the kindergarten classroom. If Ontario teachers lack awareness of the role of their values, or values in general in the pedagogical process, then there is potential for a misalignment between what teachers enact in classroom and what the curriculum document recommends (recommendations that align with research). For example, the kindergarten curriculum document states,

> The choices educators make about what to document reveal their values and what they deem important to notice about children. … [Educators] must be aware of their own subjectivity and biases—that is, they must recognize that they are capturing and representing children's learning through the lens of their particular perspective on children and on how children learn. (Ministry of Education, 2016, p. 38)

The kindergarten curriculum document calls educators to view children as capable, competent, and through an asset-based lens (Ministry of Ontario, 2016). If educators do not acknowledge and value children as capable and competent, they may look at children through the lens of what children cannot do, rather than what they can do. Educators may also see themselves as the holder of the knowledge and not as a co-learner or co-researcher. The idea of the educator as the 'holder of the knowledge' does not fit with the ideas of reform mathematics, as educators are required to give students opportunities to investigate ideas that will help them build understanding in ways that are meaningful for the student.

Additionally, as we examine our learning about pedagogical documentation, we recognize that without acknowledging values, pedagogical documentation is not much more than anecdotal evidence of student action. Without acknowledging and/or consciously considering values, teachers run the risk of unconscious biases or preconceived notions influencing what they choose to document. When teachers are mindful of their values and the potential implicit or explicit influence these values have on their engagement with pedagogical documentation, they can adopt a reflective approach and remain openly receptive to new ideas and theories that may surface. Pedagogical documentation offers the opportunity to challenge the status quo and challenge dogmas that are rooted in neoliberalism, opening the educational experience to spaces of uncertainty and infinite possibility.

Participants in the study could have indicated their values of mathematics reform in many different ways. An example could have occurred when Lee discussed the money manipulatives used in 'The Price of Apples' documentation. When referring to the circles with the numbers one, two, and three, Lee suggested that "Maybe with all those circles, maybe you have tons that are blank". This demonstrates Lee valuing the reform aspect of having tasks and/or materials with multiple entry points. The children could write what made sense to them on the circles. Lee could have anticipated some ideas that the children might have when working with the blank circles. For example, Lee might have thought the children could write all different numbers on the circles, they could write just ones, or maybe they would write twos, fives, and tens, as those are the numbers the class had been recently counting. Lee could have considered how she might have responded to those ideas as anticipating students ideas is another aspect of reform mathematics. Maybe Lee would think: "If the students respond with twos, fives, and tens, after a period of them exploring that combination, I could ask them if we could eliminate the twos and only work with the fives and tens". In doing so, Lee may be

thinking that she could provoke students to consider the values of five and ten to see if that combination provoked any other ideas about the numbers.

This study provides a strong case to explicitly examine values with in-service educators and pre-service educators. Pre-service programs and in-service professional development must explore the concept of values that educators hold in terms of how these values influence the choices that educators make, especially when considering what to document and the values inherent in reform mathematics. For example, for so long in mathematics education simply coming to the correct answer was valued. Now we understand that the value needs to be placed on the strategies and processes students use to come to their solutions. Currently pre-service programs could invite pre-service teachers who were taught to value the 'right answer' to reconsider and find a new relationship in the joy of the problem solving, used to get to that 'right answer'. Pre-service education programs must specifically discuss values and unpack how those specific values could influence practice. Additionally, programs and professional development should examine values in terms of what values are already inherent in the curriculum documents.

None of the teachers mentioned the critical aspect of conceptual understanding of mathematics reform. Traditionally, early mathematics education focused on rote memorization of facts, such as knowing that 3 + 2 = 5 without understanding the concept. Today, the curriculum emphasizes conceptual understanding as a key component of mathematics reform. For instance, students now learn that numbers can be composed and decomposed, understanding that 5 can be split into various combinations, such as 1 and 4, or 2 and 3. This shift from memorization to comprehension is evident in many aspects of the current kindergarten program and is an important aspect of mathematics reform.

Notably, many teachers in the study felt the curriculum documents were the objective foundation of their work in the classroom. Several teachers referred to the curriculum expectations as what 'had to be covered' without making a connection to the choices they could make that may subjectively, yet inadvertently, influence the outcomes of what 'had to be covered'. This is significant considering that judgement is one of the most important cognitive processes of a teacher (Clark & Yinger, 1977).

The study's compelling case for examining values extends beyond theoretical discussion to practical implications for educators like Reagan. Reagan already accepts that teachers have values that influence their practice. Reagan spoke of valuing respect in the mathematics classroom. Reagan explained to her colleagues, that if you valued respect, it is likely you would teach students to be respectful, even if it wasn't a conscious thought. If Reagan valued respect and she also valued student discourse (an aspect of math reform) Reagan may choose to teach students norms that would be embraced so that they could engage in respectful conversations in mathematics. Perhaps Reagan would provide students with sentence stems for their discussions as Bruce and Flynn (2011) discussed, as math talk guidelines. For example, when wanting students to share varying perspectives she might suggest her students use the phrase… "I would like to add onto that idea …". In this way Reagan has not only pointed to valuing respect, but she has demonstrated to students how she values respect within her value of student discourse.

Therefore, values should then be carried forward into in-service professional development where acknowledging and naming values and how they influence what is enacted in the classroom, is examined through careful reflection. By fostering a culture of deliberate consideration and critical reflection on values, educators can align their practices with the principles of democratic education, specifically mathematics education reform, ensuring that the choices they make are intentional and in line with the ethos of pedagogical documentation.

Here is an example of a learning event that could occur and then be used in professional development to discuss what values and aspects of reform mathematics are embedded in this interaction.

Imagine kindergarten students conducting a survey of their friends' favorite flowers. The educator observes and documents their process as they record names under two columns: rose and tulip. Afterward, the teacher gathers the students to discuss the results and share recorded observations. The students who conducted the survey share their processes and the other children comment and build on the ideas. This scenario exemplifies several aspects of math reform. First the teacher demonstrates the value of the students' ideas by gathering everyone around and having the 'surveyors' explain their findings. Second, the teacher engages students in a discussion about the survey and students continue the discourse. This demonstrates the aspect of valuing student discourse in mathematics reform. Third, conceptual understanding of gathering and analyzing data occurs because the context offers the students an authentic experience of collecting data from their friends, then having to make sense of the data and what it means. (Ball, 1990; Boaler, 2002; Hiebert & Wearne,1993; Leijen et al., 2023; Lesh & Zawojewski, 2007; Maas, 2011; NCTM, 2014; Smith et al., 2009). The educator participates actively, fostering a community of learners where understanding occurs because math is discussed, and together they make sense of mathematics. Pedagogical documentation captures students' ideas, guiding future provocations and decisions informed by shared values. When used in professional development, this learning event could be captured in pedagogical documentation and then discussed by educators to consider what values in general and of mathematics reform are espoused.

Pedagogical Documentation

Participants in the study conceptualized pedagogical documentation with a diverse array of definitions, features, and purposes that were partially aligned with the kindergarten curriculum

document and the literature, but also often misaligned with the kindergarten curriculum document and the literature. This may suggest that pedagogical documentation is being understood and enacted in different ways in kindergarten classrooms across Ontario.

For instance, Hope stated pedagogical documentation was pictures and Shira suggested it was a story. Lee explained that pedagogical documentation was rich samples and photos. Jean Claude suggested there was a very fine line between documentation and pedagogical documentation but did not offer a distinction between the two (e.g., suggesting how documentation might be pedagogical or how documentation could just be a collection of information). Hope stated the ten pictures in 'The Price of Apples' were pedagogical documentation and the other photos that the educators must have culled, were simply documentation. Generally, participants conceptualized pedagogical documentation as pictures or a story, or pictures that portrayed a story. These comments reveal there was not a consistent definition of pedagogical documentation among the participants. These findings are concerning as pedagogical documentation is meant to be a living record or memory of student ideas which offers a way of going back to students' theories and learning events. It can be an opportunity for educators to take up a learnable participatory disposition in mathematics, as described by Davis and Renert (2013). Without a common understanding or definition, pedagogical documentation may be used in a variety of ways that are not in line with the kindergarten curriculum document or the literature.

Jean Claude did refer to pedagogical documentation as a 'way of being' with children which he stated was all about relationships. He also noted that in many classrooms presently, mathematics education is happening to children, not with them. This reveals that Jean Claude was considering pedagogical documentation as a process that should happen with children and involves relationships. He did not elaborate further on either of these ideas. Some participants in the study did not speak of pedagogical documentation as a process that involved the child in

meaningful ways or helped foster deeper relationships between educators and students. Lee and Hope did mention that children could be involved in pedagogical documentation. For example, Lee explained that children could use pedagogical documentation to clarify understanding. Hope stated that children could be involved in the process of documenting when the teacher did not intend to use the documentation for evaluation. Otherwise, Shira described how she wanted to be effective at documenting but did not want to intrude on the students' learning. Shira also recognized that she could not be present without influencing students in the process. Shira did not speak to the idea of her presence being part of the relationship that could develop from pedagogical documentation.

Shira seemed to be torn between two thoughts. Shira did not realize that her involvement was critical in the education encounter. Shira's explanation draws me to Biesta's (2014) work. Biesta (2014) reminds us that teaching must be more than just the facilitation of learning. Shira was struggling with how much of herself she should place in the interactions with students. Shira wanted to engage with pedagogical documentation but did not want to intrude on the students' learning. But on the other hand, Shira recognized that she could not be present with students without influencing them. Referring to Biesta (2014), we must consider the idea of what the teacher brings to the learning encounter. Shira might find a generative invitation in the words of Biesta (2014): "… the role of the teacher is not of a disposable or dispensable resource, but an individual that has something to give" (p. 57).

Let's consider the teachers' perspectives on the role of the student in pedagogical documentation. Both Stacey and Hope highlighted the importance of student involvement. Stacey asked about the 'student voice' in the documentation of 'The Price of Apples,' while Hope suggested having students take the pictures. These comments indicate that Stacey and Hope value active student participation in learning encounters.

Interestingly, students being active participants in their own learning, is an important aspect of mathematics reform, which emphasizes students making sense of mathematical concepts for themselves (Kazemi & Stipek, 2001; NCTM, 2014). By questioning the presence of student actions in the documentation, Stacey and Hope were aligning with the principles of math reform, possibly without realizing it. Pedagogical documentation can thus support math reform by providing a platform for capturing student ideas and offering them up for discussion. This approach not only values student contributions but also enhances the alignment between pedagogical practices and mathematics reform (Ball & Cohen, 1999).

It is in fact difficult to say what role the students had behind the documentation we examined, other than what the documentation showed. We know the students were active in creating their own currency and prices for the fruit, as the documentation illustrated that. Yet, it is also important to consider the words of Carolyn Edwards (2012) as she discussed the relationship between educators and children. Edwards (2012) reminds us that the relationship between educators and children is one of a special partnership. Educators are not there to "facilitate" learning but more to "stimulate" learning by making problems more complex or involved. Educators do not just leave children to work on their own but cooperate with the children to help them to reach their goals. In mathematics education, this might mean that educators are solving a math problem with the children, as in the example of the survey of the flowers. The educator might ask the students how they could display the information they gathered in a way that would be clear for everyone to see the results. In a mathematics reform classroom, this would mean the educator listens to the student's suggestions considering which suggestion might advance the thinking of the class. Perhaps a student suggests a picture of a flower for each person that chose a rose or a tulip. The teacher might name the graph for the children: "That's a pictograph!" Then the educator might ask, "how could we do that?" In this scenario, many aspects of mathematics reform are being fostered. For example, the teacher

listened to the children's ideas using those ideas to advance the thinking of the entire class. The teacher also asked a question that could have multiple solutions when asking how could they display their information and how might they go about making a pictograph.

Additionally, we can recall Biesta's (2005, 2008, 2014) perspective on the role of the teacher. Biesta (2005, 2008, 2014) reminds us that the teacher has a very important role. The author states that the teacher comes to the learning encounter bringing something that is not already known by the students nor can be validated by what the students already know. The educator comes from the outside and brings something radically new to the learning encounter. Therefore, I can understand how Stacey and Hope were considering student involvement in this documentation; however, I wonder if they did not fully understand the nuanced relationship between educator and child. I think perhaps they were focused on the 'student centredness' of the documentation and not looking at the learning encounter from the multiple angles of the role of the teacher and the role of the students, in relationship with the environment, materials, and curriculum.

Implications:

The variations in participants' conceptualizations of pedagogical documentation highlight a pressing need for a collective and well-defined understanding of pedagogical documentation and its relationship to teaching. Kindergarten teachers in Ontario may be challenged to define pedagogical documentation as 'a way of being' or as a way to challenge traditional mathematics practices and consider children's ideas and theories in ways that seem unconventional. This challenge may be a result of educators believing that the expectations in the curriculum document are predetermined standards that dictate what must be 'covered'. If we are to examine pedagogical documentation as suggested by the literature, we must consider pedagogical documentation an opportunity to challenge neoliberal ideals. For example, if pedagogical documentation is being used simply as a record of task completion, a teacher may

collect student work or student ideas in mathematics, simply looking for one correct answer. If the teacher of the students in the example of the survey was looking for only one way to collect information, let's say using tallies, they may have dismissed the children's record keeping of the survey results.

Whereas, if educators are engaging with pedagogical documentation as an opportunity to challenge neoliberal ideas, then pedagogical documentation could become the documentation that captures multiple varied mathematical solutions and offers those solutions up to students for further discussion and reflection, as was done in the flower survey example. In this way the educator engages in the learning encounter with the student positioning mathematics as a learnable participatory disposition. The child and the educator engage in the process of gathering, reading, and interpreting the learning experiences as they happen, in meaningful ways. In doing so, the child helps choose the learning path and essentially is a protagonist in their learning. Here educators and children endeavour to make meaning together. This is how pedagogical documentation is deemed 'a way of being with children' and an opportunity to engage with mathematics in ways that are in line with mathematics reform. Pedagogical documentation offers an opportunity to discuss mathematical ideas, is immersed in that which is relational, and challenges pedagogies that promote standardization and competition.

Notably, teachers in Ontario have not been trained to consider pedagogical documentation as an opportunity to build relationships with children, challenge the demands of standardization and accountability, or listen hermeneutically to children's ideas and theories. As has been discussed (Davis, 1997; Rinaldi, 2006), listening to students is a critical tenant of reform mathematics. This involves paying close attention to students' ideas, reasoning, and misconceptions to assume a 'way of being' with students where mathematics is investigated together (students and educators). This approach values students' voices and recognizes the

importance of understanding their thought processes (Kazemi & Stipek, 2001). Based on the data I have gathered, it seems as though the teachers participating my study may be using pedagogical documentation to record evidence of student achievement against the curriculum expectations, rather seeing the opportunity being offered, of listening and being open to new ideas and theories that children have.

The testament of the participants of the study is important as it reveals that teachers in Ontario may not be aware of the possibilities of pedagogical documentation, in a way that is in line with the literature and ethos of a Reggio Emilia approach. This is significant, as what teachers believe and think influences their actions (Clark & Peterson, 1984). Therefore, a few recommendations could be considered for pre-service and in-service teachers.

First, pre-service and in-service teachers need a better theoretical understanding of pedagogical documentation in a way that is in line with an approach congruent with Reggio Emilia and mathematics education reform. This understanding must include an examination of the role of the teacher and student and how they exist in relationship to one another. Naturally, this reveals the view of the child as being caring, competent, and capable of creating rich theories and engaging in deep thinking. This understanding may also include an examination of values and what role values have in the use of pedagogical documentation and what the application of values communicates to students. Teachers would need to examine pedagogical documentation as an opportunity to disrupt neoliberal discourse of competition and standardization, while fostering and celebrating ideas that emerge from the process.

To support teachers' theoretical understandings of pedagogical documentation, teachers may require more concrete examples of the process of pedagogical documentation and examples of how the process involves children and their mathematical ideas. For example, the survey that kindergarten students conducted, as discussed above, could provide some lovely

documentation for teachers to view and consider, as the students' processes are described to them. These examples could also include photos or videos of how educators can use pedagogical documentation to make meaning. The documentation gathered could also be examined for the various aspects of mathematics reform it captured. Teachers must understand the process of pedagogical documentation as a way to make students' ideas and processes visible and build relationships with students and mathematics; not simply a final product of what has been 'accomplished'.

Second, pre-service and in-service teachers need more opportunities to harness and understand the potential of pedagogical documentation as a transformative tool for co-learning and co-researching with children in mathematics. Educators must understand the curriculum deeply but resist the temptation to simply list the mathematics as it relates to the curriculum. The teacher must engage in noticing what the child offers in terms of their ideas and theories, rather than notice what the teacher has in their mind as preconceived notions. Again, we can refer back to the example of the students conducting a survey to find out who likes which flower. A teacher embracing pedagogical documentation and reform mathematics would observe, listen, and document the students' ideas and theories. The teacher may offer up the documentation for many to consider, including students and other educators. The teacher may encourage the students to share their ideas of data collection, categorization, and representation. The teacher might then facilitate a discussion with everyone about the results, encouraging everyone to consider what is being highlighted and what pieces of this learning offer another path to explore.

This approach aligns with the principles of reform mathematics, which emphasize conceptual understanding and students as active participants of their learning (Kazemi & Stipek, 2001; Leijen et al., 2023). By documenting and reflecting on this activity, the teacher honors student ideas and resists neoliberal practices that focus solely on standardized outcomes.

Instead, the teacher supports students in constructing their own understanding and making mathematical connections between concepts, operations and relations connections to broader mathematical concepts (Yurekli et al., 2020). This approach brings aspects of mathematics reform forward as students build conceptual understanding, students are encouraged to talk and share their ideas, and teachers listen to students' ideas to consider what might happen next or what provocations the teacher can provide in this learning event. This collaborative, approach to learning not only meets curriculum expectations but also resists the normative practices of standardization and traditional mathematics.

Pedagogical documentation and mathematics reform can mutually reinforce each other by challenging traditional, rote-based approaches to math education and promoting the aspects of mathematics reform. Pedagogical documentation involves observing, recording, and reflecting on students' learning processes, which aligns with the core aspects of reform based mathematics like providing tasks with multiple entry points, encouraging problem-solving, listening to students' ideas, and providing ample opportunity for students to discuss and make sense of their ideas (Ball & Cohen, 1999; NCTM, 2014). By using documentation to capture and analyze students' mathematical thinking, teachers can identify and build upon students' natural curiosity and conceptual understanding, fostering a deeper engagement with mathematical ideas (Kazemi & Stipek, 2001). This process challenges the status quo of traditional math education, which often prioritizes procedural fluency and standardized testing over meaningful learning experiences (Boaler, 2002). Additionally, pedagogical documentation encourages educators to adopt a more reflective and responsive pedagogy, enabling educators to change their approach to instruction to meet students' needs and interests, thus supporting the goals of mathematics reform (Hiebert & Grouws, 2007). By integrating these practices, educators can create deeper relationships with students, honour student ideas, and foster an approach more in line with mathematics reform.

The Ontario Curriculum

The data of this study reveals without a doubt that the participants tended to approach the Ontario curriculum as a list of items that educators can 'check off'. This approach poses significant challenges when implementing pedagogical documentation in the kindergarten classroom and teaching in ways that are in line with mathematics reform. The significance of these findings is underscored by existing literature, which posits that the judgements of the participants will influence the choices they make and hence influence their practice (Clark & Yinger, 1977). Therefore, the data of the study indicates that the participants tended to look at the curriculum as a checklist, impacting their use of pedagogical documentation. This implies a potential tendency to use pedagogical documentation merely as a tool to evidence specific outcomes, rather than as a method to help co-construct meaning with the child.

The data further highlights the participants thorough understanding of the curriculum expectations, particularly in the domain of mathematics, as was evidenced in their conversation surrounding the pedagogical documentation 'The Columns of the Municipal Theatre'. Donna listed various aspects of spatial sense like the cylinders and the base of the building as a rectangular prism. Donna also identified symmetry in the building. Hannah described measurement when she saw the students hugging the columns and describing how fat they were. Hannah mentioned patterning as the children ran in and out of the columns, and added the students used positional terms to describe the position of the teacher (e.g., in front of the column). The participants identified elements from the curriculum document suggesting that they knew what had to be 'covered' from the expectations, and they had a very specific idea of the evidence that would prove the concept had been covered. However, it is crucial to note that participants primarily focused on the 'look fors' of the expectations, neglecting the opportunity to delve into children's thoughts and theories revealed through the children's actions, expressions, and dialogue. We can see the tension here with reform practices in mathematics merely from

the fact that teachers had specific ideas of what the evidence would look like to ‘cover’ a specific curriculum expectation. This suggests that teachers were not necessarily hoping for or open to multiple solutions as would occur in reform mathematics (Franke & Kazemi, 2001; Gainsberg, 2012; Smith et al., 2019). Additionally, it seems that looking for the evidence of outcomes could also dictate the types of mathematical tasks that were provided for the students. If searching for one specific outcome, teachers could provide tasks that were very narrow in their approach and/or solution, which is contrary to reform mathematics. In reform mathematics tasks are intended to offer multiple entry points and multiple solution strategies (NCTM, 2014).

Consider Hannah’s observation of students running through columns while chanting “Inside, outside, inside, outside.” This aligns with the curriculum's expectation to identify and describe repeating patterns. Hannah noted the students' understanding of patterning but didn't discuss the next steps. Reflecting on reform mathematics, Hannah could have engaged with the students in their play. Hannah might have listening to the students then asked: “Are there other ways we could run through the columns?” In doing so Hannah would be engaging with the aspect of reform mathematics that refers to educators listening to student ideas and then asking open-ended questions to advance thinking.

One can see from a review of the Ontario curriculum document why teachers feel pressure to be accountable to have evidence for every specific expectation in the curriculum document. It is clear why this may get in the way of teachers’ ability to be with children in a way that embraces children’s ideas and theories. Two main contributors to these problems may be a) the size of the kindergarten curriculum document, and b) the perceived pressure to assess curriculum.

The magnitude of the kindergarten curriculum document makes it challenging for teachers to be present with students in a way that would allow teachers to build relationships

and capture the richness of students' math understanding through pedagogical documentation, as the literature on pedagogical documentation suggests. The size of this document, as I have discussed, may put serious limitations on the way teachers are able to engage with pedagogical documentation. The document numbers the overall expectations and describes an overarching concept or skill. It is easy to see how educators could take up the expectations as a checklist as the overall expectation is listed first then a lengthy list of specific expectations follows the overall. It is understandable that teachers could or would become overwhelmed and preoccupied with specific expectations when they examine the detail and examples provided for the teacher in each specific expectation. The specific expectations continue to list, one after the other, the specificity of the skills and concepts that students are to glean over the two year program. It is clear how teachers could interpret the expectations as a checklist. It is not surprising that teachers feel pressure to collect evidence for all 126 specific expectations for each of the thirty students in their class.

Teachers in the study expressed an urgency to look for situations and events that had the potential to match the curriculum expectations, when discussing what math could occur in their classrooms. For example, Lee was concerned about having 'math' available at all centres in the classroom. Lee explained that if you could not see math in every centre, you needed to think about how you could bring it in. Lee felt that teachers needed to bring math into the centre and then it would be easier to identify as students engaged with it. Lee went as far to say that the teacher must be the 'conduit' of learning. Lee explained that it is the teacher that knows the curriculum and what it says, therefore the teacher must ensure that the student produces the curriculum expectations in their play. Lee's ideas seemed to be leaning towards the ideals of reform mathematics and pedagogical documentation as she recognized the important role of the teacher, not as the "knower of all" but as a collaborator. However, Lee's comments seemed to suggest that she still felt the pressure of having evidence for those expectations. Lee's

comments reveal that Lee felt the teacher knows the curriculum so the teacher can either identify the expectations in play or direct the play, so the expectations emerge.

Although Lee was considering carefully what the teacher could bring to the learning experience, it seemed as though Lee was approaching that from a point of responsibility and accountability (as the teacher) rather than ways in which the educator could create a learning encounter with the child. This brings us back to what Britzman (2003) suggested about the cultural myths of the teacher. In this conversation, Lee felt responsible for making learning happen. Lee was caught in the myth that everything depends on the teacher. Sadly, when this occurs, teachers feel that they must be perfect. Britzman (2003) reminds us that teachers then equate learning with social control, and they view pedagogy as a process of instilling knowledge rather than considering what constructs knowledge and our relationships to knowledge. To further this discussion, I think of the dogmas of pedagogical documentation as discussed by Giamminuti et al. (2022). Perhaps, Lee was also caught in the dogma of implementation as she described the teacher as the ‘conduit’. Giamminuti et al. (2022), describe how the dogma of implementation leads teachers to feel unquestionably tied to the power and content-driven, government mandated curriculum. In mathematics reform, this could mean that teachers are drawn to traditional teaching practices to more easily identify those curriculum outcomes. Teachers want to comply and resist any form of interesting experimentation. Pedagogical documentation, when used under this dogma, is simply a tool used by a technician to ensure that routine tasks are carried out efficiently.

There is a serious risk of teachers using pedagogical documentation as a record of task completion (Wien et al., 2011) when they approach the curriculum as a list and teach math as bite sized pieces waiting to be checked off a list once identified (Jardine, Friesen, & Clifford, 2006). Many teachers are so concerned with finding evidence (to be able to assess and evaluate), that they miss the opportunity to be with children and listen hermeneutically to their

math ideas. Being with children hermeneutically is something that educators must practice when engaging with pedagogical documentation, honouring student thinking, and encouraging multiple strategies in mathematical solutions, all aspects relating to mathematics reform practices. Rinaldi (2001) explains that when educators miss the opportunity to be with children hermeneutically, they inevitably collect pedagogical documentation as evidence of learning, and they only examine it at the end of the learning experience. Documentation collected in this way will show the salient moments of learning on the path taken by the student but determined by the teacher. As an example, Stacey preferred math centres to be very specific and have very explicit directions, to the point of having pictures for students to follow as they complete a math task. Even if Stacey collected documentation during the centre by capturing photos of the work students were doing, it would have been documentation of the path the teacher created to ensure students achieved the objectives. This suggests that the teacher is in control and is designing the learning to cover the curriculum. This is an example of a mathematics learning event that is not in line with the ideas of mathematics reform. Rinaldi (2001) explains this is a key difference between the way educators use pedagogical documentation in North American contexts versus Reggio Emilia. The intention of pedagogical documentation, as it originated in Reggio Emilia, is for pedagogical documentation to be used as an integral part of a process that is aimed at fostering learning, building relationships, and opening up the encounter to something new; it is used during the learning process for immediate reflection and interpretation.

The Ontario kindergarten document (Ministry of Education, 2016) requires teachers to measure what they observe students saying and doing in some type of assessment. Teachers measure student ‘learning’ as it is produced, against the curriculum ‘list’. When educators focus so much on the list of expectations and educators measure student learning as it is produced, they miss the opportunity to be present with students and capture the richness of mathematical understanding and theories that students demonstrate. This leads teachers to habitually look for

normative outcomes of learning, which is problematic as in doing so, educators view learning as a commodity they may check off a list once it is obtained (Jardine, Friesen, & Clifford, 2006).

The form of the current Ontario curriculum (Ministry of Education, 2016) seems to make it conducive for teachers to check off an item each time it is produced in the classroom by the students. This is evidenced as described above, with a long list of specific, detailed expectations itemizing the ‘look fors’ of the overall expectations. Biesta (2008) reminds us that we have seen a rise in the interest of measuring student outcomes in the last 30 years. Biesta (2008) suggests that teachers become so concerned with being able to check items off their list, they begin to value what is easiest to measure. The focus then diverts to valuing what is easy to measure rather than measuring what is valued. This is particularly relevant in math education, where the emphasis often falls on procedural fluency and rote memorization rather than conceptual understanding and problem-solving. Therefore, teachers end up looking for the expectations as items that they recognize easily, rather than spending time in dialogue with children understanding their mathematical ideas and theories. This approach overlooks the importance of fostering students' deep comprehension and creative engagement with mathematical concepts, which are central to reform mathematics education.

Implications:

Considering the conceptions of the study participants, a few recommendations could be considered because opportunities to engage with student ideas and theories are missed when curriculum expectations are searched for in a list-like fashion.

First, more professional development is imperative, specifically in kindergarten, to help teachers understand the importance of the big ideas and overall expectations of the curriculum document. As specific expectations serve as ways that students can demonstrate understanding of overall expectations, educators do not need to have evidence collected for

every specific expectation to be able to justify their assessment. Educators need to consider specific expectations as occurring in the classroom context, as learning occurs individually, in small groups, and in whole groups. In mathematics education, this may mean that teachers need a deeper understanding of the connections between mathematical ideas so they can identify them and help children make those connections. Educators could then use pedagogical documentation as a way of being with students, to make students' ideas visible in encounters that are emergent. The educator may then notice SOME of the specific expectations (not all), knowing that the overall expectation is being evidenced. Teachers would then have more time to spend with children having conversations about their mathematical ideas, rather than hyper-focusing on evidence for every specific expectation. Teachers need to have the freedom to listen to student ideas and consider how those ideas fit under a broader umbrella of overall expectations and how students' conceptions create deep meaning.

Second, kindergarten teachers must critically reconsider how they measure expectations. Professional development needs to help educators focus on how to measure and report progress in kindergarten. Although educators deem pedagogical documentation as the way in which educators can make student ideas visible, teachers continue to look at the expectations as isolated pieces of learning to be checked off their lists. This also returns to my comment above about teachers having the mathematical connections themselves, to help students make those connections. As Wien et al. (2011) say, educators end up using pedagogical documentation as evidence of task completion when they use it to record what has occurred.

Professional development around how educators could use pedagogical documentation as a 'way of being' with children and a way to create deeper meaning, would help refocus the purpose of the expectations of the curriculum. Pedagogical documentation offers an opportunity to sit with the student in a space of meaning making, relationship, and being open to something

new. Educators would benefit from participating in professional development that would offer them opportunities to consider what type of listening they usually engage with, in their mathematics classrooms. For example, educators could have opportunities to engage with doing mathematics with their students where they consider if they are applying evaluative, interpretive or hermeneutic listening (Davis, 1997) in the learning event. Educators could reflect upon what their normative response might be and then consider what that learning event might look like if they were to listen hermeneutically. In professional development educators may need to unpack what hermeneutic listening might feel like and look like, and even practice engaging in that kind of listening. By listening hermeneutically, educators can strategically and intentionally 'stimulate' theories and ideas, by introducing prompts or materials to support the student in discovering what was not there before.

Centres

Participants in the study spoke of the use of centres in the kindergarten classroom in ways that revealed a notable discrepancy between the intention of centres (as they appear in the literature) and the intentions of pedagogical documentation and mathematics reform. The participants used the term 'centres' frequently in conversations and the educators consistently referenced 'centres' as the areas of the classroom where learning occurred. There were very few occasions where the participants referred to learning that was not 'in a centre'. This suggests that the participants may have inadvertently perceived the 'centre' as one of the principal areas in the classroom where learning occurs. I began to wonder if the participants, without consciously knowing, viewed 'centres' as the only place where learning could occur. Did they then conceptualize learning not occurring in other spaces in the classroom? The use of 'centres' as a place where learning exclusively occurs in the kindergarten classroom, could be problematic as kindergarten is a play-based, inquiry-based program, whereas 'centres' (as described in the literature) are often used as spaces to direct and control student activities and

learning (Keiff, 2005; King-Sears, 2005). Notably, the literature on pedagogical documentation emphasizes that learning experiences should unfold organically anywhere in the learning space, rather than confining learning to a specific area of the room (i.e., a centre). In mathematics education reform, there is a focus on tasks that are rich, open-ended, and offer multiple solution possibilities (NCTM, 2014). These tasks could also occur anywhere in the classroom and are not necessarily subject to only occurring at a centre.

The kindergarten curriculum document (Ministry of Education, 2016) describes the role of the educator in an inquiry based classroom as one who questions and wonders "alongside the children, to support their learning as they exercise their natural curiosity" (p. 21). The document suggests that educators no longer prepare lessons based on pre-determined topics or themes. The document describes how inquiries should develop out of reciprocal conversations. As children express their ideas, educators should continue to consider what further questions or materials they could offer to continue to stimulate those ideas. The question arises concerning the role of a centre within this framework. If a centre is a pre-determined theme based area, with specific teacher directed tasks, it is not in line with the fluid and child-directed nature for which the curriculum advocates or methods that are in line with the Reggio Emilia approach to education. For example, if educators want to address the curriculum expectation "18.1 identify and describe informally the repeating nature of patterns in everyday content (e.g., patterns in nature such as morning-noon-night, …), using appropriate terminology (e.g., "goes before", "goes after") and gestures (e.g., pointing, nodding, using slap/claps)" they may set out very specific materials at a centre (could be a table top activity). Perhaps a card is placed out that shows a series of shapes in a row, for example, red triangle, blue square, red triangle, blue square. The task is, "Name the next two shapes." This offer is not really in line with mathematics reform and the fact that it is set out at a specific table offers even less flexibility in the task. The task is very specific with only one interpretation and only one 'correct' solution.

These are not aspects of reform mathematics. The ‘centre’ allows the educator to control exactly what is occurring and how the student must engage with the task. The participants’ descriptions of mathematics learning in their classrooms suggest a perception that learning primarily occurs at centres, potentially limiting the dynamic and evolving nature of the learning space, materials, and tasks, as outlined in the kindergarten document and supported by mathematics reform.

It is significant to note that some participants explained how they liked explicit directions for children at the centres. This illustrates the tension between the goal of the kindergarten program (as play-based and inquiry-based) and the centre which was teacher controlled and directed. Specifically, Stacey shared how she liked math centres set up with pictures accompanied by a written narration of what the students were to do at the centre. She explained that each child would be able to follow the images of another child successfully completing the task and they too would complete the task successfully (i.e., as was determined by the teacher). For example, imagine a centre that is set up with 3 dimensional shapes placed on a tabletop. There are a set number of shapes at the table and there are instructions that tell the students they must build a structure that uses 4 rectangular prisms (including a cube), one cylinder, and one pyramid. A picture accompanies the instructions that give an example of a structure including these specific shapes. The activity is very teacher directed and again, not really in line with the ideas of mathematics reform. Although it could be said that the child has choices with what kind of structure they will build (suggesting multiple ways to approach the task), it could be argued that multiple entry points are controlled by the shapes that are offered at the centre. There are only a specific number and type of shape available, and a photo is provided showing students what their structure could (or should) look like. Further to this, if students are completing the task independently at the centre, then likely student dialogue is not being encouraged, which is another important aspect of mathematics reform.

This scenario is problematic for a couple of reasons. Here we run the risk that the students may simply copy the example, which requires very little independent thinking. The children's ideas are further limited by including a specific number of shapes on the table, Additionally, this approach challenges the intentions of the kindergarten curriculum document and mathematics reform, in that the kindergarten program requires us to create learning spaces that are flexible and change in relation to the students' curiosity, interests, and thinking (Ministry of Education of Ontario, 2016) and mathematics reform supports students engaging in tasks with multiple entry points and constructing meaning in ways that make sense for them. The kindergarten document states that the space "is created and arranged as the children's learning process unfolds—it is constantly being negotiated by and with the children" (Ministry of Education of Ontario, 2016, p. 29). I propose that a centre that is so rigid and there are step by step pictures illustrating to students what they must do, does not evolve according to a students' interest or curiosity. Nor does a centre like this uphold many of the aspects of mathematics reform. If centres are used to control students' tasks or keep students 'busy' while the teachers work with small groups (Keiff, 2005; King-Sears, 2005; Pattillo & Vaughan, 1992), this would be contrary to what the kindergarten curriculum document and literature suggest, plus points to practices that uphold a neoliberalist agenda.

Further to this point, is the issue of how 'centres' fit into developmentally appropriate practice. As I have previously discussed, centres are often used to allow the educator to plan within the context of 'developmentally appropriate practice'. Yet, let us recall how developmentally appropriate practice has been found to limit a child's natural potential, be culturally narrow, reduce a child's social opportunities, and may lead to an overreliance on standardized tools to track and evaluate achievement (Meisels, 1992). This is quite contrary to the goals of pedagogical documentation and a Reggio Emilia approach to education. Another aspect in the data that made me consider the testimony of the participants occurred when Lee

stated she just likes to watch students play in the centres and document what children are saying in their play. I noted that it was unclear if Lee would interact with the children when she was just watching them play. Although it is important to observe students in their play, as I have discussed, it is important to do so hermeneutically. Also where mathematics reform is concerned, teachers have an important role to support students in expressing their ideas and then teachers can engage with the students in mathematical thinking by participating in the discourse and/or asking open questions to prompt further ideas. According to their research, Bautista et al. (2019) explained that just because there are centres in a classroom, it did not mean teachers were spending time engaging with students at the centres. It is a concern if teachers approach 'centre time' as a period without interactions with children. To engage with pedagogical documentation and mathematics in ways that the literature describes, it is critical that teachers be present and interact as a co-learner and co-researcher. Without engaging with the children, Lee's documentation could simply become a running record of what students were doing.

Implications:

Addressing the observed disparities in the use of centres in the classroom, I suggest specific recommendations and implications for further research and professional development.

There is a clear need for additional research to examine the dynamics of classroom spaces and how educators configure these spaces. Examining the role of various locations in the classroom, isolating activities, or tasks to specific areas, and understanding the reasons behind such configurations would provide valuable insights. Further to that, the way in which these spaces are being used in comparison to the important aspects of mathematics reform, is also warranted. Research should examine if teachers are in fact using 'centres', how are these spaces used. Research should also scrutinize the types of tasks offered at centres against the

aspects of mathematics reform, considering both the tasks themselves and the accompanying directions provided to students.

Targeted professional development is essential for educators to align the use of centres with the principles of play-based and inquiry-driven learning. Centres may in fact be a part of the culture of education in Ontario, however, I believe there is not a universal definition of centres either in the way they are set up or with the types of tasks offered at the centres, especially in mathematics education. Educators must gain a comprehensive understanding of the nature of play-based and inquiry-based learning and how mathematics reform is situated within that context, critically evaluating whether the use of centres supports that learning in the classroom or detracts from it. Ultimately when pedagogical documentation is being enacted in a way that fosters relationships and creates meaning with students, it does not necessarily have to occur in particular areas of the classroom delegated to specific play. Adjacent to that, mathematics tasks in line with reform aspects of mathematics, do not need to occur solely at centres. In fact, some of the most authentic mathematical problems could occur anywhere in the learning environment. Teachers should also be aware of the connections centres have to developmentally appropriate practices that are not necessarily in line with Reggio Emilia approaches, the use of pedagogical documentation, and mathematics reform. The focus for engaging with pedagogical documentation should be on offering materials that provoke thinking based on students' curiosities and interests, not based on practices that may limit student potential. Interacting with children to gain insight into their theories and mathematical ideas should be able to occur anywhere in the classroom, whether that is in the environment in general, or in a specific area.

Mathematics

During my study, the study group did discuss mathematics; however, it is evident that much of the conversation focused on identifying mathematics in general or identifying mathematics that appears in the curriculum document and discussing the tools and materials

used in mathematical activities. However, the analysis highlights a critical gap in understanding participants' comprehension of actual mathematical concepts and their perspectives on supporting students in building on students' ideas. It is imperative to assert that, in line with pedagogical documentation, the discourse seldom framed mathematics as an opportunity to actively 'be with students' or build on student ideas. This study's unique contribution lies in the revelation of the conceptions of educators hold and the unexplored aspects of educators' engagement with mathematics as an opportunity to listen to students, share learning encounters, and build on their ideas with in-the-moment actions.

When analyzing the data around the conversations about mathematics, it is important to note that most often the comments consisted of stating what mathematics was present and how it connected to or appeared in the Ontario kindergarten curriculum document, particularly emphasizing the significance of number. I would like to point out that the participants had an extensive understanding of what content is in the kindergarten curriculum document by their adept reference to mathematics in the documentation examples. Notably, Hope demonstrated that she is well aware of the Ontario kindergarten curriculum document when she stated that when students engage in open ended mathematical play like in the examples, they are more open to risk taking; risk taking is identified in the curriculum. Reagan knew by referring to the curriculum document, that students were to be posing mathematical questions and Reagan recognized that happening in the examples. Lee and Hope related to the curriculum when they discussed the students counting in groups of 2, 5 and 10, as in the curriculum mentions. Lee and Hope spoke to the fact that kindergarten expectations state that educators must offer mathematical opportunities to students to explore the concept of quantity and the purpose of numbers in a variety of contexts. Lee explained that the curriculum states students must construct and deconstruct numbers to ten, noting the importance of number sense. Interestingly, in some cases the teachers could quote the curriculum document verbatim. This is significant

because the participants, despite their profound curriculum knowledge, fall short in discussing examples in relation to actively being with students or advancing student ideas. Participants did not discuss their actions as teachers in terms of in-the-moment responses to what could occur. This observation underscores the study's distinctive contribution by pinpointing a crucial gap in the educators' discourse about mathematics in relation to pedagogical documentation.

I am acutely interested in the lack of conversation about advancing mathematical ideas during the study. If we refer to what Davis and Renert (2013) discussed regarding the mathematics teachers need to teach mathematics, these authors suggest that this knowledge would include a fluency of various realizations of concepts and processes through which teachers and students can create mathematics. These authors suggest this body of knowledge is viewed as a "learnable participatory disposition" (Davis & Renert, 2013, p. 247) with a flexible and vibrant knowledge range. The participants in my study did not talk about mathematical ideas as participatory. They did not suggest how they interacted with students to engage in mathematical 'realizations' (Davis & Renert, 2013). Therefore, I posit that the educators did not look at mathematics knowledge as being a disposition. I draw upon the work of Mason and Davis (2013) to suggest that my participants did seem to lack the connective tissue the authors spoke of, between understanding mathematics and in-the-moment pedagogy. My participants seemed to not consider mathematics as a way of being with and in front of their students (Mason & Davis, 2013). This suggests that my participants understood mathematics as a set of discrete skills and units of isolated information. Naturally then, this impacts their ability to build on student thinking, or use pedagogical documentation as a way to encounter learning with children.

Interestingly, there was much conversation in the study meetings about the use of tools and materials of mathematics, and how they were used in 'The Price of Apples' example. Although we spent a great deal of time talking about tools and materials, participants spoke of

tools and materials in terms of affordances and possibilities but never related to ‘ways of being’ with students or ways to build on student thinking, once again, revealing a noteworthy omission in the discourse. Participants shared what they observed students doing with tools and materials, for example, when Lee shared one of her students used a measurement of two metres to keep that distance from his friends. Lee explained what had happened, but she was not involved with the student; her stories did not approach the idea of listening to children to expand on their ideas. Shira commented on the girl in the documentation using coins to represent the number 133. Shira noticed the girl placed the coins in the place value positions rather than ‘counting up’ the values of the coins. Hope noted the young girl did not have unitization. Many of the comments spoke of the limitations of student capabilities. The participants were able to connect what they saw to the curriculum document; however, they did not elaborate on what they thought the children considered in the actual mathematics, nor how to co-learn with students to develop a deeper understanding. This study serves as a pioneering exploration of identifying the absence of active engagement with students’ ideas, a pivotal component of using pedagogical documentation as a way of being and advancing student ideas mathematically.

Another dominant theme that occurred when talking about mathematics was the role of the teacher. When considering ways of being with students (as described in relation to research on pedagogical documentation), the role of the teacher is important. Participants began to allude to the importance of the teacher being present in these interactions. However, I would like to note a significant gap in that the participants did not speak to the role of the teacher in the context of a co-learner or co-researcher in the learning process with mathematics. For example, Jacques referred to the role of the teacher when discussing ‘The Columns of the Municipal Theatre’. Jacques suggested that teachers should ask questions like: “Why do you think this is?” or “How do you think that happened?” Jacques thought this was an excellent example of

how educators should teach math in kindergarten. Lee suggested the teacher did not need to provide answers. Hope felt that teachers must help students make connections and express themselves. It is evident that Jacques, Lee, and Hope were leaning towards mathematical reform ideas, but did not expand any further. Many comments about 'The Columns of the Municipal Theatre' example connected to the curriculum document; however further details were not offered. For example, Jacques did not explain why he thought this was an excellent example of how to teach mathematics in kindergarten, nor did he elaborate on what he thought the children were considering as mathematics. These revelations represent a necessary paradigm shift in understand the dynamics of the educator-student relationship, positioning this study as a catalyst for a re-evaluation of the role of the teacher in fostering deeper connections with students in mathematical exploration. These revelations become a focal point of this study's contributions as it requires an acknowledgement of the teacher's role as one that is more collaborative and interactive dismantling traditional hierarchies of power, as is described in the literature on mathematics reform and pedagogical documentation.

However, there were some comments that did suggest the co-construction of experiences and learning happening with children rather than to children. Hope suggested the teacher helps the student make connections when she said: "You as the educator are a wiser, smarter more experienced person would make that connection whereas they can't express that right?" Hope also alluded to co-constructing with students when she questioned whether or not she was listening to the children, giving them control, and constructing learning together, demonstrating that Hope was considering mathematics reform, even if not realizing it. Jean Claude also recognized the importance of the teacher participating with the child. Jean Claude stated that the teacher has a necessary role to move the learning forward, but he also warned that there is a danger of the teacher 'bulldozing' the learning, like an actor taking over a scene. Jean Claude suggested that students playing alone could limit the learning potential. Jean

Claude discussed learning in terms of who was leading learning. He suggested that it is redundant to try to determine who is leading. He stated that when educator and student are really listening to each other, something occurs where the sum is greater than its parts. A special combined effect occurs that without the relationship between the people, would not be as effective. Jean Claude's comment was perhaps the closest to understanding 'being with children', co-construction of ideas, and hermeneutic listening. Jean Claude referred to a synergy when educators and students are working together, and it is not evident who is leading the learning.

It is important to note what the participants in the study had to say, as well as what they did not say. Perhaps the conversations about mathematics remained at an identifying stage and did not become more reflective of how to be with students and advance students' ideas, as the group sessions were not intended to be "teacher led lessons". Importantly, the study refrained from adopting a teacher-led lesson approach, emphasizing its unique goal of understanding teachers' current thoughts and interpretations of pedagogical documentation and mathematics.

Implications:

Addressing the observed gaps in the discourse on mathematics, specifically in the context of engaging with pedagogical documentation, calls for recommendations and implications for future research and professional development.

The study proposes possible explanations for the observed gaps in the discourse, such as teacher potentially lacking a disposition toward math from significant mathematical realizations, required to engage with more advanced mathematical thoughts. For example, perhaps the teachers did not have a web of understandings (mathematical realizations) to support students to build on their current mathematical ideas (e.g., supporting students to gain deeper experience in unitization). In conversations about mathematics in the study, the teachers

discussed enhancing children's thinking and understanding the least. Therefore, educators themselves need more mathematical encounters to form a disposition that honours a web of understandings.

I would invite educators to deepen their own understanding of mathematics through participation in mathematical realizations, to move beyond identifying mathematical content from the curriculum, to actively fostering deeper connections with students in mathematical exploration. Educators should engage with mathematics in a way that will allow them to 'notice more' (as described by Mason & Davis) and in doing so develop the understanding of possible actions in response to what they notice. I would suggest that pre-service and in-service teacher development engage teachers with mathematics whereby teachers could learn to make meaning of students' actions mindfully and respond flexibly (Mason & Davis, 2013). I invite teachers to learn to extend their understanding and that of their students, while expanding the range of the interpretive possibilities of their students' ideas. These recommendations contribute to the broader discourse on teacher preparation and professional development, suggesting a reconsideration of the experiences for teachers when engaging with mathematical pedagogy.

This study did not provide a context for teachers to be responsive to students, which may explain the lack of discussion of enhancing children's thinking and understanding. Because the study occurred over the summer, teachers did not have a classroom of students to return to between meeting sessions. The dialogue focused on situations that had already occurred in the participants' classrooms, or the situations they believed were occurring in the examples of pedagogical documentation. Teachers could not bring back the ideas discussed and put them into action in their mathematics classroom. Teachers did not have an opportunity to practically apply the ideas we discussed in the study with their students. This could have left the participants unable to consider practical ways to build on student ideas. Teachers could not receive immediate feedback from their students. Had teachers returned to the classroom

between group sessions, perhaps they may have had more opportunities to engage with students and mathematics in the ways being discussed, and then they may have seen the results of that effort firsthand. This could have resulted in a positive feedback loop where participants could return to the group, report their week's actions and findings, and then continue to build their own understanding of mathematical realizations.

Therefore, this study suggests recommendations for future research, advocating for a more dynamic engagement where teachers are actively involved in their practices during the study. As a result, the educators could implement ideas and suggestions discussed between meetings. Perhaps, a deeper understanding of how to use pedagogical documentation as a way of being and a means to build on students' mathematical ideas would be gleaned. In the example of Jean Claude, having an opportunity to go back to the classroom after having this discussion and co-learning with students, he may develop a more nuanced understanding of what he was suggesting was the 'sum greater than its parts'. In effect, experiencing something closer to what the literature explains as 'being with children' as a co-learner and co-researcher. Perhaps engaging with pedagogical documentation in this experience, Jean Claude may develop a deeper understanding of how to use pedagogical documentation to make student ideas visible and create deeper meaning with children.

This study calls for a paradigm shift in pedagogical practices, urging educators to move beyond a mere identification of mathematical content according to the curriculum, to actively engaging with students' ideas, fostering collaborative learning experiences, and embracing a more dynamic and responsive approach to pedagogical documentation and mathematics. Through its examination of teachers' thoughts and beliefs and subsequent recommendations, this study becomes a voice in the ongoing dialogue on effective pedagogy both in mathematics and early learning, marking a significant contribution to the field of education.

Chapter 9: Conclusion

This study reveals how some teachers perceive and utilize pedagogical documentation in mathematics education, urging us to reevaluate the foundational values that guide our teaching practices. The findings of the study demonstrate a significant mismatch between what is written in the literature about pedagogical documentation and mathematics reform, and what is being conceptualized by teachers.

Themes of Insights and Learning:

Key findings occurred in themes such as values, pedagogical documentation, the Ontario curriculum, centres, and mathematics. Under the theme of 'values' a tension occurred between teachers' views and the current literature. Many teachers were adamant that values did not have a place in their practice. Teachers were not aware of the significant role values play in their practice as teachers, nor in the way in which values influence their engagement with pedagogical documentation. For example, when Lee reminded us that using the word values made her feel "moralistic". The findings within the theme of pedagogical documentation unveiled diverse definitions and understanding among the participants, with most not considering the nuances and gifts of the learning encounter in mathematics when engaging with pedagogical documentation with children. Generally, participants viewed pedagogical documentation as an opportunity to make students' ideas visible, yet, in a way of gathering evidence to compare to expectations of other markers of normative outcomes. No participant considered pedagogical documentation as an opportunity to upset the status quo, challenge neoliberal discourses, or uphold ideals of mathematics reform.

It was obvious that participants in the study approached the Ontario curriculum as a list of items that were to be checked off once completed. Teachers knew the math expectations well

however, there was limited discussion about how to build on student ideas and theories. It seemed as though the teachers in the study could benefit from participation in mathematical learning encounters to experience mathematics as a shared experience of growth. Participants did recognize that the role of the teacher should be more collaborative and interactive than more traditional roles of a teacher in hierarchies of power, which is in line with mathematics reform ideas. ‘Centres’ was a dominant theme and educators spoke of ‘centres’ as spaces where educators could monitor children, and they could control tasks within a specific space, which is significant as this contradicts the literature on pedagogical documentation and mathematics reform.

Reimagining Pedagogical Documentation:

The study has revealed that educators misunderstand and question the concept of ‘values’. Recognizing and integrating values into teaching are critical for educators as these values influence the choices educators make. Values demonstrate to children what educators value. Teachers at all levels should be encouraged to view pedagogical documentation as more than a record-keeping tool as it has the potential to revolutionize the relationship between educators and students. The danger of ignoring or denying that values have any implication on our practice as teachers comes from the infiltration of neoliberal ideals like standardization, competition, and individualization, which also tend to appear in traditional practices of mathematics education. Practices of accountability and evaluation seep in relatively unnoticed. We then, inadvertently, choose practices that favour some groups and marginalize others. By consciously attending to our values, values inherent in our practices, and values inherent in the curriculum, we may engage in ethical practices and recognize the political aspect of mathematics education, and education at large. We, as teachers, can then acknowledge that values inform what we choose to document and what that communicates to the students. I would invite professional development about pedagogical documentation to explore the role of

pedagogical documentation as a dynamic opportunity to develop relationships with children, listen to their ideas and theories, and challenge oppressive practices that continue to trap students in a stage of becoming, rather than allowing them to be the capable and competent people they are.

Another broad implication lies within the context of understanding students' ideas and theories, within the context of the Ontario curriculum, without prescribing rigid paths. Through professional development, educators in Ontario need to explore how they can work within the curriculum guidelines without those guidelines becoming prescriptive. Could educators consider the emergent thoughts of students within the broad context of curriculum? Using pedagogical documentation to have encounters with students and follow students' theories, along with working with the broad context of curriculum, learning experiences with children could be more purposeful and lend themselves to deeper meaning.

In summary, the implications of this study extend beyond the specifics of pedagogical documentation or mathematics education. My research advocates for a broader transformation in teaching practices, emphasizing values, holistic pedagogy, hermeneutic listening, and the integration of documentation as a way of being with students, challenging normative practices of compliance and accountability, and creating opportunities for educators and students to experience holistic realizations in mathematics. These implications can guide educational policies, curriculum development, and professional development programs across many educational contexts.

Contributions to the Field of Mathematics in Early Childhood Education:

This study marks a pivotal contribution to the fields of early childhood education and mathematics education, through discussions with kindergarten educators. A marked revelation of my research is the unfortunate underutilization of pedagogical documentation by educators.

Seemingly, pedagogical documentation is not viewed as a vibrant medium for enriching teacher and student encounters, challenging the status quo of educational practices, or supporting aspects of mathematics reform, but rather simply as a tool for record-keeping. This underscores a missed opportunity to foster a democratic educational ethos that privileges 'being with students' over traditional norms of accountability and surveillance.

Furthermore, my research highlights a notable oversight on the part of the educators, regarding the position of 'values' in their daily practices. In pre-service teacher education and in-service teacher education, I invite educators to examine the deliberate integration and recognition of values into our pedagogy and the acknowledgement that much of what we do in mathematics education and education at large is directed and influenced by 'values'. A careful examination of our values and how our values influence our thoughts and beliefs, is essential.

My work revealed that many educators take up the Ontario curriculum as a checklist. This is notable, problematic, and proposes further examination of the balance between curriculum expectations and student interests, suggesting that educators should interpret students' ideas within the broader context of the curriculum. This finding calls for a deeper inquiry into how educators can navigate Ontario curriculum mandates while genuinely attending to students' thoughts and inquiries, and other aspects of reform mathematics. Here there are implications that can be taken up in research, pre-service teacher education, and in-service teacher education.

The discussions with educators also revealed a misalignment between current pedagogical documentation practices and the literature's review of the use of learning centres in play-based, inquiry-based education. This discrepancy raises important questions about the role of learning centres in perpetuating neoliberal agendas of control and accountability in the

mathematics classroom, warranting further investigation into their pedagogical purpose and impact.

Of greatest significance is the contribution of this study in bridging the gap between early childhood education and mathematics education, spotlighting Ontario kindergarten teachers' engagements with pedagogical documentation in math classrooms. While existing literature has explored the intersection of early childhood and mathematics education in certain contexts, this work foregrounds teachers' voices, offering fresh insights into their perceptions and practices. It is my aspiration that this study will inspire further research into the transformative potential of pedagogical documentation across all levels of mathematics education, encouraging a broader scholarly discourse on its affordances, applications, and implications for democratic education in mathematics.

Where Do We Go from Here?—Opportunities for Future Research and Professional Development

This study revealed several areas where more professional development is necessary. Besides the aforementioned recommendations for professional development as explained in the discussion, more professional development that focuses on exploring how educators view their students is needed. Again, I invite educators to engage in professional conversations regarding their beliefs about teaching and learning in mathematics, and their beliefs around student capabilities. Kindergarten educators need to explore research that showcases the capabilities and thought processes of children, highlighting the work they can achieve. Much research in this way already exists in mathematics education. Professional development of this kind could examine the depths of thought children may have and the ways children can reflect on their own ideas in mathematics. Before educators begin to engage with pedagogical documentation as an opportunity to make children's ideas and theories visible, educators must believe that young children are capable of having thoughtful ideas and theories in mathematics.

As mentioned, educators must be aware and believe in children's capability to engage with complex thoughts in mathematics. Much research and many examples already exist which could heighten educators' awareness of children's capabilities, generally and in mathematics (Ginsburg et al., 2008, Giudici et al., 2001). Educators must understand the mathematics being discussed, but also have an understanding of the way they can build upon students' ideas in mathematics. This is important because pedagogical documentation emphasizes how crucial it is to listen to and to make meaning with children.

As this study occurred over the summer break, teachers did not have an opportunity to go back to their classrooms to further examine or try out suggestions and ideas discussed in the study group. An opportunity for future research to include a blended study where teachers could engage in their classrooms with ideas discussed in the study group meetings, would be beneficial.

Although the Ontario kindergarten program contains threads tied to Reggio Emilia approaches, an entirely emergent curriculum is not possible because of the curriculum expectations laid out in the Ontario document. Further research into how to help educators understand the affordances of pedagogical documentation and the possibilities of making meaning with children in mathematics, while working with the Ontario curriculum document, is needed.

Of Consideration: Limitations of the Study

An evident bias or limitation of this study is the study group itself. The participants were unique in that they were a keen group of self-selected kindergarten teachers. The group was a relatively small group of 11 participants. These teachers volunteered to participate in the study, meeting once a week throughout their summer for ten weeks. I consider these participants a specific population of kindergarten teachers interested in meeting together and learning from

each other during their own, personal time during the summer holidays. There are not many teachers who would be willing to dedicate one evening a week during the summer to engage in professional dialogue. This fact makes this group homogeneous in their commitment to their professional learning. Although the group was diverse in many ways, they were likely very similar in their interest to learn and explore their practice.

Another consideration is the fact that I had a mentoring or teaching relationship with many of the participants. Several participants in the study had been students of mine during their Bachelor of Education. Knowing that our values dictate much of what we do in the classroom, it is safe to say that the participants who had been my students had already been exposed to my values as a teacher in their courses, and naturally they would have brought those influences (consciously or subconsciously) with them into this work. It doesn't necessarily mean that they embraced the same values as I may have shared, however, they would have been exposed to those values, and therefore, there is potential that those previous encounters may have influenced their thinking in similar ways. The same applies to Hope, with whom I had worked extensively in our time together in the same school board. We had done a great deal of professional learning together, so naturally, we had shared many opportunities to learn and impact each other's ideas.

Although phenomenography has been used as a method to analyze a variety of data sources like discussions of a study group (e.g., Marton & Booth, 1997; Trigwell & Prosser, 1996), some believe that the best data source for phenomenography is the structured interview (Tight, 2016). When a researcher conducts interviews with individual participants, the questions are standardized, and the responses are without influence of other participants. However, as I have acknowledged, there was the potential of the participants in my study to hear the thoughts of other participants, because I position the study group as a collective opportunity for learning which aligns with conceptualizations of learning that were used in this study.

I intentionally chose materials to share as provocations for conversations, rather than each session being a didactic professional development session. The materials shared represented only a small number of resources compared to the quantity of resources and materials available for learning about pedagogical documentation. The study lasting only ten weeks also limited the quantity of resources explored.

It takes time to build a community of learners. The study ran for 10 weeks, in an online environment, and in some ways that may not have been enough time for the group to build a trusting community of learners. This could have impacted the sharing and conversations that resulted in the group. Perhaps with several more weeks, the participants would have built more trusting relationships that would have promoted richer conversations and learning.

In conclusion, pedagogical documentation can offer an opportunity to build relationships and make meaning with children in the kindergarten mathematics classroom. Envisioning a future where pedagogical documentation becomes an integral part of mathematics education, we open the door to classrooms where every child's thoughts are valued, learning is a shared journey of discovery, and practices of normalization and standardization are resisted. Exciting and rich opportunities lie ahead for educators in Ontario in mathematics education, when considering how to engage with pedagogical documentation as an embodied activity. Pedagogical documentation can enhance math realizations, supporting knowledge creation through a deeply personal and contextual process between, people, the materials, and the environment.

References

Abelson, R. (1979). Differences between belief systems and knowledge systems. *Cognitive Science*, 3, 355 - 366. https://doi.org/10.1207/s15516709cog0304_4

Agathangelou, S. A., & Charalambous, Charalambos Y. (2021). Is content knowledge pre-requisite of pedagogical content knowledge? An empirical investigation. *Journal of Mathematics Teacher Education*, 24, 431 - 458. http://dx.doi.org/10.1007/s10857-020-09466-0

Aizikovitsh-Udi, E., & Star, J. (2011). The skill of asking good questions in mathematics teaching. *Procedia Social and Behavioral Sciences*, 15, 1354 - 1358. http://dx.doi.org/10.1016/j.sbspro.2011.03.291

Akerlind, G. (2005). Variation and commonality in phenomenographic research methods. *Higher Education Research and Development*, 24 (4), 321 - 334. https://doi.org/10.1080/07294360500284672

Allen, K. (2011). Mathematics as thinking, a response to “Democracy and School Math.” *Democracy and Education*, *19*(2), 1 - 7. https://democracyeducationjournal.org/cgi/viewcontent.cgi?article=1036&context=home

Alphonso, C. (2018, August 29). EQAO: Fewer than half of Ontario Grade 6 math students make the grade. *The Globe and Mail*. https://www.theglobeandmail.com/canada/article-eqao-less-than-half-of-ontario-grade-6-math-students-make-the-grade/

Andreasen, J. B., & Hunt, J. H. (2012). Using math stations for commonsense inclusiveness. *Teaching Children Mathematics*, 19(4), 238 - 246. http://dx.doi.org/10.5951/teacchilmath.19.4.0238

Anthony, G., & Walshaw, M. (2009a). *Effective pedagogy in mathematics. Educational practices series—19*. Belgium: International Academy of Education. https://unesdoc.unesco.org/ark:/48223/pf0000183624

Anthony, G., & Walshaw, M. (2009b). Mathematics education in the early years: Building bridges. *Contemporary Issues in Early Childhood,* 10(2), 107 - 121. https://doi.org/10.2304/ciec.2009.10.2.107

Argın, Y. (2019). *Okul oncesi egitimcilerinin matematik egitimine iliskin pedagojik alan bilgi duzeylerinin cesitli degiskenler acısından incelenmesi* [*Examination of pedagogical content knowledge levels of pre-school educators in regard to mathematics education in terms of various variables*]. (Unpublished master’s thesis). Gazi University, Ankara, Turkey. https://avesis.gazi.edu.tr/yonetilen-tez/7dd489b5-da8d-471c-8e02-9afee034157a/okul-oncesi-egitimcilerinin-matematik-egitimine-iliskin-pedagojik-alan-bilgi-duzeylerinin-cesitli-degiskenler-acisindan-incelenmesi

Badger, J. (2016). Patterns of students’ and teachers’ interactions in learning centres: A

case study of three teacher leaders in an elementary school. *Education*, 3-13, 44(6), 617 - 631. https://doi.org/10.1080/03004279.2014.952756

Ball, A. L., Knobloch, N. A., & Hoop, S. (2007). The instructional planning experiences of beginning teachers. *Journal of Agricultural Education*, 48(2), 56 - 65. https://doi.org/10.5032/jae.2007.02056

Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal, 90*(4), 449 - 466. https://www.jstor.org/stable/1001941

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *The Elementary School Journal*, 93(4), 373 - 397. https://doi.org/10.1086/461730

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In G. Sykes & L. Darling-Hammond (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3 - 32). Jossey-Bass. https://www.fisica.uniud.it/~stefanel/PerMarisa/ProfessionalDevelopment/ball_cohen_-_developingpractice.pdf

Ball, D. L., Goffney, I. M., & Bass, H. (2005). The role of mathematics instruction in building a socially just and diverse democracy. *The Mathematics Educator*, *15*(1), 2 - 6. https://deepblue.lib.umich.edu/bitstream/handle/2027.42/72282/j.1744-7984.2008.00140.x.pdf?sequence=1

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In *Multiple Perspectives on the Teaching and Learning of Mathematics* (pp. 83 - 104). https://mspnet-static.s3.amazonaws.com/BallBassInterweaving.pdf

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389 - 407. https://doi.org/10.1177/0022487108324554

Bandura, A. (1997). *Self-efficacy: The exercise of control*. New York, NY: Freeman.

Baroody, A. J. (2000). Does mathematics instruction for three- to five-year-olds really make sense? *Young Children*, *55*, 61 - 67. https://www.jstor.org/stable/42727827

Baroody, A. J., Feil, Y., & Johnson, A. R. (2007). An alternative reconceptualization of procedural and conceptual knowledge. *Journal for Research Mathematics Education*, 38, 2, 115 - 131. https://www.jstor.org/stable/30034952

Bath, C. (2012). 'I can't read it; I don't know': young children's participation in the pedagogical documentation of English early childhood education and care settings. *International Journal of Early Years Education*, 20(2), 190-201. http://dx.doi.org/10.1080/09669760.2012.715242

Bauer, J., Qualmann, J., Stadtmüller, G., & Bauer, H. (1998). Lebenslaufuntersuchungen bei

Alzheimer-Patienten: Qualitative Inhaltsanalyse prämorbider Entwicklungsprozesse. In A. Kruse (Ed.), *Psychosoziale Gerontologie. Band 2: Intervention* (pp. 251 - 274). Göttingen, Germany: Hogrefe.

Bautista, A., Habib, M., Eng, A., & Bull, R. (2019). Purposeful play during learning centre time: From curriculum to practice. *Journal of Curriculum Studies*, 51(5), 715 - 736. http://dx.doi.org/10.1080/00220272.2019.1611928

Begg, A. (1999). Enactivism and mathematics education. *Annual Conference of The Mathematics Education Research Group of Australasia*, 68–75. http://courses.educ.ubc.ca/cite/socials/Articles/EnactivismMathBegg.pdf

Bell, L. C. (1983). Learning centres in the classroom. *Middle School Journal*, 14(2), 17- 19. https://doi.org/10.1080/00940771.1983.11495609

Berg, B. (2008). *Qualitative research methods for the social sciences* (7th ed). Boston: Allyn & Bacon.

Bhattacharya, D., Adams, K. L., & Mittelstet, T. J. (2018). Modeling in learning centres: Exploring the concept of living and nonliving with second-grade students. *Science and Children*, 55(9), 47 - 55. http://proxy.lib.trentu.ca/login?url=https://www.proquest.com/scholarly-journals/modeling-learning-centers/docview/2062632899/se-2

Biesta, G. (2005). Against learning. Reclaiming a language for education in an age of learning. *Nordisk Pedagogik*, 25, 54 - 66. http://dx.doi.org/10.18261/ISSN1891-5949-2005-01-06

Biesta, G. (2008). Good education in an age of measurement: On the need to reconnect with the question of purpose in education. *Education Assessment, Evaluation and Accountability*, 21, 33 - 46. https://doi.org/ 10.1007/s11092-008-9064-9

Biesta, G. (2014). *The beautiful risk of education*. London: Paradigm Publishers.

Bishop, A. J. (2001). What values do you teach when you teach mathematics? *Teaching Children Mathematics*, 7(6), 346 - 349. http://proxy.lib.trentu.ca/login?url=https://www.proquest.com/scholarly-journals/what-values-do-you-teach-when-mathematics/docview/214138545/se-2

Bishop, A. J. (2005). Values and mathematics education: A developing research field. *Proceedings of the Annual Meeting of Japan Society for Science Education*, 107 - 109. https://doi.org/10.14935/jssep.29.0_107

Bishop, A. J. (2012). From culture to well-being: A partial story of values. *ZDM Mathematics Education*, 44, 3 - 8. http://dx.doi.org/10.1007/s11858-011-0379-5

Blanton, W. E., & Moorman, G. B. (1987). *The effects of knowledge on the instructional behavior of classroom reading teachers. Research Report No. 7.* Boone, NC: Appalachian State University, Centre on Excellence in Teacher Education.

Boaler, J. (1999). Participation, knowledge, and beliefs: A community perspective on

mathematics learning. *Educational Studies in Mathematics, 40*(3), 259 - 281. http://dx.doi.org/10.1023/A:1003880012282

Boaler, J. (2002). The development of disciplinary relationships: Knowledge, practice, and identity in mathematics classrooms. *For the Learning of Mathematics, 22*(1), 42 - 47. https://www.jstor.org/stable/40248383

Borg, S. (2003). Teacher cognition in language teaching: A review of research on what language teachers think, know, believe, and do. *Language Teaching*, 36(2), 81 - 109. https://doi.org/10.1017/S0261444803001903

Bozkurt, A., Akgun-Ozbek, E., Yilmazel, S., Erdogdu, E., Ucar, H., Guler, E., Sezgin, S., Karadeniz, A., Sen-Ersoy, N., Goksel-Canbek, N., Deniz Dincer, G., Ari, S., & Hakan Aydin, C. (2015). Trends in distance education research: A content analysis of journals 2009 - 2013*. International Review of Research in Open and Distributed Learning*, 16(1), 330 - 363. https://doi.org/10.19173/irrodl.v16i1.1953

Brown, C. A., & Cooney, T. J. (1982). Research on teacher education: A philosophical orientation. *Journal of Research and Development in Education*, 15, 13 - 18. https://eric.ed.gov/?id=EJ267692

Brown, C. A., Stein, M. K., & Forman, E. A. (1996). Assisting teachers and students to reform the mathematics classroom. *Educational Studies in Mathematics*, *31*(1/2), 63 - 93. http://www.jstor.org/stable/3482935

Brackett, M. A., Reyes, M. R., Rivers, S. E., Elbertson, N. A., & Salovey, P. (2012). Assessing teachers' beliefs about social and emotional learning. *Journal of Psychoeducational Assessment*, 30(3), 219 - 236. http://dx.doi.org/10.1177/0734282911424879

Bruce, C. & Flynn, T. (2011). Which is greater: Once half or two fourths? An examination of how two grade 1 students negotiate meaning. *Canadian Journal of Science, Mathematics, and Technology Education*, 11(4), 309 - 327. http://dx.doi.org/10.1080/14926156.2011.570475

Buldu, M. (2010). Making learning visible in kindergarten classrooms: Pedagogical documentation as a formative assessment technique. *Teaching and Teacher Education*, 26, 1439 - 1449. http://dx.doi.org/10.1016/j.tate.2010.05.003

Burman, E. (2017). *Deconstructing developmental psychology, 3rd Edition*. New York, New York: Routledge.

Cagliari, P., Castagnetti, M., Giudici, C., Rinaldi, C., Vecchi, V., & Moss, P. (Eds.) (2016). *Loris Malaguzzi and the schools of Reggio Emilia*. New York, New York: Routledge.

Canada Ministers of Education Council Report. (2010). *World conference on early childhood care and education*, *Moscow, Russian Federation, September* 27 - 29, 2010. http://www.cmec.ca/publications/lists/publications/attachments/256/wcecce-moscow-sept-2010-en.pdf.

Cannella, G.S., & Viruru, R. (2004). *Childhood and post colonization: Power, education, and contemporary practice (1st ed.)*. Routledge. https://doi.org/10.4324/9780203463536

Carpenter, T. P., Ansell, E., Franke, M. L., Fennema, E., & Weisbeck, L. (1993). Models of problem solving: A study of kindergarten children's problem-solving processes. *Journal for Research in Mathematics Education*, *24*(5), 428 - 441. https://doi.org/10.2307/749152

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal*, 26(4), 499 - 531. http://dx.doi.org/10.3102/00028312026004499

Chapman, O. (2016). Approaches and challenges in supporting mathematics teachers' change. *Journal of Mathematics Teacher Education*, *19*(1), 1 - 5. http://dx.doi.org/10.1007/s10857-016-9342-2

Clark, C., & Lampert, M. (1986). The study of teacher thinking: Implications for teacher education, *Journal of Teacher Education*, September-October, 27 - 31. https://doi.org/10.1177/002248718603700506

Clark, C., & Yinger, J. (1977). Research on teacher thinking. *Curriculum Inquiry*, 7(4), 279 - 304. https://doi.org/10.1080/03626784.1977.11076224

Clark, C. M., & Peterson, P. L. (1984). *Teachers' Thought Processes (Occasional Paper No. 72).* Michigan State University, East Lansing Institute for Research on Teaching. https://edwp.educ.msu.edu/research/wp-content/uploads/sites/10/2020/11/op072.pdf

Clark, L., DePiper, J. N., Jones Frank, T., Nichio, M., Campbell, P. F., Smith, T., Griffin, M., Rust, A., Conant, D., & Choi, Y. (2014). Teacher characteristics associated with mathematics teachers' beliefs and awareness of their students' mathematical dispositions. *Journal for Research in Mathematics Education*, 45 (2), 246 - 284. http://dx.doi.org/10.5951/jresematheduc.45.2.0246

Clarke, M. (2012). The (absent) politics of neo-liberal education policy. *Critical Studies in Education*, *53*(3), 297 - 310. https://doi.org/10.1080/17508487.2012.703139

Clements, D. H., Copple, C., & Hyson, M. (2002). Early childhood mathematics: Promoting good beginnings. *National Association for the Education of Young Children*, 1 - 21. https://doi.org/10.1007/s10857-007-9068-2

Cobb, J., & Meter, V. V. (2000). Preservice teachers' assessment of the use of centers as an aspect of developmentally appropriate practice in first grade classrooms. *Journal of Early Childhood Teacher Education*, 21 (2), 169 - 177. http://dx.doi.org/10.1080/0163638000210208

Cochran-Smith, M., & Lytle, S. L. (1999). Relationships of knowledge and practice: Teacher learning in communities. *Review of Research in Education*, 24, 249 - 305. http://dx.doi.org/10.2307/1167272

Colgan, L. (2016). Hey, it's elementary: Play and learning: The current kindergarten conundrum. *Ontario Mathematics Gazette*, *55*(1), 20 –.

http://proxy.lib.trentu.ca/login?url=https://www.proquest.com/scholarly-journals/hey-elementary-play-learning-current-kindergarten/docview/1829450924/se-2?accountid=14391

Conners, R.D. (1978). *An analysis of teacher thought processes, beliefs, and principles during instruction*. Unpublished doctoral dissertation, the University of Alberta, Canada. https://doi.org/10.7939/R3707WX5Z

Convery, A. (1999). Listening to teachers' stories: Are we sitting too comfortably? *International Journal of Qualitative Studies in Education*,12(2), 131 - 146. http://dx.doi.org/10.1080/095183999236213

Copur-Gencturk, Y., & Tolar, T. (2022). Mathematics teaching expertise: A study of the dimensionality of content knowledge, pedagogical content knowledge, and content-specific noticing skills. *Teaching and Teacher Education*, 114, 1 - 13. http://dx.doi.org/10.1016/j.tate.2022.103696

Creswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five approaches (Second Edition).* Sage Publications, Inc.

Creswell, J. W. (2015). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research (Fifth Edition).* Upper Saddle River, New Jersey: Pearson Education Inc.

Cross, D. I. (2009). Alignment, cohesion, and change: Examining mathematics teachers' belief structures and their influence on instructional practices. *Journal of Math Teacher Education*, 12, 325 - 346. http://dx.doi.org/10.1007/s10857-009-9120-5

Dağli, H., & Dağlioğlu, H. E. (2021). The relationship between preschool teachers' pedagogical content knowledge in mathematics and children's' math ability and liking. *Research in Pedagogy*, 11(2), 359 - 376. http://dx.doi.org/10.5937/IstrPed2102359D

Dahlberg, G. (2012). Pedagogical documentation: A practice for negotiation and democracy. In Edwards, C. P., Gandini, L., & Forman, G. E., (eds.) *The hundred languages of children: The Reggio Emilia experience in transformation* (pp. 225 - 232). Santa Barbara, Calif: Praeger.

Dahlberg, G & Moss, P. (2005). *Ethics and politics in early childhood education*. London: Routledge.

Dahlberg, G., Moss, P., & Pence, A. (1999). *Beyond quality in early childhood education and care: Postmodern perspectives.* Philadelphia, PA.: Falmer Press, Taylor & Francis Inc.

Danniels, E., & Pyle, A. (2022). Inclusive play-based learning: Approaches from enacting kindergarten teachers. *Early Childhood Education Journal*, 1 - 11. https://doi.org/10.1007/s10643-022-01369-4

Davies, B. (2014a). *Listening to children, being and becoming*. New York, NY: Routledge.

Davies, B. (2014b). Reading anger in early childhood intra-actions: A diffractive analysis.

Qualitative Inquiry, 20(6), 734 - 741. http://dx.doi.org/10.1177/1077800414530256

Davies, B., & Bansel, P. (2007). Neoliberalism and education. *International Journal of Qualitative Studies in Education*, *20*(3), 247 - 259. https://doi.org/10.1080/09518390701281751

Davis, A. B., Sumara, D. J., & Kieren, T. E. (1996). Cognition, co-emergence, curriculum. *Journal of Curriculum Studies*, *28*(2), 151 - 169. http://dx.doi.org/10.1080/0022027980280203

Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching, *Journal for Research in Mathematics Education*, 28(3), 355 - 376. http://dx.doi.org/10.2307/749785

Davis, B. A. (1994). Mathematics teaching: Moving from telling to listening. *Journal of Curriculum and Supervision*, 9(3), 267 - 83. https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=ce8d3d92b3b407cf6d9011bfb765fd34b6010725

Davis, B, & Renert, M. (2013). Profound understanding of emergent mathematics; Broadening the construct of teachers' disciplinary knowledge. *Educational Studies in Mathematics*, 82(2), pp. 245 - 265. http://dx.doi.org/10.1007/s10649-012-9424-8

Davis, B., & Sumara, D. (2007). Complexity science, ecology, and enactivism. In J. L. Kincheloe & R. A. Horn (Eds.), *The Praeger handbook of education and psychology: Volumes 1 - 4* (Vol. 3, pp. 463 - 473). Praeger.

de Bilde, J., Boonen, T., Speybroeck, S., De Fraine, B., Van Damme, J., & Goos, M. (2015). Experiential education in kindergarten: Associations with school adjustment and the moderating role of initial achievement. *The Elementary School Journal*, *116*(2), 173 - 197. https://doi.org/10.1086/683802

De Lissovoy, N., & Cedillo, S. (2017). Neoliberalism and power in education. In: Peters, M.A. (eds) *Encyclopedia of Educational Philosophy and Theory*. Springer, Singapore. https://doi.org/10.1007/978-981-287-588-4_155

De Luca, C., Pyle, A., Roy, S., Chalas, A., & Danniels, E. (2019). Perspectives on kindergarten assessment: Toward a common understanding. *Teachers College Record*, *121*(3), 1 - 58. https://doi.org/10.1177/016146811912100302

De Luca, C., Pyle, A., Valiquette, A., & LaPointe-McEwan, D. (2020). New directions for kindergarten education: Embedding assessment in play-based learning. *The Elementary School Journal*, *120*(3), 455 - 479. https://doi.org/10.1086/707008

De Wever, B., Schellens, T., Valcke, M., & Van Keer, H. (2006). Content analysis schemes to analyze transcripts of online asynchronous discussion groups: A review. *Computers & Education*, 46(1), 6 - 28. http://dx.doi.org/10.1016/j.compedu.2005.04.005

Dewey, J. (1938). *Experience and education*. New York, NY: Macmillan

Drake, C. (2006). Turning points: using teachers' mathematics life stories to understand the implementation of mathematics education reform. *Journal of Mathematics Teacher Education*, *9*, 579 - 608. http://doi.org/10.1007/s10857-006-9021-9

Drisko, J. W., & Maschi, T. (2015). Chapter 4 qualitative content analysis. In *Content Analysis* (Ser. Pocket Guide to Social Work Research, pp. 81 - 120). essay, Oxford University Press.

Dyson, N., Jordan, N. C., Beliakoff, A., & Hassinger-Das, B. (2015). A kindergarten number sense intervention with contrasting practice conditions for low-achieving children. *Journal for Research in Mathematics Education*, *46*(3), 331 - 370. https://doi.org/10.5951/jresematheduc.46.3.0331

Edwards, C. (1995). Democratic participation in a community of learners: Loris Malaguzzi's philosophy of education as relationship. *Faculty Publications, Department of Child, Youth, and Family Studies*, (October). http://digitalcommons.unl.edu/famconfacpub/15

Edwards, C. (2012). Teacher and learner, partner and guide: The role of the teacher. In Edwards, C. P., Gandini, L., & Forman, G. E., (eds.) *The hundred languages of children: The Reggio Emilia experience in transformation* (pp. 147 - 172). Santa Barbara, Calif: Praeger.

Edwards, C., Gandini, L., & Forman, G. (2012). Introduction: Background and Starting Points. In Edwards, C. P., Gandini, L., & Forman, G. E., (eds.) *The hundred languages of children: The Reggio Emilia experience in transformation* (pp. 5 - 26). Santa Barbara, Calif: Praeger.

Emerson, A. M., & Linder, S. M. (2021). A review of research of the Reggio Inspired approach: An integrative re-framing. *Early Years*, *41*(4), 428 - 442. https://doi.org/10.1080/09575146.2019.1591350

English, L. D., & Mulligan, J. T. (2013). Perspectives on reconceptualizing early mathematics learning: Introduction. In English, Lyn D., Mulligan, Joanne T. (eds.) *Reconceptualizing Early Mathematics Learning*, 8, (pp. 1 - 4). Dordrecht: Springer Science & Business Media.

Entwistle, N. (2004). Chapter 2: Contrasting perspectives on learning. In Marton, F., Hounsell, D., and Entwistle, N. (Eds.), *The Experience of Learning: Implications for teaching and studying in higher education. (2nd ed., pp. 39 - 58).* Scottish Academic Press. http://www.etl.tla.ed.ac.uk/docs/ExperienceOfLearning/EoL.html

Erfjord, I., Hundeland, P. S., & Carlsen, M. (2012). Kindergarten teachers' accounts of their developing mathematical practice. *ZDM*, *44*(5), 653 - 664. https://doi.org/10.1007/s11858-012-0422-1

Erdogan, A. (2012). Preservice mathematics teachers' conceptions of and approaches to learning: A phenomenographic study. *Energy Education Science and Technology Part B: Social and Educational Studies*, 4(1), 21 - 30. https://files.eric.ed.gov/fulltext/ED572719.pdf

Errington, E. (2004). The impact of teacher beliefs on flexible learning innovation: Some

practices and possibilities for academic developers. *Innovations in Education and Teaching International*, 41(1), 39 - 47. http://dx.doi.org/10.1080/1470329032000172702

Ernest, P. (1989). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching, 15*(1), 13 - 33. http://dx.doi.org/10.1080/0260747890150102

Ernest, P. (1991). *The Philosophy of mathematics education*. London: Falmer Press.

Erturk Kara, H.G. (2018). The role of in-class physical arrangements in supporting preschool children's behavior regulation skill. *International Journal of Research in Education and Science (IJRES), 4*(1), 14 - 29. http://dx.doi.org/10.21890/ijres.369209

Everton, T., Galton, M., & Pell, T. (2002). Educational research and the teacher. *Research Papers in Education*, 17(4), 373 - 401. http://dx.doi.org/10.1080/0267152022000031388

Fang, Z. (1996). A review of research on teacher beliefs and practices. *Educational Research*, 38(1), 47 - 65. http://dx.doi.org/10.1080/0013188960380104

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*, 27(4), 387 - 402. http://dx.doi.org/10.2307/749875

Fielding, M., & Moss, P. (2012, August). Radical democratic education. In *2012 Annual Meeting Theme: 107th ASA Annual Meeting. ASA* (pp. 1 - 37). https://www.sscc.wisc.edu/soc/faculty/pages/wright/ASA/Fielding%20and%20Moss%20Real%20Utopia%20Proposal%20--%20democratic%20education%20--%20revised%20April%202012.pdf

Fives, H., & Buehl, M. M. (2012). Spring cleaning for the "messy" construct of teachers' beliefs: What are they? Which have been examined? What can they tell us? In K. R. Harris, S. Graham, T. Urdan, S. Graham, J. M. Royer, & M. Zeidner (Eds.), *APA educational psychology handbook, Vol. 2. Individual differences and cultural and contextual factors* (pp. 471 - 499). American Psychological Association. https://doi.org/10.1037/13274-019

Forman, G., & Fyfe, B. (2012). Negotiated learning through design, documentation, and discourse. In Edwards, C. P., Gandini, L., & Forman, G. E., (eds.) *The hundred languages of children: The Reggio Emilia experience in transformation* (pp. 247 - 272). Santa Barbara, Calif: Praeger.

Forrest, L., & Hechter, R. (2017). Waiting for the monarch. *Science and Children*, *54*(7), 34 -. https://doi.org/10.2505/4/sc17_054_07_34

Freire, P. (2002). *Pedagogy of the oppressed*. New York: Continuum.

Fuson, K. C., Clements, D. H., & Sarama, J. (2015). Making early mathematics education work for all children. *Kappan,* 97(3), 63 - 68. http://dx.doi.org/10.1177/0031721715614831

Fyfe, B. (2012). The relationship between documentation and assessment. In Edwards, C.P., Gandini, L., & Forman, G. E., (eds.) *The hundred languages of children: The Reggio Emilia experience in transformation* (pp. 273 - 292). Santa Barbara, Calif: Praeger.

Gadamer, H.G. (1990). *Truth and method*. New York: Continuum.

Gainsburg, J. (2012). Why new mathematics teachers do or don't use practices emphasized in their credential program. *Journal of Mathematics Teacher Education*, *15*, 359 - 379. http://doi.org/10.1007/s10857-012-9208-1

Gandini, L. (2012). History, ideas, and basic principles: An interview with Loris Malaguzzi. In Edwards, C.P., Gandini, L., & Forman, G. E., (eds.) *The hundred languages of children: The Reggio Emilia experience in transformation* (pp. 273 - 292). Santa Barbara, Calif: Praeger.

Ghafouri, F. (2014). Close encounters with nature in an urban kindergarten: a study of learners' inquiry and experience. *Education 3-13*, *42*(1), 54 - 76. https://doi.org/10.1080/03004279.2011.642400

Giamminuti, S., Merewether, J., & Blaise, M. (2022). Pedagogical documentation and the refusal of method: Troubling dogmas and inviting collective obligations. *European Early Childhood Education Research Journal*, 30(2), 213 - 226. http://dx.doi.org/10.1080/1350293X.2022.2046834

Ginsburg, H., & Ertle, B. (2008). Knowing the mathematics in early childhood mathematics. In O.N. Saracho & B. Spodek (Eds.), *Contemporary perspectives in mathematics in early childhood education* (pp. 45 - 66). Charlotte, NC: Information Age Publishing.

Ginsburg, H. P., Lee, J.S., & Boyd, J. S. (2008). Mathematics education for young children: What it is and how to promote it. *Social Policy Report Giving Child and Youth Development Knowledge Away,* XXII(I), 3 - 22. https://doi.org/10.1002/j.2379-3988.2008.tb00054.x

Ginsburg, H.P., & Golbeck, S. L. (2004). Thoughts on the future of research on mathematics and science learning and education. *Early Childhood Research Quarterly*, 19, 190 - 200. https://www.sciencedirect.com/science/article/pii/S088520060400016X?via%3Dihub

Goodson, I. F. (1991). Sponsoring the teacher's voice: Teachers' lives and teacher development. *Cambridge Journal of Education*, 21(1), 35 - 38. http://dx.doi.org/10.1080/0305764910210104

Gooya, Z., Fadaie, M. R., & Agah, Z. (2013). Mathematics teachers' approaches to "listening" in mathematics classes: A phenomenographic approach. *Theory and Practice in the Curriculum*, *1*(1), 27 - 44. https://www.jcstpicsa.ir/article_190676.html?lang=en

Gottfried, & Kirksey, J. J. (2019). General education teachers' math instructional practices in kindergarten classrooms with and without students with emotional and behavioral disabilities. *Teaching and Teacher Education*, *77*, 309 - 320. https://doi.org/10.1016/j.tate.2018.10.019

Graneheim, U. H., & Lundman, B. (2004). Qualitative content analysis in nursing research:

Concepts, procedures, and measure to achieve trustworthiness. *Nurse Education Today*, 24(2), 105 - 122. http://dx.doi.org/10.1016/j.nedt.2003.10.001

Graue, M. E. (1993). *Ready for what?: Constructing meanings of readiness for kindergarten*. Albany, N.Y: State University of New York Press.

Greenes, C., Ginsburg, H. P., & Blafanz, R. (2004). Big math for little kids. *Early Childhood Research Quarterly*, 19, 159 - 166. https://www.sciencedirect.com/science/article/pii/S0885200604000122

Gresham, G., & Burleigh, C. (2019). Exploring early childhood preservice teachers' mathematics anxiety and mathematics efficacy beliefs. *Teaching Education*, 30(2), 217-241. http://dx.doi.org/10.1080/10476210.2018.1466875

Gudmundsdottir, S., & Shulman, L. (1987). Pedagogical content knowledge in social studies, *Scandinavian Journal of Educational Research*, 31(2), 59 - 70. https://doi.org/10.1080/0031383870310201

Gutstein, E. (2007). "And that's just how it starts": Teaching mathematics and developing student agency. *Teachers College Record, 109*(2), 420 - 448. http://dx.doi.org/10.1177/016146810710900203

Han, F., & Ellis, R. A. (2019). Using phenomenography to tackle key challenges in science education. *Frontiers in psychology*, *10*,(1414), 1 - 10. https://doi.org/10.3389/fpsyg.2019.01414

Handal, B., & Herrington, A. (2003). Mathematics teachers' beliefs and curriculum reform. *Mathematics Education Research Journal*, *15*(1), 59 - 69. https://doi.org/10.1007/BF03217369

Hansson Halleröd, S. L., Anckarsäter, H., Råstam, M., & Hansson Scherman, M. (2015). Experienced consequences of being diagnosed with ADHD as an adult—a qualitative study. *BMC psychiatry*, *15*(31), 1 - 13. https://doi.org/10.1186/s12888-015-0410-4

Hara, N., Bonk, C. J., & Angeli, C. (2000). Content analysis of online discussion in an applied educational psychology course. *Instructional Science*, 28, 115 - 152. http://dx.doi.org/10.1023/A:1003764722829

Hargreaves, A., & Sahlberg, P. (2018, November 14). For the sake of the kids, embrace math. *The Conversation*. https://theconversation.com/for-the-sake-of-kids-embrace-math-106433

Harr, N., Eichler, A., & Renkl, A. (2014). Integrating pedagogical content knowledge and pedagogical/psychological knowledge in mathematics. *Frontiers in Psychology*, 5, 1 - 10. http://dx.doi.org/10.3389/fpsyg.2014.00924

Harris, H. A. (2021). Parental choice and perceived benefits of Reggio Emilia inspired programs. *Early Child Development and Care*, *191*(1), 123 - 135. https://doi.org/10.1080/03004430.2019.1608194

Harris, K., Crabbe, J. J., & Harris, C. (2017). Teacher discourse strategies used in

kindergarten inquiry-based science learning. *Asia-Pacific Forum on Science Learning and Teaching*, *18*(2), 1 - 30. https://www.eduhk.hk/apfslt/download/v18_issue2_files/harris.pdf

Helenius, O., Johansson, M. L., Lange, T., Meaney, T., Riesbeck, E., & Wernberg, A. (2014). 'When is preschool children's play mathematical?' In Helenius, O., Johansson, M. L., Lange, T., Meaney, T., & Wernberg, A., (eds.) *Mathematics Education in the Early Years Results from the POEM2 Conference* (pp. 139 - 156). Switzerland: Springer International Publishing. https://ltu.diva-portal.org/smash/get/diva2:1013541/FULLTEXT01.pdf

Hickey, P. J. (2016). "They always keep us in line": Neoliberalism and elementary emergent bilinguals. *Journal for Critical Education Policy Studies (JCEPS)*, *14*(2), 14 - 40. http://www.jceps.com/wp-content/uploads/2016/08/14-2-1.pdf

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371 - 404). Information Age Publishing.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal, 30*(2), 393 - 425. http://dx.doi.org/10.3102/00028312030002393

Hiebert, J., Carpenter, T.P., Fennema, E., Fuson, K., Humna, P., Murray, H., Olivier, A., & Wearne, D. (1996). Reform in curriculum and instruction. *Educational Researcher*, *25*(4), 12 - 21. https://doi.org/10.3102/0013189X025004012

Hirvonen, R., Tolvanen, A., Aunola, K., & Nurmi, J. E. (2012). The developmental dynamics of task-avoidant behavior and math performance in kindergarten and elementary school. *Learning and Individual Differences*, *22*(6), 715 - 723. https://doi.org/10.1016/j.lindif.2012.05.014

Hong, Ye, J.-H., Ho, Y.-J., & Ho, H.-Y. (2020). Developing an inquiry and hands-on teaching model to guide STEAM lesson planning for kindergarten children. *Journal of Baltic Science Education*, *19*(6), 908 - 922. https://doi.org/10.33225/jbse/20.19.908

Hope-Southcott, L. (2013). The use of play and inquiry in a kindergarten drama centre: A teacher's critical reflection. *Journal of Childhood Studies (Prospect Bay)*, *38*(1), 39 - 46. https://doi.org/10.18357/jcs.v38i1.15437

Hsieh, H., & Shannon, S. E. (2005). Three approaches to qualitative content analysis. *Qualitative Health Research*, 15(9), 1277 - 1288. http://dx.doi.org/10.1177/1049732305276687

Hunting, R., & Pearn, C. (2009). 'Mathematical thinking of young children through the eyes of preschool practitioners'. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia,* 1 (pp. 654 - 657). Palmerston North, NZ: Merga. https://www.academia.edu/6364067/Mathematical_Thinking_of_Young_Children_Through_the_Eyes_of_Preschool_PractitionersBNsITGxNYxnPOzp1dqTJBhuXFLK5VySnGJzw5FEt6EGoICQHRHWVOgwYK1Ni5ZAJsoRzC2xiT6~~8LZBIaJpT5V0FJ63qQZOUHV

O-314fnt8MAhA6UkfQ~QuNqZB008UXyOAkx8ErH2vLL94b7GSJ4KndlikNkYRoICr9uqeqkC5AfWAhdObSP1xhVzwu7Wm63MB3s9SvtzJCw5VAB9vjEAIC-Q6XFP5lvZZHdNDeq63W7kFfn6A9jJ1sfZR~gcioK4MV1ZP5XpcCRCsC3ZXyIuihYljiJkXsd1StvKd7jl75sQiiNJKpdsszdI4k0NM7HUS2disG0akmWzmw__&Key-Pair-Id=APKAJLOHF5GGSLRBV4ZA

Hursh, D. (2007). Assessing No Child Left Behind and the rise of neoliberal education policies. *American Educational Research Journal*, *44*(3), 493 - 518. https://doi.org/10.3102/0002831207306764

Hustedt, J. T., Buell, M. J., Hallam, R. A., & Pinder, W. M. (2018). While kindergarten has changed, some beliefs stay the same: Kindergarten teachers' beliefs about readiness. *Journal of Research in Childhood Education*, *32*(1), 52 - 66. https://doi.org/10.1080/02568543.2017.1393031

Inan, H.Z., & Dogan-Temur, O. (2010). Understanding kindergarten teachers' perspectives of teaching basic geometric shapes: a phenomenographic research. *ZDM Mathematics Education* **42**, 457 - 468. https://doi.org/10.1007/s11858-010-0241-1

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, 41(2) 169 - 202. http://www.cadrek12.org/sites/default/files/NoticingJRME2010.pdf

Jardine, D, Clifford, P, & Friesen, S. (2006). *Curriculum in abundance.* Lawrence Erlbaum Associates: Mahwah, New Jersey.

Jenßen, L., Dunekacke, S., Eid, M., Szczesny, M., Pohle, L., Koinzer, T., Eilerts, K., & Blomeke, S. (2022). From teacher education to practice: Development of early childhood teachers' knowledge and beliefs in mathematics. *Teaching and Teacher Education*, 114, 1 - 13. http://dx.doi.org/10.1016/j.tate.2022.103699

Johnson, L., McHugh, S., Eagle, J. L., & Spires, H. A. (2019). Project-based inquiry (PBI) global in kindergarten classroom: Inquiring about the world. *Early Childhood Education Journal*, *47*(5), 607 - 613. https://doi.org/10.1007/s10643-019-00946-4

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology*, 45(3), 850 - 867. http://dx.doi.org/10.1037/a0014939

Jung, H. Y., & Reifel, S. (2011). Promoting children's communication: A kindergarten teacher's conception and practice of effective mathematics instruction. *Journal of Research in Childhood Education*, *25*(2), 194 - 210. https://doi.org/10.1080/02568543.2011.555496

Jurdak, M. (1999). The role of values in mathematics education, *Humanistic Mathematics Network Journal*, 21(17), 39 - 45. http://dx.doi.org/10.5642/hmnj.199901.21.17

Kagan, D.M. (1992). Professional growth among preservice and beginning teachers. *Review of Educational Research*, 62(2), 129 - 169. http://dx.doi.org/10.3102/00346543062002129

Kahlenberg, R. D., & Potter, H. (2014-2015). Why teacher voice matters. *American Educator*, 38(4) pp. 6 - 7. https://files.eric.ed.gov/fulltext/EJ1049443.pdf

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal, 102*(1), 59 - 80. http://dx.doi.org/10.1086/499693

Keddie, A. (2016). Children of the market: Performativity, neoliberal responsibilisation and the construction of student identities. *Oxford Review of Education*, 42(1), 108 - 122. http://dx.doi.org/10.1080/03054985.2016.1142865

Kelchtermans, G. (2009). Who I am in how I teach is the message: Self-understanding, vulnerability, and reflection. *Teachers and Teaching: theory and practice*, 15(2), 257 - 272. http://dx.doi.org/10.1080/13540600902875332

Kennedy, M. M. (2016). How does professional development improve teaching? *Review of Education Research*, 86(4), 945 - 980. http://dx.doi.org/10.3102/0034654315626800

Kenny, M., & Fourie, R. (2014). Tracing the history of grounded theory methodology: From formation to fragmentation. *The Qualitative Report,* 19(103), 1 - 9. https://doi.org/10.46743/2160-3715/2014.1416

Keung, C. P. C., & Cheung, A. C. K. (2019). Towards holistic supporting of play-based learning implementation in kindergartens: A mixed method study. *Early Childhood Education Journal*, *47*(5), 627 - 640. https://doi.org/10.1007/s10643-019-00956-2

Kieff, J. (2005). A 1st-grade teacher's survival guide to the implementation and management of literacy centres during guided reading. *Childhood Education*, *81*(5), 292-K – 292-N. Olney: Association for Childhood Education International (ACEI) Subscriptions. https://link-gale-com.proxy1.lib.trentu.ca/apps/doc/A134315421/AONE?u=ocul_thomas&sid=bookmark-AONE&xid=66d5b8a9

Kieren, T. E. (1997). Theories for the classroom: Connections between research and practice. *For the Learning of Mathematics*, *17*(2), 31 - 33. https://www.jstor.org/stable/40248236

King-Sears, M. E. (2005). Scheduling for reading and writing small group instruction using learning centre designs. *Reading & Writing Quarterly,* 21(4), 401 - 405. http://dx.doi.org/10.1080/10573560500180558

Kinnunen, P., & Simon, B. (2012). Phenomenography and grounded theory as research methods in computing education research field. *Computer Science Education*, 22(2), 199 - 218. http://dx.doi.org/10.1080/08993408.2012.692928

Koehler, M. J., & Mishra, P. (2009). What is technological pedagogical content knowledge? *Contemporary Issues in Technology and Teacher Education*, *9*(1), 60 - 70. https://citejournal.org/wp-content/uploads/2016/04/v9i1general1.pdf

Krauss, S., Brunner, M., Kunter, M., Baumert, J., Blum, W., Neubrand, M., & Jordan, A.

(2008). Pedagogical content knowledge and content knowledge of secondary mathematics teachers. *Journal of Education Psychology*, 100(3), 716 - 725. http://dx.doi.org/10.1037/0022-0663.100.3.716

Krippendorff, K. (2019). *Content analysis: An introduction to its methodology* (4th ed.). SAGE Publications Inc. https://doi.org/https://doi.org10.4135/9781071878781

Kroeger, J., & Cardy, T. (2006). Documentation: a hard to reach place. *Early Childhood Education Journal*, 33(6), 389 - 398. http://dx.doi.org/10.1007/s10643-006-0062-6

Lampert, M. (2000). Knowing teaching: The intersection of research on teaching and qualitative research. *Harvard Educational Review*, *70*(1), 86 - 99. https://websites.umich.edu/~mlampert/lampert%20pdfs/Lampert_2000.pdf

Land, N., Vinitmilla, C.D., Pacini-Ketchabaw, V., & Angus, L. (2022). Propositions toward education pedagogists: Decentering the child. *Contemporary Issues in Early Childhood*, 22(2), 109 - 121. http://dx.doi.org/10.1177/1463949120953522

Latchford, T. (2018, September 1). Is 7x7 the same as 7x6+7? EQAO math scores continue to decline. *YorkRegion.com*. https://www.yorkregion.com/news-story/8873971-is-7x7-the-same-as-7x6-7-eqao-math-scores-continue-to-decline/

Leatham, K. R., Peterson, B. E., Stockero, S. L., & Zoest, L. R. Van. (2015). Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. *Journal for Research in Mathematics Education*, *46*(1), 88 - 124. https://doi.org/10.5951/jresematheduc.46.1.0088

Lee, J. (2010). Exploring kindergarten teachers' pedagogical content knowledge of mathematics. *International Journal of Early Childhood*, 42, 27 - 41. http://dx.doi.org/10.1007/s13158-010-0003-9

Lee, J. E. (2017). Preschool teachers' pedagogical content knowledge in mathematics. *International Journal of Early Childhood*, 49, 229 - 243. http://dx.doi.org/10.1007/s13158-017-0189-1

Lee, J. S., & Ginsburg, H.P. (2007). What is appropriate mathematics education for four-year-olds? Pre-kindergarten teachers' beliefs. *USA Journal of Early Childhood Research*, 5(1),2 - 31. http://dx.doi.org/10.1177/1476718X07072149

Lee, J. S., & Ginsburg, H. P. (2009). Early childhood teachers' misconceptions about mathematics for young children in United States. *Australasian Journal of Early Childhood*, 34(4), 37 - 45. https://earlyyearsliteracymath.wikispaces.com/file/view/Math+Reading.pdf.

Lenz Taguchi, H. (2010). *Going beyond the theory/practice divide in early childhood education*. New York, New York: Routledge.

Lesh, R., & Zawojewski, J. S. (2007). Problem solving and modeling. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 763 - 804). Information Age Publishing.

Lestari, L., & Surya, E. (2017). The effectiveness of realistic mathematics education approach on ability of students' mathematical concept understanding. *International Journal of Sciences: Basic and Applied Research*, 34(1), 91 - 100. https://www.gssrr.org/index.php/JournalOfBasicAndApplied/article/view/7545/3511

Liljedahl, P. (2020). *Building thinking classrooms in mathematics, grades K-12: 14 teaching practices for enhancing learning*. Corwin.

Ljunggren, C., Carlson, E., & Isma, G. E. (2021). Drama with a focus on professional communication—A phenomenographic study. *Nurse education in practice*, *52*, 103022. https://doi.org/10.1016/j.nepr.2021.103022

Lopez, R., & Mac Kenzie, J. (1993). A learning center approach to individualized instruction for gifted students. *Critical issues in gifted education*, *3*, 282 - 295.

Lu, Y., Li, Q., Patrick, H., & Mantzicopoulos, P. (2021). "Math gives me a tummy ache!" Mathematics anxiety in kindergarten. *The Journal of Experimental Education*, *89*(2), 362 - 378. https://doi.org/10.1080/00220973.2019.1680518

Ma, L. (2010*). Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Taylor & Francis.

MacDonald, K., & Breunig, M. (2018). Back to the Garten: Ontario kindergarteners learn and grow through schoolyard pedagogy. *Journal of Outdoor and Environmental Education*, *21*(2), 133 - 151. https://doi.org/10.1007/s42322-018-0011-z

MacDonald, M. (2006). Toward formative assessment: The use of pedagogical documentation in early elementary classrooms. *Early Childhood Research Quarterly*, 22, 232 - 242. http://dx.doi.org/10.1016/j.ecresq.2006.12.001

Malaguzzi, L. (1993). Your image of the child: Where teaching begins. *Child Care Information Exchange, 96.* www.childcareexchange.com.

Malaguzzi, L., & Gandini, L. (1993). For an education based on relationships. *Young Children*, *49*(1), 9 - 12. https://www.jstor.org/stable/42725534

Marcos, J. J. M., & Tillema, H. (2006). Studying studies on teacher reflection and action: An appraisal of research contributions. *Educational Research Review*, 1, 112 - 132. http://dx.doi.org/10.1016/j.edurev.2006.08.003

Martin, E., Prosser, M., Trigwell, K., Ramsden, P., & Benjamin, J. (2000). What university teachers teach and how they teach it. *Instructional Science*, 28, 387 - 412. http://dx.doi.org/10.1023/A:1026559912774

Marton, F. (1981). Phenomenography—describing conceptions of the world around us. *Instructional Science*, 10(2), 177 - 200. http://dx.doi.org/10.1007/BF00132516

Marton, F. (1986). Phenomenography—A research approach to investigating different understandings of reality. *Journal of Thought*, 21(3), 28 - 49. https://www.jstor.org/stable/42589189

Marton, F., & Booth, S. (1997). *Learning and Awareness*. New Jersey: Lawrence Erlbaum Associates.

Marton, F., & Neumann, D. (1989). Constructivism and constitutionalism. Some implications for elementary mathematics education. *Scandinavian Journal of Educational Research*, 33(1), 35 - 46. http://dx.doi.org/10.1080/0031383890330103

Marton, F., & Säljö, R. (2004). Chapter 3: Approaches to learning. In Marton, F., Hounsell, D., and Entwistle, N. (Eds.), *The Experience of Learning: Implications for teaching and studying in higher education. (2nd ed., pp. 39 -58).* Scottish Academic Press. http://www.etl.tla.ed.ac.uk/docs/ExperienceOfLearning/EoL.html

Marton, F., & Svensson, L. (1979). Conceptions of research in student learning. *Higher Education 8*(4), 471 - 486. https://www.jstor.org/stabel/3446162

Mason, J, & Davis, B. (2013). The importance of teachers' mathematical awareness for in-the-moment pedagogy. *Canadian Journal of Science, Mathematics and Technology Education*, 13(2), 182 - 197. http://dx.doi.org/10.1080/14926156.2013.784830

Maturana, H. & Varela, F. (1987). *The Tree of Knowledge: the biological roots of human understanding.* Boston: Shambhala

McCormick Smith, M., & Chao, T. (2018). Critical science and mathematics early childhood education: Theorizing Reggio, play, and critical pedagogy into an actionable cycle. *Education Sciences*, *8*(162), 1 - 16. MDPI AG. http://dx.doi.org/10.3390/educsci8040162

McCray, J. S., & Chen, J. (2012). Pedagogical content knowledge for preschool mathematics: Construct validity of a new teacher interview*. Journal of Research in Childhood Education*, 26, 291 - 307. http://dx.doi.org/10.1080/02568543.2012.685123

McDonald, J. A., Merkley, R., Mickle, J., Collimore, L., Hawes, Z., & Ansari, D. (2021). Exploring the implementation of early math assessments in kindergarten classrooms: A research-practice collaboration. *Mind, Brain and Education*, *15*(4), 311 - 321. https://doi.org/10.1111/mbe.12293\

McGarvey, L. (2014). Intentional play-spaces for teaching and learning mathematics with young children. *For the Learning of Mathematics, 34*(2), 9 - 11. http://www.jstor.org/stable/43894888.

McGarvey, L. M, & Jamison, N.M. (2015). Book Review. [Review of the book *Reconceptualizing Early Mathematics Learning, Advances in Mathematics Education*,edited by Lyn D. English & Joanne T. Mulligan]. *Mathematical Thinking and Learning*, 17, 91 - 97. https://doi.org/10.1080/10986065.2015.996107

McLellan, S. (2010). Pedagogical documentation as research in early mathematics. *The Alberta Journal of Education Research*, 56(1), 99 - 101. http://dx.doi.org/10.55016/ojs/ajer.v56i1.55387

Meisels, S. J. (1992). Doing harm by doing good: Latrogenic effects of early childhood and

enrollment and promotion policies. *Early Childhood Research Quarterly*, 7(2), 155 - 174. https://doi.org/10.1016/0885-2006(92)90002-G.

Melville, W., Kajander, A., Kerr, D., & Holm, J. (2013). Uncertainty and the Reform of Elementary Mathematics education. *ISRN Education*, *2013*, 1–8. http://dx.doi.org/10.1155/2013/845164

Merewether, J. (2018). Listening to young children outdoors with pedagogical documentation. *International Journal of Early Years Education*, *26*(3), 259 - 277.https://doi.org/10.1080/09669760.2017.1421525

Merriam-Webster. (2023). Abductive Reasoning. In *Merriam-Webster.com dictionary*. Retrieved June 2, 2023, from https://www.merriam-webster.com/words-at-play/deduction-vs-induction-vs-abduction

Merriam-Webster. (2023). Belief. In *Merriam-Webster.com dictionary*. Retrieved May 20, 2023, from https://www.merriam-webster.com/dictionary/belief

Merriam-Webster. (2023). Theory. In *Merriam-Webster.com dictionary*. Retrieved May 20, 2023, from https://www.merriam-webster.com/dictionary/theory

Merriam-Webster. (2023). Thinking. In *Merriam-Webster.com dictionary*. Retrieved May 20, 2023, from https://www.merriam-webster.com/dictionary/thinking

Merriam-Webster. (2023). Thought. In *Merriam-Webster.com dictionary*. Retrieved May 20, 2023, from https://www.merriam-webster.com/dictionary/thought

Merriam-Webster. (2024). Transcend. In *Merriam-Webster.com dictionary*. Retrieved March 14, 2024, from https://www.merriam-webster.com/dictionary/transcends

Metz, M. L., & Simmt, E. S. M. (2015). Researching mathematical experience from the perspective of an empathic second-person observer. *ZDM Mathematics Education*, *47*(2), 197 - 209. https://doi.org/10.1007/s11858-014-0621-z

Mhlolo, M. K., & Schäfer, M. (2012). Towards empowering learners in a democratic mathematics classroom: To what extent are teachers' listening orientations conducive to and respectful of learners' thinking? *Pythagoras*, *33*(2). https://doi.org/10.4102/pythagoras.v33i2.166

Miller, T. (2018). Developing numeracy skills using interactive technology in a play-based learning environment. *International Journal of STEM Education*, *5*(39), 1 - 11. https://doi.org/10.1186/s40594-018-0135-2

Ministry of Education of Ontario. (2005). *Math Curriculum* [Program of studies]. https://www.edu.gov.on.ca/eng/document/curricul/elementary/math1-8e.pdf

Ministry of Education of Ontario. (2016). *Kindergarten Program* [Program of studies]. https://www.dcp.edu.gov.on.ca/en/curriculum/kindergarten

Moran, M. J., Desrochers, L., & Cavicchi, N. M. (2007). Progettazione and documentation as sociocultural activities: Changing communities of practice. *Theory into Practice*, *46*(1), 81 - 90. https://doi.org/10.1207/s15430421tip4601_11

Moses, L., Serafini, F., & Loyd, S. (2016). Looking closely at informational texts: Considering the role(s) of mentor texts for kindergarten children. *Journal of Research in Childhood Education*, *30*(4), 529 - 539. https://doi.org/10.1080/02568543.2016.1215361

Moss, P. (2005). It's your choice. *Nursery World*, 26 - 27. https://www.nurseryworld.co.uk/content/news/its-your-choice

Moss, P. (2016). Loris Malaguzzi and the schools of Reggio Emilia: Provocation and hope for a renewed public education. *Improving Schools*, 19(2), 167 - 176. http://dx.doi.org/10.1177/1365480216651521

Moss, P. (2018). 'What would Loris Malaguzzi have to say?', *European Journal of Education*, 53(1), 82 - 85. http://dx.doi.org/10.1111/ejed.12256

Moss, P., & Petrie, P. (2002). *From children's services to children's spaces: Public policy, children and childhood*. Taylor & Francis Group: ProQuest Ebook Central. http://ebookcentral.proquest.com/lib/ocultrent-ebooks/detail.action?docID=235203

Muñez, D., Bull, R., & Lee, K. (2021). Socioeconomic status, home mathematics environment and math achievement in kindergarten: A mediation analysis. *Developmental Science*, *24*(6), e13135–n/a. https://doi.org/10.1111/desc.13135

Murcia, K., Clifford, E., Cross, E., Pepper, C., & Drury, S. (2020). Young children's inquiry: Questioning for science and mathematics learning connections in a construction project. *Teaching Science (Deakin West, A.C.T.)*, *66*(4), 20 - 29. https://doi.org/10.3316/informit.619210919478954

Nathaniel, A. (2021). Classic Grounded Theory: What it is and What it's not, *The Grounded Theory Review: An International Journal*, 20(1), 1 - 7. https://groundedtheoryreview.com/wp-content/uploads/2021/06/Nathaniel-GT-What-it-is-What-it-is-not-volume20-issue01-June2021.pdf

National Council of Teachers of Mathematics, Inc. (2000). *Principles and Standards for School Mathematics*, Reston, VA: Library of Congress. www.nctm.org.

National Council of Teachers of Mathematics. (2014). *Principles to actions*. National Council of Teachers of Mathematics. www.nctm.org.

Nations, J. E. (1976). *Learning centres in the classroom*. Washington: National Education Association

Nespor, J. K. (1985). *The role of beliefs in the practice of teaching: Final report of the teacher beliefs study* (R & D Report 8024). R&D Centre for Teacher Education, The University of Texas at Austin. https://files.eric.ed.gov/fulltext/ED270446.pdf

Neuman, D. (1997). Chapter 5: Phenomenography: Exploring the roots of numeracy. *Journal*

for Research in Mathematics Education. Monograph, *9*, 63 - 177. https://doi.org/10.2307/749947

Nieminen, J. H., Chan, M. C. E., & Clarke, D. (2022). What affordances do open-ended real-life tasks offer for sharing student agency in collaborative problem-solving? *Educational Studies in Mathematics, 109*(1), 115 - 136. http://dx.doi.org/10.1007/s10649-021-10074-9

Nisbet, S., & Warren, E. (2000). Primary school teachers' beliefs relating to mathematics, teaching and assessing mathematics and factors that influence these beliefs. *Mathematics Teacher Education and Development*, 2, 34 - 47. https://www.researchgate.net/publication/29454278_Primary_school_teachers'_beliefs_relating_to_teaching_and_assessing_mathematics_and_factors_that_influence_these_beliefs#fullTextFileContent

Nordtømme, S. (2012). Place, space and materiality for pedagogy in a kindergarten. *Education Inquiry*, *3*(3), 317 - 333. https://doi.org/10.3402/edui.v3i3.22037

Norén, E. (2015). Agency and positioning in a multilingual mathematics classroom. *Educational Studies in Mathematics, 89*, 167 - 184. http://dx.doi.org/10.1007/s10649-015-9603-5

Oktay, J. S. (2012). Grounded theory. *Oxford Scholarship online.* https://doi.org/10.1093/acprof:oso/9780199753697.001.0001

Pang, E., & Simoncelli-Bulak, L. (2017). Benefits of play-based learning in the kindergarten classroom. *Success in High-Need Schools Journal*, 13(1), 23 - 28. https://www.northcentralcollege.edu/sites/default/files/documents/2017-05/Volume%2013%20Issue%201_4.pdf

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research*, 62(3), 307 - 332. http://dx.doi.org/10.3102/00346543062003307

Pattillo, J., & Vaughan, E. (1992). Learning centres for child-centered classrooms. *NEA Early Childhood Education Series*. NEA Professional Library. https://files.eric.ed.gov/fulltext/ED344675.pdf

Pawley, A. L., Schimpf, C., & Nelson, L. (2016). Gender in engineer research: A content analysis of research in JEE 1998-2012. *Journal of Engineering Education*, 105(3), 508 - 528. http://dx.doi.org/10.1002/jee.20128

Pecaski McLennan, D. (2018). Counting kindness: A food drive inspires rich mathematical thinking. *YC Young Children*, *73*(1), 63 - 68. https://www.proquest.com/docview/2008817223?sourcetype=Scholarly%20Journals

Pecaski McLennan, D., & Bombardier, J. (2015). Paper shoes aren't for dancing. *YC Young Children*, *70*(3), 70 –. https://www.jstor.org/stable/ycyoungchildren.70.issue-3

Pecaski McLennan, D. M. (2020). Joyful number talks in kindergarten. *Journal of Teaching and Learning (Windsor)*, *13*(2), 43 - 54. https://doi.org/10.22329/jtl.v13i2.5684

Pedaste, M., Maeots, M., Siiman, L. A., de Jong, T., van Riesen, S. A. N., Kamp, E. T., Manoli, C. C., Zacharia, Z. C., & Tsourlidaki, E. (2015). Phases of inquiry-based learning: Definitions and the inquiry cycle. *Education Research Review*, 14, 47 - 61. http://dx.doi.org/10.1016/j.edurev.2015.02.003

Perry, B., & Dockett, S. (2005). "I know that you don't have to work hard": Mathematics learning in the first year of primary school. In Chick, H. L. & Vincent, J. L. (Eds.). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, 4, 65 - 72. https://www.emis.de/proceedings/PME29/PME29RRPapers/PME29Vol4PerryDockett.pdf

Perry, B., Marcus, A., Dockett, S., & MacDonald, A. (2016). Children noticing their own and others' mathematics in play. In White, B., Chinnappan, M. & Trenholm, S. (Eds.). *Opening up mathematics education research (Proceedings of the 39th annual conference of the mathematics Education Research Group of Australasia)*, 439 - 446. https://files.eric.ed.gov/fulltext/ED572326.pdf

Posner, G. J., Strike, K. A., Hewson, P. W., & Gertzog, W. A. (1982). Accommodation of a scientific conception: Toward a theory of conceptual change. *Science Education*, 66, 211 – 227. http://dx.doi.org/10.1002/sce.3730660207

Prosser, M., & Trigwell, K. (1999). *Understanding learning and teaching, The experience in higher education*. Buckingham: Open University Press.

Pui-Wah, D. C., & Stimpson, P. (2004). Articulating contrasts in kindergarten teachers' implicit knowledge on play-based learning. *International Journal of Educational Research*, *41*(4–5), 339 - 352. https://doi.org/10.1016/j.ijer.2005.08.005

Putnam, R. (1993). *Making democracy work: Civic traditions in modern Italy*. Princeton, NJ: Princeton University Press.

Pyle, A., & Bigelow, A. (2014). Play in kindergarten: An interview and observational study in three Canadian classrooms. *Early Childhood Education Journal*, *43*(5), 385 - 393. https://doi.org/10.1007/s10643-014-0666-1

Pyle, A., & Danniels, E. (2017). A continuum of play-based learning: The role of the teacher in play-based pedagogy and the fear of hijacking play. *Early Education and Development, 28*(3), 274 - 289. https ://doi.org/10.1080/10409 289.2016.12207 71

Pyle, A., DeLuca, C., & Danniels, E. (2017). A scoping review of research on play-based pedagogies in kindergarten education. *Review of Education (Oxford)*, *5*(3), 311 - 351. https://doi.org/10.1002/rev3.3097

Pyle, A., Poliszczuk, D., & Danniels, E. (2018). The challenges of promoting literacy integration within a play-based learning kindergarten program: Teacher perspectives and implementation. *Journal of Research in Childhood Education*, *32*(2), 219 - 233. https://doi.org/10.1080/02568543.2017.1416006

Pyle, A., Pyle, M. A., Prioletta, J., & Alaca, B. (2020). Portrayals of play-based learning:

Misalignments among public discourse, classroom realities, and research. *American Journal of Play*, *13*(1), 53+. https://link.gale.com/apps/doc/A666097602/AONE?u=ocul_thomas&sid=bookmark-AONE&xid=19821336

Ricard, R. J., Brown, A., & Sanders, J. (2002). What's appropriate about developmentally appropriate practices? *Observing Early Childhood Development Center Classroom Environments. ERIC Document Reproduction Service, ED468861*. https://files.eric.ed.gov/fulltext/ED468861.pdf

Richland, L. E., Stigler, J. W., & Holyoak, K. J. (2012). Teaching the conceptual structure of mathematics. *Educational Psychologist*, *47*(3), 189 - 203. http://doi.org/10.1080/00461520.2012.667065

Rinaldi, C., Gambetti, A., & Gandini, L. (2016). *The hundred languages in ministories: Told by teachers and children from Reggio Emilia.* Davis Publications.

Rinaldi, C. (2001). Documentation and assessment: What is the relationship? In Giudici, C., Rinaldi, C., and Krechevsky, M., (eds.) *Making learning visible: children as individual and group learners* (1st ed.). Project Zero Harvard Graduate School of Education; Reggio Children International Centre for the Defense and Promotion of the Rights and Potential of all Children.

Rinaldi, C. (2004). The relationship between documentation and assessment. *Innovations in Early Childhood Education: The International Reggio Exchange*, 11(1), 1 - 4. https://www.reggioalliance.org/downloads/relationship:rinaldi.pdf

Rinaldi, C. (2012). The pedagogy of listening: The listening perspective from Reggio Emilia. In Edwards, C. P., Gandini, L., & Forman, G. E., (eds.) *The hundred languages of children: The Reggio Emilia experience in transformation* (pp. 233 - 246). Santa Barbara, Calif: Praeger.

Rinaldi, C. (2021). *In dialogue with Reggio Emilia*. London: Routledge.

Ritchhart, R., & Perkins, D. (2008). Making thinking visible. *Educational Leadership*, *65*(5), 57 – 61. https://doi.org/10.2214/AJR.09.3938

Robert-Holmes, G., & Moss, P. (2021). *Neoliberalism and early childhood education*. London: Routledge.

Rogoff, B. (2003). *The cultural nature of human development*. New York: Oxford.

Russo, J., Bobis, J., Downton, A., Hughes, S., Livy, S., McCormick, M., & Sullivan, P. (2020). Elementary teachers' beliefs on the role of struggle in the mathematics classroom. *Journal of Mathematical Behavior*, 58, 1 - 16. http://dx.doi.org/10.1016/j.jmathb.2020.100774

Ryerson, R. (2017). Creating possibilities: studying the student experience. *Educational Research*, *59*(3), 297 - 315. https://doi.org/10.1080/00131881.2017.1343091

Säljö, R. (1981). Learning approach and outcome: Some empirical observations. *Instruction*

Science, 10(1), 47 - 65. https://www.jstor.org/stable/23368266

Salmon, A. K. (2010). Making thinking visible through action research. *Early Childhood Education Journal*, *39*(1), 15 - 21. https://www.academia.edu/4841813/Making_Thinking_Visible_Through_Action_Research

Saracho, O. N., & Evans, R. (2021). Early childhood education pioneers and their curriculum programs. *Early child development and care*, *191*(7-8), 1144 - 1151. ABINGDON: Routledge. http://dx.doi.org/10.1080/03004430.2021.1917268

Sato, K., & Takahashi, K. (2004). *Teacher beliefs, practices and student outcomes: The power of alternative assessmen*t. Paper presented at 2004 Ontario Conference on Applied Linguistics. Kingston, Ontario: Queen's University.

Schoenfeld, A. H. (2004). The math wars. *Educational Policy*, *18*(1), 253 - 286. http://doi.org/10.1177/0895904803260042

Schutz, P.A., Hong, J., & Cross Francis, D. (2020). *Teachers' goals, beliefs, emotions, and identity development: Investigating complexities in the profession* (1st ed.). Routledge. https://doi-org.ezproxy.library.yorku.ca/10.4324/9780429456008

Schulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4 - 14. https://www.wcu.edu/WebFiles/PDFs/Shulman.pdf

Scott, C. L. (2015). The futures of learning 3: What kind of pedagogies for the 21st century? *UNESCO Education Research and Foresight, Working Papers*, *15*, 1 - 21. https://unesdoc.unesco.org/ark:/48223/pf0000243126

Seah, W. T., Andersson, A., Bishop, A., & Clarkson, P. (2016). What would the mathematics curriculum look like if values were the focus? *For the Learning of Mathematics*, March, 36(1), 14 – 20. https://www.jstor.org/stable/44382694

Seah, W. T. (2003). Understanding mathematics classroom experiences through the values lens. In *81st Annual Meeting of the National Council of Teachers of Mathematics (Research Presession), San Antonio, TX*. https://www.researchgate.net/profile/Wee-Tiong-Seah-2/publication/267856009_UNDERSTANDING_MATHEMATICS_CLASSROOM_EXPERIENCES_THROUGH_THE_VALUES_LENS/links/546bda0e0cf2f5eb1809269a/UNDERSTANDING-MATHEMATICS-CLASSROOM-EXPERIENCES-THROUGH-THE-VALUES-LENS.pdf

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing.* Cambridge University Press. https://doi.org/10.1017/CBO9780511499944

Sigel, I. E. (1985). A conceptual analysis of beliefs. In I. E. Sigel (Ed.), *Parental belief systems*: *The psychological consequences for children* (pp. 345 - 371). Hillsdale, NJ: Erlbaum

Simmt, E., & Kieren, T. (2015). Three "Moves" in enactivist research: a reflection. *ZDM Mathematics Education*, *47*(2), 307 - 317. https://doi.org/10.1007/s11858-015-0680-9

Sin, S. (2010). Considerations of quality in phenomenographic research. *International Journal of Qualitative Methods*, *9*(4), 305 - 319. https://doi.org/10.1177/160940691000900401

Singer-gabella, M., & Shahan, E. (2016). Learning to leverage student thinking. *Elementary School Journal*, *116*(3), 411 - 436. https://doi.org/10.1086/684944

Smith III, J. P. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education*, 27 (4), 387 - 402. http://dx.doi.org/10.5951/jresematheduc.27.4.0387

Smith, M.S., Hughes, E.K., Engle, R.A., & Stein, M.K. (2009). Orchestrating Discussions. *Mathematics Teaching in the Middle School*, 14(9), pp. 548 - 556. http://dx.doi.org/10.5951/MTMS.14.9.0548

Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions*. Reston, VA: National Council of Teachers of Mathematics.

Stacey, S. (2015). *Pedagogical documentation in early childhood.* St. Paul, MN: Red Leaf Press.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*(4), 313 - 340. http://dx.doi.org/10.1080/10986060802229675

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation*, 2:1, 50 - 80. http://dx.doi.org/10.1080/1380361960020103

Stein, M. K., & Nelson, B. S. (2003). Leadership content knowledge. *Education Evaluation and Policy Analysis*, 25(4), 423 - 448. http://dx.doi.org/10.3102/01623737025004423

Stemhagen, K. (2011). Democracy and school math: Teacher belief-practice tensions and the problem of empirical research on educational aims. *Democracy and Education*, *19*(2), 1 - 13. https://democracyeducationjournal.org/cgi/viewcontent.cgi?article=1015&context=home

Strauss, A., & Corbin, J. (1990). *Basics of qualitative research: Grounded theory procedures and techniques*. Newbury Park, CA: Sage Publications

Sullivan, P. (2018). Supporting teachers in improving their knowledge of mathematics. *Journal of Mathematical Behavior*, 51, 161 - 166. http://dx.doi.org/10.1016/j.jmathb.2017.08.006

Sumara, D. J., & Davis, B. (2005). Enactivist theory and community learning: toward a complexified understanding of action research. *Educational Action Research*, *5*(3), 403 - 422. https://doi.org/10.1080/09650799700200037

Tan, M., & Lan, O. S. (2011). Teaching mathematics and science in English in Malaysian classrooms: The impact of teacher beliefs on classroom practices and student learning.

Journal of English for Academic Purposes, 10, 5 - 18. http://dx.doi.org/10.1016/j.jeap.2010.11.001

Taylor, C. (1991). *The malaise of modernity*. Concord: Anansi.

Taylor, M. E., & Boyer, W. (2019). Play-based learning: Evidence-based research to improve children's learning experiences in the kindergarten classroom. *Early Childhood Education Journal*, *48*(2), 127 - 133. https://doi.org/10.1007/s10643-019-00989-7

Thompson, C. J., & Nguyen, G. N. T. (2021). Preschool/pre-kindergarten education promoting student performance in later years: A qualitative perspective. *International Journal of Emerging Issues in Early Childhood Education (Online)*, *3*(1), 44 - 58. https://doi.org/10.31098/ijeiece.v3i1.538

Tian, F., & Huang, J. (2019). Early childhood teachers' pedagogical content knowledge in mathematics: A research report from China. *Universal Journal of Educational Research*, 7(11), 2258 – 2261. http://dx.doi.org/10.13189/ujer.2019.071102

Tight, M. (2016). Phenomenography: The development and application of an innovative research design in higher education research. *International Journal of Social Research Methodology*, 19(3), 319-338. https://doi.org/10.1080/13645579.2015.1010284

Tobin, J. J., Wu, D. Y., & Davidson, D. H. (1989). *Preschool in three cultures: Japan, China, and the United States*. Yale University Press.

Trehearne, M. P. (2003). Oral language in the classroom. *Comprehensive literacy resource for kindergarten teachers*. Vernon Hills, IL: ETA Cuisenaire.

Trigwell, K., & Prosser, M. (1991). Improving the quality of students learning: the influence of learning context and student approaches to learning on learning outcomes. *Higher Education*, 22, 251 - 266. https://www.jstor.org/stable/3447175

Trigwell, K., & Prosser, M. (1996). Changing approaches to teaching: A relational perspective. *Studies in Higher Education*, 21(3), 275-284. http://dx.doi.org/10.1080/03075079612331381211

Tronto, J. (2017). 'There is an alternative' *Homines curans* and the limits of neoliberalism'. *International Journal of Care and Caring*, 1(1), 27 - 43. http://dx.doi.org/10.1332/239788217X14866281687583

Tsai, P., Chai, C. S., Hong, H., & Koh, J. H. L. (2017). Students' conceptions of and approaches to knowledge building and its relationship to learning outcomes. *Interactive Learning Environments*, 25(6), 749 - 761. http://dx.doi.org/10.1080/10494820.2016.1178653

Tsui, A. (2003). *Understanding expertise in teaching: Case studies of second language teachers.* Cambridge: Cambridge University Press.

Turner, T., & Wilson, D. G. (2009). Reflections on documentation: A

discussion with thought leaders from Reggio Emilia. *Theory into Practice*, 49(1), 5 - 13. http://dx.doi.org/10.1080/00405840903435493

Varela, F. (2000). *Three gestures of becoming aware*/Interviewer: O. Scharmer. https://achrafkassioui.com/library/Francisco%20Varela%20-%20Three%20Gestures%20of%20Becoming%20Aware.pdf

Varela, F., & Shear, J. (1999). *The view from within: First person approaches to the study of consciousness*. Bowling Green: Imprint Academic.

Vicini, S. (1993). *Subjektive beratungstheorien. Bernische erziehungsberaterinnen reflektieren ihre Praxis.* Bern: Lang.

Vintimilla, C. (2014). 'Neoliberal fun and happiness in early childhood education'. *Journal of the Canadian Association for Young Children*, 39(1), 79 - 87. https://doi.org/10.18357/jcs.v39i1.15246

Vintimilla, C. D. (2018). Encounters with a pedagogista. *Contemporary Issues in Early Childhood*, *19*(1), 20 - 30. https://doi.org/10.1177/1463949116684886

Vodopivec, Lepicnik. (2012). The Reggio Emilia concept or different perspective on preschool education in kindergartens. *Euromentor Journal*, *3*(2), 1 –. https://euromentor.ucdc.ro/en/2012/vol3n22012/en/1_the-reggio-emilia-concept-or-different.pdf

Voss, T., Kunter, M., & Baumert, J. (2011). Assessing teacher candidates' general pedagogical/psychological knowledge: Test construction and validation. *Journal of Educational Psychology*, 103(4), 952 - 969. http://dx.doi.org/10.1037/a0025125

Walshaw, M., & Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classrooms. *Review of Educational Research, 78(3),* 516 - 551. http://dx.doi.org/10.3102/0034654308320292

Wang, M. C., Haertel, G. D., & Walberg, H. J. (1990). What influences learning? A content analysis of review literature*. The Journal of Education Research*, 84(1), 30 - 43. http://dx.doi.org/10.1080/00220671.1990.10885988

Weber, R. (1990). *Basic content analysis*. SAGE Publications, Inc., https://doi.org/10.4135/9781412983488

Whyte, K. L., Stein, M. A., Kim, D., Jou, N., & Coburn, C. E. (2018). Mathematics in early childhood: Teacher educators' accounts of their work. *Journal of Early Childhood Teacher Education*, *39*(3), 213 - 231. https://doi.org/10.1080/10901027.2017.1388306

Wien, C. A., Guyevskey, V., & Berdoussis, N. (2011). Learning to document in Reggio-inspired education. *Early Childhood Research and Practice*,13(2). https://ecrp.illinois.edu/v13n2/wien.html

Wilson Smith, T., & Colby, S. A. (2010). Teaching for deep learning. *The ClearingHouse: A Journal of Educational Strategies, Issues and Ideas*, 80(5), 205 - 210. https://www.jstor.org/stable/30189920

Wilson, S. M., & Peterson, P. L. (2006). *Theories of learning and teaching: What do they mean for educators?* Best practices, National Education Association Working Paper. https://files.eric.ed.gov/fulltext/ED495823.pdf

Wood, T., Cobb, P., & Yackel, E. (1990). The contextual nature of teaching: Mathematics and reading instruction in one second-grade classroom. *The Elementary School Journal*, 90 (5), 496 - 513. http://dx.doi.org/10.1086/461629

Wright, E., & Osman, R. (2018). What is critical for transforming higher education? The transformative potential of pedagogical framework of phenomenography and theory of learning for higher education. *Journal of Human Behavior in the Social Environment*, *28:3,* 257 – 270. http://dx.doi.org/10.1080/10911359.2017.1419898

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, 27(4), 458 - 477. http://dx.doi.org/10.5951/jresematheduc.27.4.0458

Yin, H., Keung, C. P. C., & Tam, W. W. Y. (2021). What facilitates kindergarten teachers' intentions to implement play-based learning? *Early Childhood Education Journal*, *50*(4), 555 - 566. https://doi.org/10.1007/s10643-021-01176-3

Youmans, A., Schroeter, E., & Colgan, L. (2020). Supporting early geometry and spatial reasoning in kindergarten: A collaborative inquiry. *Ontario Mathematics Gazette*, *58*(4), 30 - 35. https://www.oame.on.ca/main/files/gazettefiles/Gz58i4.pdf

Yurekli, B., Stein, M.K., Correnti, R., & Kisa, Z. (2020). Teaching mathematics for conceptual understanding: Teachers' beliefs and practices and the role of constraints. *Journal for Research in Mathematics Education*, 51(2), 234 - 247. http://dx.doi.org/10.5951/jresematheduc-2020-0021

Zhang, L. J., & Sun, Q. (2022). Developing and validating the English teachers' cognitions about grammar teaching questionnaire (TCAGTQ) to uncover teacher thinking. *Frontiers in Psychology*, 13, 1 - 10. https://www.frontiersin.org/journals/psychology/articles/10.3389/fpsyg.2022.880408/full

Zhang, Y. (2015). *Pedagogical content knowledge in early mathematics: What teachers know and how it associates with teaching and learning. (*Unpublished doctoral dissertation), Loyola University, Chicago. https://ecommons.luc.edu/luc_diss/1499/

Appendix A

Price of Apples

The Price of Apples

Protagonists: Daniela, Marco, and Tommaso (5–6).

1. The children print the money that they will use as buyers and sellers in the classic game of setting up a store. We agree with them that on the paper money there will be printed only a few numbers: 1, 2, 5, and 10, values that correspond to the currency we use.

2. Simultaneously, the children discuss what to sell and which price to give to each item.

3. The tools for the salesperson.

4. Organization of the store.

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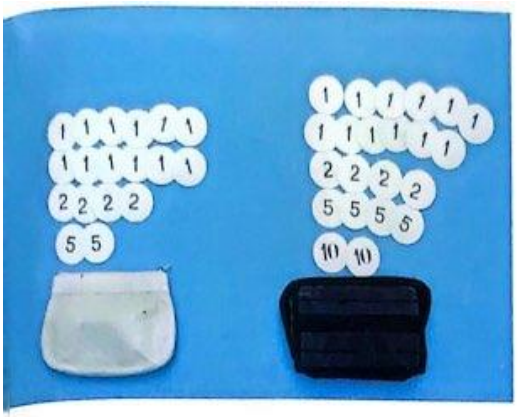
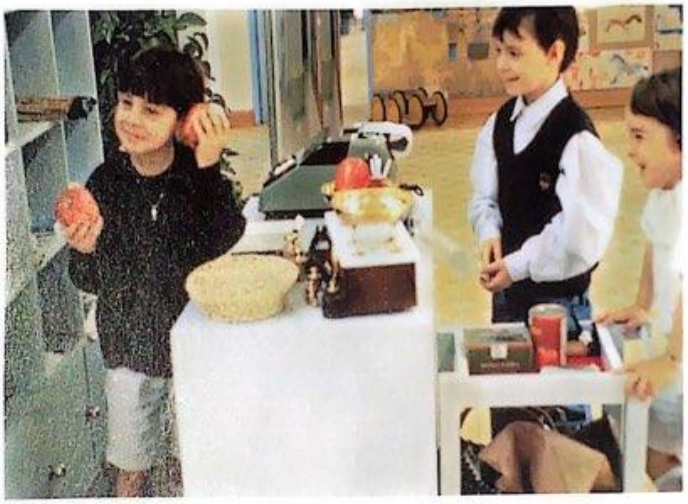


5. The change purses that will be available for the buyers. They are prepared with two levels of difficulty: one contains a total value of 30 and the other of 60.

6. Daniela is shopping and has asked to pay only when she has finished, while some other children prefer to pay separately for each item. Her change purse has a total value of 60. At a certain point, she asks for two apples.

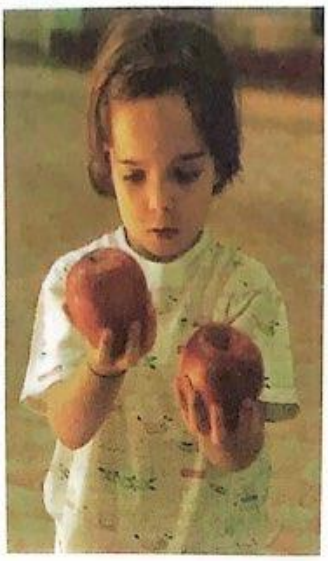
7. Tommaso takes an apple from the basket and looks for the [illegible] with the price but it is not there. How to determine the price?



8.

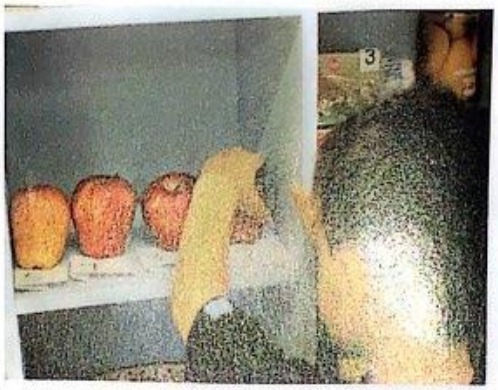
9.

10. Daniela takes both apples in her hands and feels the difference in weight: "Maybe to set the price one has to know the weight." She then changes her previous request and says, "I would like to buy the heaviest apple of all."



11. Tommaso uses the scale to find out which is the heaviest apple by comparing two apples at a time.

12. He gradually eliminates the lighter ones.



13.

14. He establishes a gradual scale of weight and a diminishing price from 5 to 2. Tommaso says, "Here is the heaviest." Now the shopping can continue.

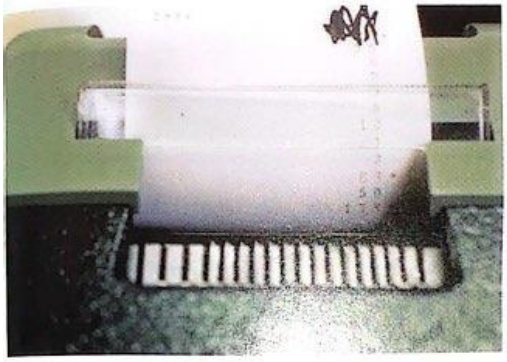
(Rinaldi et al., 2016, p. 52)

Appendix B

Thoughts about Calculating Cost

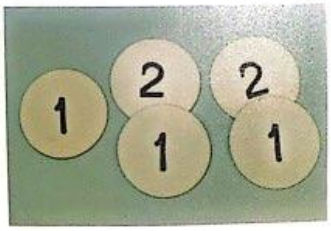
Thoughts About Calculating Cost

Protagonists: Daniela, Marco, and Tommaso (5 – 6).



1. After selecting further items [illegible] shopping is finished and she has to pay [illegible].

2. Daniela's thought process is as follows: constructing the total, 133, number by number.
1, 2 + 1 = 3, 2 + 1 = 3.



3.

4. Tommaso thinks that Daniela wants to tease him: "Are you joking? You have to pay now! This is 7, not 133." Daniela has a problem, but then she seems to understand. She starts to count how much money she has in her change purse. She has 60 so she asks Marco to lend her some money and he does so. So they put their money together. The seller and buyer together recalculate out loud several times the total of their respective money, and by helping themselves by counting on their fingers, they arrive at a total of 131 instead of 133.

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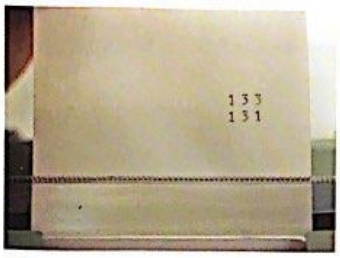
5.

6. Tommaso and Marco verify with the adding machine how much money Daniela has. Although they have favored her in making the sum, Tommaso is still missing 2. What to do?



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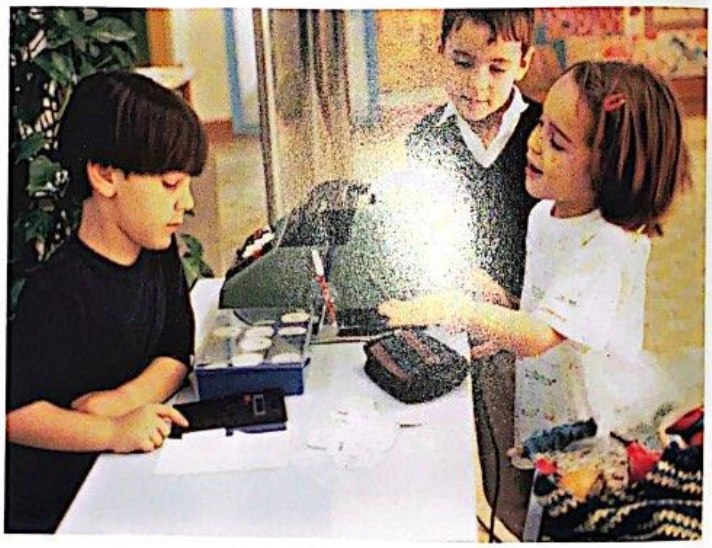
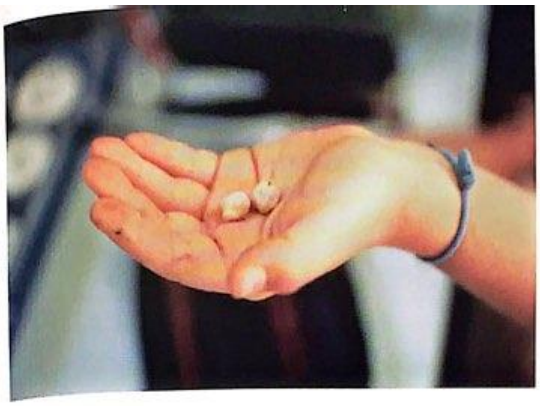


8. Daniela has an idea: "I can return to you two chickpeas so that we are even."



9. Tommaso refuses the exchange: two chickpeas don't seem to have the same value as two coins.

10. Marco and Daniela are sure that the only solution is to return part of what Daniela bought. They find something that has the number 2.

11.

12. This time Tommaso accepts and gives the receipt to her.

13. The shopping is concluded and the game can start over again.

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(Rinaldi et al., 2016, p. 55)

Appendix C

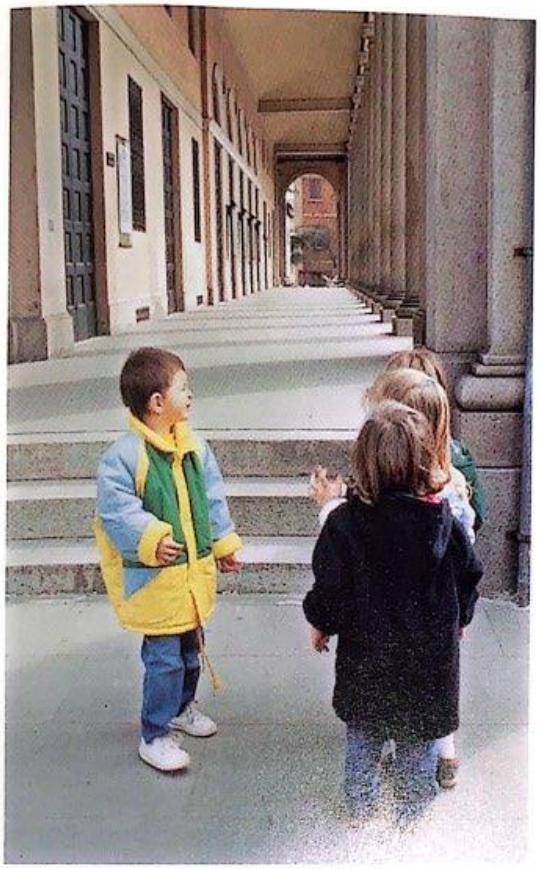
The Columns of the Municipal Theatre

The Columns of the Municipal Theater

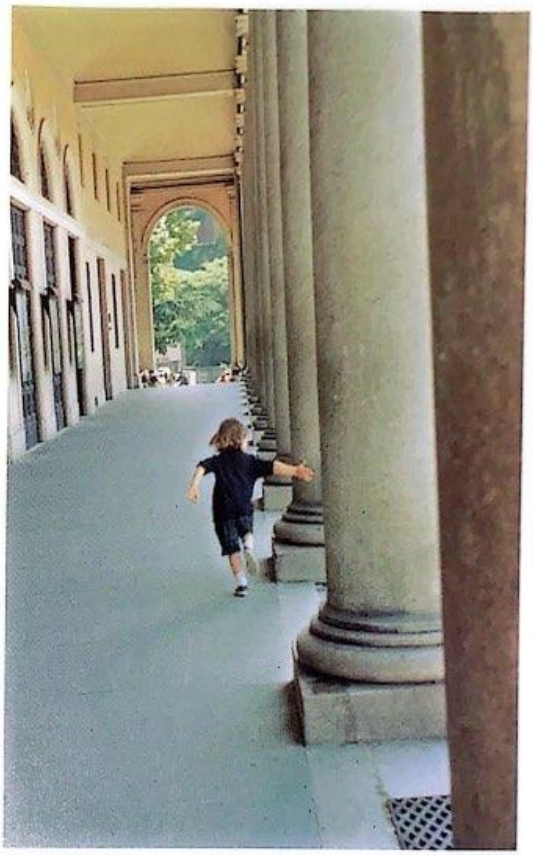
Protagonists: A group of children (3 –3.7).

1. The group of children stop in front of the colonnade of the Municipal Theater. For many adults, it is difficult to understand architecture. Will it be possible to accompany the children in experiencing it in this case, and sustain their interest since they are so young?

2. The children are attracted by the columns: "There are so many of these things!"



3. We move to the side of the colonnade in order to completely change the point of view. The children look and after a brief silence, three of them place themselves one behind the other and say: "This is the way they are placed," and one of them, Federico, closes one of his eyes to see better.

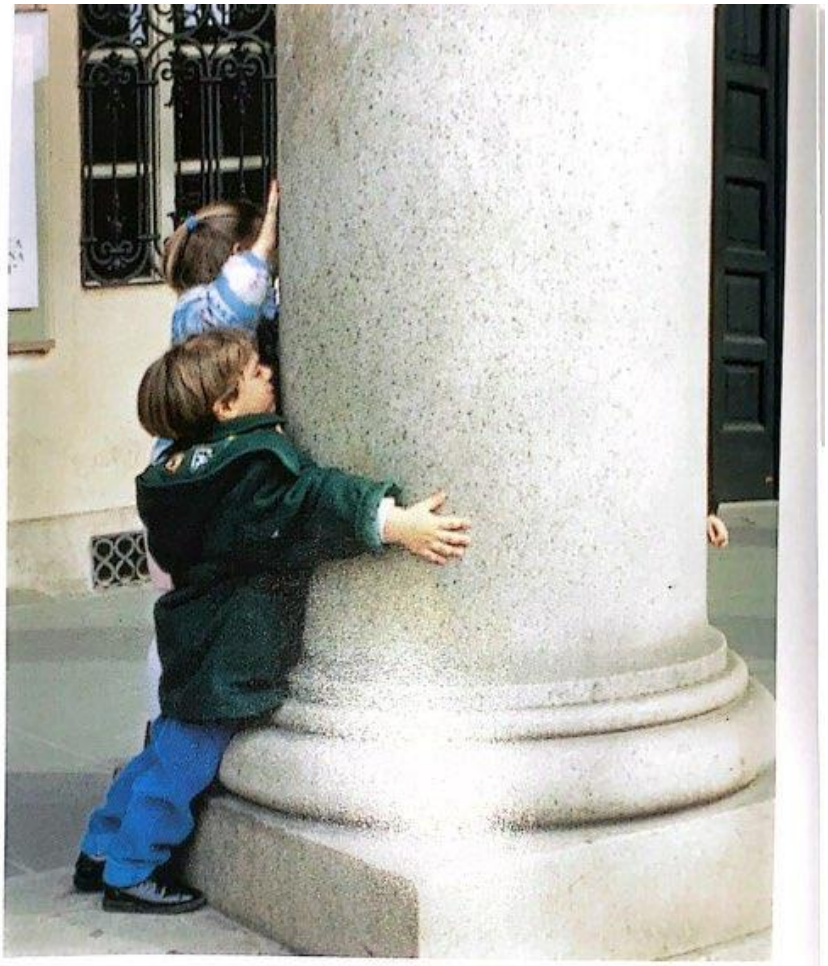


4. Then, Alessandro runs toward the columns and arrives all the way to the end of the portico, touching each column with his hand as if to perceive physically the rhythmic sequence of the placement of the columns.

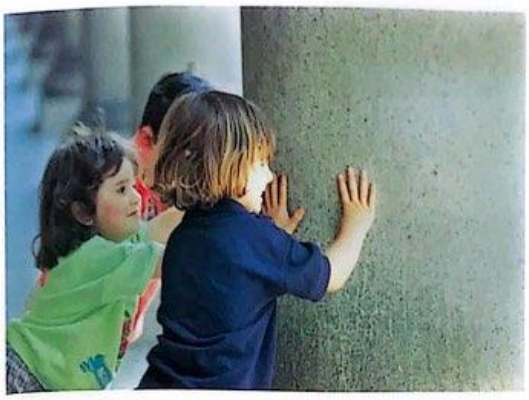
5. The teacher hides behind a column and says: "Where am I?" The children respond, "We cannot see you because you are hiding behind!"

6. "They are very, very tall!"

7. "They are planted on top!"



8. "They are so fat!"



9. "They are very strong. To move them, one needs a bulldozer or a dinosaur!" Teacher: "What are the columns for?" Children: "They are for holding everything up." "If there is one missing, the others have a bit of a hard time, but if several are missing, the ones left explode with the effort!"

10. "But if they crumble, then they can be born again." "Yes, like hair and teeth..." "But someone very old remains bald forever."

11. Before going back to school, the children run several times through the portico doing a slalom between one column and the next, saying aloud rhythmically: "Inside-outside, inside-outside, inside-outside..."

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(Rinaldi et al., 2016, p. 16)