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SPEAKERS

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So let's now come to the final example of this module. This is perhaps the most involved of the examples we've done so far. But it's also a very relevant example given that I'm recording this in the midst of, of a once in a lifetime pandemic. And all of you must be seeing in the news, things like experts predicting based on their models, that case numbers will increase, case numbers will decrease. This is going to be the impact without this policy and with this policy, and so on. Right, so most of these expert predictions come from what are called epidemiological models. So these are essentially mathematical models, which try and model how infection spread. So what I'm going to do in this example, is to take a very simplified version of one of these models, and present it to you. And the aim will be to show you that how, at the end, the more the spread of infections can be modeled by a relatively simple mathematical formula. And once you have that, then you can use that sort of mathematical model to talk about the impact of various policies, like lockdown, strict lockdowns, not so strict lockdowns, vaccinations on how infection spread. So let's get started.

So let's start with the situation that suppose in a large population, there is initially one person who gets infected. So this is the so called patient zero, right? So then, then starting from here, how do infection spread? So typically, infection spread when people interact with other people, and, and they pass on their infections. So how do we model this mathematically? So let's say that each person interacts with N people every day. So, sociological, sociologists have actually studied this and they found that the number of interactions an average person has in a day varies from somewhere between 10 to 25. Of course, this depends on where the person lives, his or her gender, age, and various other factors. Right, but what we'll assume for the purpose of a simple model, that everyone interacts with N people everyday. Okay, so we've kept it the same for everyone. So that's again a simplification, but we'll do a lot of simplifications like this in order to come up with a simple model.

Secondly, in each such interaction, the probability of the person the infected person passing on this infection is given by P. Now this P depends on the contagiousness of the of the virus, right. So if this probability is small, let's say P is point one, that means in each interaction, there is only a 10% chance that an infected person passes on the infection. On the other hand, if it's a virus, which is highly contagious, then this P is likely to be pretty high. Because if P is point seven, or point eight, that means in every interaction, there's a 70 to 80% chance of the infection being passed on to the person you're interacting with. But right now, let's assume that this is a, is P. Okay, this particular probability, now we'll be doing probability in the second course. But right now, I'm not going to go too deep into it, but just think of this probability as that if a person if an infected person meets some other people, right, then he or she is likely to pass on the infection to a fraction P of them. Okay, so let's just think of it in that sense, that say if P is point seven, right, that means if a person interacts with 100 people, then, an infected person interacts with 100 people, then he or she is likely to pass on the infection to 70% of them, or 70 of them out of 100, okay.

Okay, so now, now let's see, starting from that one person that patient zero, how do infection spread? So on the first day, that's day T equal to one, the number of infected people is one, right that patient zero. Now, in day T equal to two, right, and let's assume that people remain in fact, contagious just for one day, okay? And after that they no longer remain a contagious, right? So but this one particular day, right, so just day T equal to two, this patient zero, right, so this one person infected person, he interacts with N people, right. And now, because the probability of passing on the infection is P, that means he passes on this infection to a fraction P of them, right? So what that means is that of the N people that he meets, right, he infects a fraction P of them. So the total number of infected people in day T equal to two then becomes N times P. Right, so, we started with one person, right, he interacts with N of them, and he passes on the infection to a fraction P of them. So, that means the total number of infected people in day T equal to two is N times P.

And then we'll figure out how that moves on to the next day and the next day, and so on. Right, so that's the model of the spread of infections. So on day T equal to two, the number of new infections is N times P, right? So the this new, this patient zero, the originally infected person has remained contagious for day T equal to two and passed on the infections to N times P people, right? Now let's go to period T equal to three. Right? So this patient zero is no longer contagious. But what about this N P people? Right? So now if you take any of them, right, so each of them they meet N people, right? And the probability of passing on the infection is P. So each of them, in fact, another N times P people, right? So this is one, the next person also infects NP, the next person infects NP and so on. Right, so we started with NP infections, right? Each of them pass it on to another NP people, right. So the total number of N, of infected people at the end of day three, is NP times NP. Right, because each of them passes it on to a further NP people. Right, so the total number of infections at the end of day T equal to three is NP squared.

So this is the basic model of infection spread. So on day T, on day one, it was one, on day T equal to two it was NP on day T equal to three, it's NP squared. And hope I'm hoping that by this time, you're getting the hang of things, because what's going to happen in a T equal to four is the same thing that happened in day T equal to three, this NP squared people, right, each of them are going to meet with N. And they're going to pass it to a fraction P of them. This person is going to pass it to a fraction P of the people he meets and so on, right. So at the end of day T equal to four, each of these NP square people are going to pass on to another NP people. Right, so that means the number of new infections on day T equal to four is going to be NP cubed.

And I'm hoping that you're seeing a pattern now, right? We started with one on day T equal to one.

On day T equal to two, it's NP. Day T equal to three, it's NP squared. T equal to four it's NP cubed. And it's going to go on, so on. Alright, so, so you see what's what's important, right? So what's important here is this number N times P, right? Because every time it's one, then it's N times P, then it's N times P squared, it's N times P cubed, and so on. So this is an important number, this N times P. And you may have heard of this in the popular media, it's called the R naught, or the reproduction number. And what it means is that it's the number of new infections that's caused by one infected person. So it's called R naught or the reproduction number. So if you look at Ontario, and I've borrowed this graph from the COVID-19 science table, and these are the number for the reproduction number, the R naught for Ontario. And I plotted this from the period December 2020 to June 2021. Right? And you see what's happening to this R, R naught, right. So remember, in December, there was the second wave of infections, right? So the R naught, it was initially about one, but then it started climbing up, and it went up to about 1.2, and about 1.3, right. So this was also when the number of infections were steadily rising. But again, by January, the number of infections came much more under control. And here you see the R naught number is much lower than one. But then, again, the third wave hit. So this was starting about February, and went up to April, and you see here the R naught number, right, that climbed up steadily above one.

Right. And then again, from from about mid-April onwards, right, the number of infections started really dropping. And this is the summer of 2021, the number of infections is really plummeted, and the R naught number here is also very low. Okay, but going back to our model, right, so again, just to reiterate, right, you have one, then day, day two you have NP, day three it's NP squared, right. On day four, it's going to be NP cubed. Right, and you see that, you look at this pattern and now you can predict that on day T, day T, right, the general T, the number of new infections will be NP to the power T minus one. Right, they say, on day T equal to two the number of infections is P to the power one, which is two minus one. On day T equal to three, it's NP squared, which is NP to the power three minus one, on T equal to four, it's NP cubed, which is NP to the power four minus one, and so on. So on day T, the number of new infections is NP to the power t minus one, right? So this here, right is a mathematical expression. It's a mathematical formula, right? And as T changes, right, so this is our variable here, as T changes, we can see how the number of infections are going to be changing. And this depends on two parameters, right N and on P. Right, for a given society, under given circumstances, these two things are fixed, right. N remember, is the number of interactions that, that a person has on a daily basis. P is the, is the probability that an infected person passes it on, passes on the infection to another person.

Right, given N and P, right, this model tells you how the number of infections spread over time. Right? Because if I gave you an N and a P, and I said, Okay, what will it be in day one, day two, day three, day 100, right? You can, you can calculate it using this mathematical formula. And that's what we're going to do next. We're going to use Excel and Google Sheets to sort out how it will look under different circumstances