Equations and Inequalities II

SUMMARY KEYWORDS

equation, equal, solution, inequality, absolute value, question, negative, solve, answer, sides, satisfy, hollow, multiply, unknown variable, correct, greater, possibility, line, contradiction, number

SPEAKER

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Here's our first question where we have an absolute value. And the question says, graph the solution to the inequality on the number line, or what I would call the real number line. When I have a question like this, I like to transform the inequality into an equality. So I've got the absolute value of x plus three is equal to four. And my first question to myself is, what values of x satisfy the quality. And looking at this, I can see that if x is equal to one, then x plus three is equal to four. with absolute values, there's often more than one solution. And when we talk about functions, we're going to see that with some special functions later, but there's also the fact that we could have more than one solution. So looking at this, if x, and I'm just sort of plugging numbers into my mind, and guess saying, if x is equal to negative seven, then x plus three is also equal to four. Because I have, of course, its absolute value. So I have negative seven plus three, I've got the absolute value of negative four is equal to four. Right, so they're using the absolute value operator. Now, let me clear this up a little bit. Now notice that if x is greater than one, the equation or the inequality fails to halt. And if x is less than negative seven, it also fails to hold. I should also point out that the way I've written the axis is not entirely correct. x should be greater than or equal to one, and x should be less than or equal to negative seven. Because if x is actually equal to one that this inequality doesn't hold, because it's less than sign, not a less than or equal to sign. Now looking at the real number line, I could have this, you know, going off to infinity over here, and off to negative oops, not at eight. That's not what I want. I can have one here, and a negative zero, or sorry, excuse me a negative seven there. And as long as x falls between these two values, the absolute inequality holds. So how am I going to answer this question on ALEKS? Here's the question on ALEKS. I'm going to use the circle that is hollow. And I'm going to identify the two points of interest. The two points of interest are one and negative seven. That I'm going to press this line icon and it looks like it's automatically filled in the correct interval where this inequality is satisfied. And now I will. Oops. And now I'm plus the check Let's see if we get the right answer. And we did. So I have to remember that it's an open interval, or open set. And so we're using the dot that's hollow on both the upper value and the highest value and the lowest via this question is to keep you working on your algebra skills when it comes to doing this type of question where you have an expression, and you're being asked to solve this equation for y two, and what does that mean, they want us to isolate y two on one side of the equation. A economists, undergraduate majoring in economic economics, you're going to be asked to do this all the time. If you can do it well, if you can do it easily. If you practice it so that you can do it before the test you're good at have a great undergraduate career, you're going to be very relaxed, and everything is going to be easy. So let's go ahead and let's try and solve this equation for y two. The first thing I want to do is I want to get rid of this denominator. And how am I going to do that, I'm going to multiply both sides of the equation by x two minus x one. And so I'm going to have m times x two minus x one is equal to y two minus y one. And I'm pretty much there. I'm going to add y one to both sides of the equation. And there's my answer. Now, let's go ahead and put it into ALEKS and see if it's correct. So I am here on ALEKS. You can see the question is the exact same question we solved on the slides. So let's go ahead and input the answer that we came up with got y to z equal to m. x, and I'll use this button here. And then I press the right arrow to move over x. And press that button again, one and then the right arrow on my keyboard to get it to go like that. And the last bit is to add y one. And let's see if we have the right answer. And we do we have the right answer. So pretty straightforward. Just showing you that really, you can multiply both sides of the equation by more than one term, right, we multiply both sides of the equation by x two minus x one. As long as we do that properly, there's no problems with doing that. I'd like you to take a look at the question in front of you. It says at the top for each equation, choose the statement that describes its solution, if applicable, give the solution. Now you can see that you've got three different boxes to choose from. One is that there's no solution. The second box as well. Maybe the unknown variable has a value. That's the solution, something like w is equal to two. And the next option is that all real numbers are solutions. So let's take a let's go back to the slides now. And let's talk about the possible answers to a question like this. So looking at your slides, I'll let you sort of read it yourself. But you can see that when we solve an equation like we were given on ALEKS, there are really three possibilities that are gonna come up. The first possibility is that we're going to get a contradiction. We're gonna get something like zero was equal to four, or three is equal to negative two, something like that. If we end up with that result, that means there is no solution. There's no actual solution to that equation. Two is the most common one. That's the one that you're used to and you expect is that the answer is going to be something like x is equal to two and x is equal to eight or something like that. And you've seen lots of questions like that. The final possibility is that You're going to get an identity, something like zero is equal than zero, or three is equal to three. What that means is that any value for the unknown variable, so suppose the unknown variables x, so any value of x will satisfy that equation, the value of x does not matter. And any value of x will solve the equation. And that's what it says here, all real numbers are solutions. Now, here's the problem that we're given. So the first thing I'm going to do, I'm going to start with the equation on the left, and I'm going to solve for w, and I'm going to see what I get. So let's multiply four inside the bracket. So I've got four w, minus four minus one is equal to four w minus six. Now notice, if I subtract four w on both sides of the equation, they're just going to cancel each other out. Or, more specifically, they're going to sum to zero. Now I've got minus five is equal to minus six, Hmm, well, I know that can't be true. So unless I've made a mistake, this is a contradiction. And there is no solution. Now let's take a look at the equation on the right hand side, I'm going to start with the same process, I've got four x plus four, plus x is equal to three x minus six plus two. So I'm working out the bracket. Now I'm going to collect like terms, and I've got five x plus four is equal to three x minus four. And if I subtract four on both sides of the equation, and I subtract three x on both sides of the equation, I get two x is equal to minus eight, and x is equal to negative four. And so you can see that there is a unique solution, there is one solution to the equation on the right equation on the left, there is no solution. Our last question is short. And to the point, we're going to look at an absolute value equation, this time similar to the one we just did. There could be more than one solution, there could be one solution. Or there could be no solution at all. So to solve this, I'm going to isolate the absolute value of u. And I can do that by dividing both sides by 2am. I done? Is there a solution? Well, one thing to remember is that absolute value denotes distance. On the real number, line. Negative distance negative distance is impossible. This means that the absolute value of u, which is equal to negative two has no solution. It's not possible for an absolute value to be less than zero. And so there's no solution here. I wanted to illustrate to you an example, with absolute value, where there is no solution so the solution is not Yeah, there's no you that satisfies an answer of negative two